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The slippy nature of wetting flows

Jacco H. Snoeijer

Physics of Fluids Group, Faculty of Science and Technology, University of Twente, 7500 AE Enschede, The Netherlands

Corresponding author: Jacco H. Snoeijer, j.h.snoeijer@utwente.nl

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The hydrodynamics of wetting involves a singularity of viscous stress, and its microscopic regularisation ultimately determines the speed at which contact lines move over a surface. In a recent paper, Luo & Gao (*J. Fluid Mech.*, vol. 1019, 2025, A52) explore a new analytical solution, based on which they construct a model for 'slippery wedge flow'. This lucid approach provides an accurate description of viscous wetting flows in the presence of slip, without the usual restriction to small contact angles, and offers a quantitative multiscale formalism for slippery contact lines.

Key words: contact lines, capillary flows, drops

1. Introduction

Wetting flows are encountered whenever liquid drops or films are deposited on a solid surface. Yet, the hydrodynamics of wetting flows is extremely delicate, as was first appreciated in a seminal paper by Huh & Scriven (1971). The displacement of the three-phase contact line induces a fluid motion that closely resembles the flow of a viscous wedge (figure 1*a*). In the absence of wall slip, the wedge flow can be solved analytically and reveals a divergence of the viscous stress at the contact line. This singularity of stress is not integrable and leads to a logarithmic divergence of the force required to displace a contact line. This insight led to Huh & Scriven's famous quote that 'not even Herakles could sink a stone', which was immediately followed by the disclaimer 'if the physical model were entirely valid'. What is truly remarkable is that, besides identifying the moving contact line singularity, Huh & Scriven already pointed out a multitude of microscopic mechanisms that could resolve the issue. Among these are wall slip, long-ranged interactions, non-Newtonian rheology or the breakdown of the continuum description.

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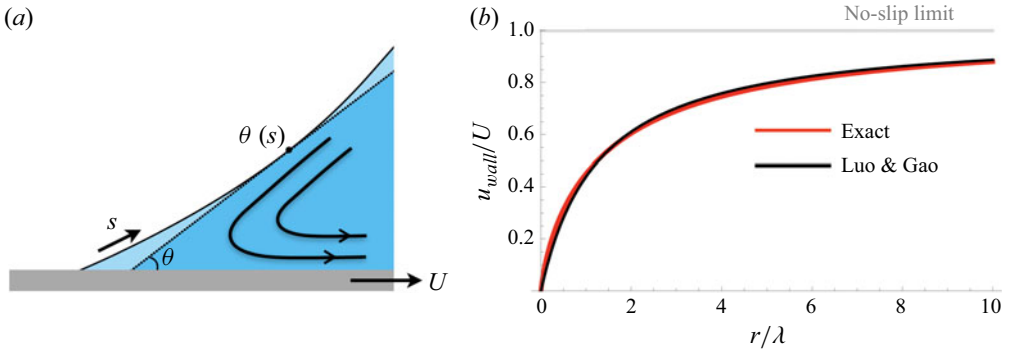


Figure 1. (*a*) Flow near a moving contact line can be approximated by a wedge flow (dark blue region). The flow is sketched in the frame co-moving with the contact line. The visco-capillary action leads to viscous bending, where the wedge angle θ slowly varies with the interface curvilinear coordinate s. (*b*) Fluid velocity at the wall u_{wall} as a function of the distance to the contact line r, for a wedge flow with slip length λ and $\theta = 90°$. The slippery wedge flow proposed by Luo & Gao (2025) (black) closely follows the exact solution (red) by Hocking (1976).

Each of these mechanisms would be pursued, and enthusiastically disputed, in the subsequent decades.

Away from the singular region, say beyond 10 nm from the contact line, the creeping flow is intricate but well understood (Bonn *et al.* 2009; Snoeijer & Andreotti 2013). The flow generates a normal stress that by capillary action leads to a gradual change of the contact angle θ. The resulting visco-capillary balance naturally involves the capillary number $Ca = \eta U/\gamma$, based on viscosity η, surface tension γ and contact line speed U. The interface angle θ, as a function of the distance s along the interface (see figure 1*a*), satisfies the generalised lubrication equation (Snoeijer 2006; Chan *et al.* 2013):

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}s^2} = \frac{Ca\, F(\theta)}{h^2}. \tag{1.1}$$

(The equation follows from an expansion with $Ca \ll 1$, but unlike the classical lubrication theory allows for arbitrary contact angles and can include the viscosity of the outer phase.) The geometric function $F(\theta)$ actually follows from Huh & Scriven's analytical wedge solution. Since h is the interface height, the moving contact line singularity is obvious for $h \to 0$. Still, (1.1) can be safely integrated in the direction away from the contact line, bridging the decades between 10 nm and macroscopic scales. This bridging of scales is perhaps best illustrated by the famous Stokes flow expansion by Voinov (1976) and Cox (1986), which for moderate θ and negligible outer viscosity yields,

$$\theta^3 = \theta_m^3 + 9Ca\left(\ln\frac{s}{L_m} + Q\right). \tag{1.2}$$

This is indeed an asymptotic solution to (1.1) that, strictly speaking, is not universal but applicable when interface curvature vanishes at large scales. This Cox–Voinov law is often used to interpret experiments under partial wetting conditions (Podgorski, Flesselles & Limat 2001). It relates the dynamic contact angle at experimentally accessible scales to the microscopic angle θ_m at the regularisation scale L_m. Yet, the actual closure of the problem requires an explicit treatment of the microscopic region: the theory becomes accurate or predictive only once the microscopic angle θ_m and the integration constant Q are known.

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2. Navier slip regularisation: motivation, challenge, solution

In their recent paper, Luo & Gao (2025) set out to provide a closed-form theory for moving contact lines, using the Navier slip boundary condition as a regularisation mechanism. The physical reality of Navier slip and the experimental determination of the slip length have been unambiguously established since the early 2000s (Bocquet & Charlaix 2010). Remarkably, for simple liquids it was found that the continuum description remains valid down to the scale at which slip occurs, which can be as small as a few molecular sizes. These developments provide a solid foundation for slip in wetting flows.

In spite of this long history, an explicit theory of moving contact lines with slip has remained elusive. The difficulty can be traced back to the nature of Huh & Scriven's wedge flow solution. In a polar coordinate system (r, φ) that is co-moving with the contact line, wedge flow is described by a scale-invariant streamfunction:

$$\psi(r, \varphi) = U r^a f(\varphi), \tag{2.1}$$

where a and $f(\varphi)$ follow from boundary conditions. Without slip the fluid velocity along the plate is uniform, which fixes the exponent to $a = 1$. The introduction of a slip length λ, however, breaks the scale invariance and is not compatible with (2.1). Explicit solutions with the Navier slip boundary conditions are only available for special cases. Figure 1(*b*) shows the fluid velocity at the wall in the presence of slip for the special case of $\theta = 90°$ (Hocking 1976). The wall velocity is not uniform: a stagnant zone appears near the contact line and the no-slip limit is only recovered at distances $r \gg \lambda$.

The key idea by Luo & Gao (2025) is to introduce an auxiliary 'slippery wedge flow' problem that is enforced to be compatible with (2.1). To achieve this, they introduce a variable slip length $\lambda = \beta r$, where β is a constant. This modified boundary condition indeed admits a family of solutions with $a = 1$, parametrised by β. Subsequently, the analysis returns to the true Navier slip boundary condition with constant λ: for a given distance to the contact line, the effective value of β is estimated such that locally the Navier slip boundary condition is satisfied. Strictly speaking, this local approximation scheme does not provide an exact solution. However, Luo & Gao verified against direct numerical simulations that this scheme provides an extremely accurate approximation of the flow. Here, I further illustrate the excellent quality of the proposed slippery wedge flow by a direct comparison of the wall velocity with the exact solution for $\theta = 90°$ (figure 1*b*).

Their discovery of an accurate representation of the slip flow enabled Luo & Gao to derive closed-form expressions for Q in (1.2), for arbitrary contact angles and arbitrary viscosity ratios. In addition, they showed how to consistently introduce slip in the generalised lubrication equation. When the outer viscosity is negligible, this amounts to a simple modification of (1.1):

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}s^2} = \frac{Ca\, F(\theta)}{h\,[h + c\lambda]}, \tag{2.2}$$

a form that was previously hypothesised by Chan *et al.* (2013, 2020). Luo & Gao's slippery wedge flow gives the simple prediction $c = F(\theta)$, which compares well with previous numerical determination. Only for very large angles ($\theta \to 180°$) does the slippery wedge flow lose its accuracy; in this limit one can resort to asymptotic expressions (Chan *et al.* 2020).

3. Future

The Cox–Voinov theory has been a cornerstone for the hydrodynamics of wetting flows. This formalism can now be used without adjustable parameters owing to the closed-form

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expressions derived by Luo & Gao. This paves the way for fully quantitative numerical schemes where the expensive nanoscale resolution can be replaced by the asymptotic Cox–Voinov solution as an effective boundary condition, or even resolve the smallest scales down to $h \to 0$ using (2.2).

Looking ahead, there are numerous developments that reach beyond the viscous-wedge description of wetting. From a hydrodynamic perspective, technologies often operate at velocities for which neither the Reynolds number nor the capillary number are small, or can involve non-Newtonian fluids. In the last few decades attention has shifted from the liquid to the substrate, with the aim of designing surfaces with specific functionality (Quéré 2008; Hardt & McHale 2022). Examples are patterned substrates to tune hydrophobicity, lubricant-infused substrates to enhance slipperyness or adaptive substrates that respond to the presence of the wetting fluid. The latter typically involves polymeric substrates that can deform elastically (Style *et al.* 2017; Andreotti & Snoeijer 2020), and the resulting contact line speed can be tuned by the degree of cross-linking or swelling (Hourlier-Fargette *et al.* 2017). Additional microscopic phenomena like phase separation (Hauer *et al.* 2023; Qian *et al.* 2024) or spontaneous charging (Li *et al.* 2023) can start to play a dominant role. As such, moving contact lines will continue to serve as macroscopic probes of nanoscale physics.

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