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Nonlinear third harmonic radiation in thermal free electron laser with modified wiggler

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Simulation of free electron laser (FEL) with modified wiggler consisting of a conventional planar wiggler with third harmonic field component is presented. A set of self consistent nonlinear differential equations is derived and solved numerically by Runge–Kutta method. The optimum amplitudes of fundamental and third harmonic wiggler filed are obtained to increase the third harmonic radiation in comparison with conventional wiggler and also to have lower electron beam energy compared to conventional wiggler with the same wavelength. For the thermal effect the axial energy spread of electron beam, without any spread in the transverse velocity, is assumed. A peculiar region is found in which there is a sharp increase of the radiation amplitude. Thermal effect of the electron beam was found to be irregular in this peculiar region.

1. Introduction

Harmonic radiation in FEL has been the most important method to achieve shorter wavelength with lower energy beam (Colson 1981; Freund et al. 2000; Yu et al. 2003; McNeil et al. 2006; Bazouband and Maraghechi 2012; Giannessi et al. 2012). In the compact FEL, which operates in long wavelengths, the low current/voltage electron beam is employed to reduce the size and cost of the required electron source. For this reason, these FEL devices have been subject of much attention because of their many realistic applications in medicine and industry (Kong 1997; Sung et al. 2006; Doria et al. 2009). Although harmonic gain in FEL is much smaller than that of the fundamental, several gain and efficiency enhancement techniques have been suggested (Freund and Gold 1984; Freund and Pasour 2003; Wang 2007; Rouhani and Maraghechi 2009; Jafari Bahman and Maraghechi 2012; Zahedian et al. 2012; Bazouband and Maraghechi 2013). Adding the harmonic wiggler field to the fundamental with the appropriate ratio and phase can increase the gain of harmonic (Asakawa et al. 1992) that is considered in this paper and this nonconventional wiggler is known as modified wiggler. It needs to be mentioned that bunching of the beam is directly caused by the pondermotive wave generated by beating of the radiation with the wiggler field. The electron beam will be bunched at its fundamental resonance as well as at its harmonics. In the modified wiggler, appropriate controlling of the amplitude and phase of harmonic wiggler field increase the bunching of harmonic and improves its radiation. Due to the simplicity of this harmonic wiggler which consists of conventional planar wiggler and high permeability shims, various kinds of modified wiggler field can be developed easily and economically (Asakawa et al. 1995; 1996).

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Suitable wiggler schemes known as nonconventional wigglers, to provide both the saturated power at fundamental and its harmonics, have been suggested for various purposes within the framework of FEL studies. The effects of harmonic wiggler field components on FEL operation has been investigated (Mark and Elliott 1987; Iracane et al. 1994). Spontaneous emission and small signal gain of FEL with different types of harmonic wigglers have been studied analytically (Asakawa et al. 1992; Marshall et al. 1999; Sircilli et al. 2001; Mishra et al. 2009) and spectral properties of two-harmonic undulator radiation have been presented (Dattoli and Voykov 1993; Mishra et al. 2009) .The important influence of beam energy spread on the small signal gain spectrum have been included (Hussain et al. 2009). The FEL experiment of the 5th harmonic generation with a modified wiggler has been introduced (Nakao et al. 1998). In Dattoli et al. (2002), undulator configuration in which the on axis field oscillates with different orthogonal polarizations was considered in high gain FEL. Also to enhance harmonic power, the possibility of considering parallel rather than orthogonal wiggler field contributions has been taken into account. The use of the harmonic undulator for reducing betatron oscillation effects in a FEL has been examined (Gupta and Mishra 2006). Variable polarized undulator scheme having a harmonic field component was discussed in Ref. (Gupta and Mishra 2007) and linear and helical polarized cases were analyzed. One interesting characteristic was noted that at helical case the intensity is not zero for higher harmonics as the scheme provides electron longitudinal velocity modulation due to its harmonic field component, which is absent in a simple helical wiggler field.

Several suggestions that have been proposed in the past to study the harmonic wiggler, have also analyzed the linear harmonic radiation. In this paper, for the first time, the generation of the nonlinear third harmonic radiation affected by nonconventional wiggler is simulated numerically in one dimension by solving the self consistent nonlinear differential equations governing the evolution of radiation and electron beam. Also parameters for the best gain of third harmonic are assumed and the electron beam is injected with axial energy spread.

The organization of the paper is as follows. In Sec. 2, equations describing the nonlinear evolution of the one-dimensional FEL with modified wiggler are derived. In Sec. 3, a numerical investigation is conducted to obtain optimum parameters and increase the radiation. In Sec. 4, concluding remarks are made.

2. Basic equations

Consider an electron beam that moves along the axis of the modified planar wiggler consisting of two harmonic field components whose field is given by

$$\mathbf{B} = (B_{w1}\sin(k_{w1}z) + B_{w3}\sin(k_{w3}z))\,\hat{\mathbf{e}}_y, \tag{1}$$

where $k_{w1} = 2\pi/\lambda_w$, $k_{w3} = 3k_{w1}$ and λ_w is the wiggler wavelength. B_{w1} and B_{w3} are the peak field strength of fundamental and third harmonic.

The vector and scalar potential of radiation and the space charge wave are written as

$$\delta\mathbf{A}(z,t) = \sum_{h=1,3}\delta A_h(z)\cos\alpha_{+h}\widehat{e}_x, \qquad \alpha_{+h} = \int_0^z k_{+h}(z')dz' - h\omega t, \tag{2a}$$

$$\delta\phi(z,t) = \sum_{h=1,3}\delta\phi_h(z)\cos\alpha_h, \qquad \alpha_h = \int_0^z k_h(z')dz' - h\omega t, \tag{2b}$$

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where k_{+h} and k_h are the wave number of radiation and space-charge waves of hth harmonic, respectively. The amplitudes and wave numbers of the radiation and space-charge fields are assumed to vary slowly with z. By substituting the above mentioned fields as well as the radiation fields $\delta\vec{E} = -c^{-1}\,\partial\delta\vec{A}/\partial t - \hat{e}_z\partial\delta\phi\,/\partial z$ and $\delta\vec{B} = \hat{e}_y\partial\delta A_x/\partial z$ in the relativistic equation of motion,

$$\frac{d\,(m\gamma\vec{v})}{dt} = -e\left\{\delta\vec{E} + \frac{\vec{v}}{c}\times(\vec{B}_w + \delta\vec{B})\right\}, \tag{3}$$

equations governing the dynamics of electrons can be simplified as

$$\begin{aligned}
\frac{du_x}{dz} &= \frac{e}{mc}\sum_h\left[\frac{d\delta A_h}{dz}\cos\alpha_{+h} - \delta A_h k_{+h}\sin\alpha_{+h}\right] + \frac{e}{mc}\left(B_{w1}\sin k_{w1}z + B_{w3}\sin k_{w3}z\right)\\
&\quad + \frac{e}{mv_z}\sum_h\frac{h\omega}{c}\delta A_h\sin\alpha_{+h},\\
\frac{du_y}{dz} &= 0.0,\\
\frac{du_z}{dz} &= \frac{e}{v_z}\sum_h\left[\frac{d\delta\phi_h}{dz}\cos\alpha_h - \delta\phi_h k_h\sin\alpha_h\right] - \frac{e}{mc}\frac{v_x}{v_z}\\
&\quad \times\sum_h\left[\frac{d\delta A_h}{dz}\cos\alpha_{+h} - \delta A_h k_{+h}\sin\alpha_{+h}\right] - \frac{e}{mc}\frac{v_x}{v_z}\left(B_{w1}\sin k_{w1}z + B_{w3}\sin k_{w3}z\right),
\end{aligned} \tag{4}$$

where $\mathbf{u} = \gamma\mathbf{v}$ denotes a velocity like variable and γ is the dimensionless electron beam energy. The transverse electron motion is described as

$$\frac{dx}{dz} = \frac{v_x}{v_z}, \qquad \frac{dy}{dz} = \frac{v_y}{v_z}. \tag{5}$$

In addition, the phase of electromagnetic wave and space-charge wave evolves according to the following differential equations

$$\frac{d\alpha_{+h}}{dz} = k_{+h}(z) - \frac{h\omega}{v_z}, \qquad \frac{d\alpha_h}{dz} = k_h(z) - \frac{h\omega}{v_z}. \tag{6}$$

By substituting fields of (2) into the Maxwell's equations, the dynamical equations which govern the evolution of the hth harmonic radiation are derived as follow (Freund and Antonsen 1996; Rouhani and Maraghechi 2010; Bazouband and Maraghechi 2012):

$$\begin{aligned}
\frac{d\delta a_h}{dz} &= \Gamma_{+h}\delta a_h,\\
\frac{d\Gamma_{+h}}{dz} &= -\Gamma_{+h}^2 - \frac{h^2\omega^2}{c^2} + k_{+h}^2 + \frac{2\omega_b^2}{c^2\delta a_h}\beta_{z0}\left\langle\frac{v_x}{|v_z|}\cos\alpha_{+h}\right\rangle,\\
\frac{dk_{+h}}{dz} &= -\frac{2\omega_b^2}{c^2\delta a_h}\beta_{z0}\left\langle\frac{v_x}{|v_z|}\sin\alpha_{+h}\right\rangle - 2k_{+h}\Gamma_{+h},
\end{aligned} \tag{7a}$$

$$\begin{aligned}
\frac{d\delta\phi_h}{dz} &= \frac{-2\omega_b^2}{h\omega c}\beta_{z0}\left\langle\sin\alpha_h\right\rangle,\\
k_h &= \frac{-2\omega_b^2}{h\,\omega\,c\,\delta\phi_h}\beta_{z0}\left\langle\cos\alpha_h\right\rangle,
\end{aligned} \tag{7b}$$

where $\omega_b^2 = 4\pi n_e e^2/m_e$ and $\delta a_h = \delta A_h/m_0c^2$ is the dimensionless amplitude of vector potential that describes the radiation field of hth harmonic. The growth rate of the

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hth harmonic (Γ_{+h}) is defined at the first line of (7a). The averaging operator $\langle \ldots \rangle$ is defined over an ensemble of electrons injected into the wiggler within one wave period. For this particle averages, a choice of 61 particles in Simpson technique was found to be adequate in the case of cold beam, which is increased to $16 \times 16 = 256$ particles in Gaussian quadrature technique for nonzero energy spread (Bazouband and Maraghechi 2012; 2013).

Equations (4)–(7) form a set of 7N + 5Q self-consistent first order nonlinear differential equations that describes the FEL interaction and are solved simultaneously by the fourth order Runge–Kutta method subject to the appropriate initial conditions. N is the number of electrons and Q is the number of harmonics. It should be noted that the present nonlinear formalism and simulation are based upon taking the average of Maxwell's equations over a wave period to remove the fast time-scale oscillations. This is a steady-state simulation in which the slippage of radiation over the electron bunch is ignored; therefore, time plays no role in this formalism. It is the evolution of the amplitudes of radiation and pondermotive wave, along the wiggler axis, that we study.

The quasi-steady state (Rouhani and Maraghechi 2010) solution of equation of motion in the absence of radiation and space-charge field is as follows

$$\bar{u}_x = -\left(\bar{\boldsymbol{\Omega}}_{w1}\cos k_{w1}z + \bar{\boldsymbol{\Omega}}_{w3}\cos k_{w3}z\right), \qquad \bar{u}_y = 0, \tag{8}$$

where $\bar{\boldsymbol{\Omega}}_{wi} = eB_{wi}/mc^2k_{wi}(i = 1, 3)$ is the dimensionless amplitude of wiggler fields and $\bar{\mathbf{u}} = \mathbf{u}/c$. By substituting these results in the energy conservation relation $\gamma_0^2 = 1 + \bar{u}_x^2 + \bar{u}_y^2 + \bar{u}_z^2$, the average axial velocity of an electron over a wiggler period is obtained as

$$\beta_z = \sqrt{1 - \frac{1}{\gamma_0^2}\left(1 + \frac{1}{2}\bar{\boldsymbol{\Omega}}_{w1}^2 + \frac{1}{2}\bar{\boldsymbol{\Omega}}_{w3}^2\right)}. \tag{9}$$

By using β_z for the given parameters of γ_0, $\bar{\boldsymbol{\Omega}}_{w1}$, and $\bar{\boldsymbol{\Omega}}_{w3}$, linearly polarized electromagnetic dispersion relation (Freund and Antonsen 1996)

$$\bar{\omega}^2 - \bar{\kappa}_+^2 = \frac{\bar{\omega}_b^2}{\gamma_0}, \tag{10}$$

together with the dispersion relation for negative energy space-charge wave,

$$\bar{\omega} - \beta_z(1 + \bar{\kappa}_+) = \frac{-\bar{\omega}_b}{\gamma_0^{1/2}\gamma_{z0}}, \tag{11}$$

should be solved simultaneously to find the initial values of $\bar{\kappa}_+$ and $\bar{\omega}$ where $\bar{\kappa}_+ = \kappa_+/\kappa_w$ and $\bar{\omega} = \omega/c\kappa_w$ are dimensionless parameters. The phase matching condition $\bar{\kappa} = \bar{\kappa}_+ + 1$ gives the initial value of $\bar{\kappa}$. As mentioned in (2b) κ_+ and κ are the wave number of radiation and space-charge waves of fundamental frequency, respectively. It should be noted that β_{z0} and β_z are equilibrium axial velocities for the zero and final wiggler field, respectively, in the absence of radiation which is related to the electron beam energy, γ_{z0} and γ_0, for the zero and final wiggler field according to (9).

3. Numerical results

Equations (4)–(7) are the set of equations which describe the FEL interaction in one-dimension with modified planar wiggler. The appropriate initial conditions to solve these 7N + 5Q number of self-consistent first order nonlinear differential

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equations are as follow: The electron beam in the case of cold beam is injected axially ($\bar{u}_{z0} = \sqrt{\gamma_0^2 - 1}$) and the transverse displacement and the velocity of electron beam at the entry plane are taken to be zero. Initial amplitudes of radiation are arbitrary but small, $\delta a_1(z = 0) = 10^{-7}$ and $\delta a_3(z = 0) = 0.01\delta a_1$. Since the wiggler field at the entrance increases adiabatically from zero, the initial growth rates of vector potentials are zero ($\Gamma_+(z = 0) = \Gamma_{+3}(z = 0) = 0$). The planar wiggler with a period $\lambda_w = 3.3$ cm and a magnetic field that is increasing from zero to a constant value over $N_w = 10$ wiggler periods is assumed. Also a 3.5-MeV/43-A electron beam with an initial radius of 0.02 cm is considered.

Before going to the numerical solution it should be mentioned that the aim of figures is to find the optimum amplitude for harmonic and fundamental wiggler field to have higher third harmonic radiation with lower beam energy relative to the conventional wiggler.

The variation of the normalized amplitude of fundamental and third harmonic vector potentials with axial distance is shown in Fig. 1 for the fundamental and third harmonic radiation and they are compared for different values of third harmonic wiggler field. In Fig. 1(a) it is seen that fundamental radiation for $\bar{\Omega}_{w3} = 0.4\,\bar{\Omega}_{w1}$ is increased relative to the conventional wiggler ($\bar{\Omega}_{w3} = 0.0$) but for higher values of harmonic wiggler field, the fundamental radiation does not have any growth at all. However, according to Fig. 1(b) the third harmonic radiation is increased about 43% for $\bar{\Omega}_{w3} = 1.6\,\bar{\Omega}_{w1}$. In fact the electron beam will be bunched at its fundamental resonance as well as at its harmonics and by increasing the harmonic wiggler field, the bunching of fundamental resonance reduces and improves the bunching of harmonics. When the ratio of harmonic to the fundamental field is greater than one the third harmonic radiation relative to conventional wiggler is increased. In this case if the negative ratio of third harmonic wiggler field is regarded, the amount of wiggler field according to (1) will be negative. So we did not consider the controlling of the phase of the harmonic for $\bar{\Omega}_{w3}/\bar{\Omega}_{w1} < 0$.

Figure 2(a) shows the variation of the maximum of the third harmonic amplitude of the vector potential, known as saturated third harmonic radiation, versus dimensionless fundamental wiggler field ($\bar{\Omega}_{w1}$) for different ratio of third harmonic wiggler field ($\bar{\Omega}_{w3}/\bar{\Omega}_{w1}$). It shows that the amplitude of the third harmonic radiation increases with increasing the third harmonic wiggler field when $\bar{\Omega}_{w1}$ is larger than unity. However, for lower $\bar{\Omega}_{w1}$, this is not the case and a more complicated behavior is observed that is due to the electron trajectory is distorted from the sinusoidal trajectory. It is seen that in the case of the modified wiggler field with $\bar{\Omega}_{w3}/\bar{\Omega}_{w1} = 1.8$ and $\bar{\Omega}_{w1} = 1.0$, the improvement of about 70% for the third harmonic radiation is achieved compared to the conventional wiggler. According to dispersion relation of (9)–(11), resonance frequency depends on the amplitudes of wiggler fields. The general formula showing the dependence of the resonance wavelength on the wiggler fields has been provided (Asakawa et al. 1992) analytically by single particle analysis of spontaneous emission, while here the space charge effects are considered and so the result will be more exact for the FEL amplification. In Fig. 2(b), the variation of fundamental resonance wavelength (λ) versus the fundamental wiggler field is shown for various types of harmonic wiggler field. This shows that λ increases by increasing $\bar{\Omega}_{w1}$ as well as with $\bar{\Omega}_{w3}/\bar{\Omega}_{w1}$ and it is evident that curves are quadratic in nature in accordance with the result of (Asakawa et al. 1992). Therefore, $\bar{\Omega}_{w3}$ may be increased up until the wavelength of the third harmonic radiation of the modified wiggler, i.e. $\lambda/3$, to remain less than that of fundamental resonance of the conventional wiggler.

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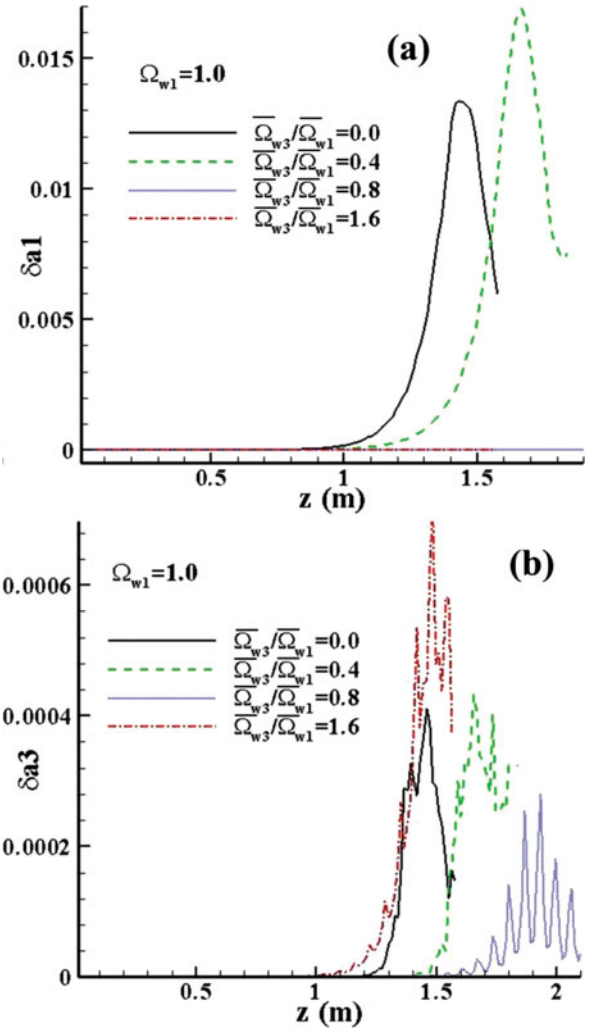


FIGURE 1. Evolution of vector potential amplitude with axial position for different values of third harmonic wiggler field for (a) fundamental and (b) third harmonic radiation.

This can be demonstrated by giving some numbers. Electron beam energy in Fig. 2 is 3.5 MeV ($\gamma = 7$). In the case of conventional wiggler $\bar{\boldsymbol{\Omega}}_{w1} = 1.0$ ($\bar{\boldsymbol{\Omega}}_{w3} = 0.0$) the fundamental resonance wavelength is $\lambda = 0.055\,\text{cm}$ and by increasing the ratio of harmonic wiggler field to $\bar{\boldsymbol{\Omega}}_{w3}/\bar{\boldsymbol{\Omega}}_{w1} = 1.8$, the radiation wavelength will increase to $\lambda = 0.12\,\text{cm}$ with $\lambda_3 = 0.04\,\text{cm}$. The required energy to tune the conventional FEL with the third harmonic resonance wavelength $\lambda_3 = 0.04\,\text{cm}$ is about $\gamma = 8.1$. The third harmonic wiggler field $\bar{\boldsymbol{\Omega}}_{w3}$ is actually larger than the fundamental field and it looks as if we have a wiggler tuned to the third harmonic, however, if we want λ_3 to be the resonance frequency of a conventional wiggler to have lower wavelength, it requires higher energy with $\gamma = 8.1$. Therefore, in the case of a modified wiggler field, the operation at the third harmonic wavelength $\lambda_3 = 0.04\,\text{cm}$ is achieved by using electron beam with 13% less energy compared to the conventional wiggler tuned to the same wavelength $\lambda = 0.04\,\text{cm}$. In Asakawa et al. (1996) and Nakao et al. (1998), the linear FEL gain of modified wiggler with the harmonic wiggler field component

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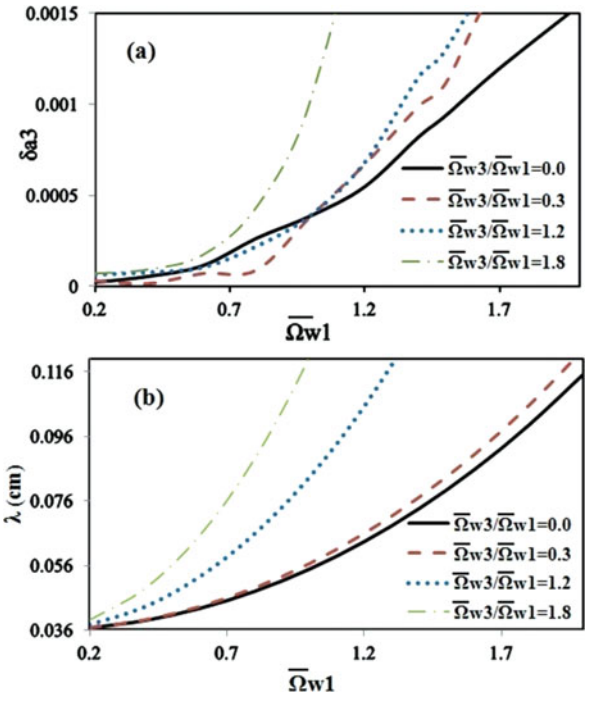


FIGURE 2. Variation of (a) the saturated amplitude of third harmonic radiation and (b) the fundamental resonance wavelength, as a function of fundamental wiggler field for different values of third harmonic wiggler field.

smaller than the fundamental and the decrease in the fundamental wiggler field in order to shorten the wavelength is reported, but small relative gain is shown. In the present study, we are going to have optimum increase of the third harmonic gain relative to conventional wiggler with less energy compared with the conventional wiggler at the same wavelength.

As shown in Fig. 2 it seems that the amplitude $\bar{\boldsymbol{\Omega}}_{w1} = 1.0$ is a good choice for our purpose. So in Fig. 3 variation of saturated amplitude and the saturation length of third harmonic radiation versus ratio of wiggler fields $\bar{\boldsymbol{\Omega}}_{w3}/\bar{\boldsymbol{\Omega}}_{w1}$ is shown for $\bar{\boldsymbol{\Omega}}_{w1} = 1.0$. As mentioned before, third harmonic radiation is increased by increasing the third harmonic wiggler field but amplitudes corresponding to $\bar{\boldsymbol{\Omega}}_{w3}/\bar{\boldsymbol{\Omega}}_{w1} > 2.0$ are not useful because, in that case, longer wavelength and higher beam energy is required compared to the conventional wiggler tuned at the third harmonic. At $\bar{\boldsymbol{\Omega}}_{w3}/\bar{\boldsymbol{\Omega}}_{w1} = 1.8$, a significant increase of the radiation amplitude (≈70%) and a decrease of the saturation length are obtained in comparison with conventional wiggler $\bar{\boldsymbol{\Omega}}_{w3} = 0.0$. Also the energy of the electron beam is reduced compared to the conventional wiggler tuned at the third harmonic.

At the following figures, the thermal electron beam with axial energy spread in the form of Gaussian distribution function (Bazouband and Maraghechi 2013) is considered and the fundamental wiggler field amplitude $\bar{\boldsymbol{\Omega}}_{w1} = 1.0$ is chosen. In Fig. 4, evolution of the third harmonic radiation versus axial distance z is shown for the axial energy spread $\Delta\gamma_{z0}/\gamma_{z0} = 0.03$. Here, two values of third harmonic wiggler field are compared with the conventional wiggler. It can be seen that the saturated power first decreases from 3.0×10^{-4} to 2.5×10^{-4} as $\bar{\boldsymbol{\Omega}}_{w3}/\bar{\boldsymbol{\Omega}}_{w1}$ is increased from 0 to 0.4 and then it increases to 7.8×10^{-4} as $\bar{\boldsymbol{\Omega}}_{w3}/\bar{\boldsymbol{\Omega}}_{w1}$ is increased to 1.8. Similar the

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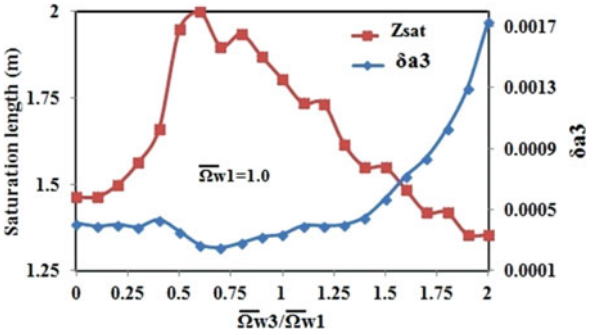


FIGURE 3. Variation of the saturated amplitude and saturation length (*Zsat*) of third harmonic radiation with the ratio of third harmonic wiggler field for $\bar{\boldsymbol{\Omega}}_{w1} = 1.0$.

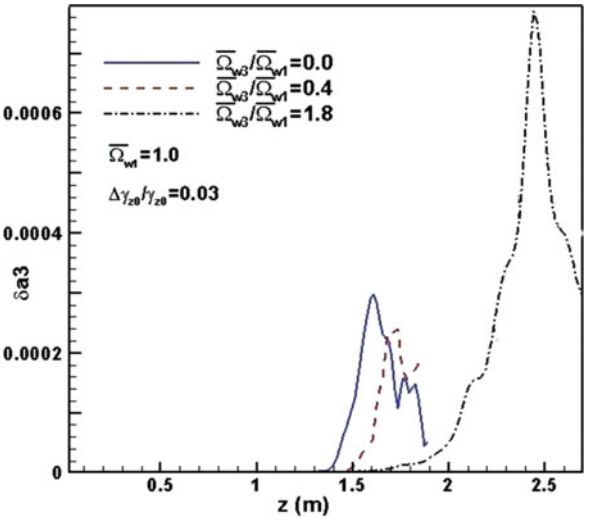


FIGURE 4. Evolution of the third harmonic radiation amplitude along the axial distance for two ratios of third harmonic wiggler field and comparison with conventional wiggler in the presence of energy spread $\Delta\gamma_{z0}/\gamma_{z0} = 0.03$ and $\bar{\boldsymbol{\Omega}}_{w1} = 1.0$.

cold beam case in Fig. 1(b), the third harmonic radiation is improved for the ratio of the harmonic to the fundamental field larger than one. The variation of resonance wavelength with the wiggler field components in the presence of thermal effect is the same as the cold beam case as shown in Fig. 2(b).

Thermal effect on the generation of third harmonic radiation with modified wiggler is also studied. Variation of δa_3 and saturation length versus third harmonic wiggler filed is shown in Fig. 5 for the $\bar{\boldsymbol{\Omega}}_{w1} = 1.0$ and $\Delta\gamma_{z0}/\gamma_{z0} = 0.03$. Figure 5(a) compares the amplitude of the third harmonic radiation of a warm beam with that of the cold beam ($\Delta\gamma_{z0} = 0.0$). As expected, energy spread decreases the radiation but at the harmonic wiggler field $\bar{\boldsymbol{\Omega}}_{w3} \approx 0.5\bar{\boldsymbol{\Omega}}_{w1}$, a peculiar point is seen that behaves differently. Although energy spread decreases the amplitude of radiation, applying modified wiggler with third harmonic wiggler field at $\bar{\boldsymbol{\Omega}}_{w3} = 1.8\bar{\boldsymbol{\Omega}}_{w1}$ compensates this negative effect and so the amplitude of the third harmonic at $\bar{\boldsymbol{\Omega}}_{w3} = 1.8\bar{\boldsymbol{\Omega}}_{w1}$ is higher than the conventional wiggler with a cold beam. Variation of the saturation length of third harmonic with the third harmonic wiggler field is shown in Fig. 5(b). It can be seen that the energy spread increases the saturation length compared to a cold

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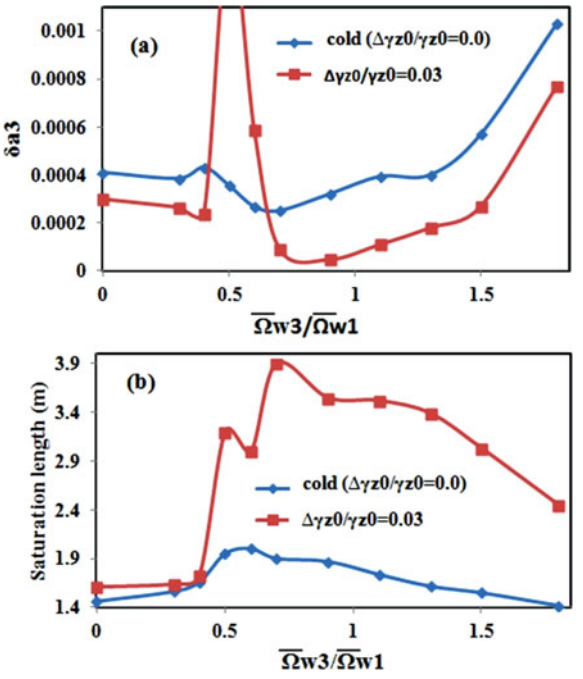


FIGURE 5. Variation of (a) the saturated amplitude and (b) the saturation length of third harmonic radiation as a function of third harmonic wiggler field for thermal case ($\Delta\gamma_{z0}/\gamma_{z0} = 0.03$) and comparison with the cold beam, $\Delta\gamma_{z0} = 0.0$.

beam. For $0 < \bar{\boldsymbol{\Omega}}_{w3}/\bar{\boldsymbol{\Omega}}_{w1} < 0.4$ where δa_3 remains almost unchanged the saturation length also remains constant. Saturation length increases around the peculiar point $\bar{\boldsymbol{\Omega}}_{w3} \approx 0.5\bar{\boldsymbol{\Omega}}_{w1}$, for a warm beam, and then gradually falls.

Finally by choosing the optimum amplitudes of fundamental and third harmonic wiggler field $\bar{\boldsymbol{\Omega}}_{w1} = 1.0$ and $\bar{\boldsymbol{\Omega}}_{w3} = 1.8\bar{\boldsymbol{\Omega}}_{w1}$, the variation of the saturated amplitude and saturation length of third harmonic radiation with axial energy spread ($\Delta\gamma_{z0}/\gamma_{z0}$) is shown in Fig. 6. In Fig. 6(a) conventional wiggler ($\bar{\boldsymbol{\Omega}}_{w3} = 0.0$) is compared with the modified wiggler ($\bar{\boldsymbol{\Omega}}_{w3} = 1.8\bar{\boldsymbol{\Omega}}_{w1}$). In both cases increasing the energy spread decreases the amplitude of radiation but modified wiggler has higher radiation amplitude than conventional wiggler. The thermal spread provides a wider range of axial velocities. In fact, the bulk axial velocity decreases and the central frequency of the gain band also decreases. Thus the beam energy spread drives the electrons away from the resonance condition, causing a reduction in the FEL gain. Furthermore, the harmonic field brings back the FEL to the resonance condition causing the gain improvement of the harmonic. Saturation length of the third harmonic is also plotted in Fig. 6(b) versus energy spread for modified and conventional wiggler. Increasing the energy spread in both of them increases the saturation length but close to cold beam ($\Delta\gamma_{z0}/\gamma_{z0} < 0.01$) saturation length of modified wiggler is slightly smaller than the conventional wiggler. However, for larger energy spread, saturation length of the modified wiggler is larger than that of the cold beam.

In order to investigate the peculiar region $\bar{\boldsymbol{\Omega}}_{w3} \approx 0.5\,\bar{\boldsymbol{\Omega}}_{w1}$, further in Fig. 5, variation of saturation length and radiation amplitude with energy spread is plotted in Fig. 7 for $\bar{\boldsymbol{\Omega}}_{w3} = 0.6\,\bar{\boldsymbol{\Omega}}_{w1}$. In normal circumstances like in Fig. 6 we expect the radiation amplitude to decrease and the saturation length to increase with increasing

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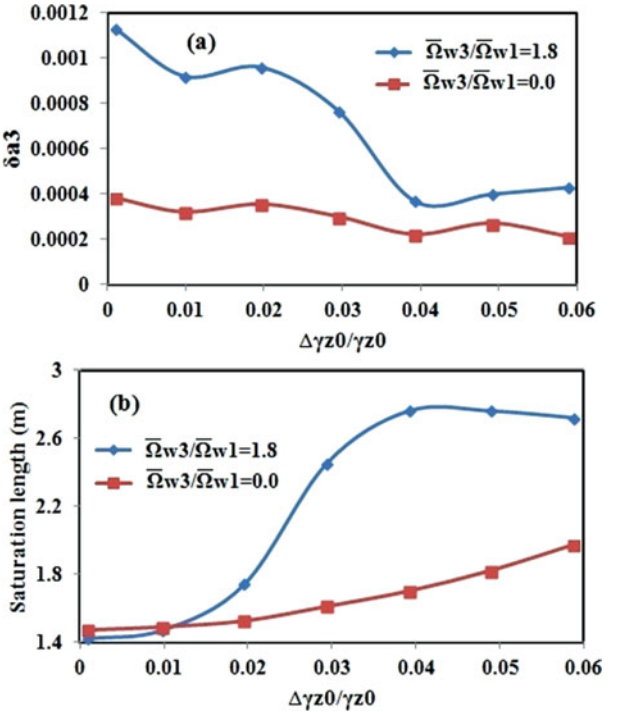


FIGURE 6. Variation of (a) the saturated amplitude and (b) the saturation length of third harmonic radiation with axial energy spread for modified wiggler ($\bar{\boldsymbol{\Omega}}_{w3} = 1.8\bar{\boldsymbol{\Omega}}_{w1}$) and comparison with the conventional wiggler ($\bar{\boldsymbol{\Omega}}_{w3} = 0.0$). The amplitude of fundamental wiggler field is $\bar{\boldsymbol{\Omega}}_{w1} = 1.0$.

energy spread. This is not what we see near the peculiar region in Fig. 7 in which there are sharp increases of both saturation length and radiation amplitude up to $\Delta\gamma_{z0}/\gamma_{z0} \approx 0.02$ and a relatively sharp decline beyond that point. This shows that we have abnormal behavior near this peculiar region of the proposed modified wiggler. In the conventional FEL, the pondermotive force arising from the coupling between the wiggler magnetic field B_{w1} and the transverse motion of electrons in the radiation fields, produces bunching in the electron beam that drives at the fundamental wavelength. The mechanism for generating the third harmonic is due to the coupling of the longitudinal motion of electrons at $2k_{w1}z$ with the transverse wiggler motion of electrons.

Enhancement of the third harmonic in the modified wiggler, is due to the micro-bunching of the electron beam at the third harmonic arising from the coupling between wiggler magnetic field B_{w3}, and the transverse motion of electrons in the radiation fields. This mechanism for the generation of the third harmonic, in the modified wiggler, is in addition to the conventional mechanism due to the longitudinal oscillation as described above. Existence of two separate resonant mechanisms for generating the third harmonic suppresses the growth of fundamental by transferring the kinetic energy of the electron beam into the third harmonic radiation leading to its enhanced growth. In the conventional FEL, increasing the energy spread decreases the power for both the fundamental and the third harmonic. We have found that in the modified wiggler there is a window of parameters in which an increase in the energy spread suppresses the fundamental radiation more strongly making the kinetic

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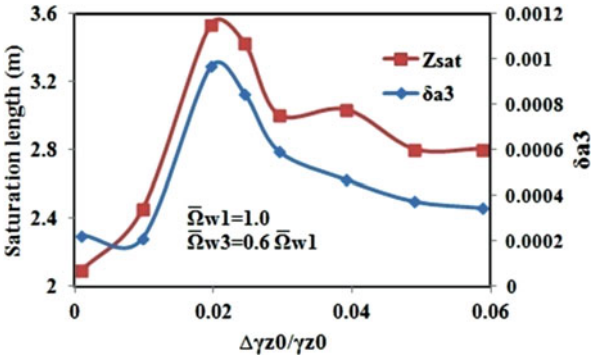


FIGURE 7. The saturated amplitude and the saturation length of third harmonic radiation versus axial energy spread at peculiar wiggler field $\bar{\boldsymbol{\Omega}}_{w3} = 0.6\,\bar{\boldsymbol{\Omega}}_{w1}$ with $\bar{\boldsymbol{\Omega}}_{w1} = 1.0$.

energy of the electron beam to be more available to the third harmonic. Presence of two separate mechanisms makes farther growth of the third harmonic possible with an increase of the energy spread. Although in Ref. (Hussain et al. 2009) beam energy spread is considered in order to analyze the spectrum broadening and small signal gain reduction in harmonic undulator FEL, any peculiar region with abnormal behavior has not been discussed.

4. Conclusion

The third harmonic radiation is improved by adding harmonic wiggler fields to the fundamental in appropriate ratio. Optimum parameters to have maximum improvement of the third harmonic radiation and reduced saturation length of interaction compared to conventional wiggler and also to have lower electron beam energy compared to conventional wiggler with the same wavelength are obtained. Thermal effect of the electron beam on this problem is also studied. Saturated amplitude and saturation length of nonlinear third harmonic radiation with modified planar wiggler are studied in the presence of axial energy spread and are compared with conventional wiggler and cold beam. A peculiar region is found in which there is a sharp increase of the radiation amplitude. Thermal effect was found to be irregular in this peculiar region.

REFERENCES

Asakawa, M. et al. 1995 Development of a modified wiggler for higher harmonic lasing of a free-electron laser. *Nucl. Instrum. Methods Phys. Res.* A **358**, 399–402.

Asakawa, M., Mima, K., Nakai, S., Imasaki, K. and Yamanaka, C. 1992 Higher harmonic generation in a modified wiggler magnetic field. *Nucl. Instrum. Methods Phys. Res. A* **318,** 538–545.

Asakawa, M. et al. 1996 Multi-order harmonic lasing with a modified wiggler. *Nucl. Instrum. Methods Phys. Res. A* **375**, 416–419.

Bazouband, F. and Maraghechi, B. 2012 Improvement of nonlinear harmonics in free electron laser with planar wiggler. *Phys. Plasmas* **19**, 113106.

Bazouband, F. and Maraghechi, B. 2013 Efficiency enhancement of nonlinear odd harmonics in thermal free electron laser. *J. Appl. Phys.* **113**, 17 310–17 318.

Colson, W. B. 1981 Free-electron lasers operating in higher harmonics. *Phys. Rev. A* **24**, 639–641.

Dattoli, G., Giannessi, L., Ottaviani, P. L., Freund, H. P., Biedron, S. G. and Milton, S. V. 2002 Two harmonic undulators and harmonic generation in high gain free electron lasers. *Nucl. Instrum. Methods Phys. Res. A* **495**, 48–57.

https://doi.org/10.1017/S0022377814001159 Published online by Cambridge University Press

Dattoli, G. and Voykov, G. 1993 Spectral properties of two-harmonic undulator radiation. *Phys. Rev. E* **48**, 3030–3039.

Doria, A., Gallerano, G. P., Giovenale, E., Messina, G., Petralia, A. and Spassovsky, I. 2009 The ENEA activity on compact far infrared FELs. In: *Vacuum Electronics Conf. IVEC'09. IEEE Int*.

Freund, H. P. and Antonsen, J. M. 1996 *Principle of Free Electron Laser*, London: Chapman and Hall.

Freund, H. P., Biedron, S. G. and Milton, S. V. 2000 Nonlinear harmonic generation in free-electron lasers. *IEEE J. Quantum Electron.* **36**, 275–281.

Freund, H. P. and Gold, S. H. 1984 Efficiency Enhancement in free-electron lasers using a tapered axial guide field. *Phys. Rev. Lett*. **52**, 926–929.

Freund, H. P. and Pasour, J. 2003 Efficiency enhancement in free-electron lasers by means of concurrent rf acceleration. *Phys. Rev. Lett*. **91**, 094–801.

Giannessi, L. et al. 2012 High-order-harmonic generation and superradiance in a seeded free-electron laser. *Phys. Rev. Lett*. **108,** 164 801.

Gupta, V. and Mishra, G. 2006 Harmonic undulator free electron laser and betatron oscillations. *Nucl. Instrum. Methods Phys. Res. A* **556**, 350–356.

Gupta, V. and Mishra, G. 2007 Variable polarized harmonic-undulator free-electron laser. *Nucl. Instrum. Methods Phys. Res. A* **574**, 150–157.

Hussain, J., Gupta, V. and Mishra, G. 2009 Effect of two-peak beam energy spread on harmonic undulator radiation and free-electron laser gain. *Nucl. Instrum. Methods Phys. Res. A* **608**, 344–350.

Iracane, D., Touati, D. and Chaix, P. 1994 Effect of the wiggler harmonics on the free electron laser dynamics. *Nucl. Instrum. Methods Phys. Res. A* **341**, 220–224.

Jafari Bahman, F. and Maraghechi, B. 2012 Efficiency enhancement in free-electron laser amplifier with one dimensional helical wiggler and ion-channel guiding. *Phys. Plasmas* **19**, 013 107–013 114.

Kong, M. G. 1997 A low voltage waveguide free electron laser based on a longitudinal interaction mechanism. *Opt. Commun.* **141**, 48–52.

Mark, J. S. and Elliott, C. J. 1987 The effects of harmonic wiggler field components on free-electron laser operation. *IEEE J. Quantum Electron.* **23**, 1552–1557.

Marshall, T. C., Shao, Y. and Parsa, Z. 1999 Intensification of harmonic spontaneous radiation with a novel wiggler. In: *Accelerator Concepts: Eighth Workshop*, Maryland: American Institute of Physics.

McNeil, B. W. J., Robb, G. R. M., Poole, M. W. and Thompson, N. R. 2006 Harmonic lasing in a free-electron-laser amplifier. *Phys. Rev. Lett*. **96**, 084 801.

Mishra, G., Gehlot, M. and Hussain, J. K. 2009 Spectral properties of bi-harmonic undulator radiation. *Nucl. Instrum. Methods Phys. Res. A* **603**, 495–503.

Nakao, N. et al. 1998 FEL experiment of the 5th harmonic generation with a modified wiggler. *Nucl. Instrum. Methods Phys. Res. A* **407**, 374–379.

Rouhani, M. H. and Maraghechi, B. 2009 Efficiency enhancement in a single-pass Raman free electron laser. *Phys. Plasmas* **16**, 093 110–093 119.

Rouhani, M. H. and Maraghechi, B. 2010 Simulation of a prebunched free-electron laser with planar wiggler and ion channel guiding. *Phys. Plasmas* **17**, 023 104.

Sircilli, F., Paes, A. C. J., Serrao, V. A. and De Polli, Y. C. 2001 Design of a hybrid undulator for free-electron lasers. *IEEE Trans. Magn.* **37**, 3810–3813.

Sung, C., Tochitsky, S. Y., Reiche, S., Rosenzweig, J. B., Pellegrini, C. and Joshi, C. 2006 Seeded free-electron and inverse free-electron laser techniques for radiation amplification and electron microbunching in the terahertz range. *Phys. Rev. ST Accel. Beams* **9**, 120 703.

Wang, X. J., Watanabe, T., Shen, Y., Li, R. K., Murphy, J. B., Tsang, T. and Freund, H. P. 2007 Efficiency enhancement using electron energy detuning in a laser seeded free electron laser amplifier. *Appl. Phys. Lett*. **91**, 181 115–181 113.

Yu, L. H. et al. 2003 First Ultraviolet High-Gain Harmonic-Generation Free-Electron Laser. *Phys. Rev. Lett*. **91**, 074–801.

Zahedian, M., Maraghechi, B. and Rouhani, M. H. 2012 Efficiency enhancement of a two-beam free-electron laser using a nonlinearly tapered wiggler. *Chinese Phys. B* **21**, 034 101.

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