# 14 Further reading

In this chapter, we list references to other reviews and we highlight a few subjects that were not covered in the main text.

**Reviews on on-shell methods for scattering amplitudes**

- *Introduction to on-shell methods*
  The QFT textbooks by Srednicki [2], Zee [276], and Schwartz [277] provide brief introductions to the spinor helicity formalism and on-shell recursion relations. In addition, the book [278] offers a comprehensive introduction to on-shell recursion and to loop-integrals and also covers some aspects of scattering amplitudes in $\mathcal{N} = 4$ SYM.
  The following reviews cover various aspects of on-shell methods (the most recent reviews are listed first):
  "A brief introduction to modern amplitude methods" [11].
  "Scattering amplitudes: the most perfect microscopic structures in the universe" [4].
  "Hidden simplicity of gauge theory amplitudes" [52].
  "A first course on twistors, integrability and gluon scattering amplitudes" [279].
  "On-shell methods in perturbative QCD" [280].
  "Calculating scattering amplitudes efficiently" [3].
  "Multiparton amplitudes in gauge theories" [1].
- *Numeric methods and applications in phenomenology*
  "Susy theories and QCD: Numerical approaches" [70].
  "One-loop calculations in quantum field theory: from Feynman diagrams to unitarity cuts" [77].
  "Simplifying multi-jet QCD computation" [281].
  "Loop amplitudes in gauge theories: Modern analytic approaches" [71].
- *Gravity*
  "Introduction to the effective field theory description of gravity" [282].
  "Perturbative quantum gravity and its relation to gauge theory" [188].
  "Ultraviolet behavior of $\mathcal{N} = 8$ supergravity" [283].

**Less supersymmetry:** $1 \leq \mathcal{N} < 4$

An obvious arena for on-shell methods is theories with non-maximal supersymmetry. Scattering amplitudes in $\mathcal{N} = 1$ supersymmetric theories have been of particular interest in particle phenomenology and on-shell methods were used in such studies [48, 67, 284–286]. The superamplitude and on-shell superspace formalism generalizes to $1 \leq \mathcal{N} < 4$ SYM [87] and has also been used for SYM coupled with matter [85, 287]. It is expected

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that the on-shell diagram approach to planar amplitudes of $\mathcal{N} = 4$ SYM has a natural extension to $\mathcal{N} < 4$ theories [120].

**Amplitudes with massive particles**

We have focused on amplitudes with massless particles, but there are also efficient on-shell methods available for amplitudes involving massive particles. The spinor helicity formalism for massless particles, introduced in Section 2, can be generalized to massive particles. There are two approaches to this. In the first, one studies the eigenvectors of the momentum matrix $p_{\alpha\dot{\beta}} = p_\mu \sigma^\mu_{\alpha\dot{\beta}}$ to directly get solutions to the massive Dirac equation. In the second approach, the time-like momentum $p_i$ is decomposed along two light-like directions by introducing a null reference vector $q_i$ for each state: $p_i^\mu = p_{i\perp}^\mu - \frac{m_i^2}{2q_i \cdot p_i} q^\mu$. The familiar spinor helicity formalism can then be used for $q_i = -|q_i\rangle[q_i|$ and $p_{i\perp} = -|i_\perp\rangle[i_\perp|$. Helicity is only a Lorentz-invariant quantity for massless particles, but $q_i$ breaks Lorentz-invariance and can therefore be used to define a helicity basis in which we can calculate helicity amplitudes. For an introduction to both approaches, see [288]. The papers [135, 289, 290] studied applications of BCFW recursion relations with massive particles. A recent discussion of the latter approach, as well as applications to CSW-like recursion relations, was given in [26]. Finally, let us mention that there are simple amplitudes with a pair of massive particles. Examples of such "towers" of amplitudes – in a sense massive versions of Parke–Taylor – were presented for scattering processes on the Coulomb branch of $\mathcal{N} = 4$ SYM in [140, 203] (see also [291–293]).

**Extensions of recursion relations**

Attempts have been made to generalize various forms of recursion relations beyond gauge theory and gravity, for example to string theory amplitudes [294, 295] and to non-linear sigma models [6, 296]. Recursion relations can also be utilized to obtain rational functions that appear at 1-loop [297]. A review of various recursion relations is given in [298].

**Triality: Wilson loop, correlation function, amplitude**

In Section 5.3 we discussed the emergence of dual superconformal symmetry in planar $\mathcal{N} = 4$ SYM. It states that the scattering amplitude in the dual coordinates is superconformal covariant. Could we define $\mathcal{N} = 4$ SYM directly in these new coordinates? In dual coordinates, the kinematic setup for the on-shell momenta is a polygon with null edges. A similar physical quantity is a Wilson-loop specialized to a null-polygon contour. So could the amplitude in momentum space be dual to a null-polygon Wilson-loop in the dual space? Indeed it is. This duality has been established at strong coupling by Alday and Maldacena [107] as well as at weak coupling by Drummond, Korchemsky, and Sokatchev [299].

At strong coupling, the duality can be understood as a consequence of T-duality in string theory [300, 301]. At weak coupling, evidence for such duality was first reported by [299, 302]. It was later proven by defining the action of $\mathcal{N} = 4$ SYM directly in supertwistor space [303]. Remarkably, using the duality, the first computation of the 6-point 2-loop MHV amplitude was done by computing the 6-edged Wilson loop [110]. The amplitude/Wilson-loop duality was first established between the bosonic Wilson-loop and the ratio of the

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MHV scattering amplitude, divided by the MHV tree amplitude. It can be generalized to $\mathrm{N}^K$MHV amplitudes by supersymmetrizing the bosonic Wilson-loop [303–305].

Another extension is the realization that the super Wilson-loop in $\mathcal{N} = 4$ SYM is also dual to the correlation function of operators with light-like separation [306]. This can also be proven in supertwistor space [307]. For a review on the (MHV)amplitude/(bosonic)Wilson-loop duality see [308–310], for the general amplitude/super-Wilson loop duality in the framework of supertwistor space, see [311].

Based on an operator-product-expansion approach first developed in the perturbative computation of null Wilson-loops [312], a non-perturbative formulation of the $S$-matrix/Wilson-loop for planar $\mathcal{N} = 4$ SYM has been proposed in [313–315]. Its perturbative expansion gives important predictions for the explicit loop amplitude, and as a result the integrated functional form of the 6-point 3- and 4-loop MHV amplitude for planar $\mathcal{N} = 4$ SYM [316, 317] was found.

**Twistors**

Standard reviews of twistor space include [318–320]. For amplitude-friendly reviews, we suggest [321] as well as [54, 60].

**What's next?**

We hope you have found this book useful. We look forward to seeing what the future will bring for studies of scattering amplitudes.

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