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EFFECT OF A SHIFT IN THE TREND FUNCTION ON DICKEY–FULLER UNIT ROOT TESTS

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This article analyzes the asymptotic behavior of the Dickey–Fuller unit root tests when the variable is generated under the breaking trend hypothesis. Our results show that the asymptotic behavior of these statistics allows for the rejection of the unit root hypothesis. This asymptotic finding contrasts with the results that can be found in the literature devoted to the analysis of the integration order of a variable in the presence of a structural break. However, some Monte Carlo exercises show that the argument of Perron (1989, *Econometrica* 57, 1361–1401) that the tests are biased in favor of nonrejection of the unit root hypothesis remains valid for sample sizes of practical interest.

1. INTRODUCTION

Since Dickey and Fuller (1979) settled the basis of unit root tests, a large volume of literature has been devoted to the analysis of the main characteristics of these statistics. One of the most interesting topics related to this theme is the study of the behavior of the unit root tests in variables that exhibit a structural break. This issue has a long tradition in the literature and has been widely discussed. Among others, we can cite the works of Perron (1989), Rappoport and Reichlin (1989), Christiano (1992), Banerjee, Lumsdaine, and Stock (1992), and Zivot and Andrews (1992) with respect to breaking trend variables and Perron (1990) and Perron and Vogelsang (1992) with respect to changing mean variables.

From an analytical point of view, the most influential contribution is found in Perron (1989). This author showed that the autoregressive parameter estimator goes asymptotically to 1 when the variable is generated according to the breaking trend model. Similarly, the normalized estimator goes to a value that will never permit the rejection of the unit root null hypothesis. Thus, the main conclusion that can be inferred is that these statistics are not capable of rejecting their null

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hypothesis asymptotically. As Rappoport and Reichlin (1989) stated, we can consider that the nonstationary model (difference stationary [DS], according to the terminology employed by Nelson and Plosser, 1982) becomes a default specification of the true data-generating process and that the Dickey–Fuller tests lead us to erroneously overaccept the unit root null hypothesis.

The aim of this paper is to deepen the study of the effect caused by a break in the trend function on the Dickey–Fuller unit root tests by extending the results of Perron (1989) for the most commonly used Dickey–Fuller test, namely, the pseudo *t*-ratio. This main objective also directs us toward the finding that the asymptotic behavior of the normalized estimator for the changing growth model is not accurately determined in Perron (1989). Finally, we show that the functioning of the Dickey–Fuller statistics is different for the changing growth model than for the mixed model (which combines a break in the intercept with a break in the trend), a result that is in contrast to Perron (1989), where it is supposed that these effects are identical. All these results are reflected in two theorems whose proofs are included in the Appendix.

2. ANALYSIS OF THE EFFECT OF A SHIFT IN THE TREND FUNCTION ON UNIT ROOT TESTS

Let us suppose that y_t is generated by a trend stationary model such that it exhibits a break in period T_B, with $T_B = \lambda T$ and where the parameter λ, habitually described as the break fraction, controls the time at which the change in the parameters of the trend occurs. Under these characteristics, we consider the following two alternative models:

$$y_t = \mu_1 + \beta_1 t + (\beta_2 - \beta_1)(t - T_B)DU_t + u_t, \quad \textbf{(1)}$$

$$y_t = \mu_1 + (\mu_2 - \mu_1)DU_t + \beta_1 t + (\beta_2 - \beta_1)tDU_t + u_t, \quad \textbf{(2)}$$

where t is a deterministic trend and DU_t is a dummy variable that takes the value 1 if $t > T_B$ and 0 otherwise. Model (1) is known as the changing growth model. Model (2) presents a sudden change in the level, followed by a change in the growing path, and we shall refer to this as the mixed model.

Let us now suppose that we are interested in analyzing the integration order for y_t. This can be done by calculating either the pseudo *t*-ratio or the normalized estimator of the autoregressive parameter in the model

$$y_t = \mu + \beta t + \rho y_{t-1} + u_t. \quad \textbf{(3)}$$

We are now going to study the asymptotic behavior of the statistics τ_τ and $T(\hat{\rho} - 1)$ when the variable y_t is generated under either model (1) or model (2). Following Perron (1989), to carry out this process we consider that both the prebreak and postbreak samples increase at the same rate as the total number of observations. Consequently, we assume that $T_B = \lambda T \; \forall T$. The limits are taken as T increases to

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infinity in a sequence that ensures an integer value of T_B for a given λ. The results we have obtained are presented in the following theorems. The first relates to the changing growth model.

THEOREM 1. *Let* $\{y_t\}_0^T$ *be a sample of size* $T + 1$ *generated under model* (1), *with the innovations* $\{u_t\}$ *satisfying Perron* (1989) *Assumption* 1. *Let "*$\rightarrow$*" denote convergence in probability and let* $T_B = \lambda T$, *for all* T *and* $0 < \lambda < 1$. *Thus, as* T *goes to infinity*,

(a)

$$T(\hat{\rho} - 1) \rightarrow -\frac{3(1 - 2\lambda)}{2(1 - \lambda)\lambda},$$

(b)

$$T^{-1/2}\tau_\tau \rightarrow -(1 - 2\lambda)\sqrt{\frac{3(\beta_2 - \beta_1)^2(1 - \lambda)\lambda}{8(\sigma_u^2 - \gamma) + (\beta_2 - \beta_1)^2(1 - \lambda)\lambda}},$$

with

$$\sigma_u^2 = \lim_{T\rightarrow\infty} T^{-1} \sum E(u_t^2),$$

$$\gamma = \lim_{T\rightarrow\infty} T^{-1} \sum E(u_t u_{t-1}).$$

Proof. See Appendix.

Some of these findings contrast with those reported in Perron (1989). Let us begin by studying part (a). According to Perron (1989, p. 1371), the limit value of the normalized autoregressive parameter would be

(a′)

$$T(\hat{\rho} - 1) \rightarrow \frac{3(1 - 4\lambda + 5\lambda^2 - 2\lambda^3)}{2(3 - 4\lambda + 3\lambda^2 - 3\lambda^3 + \lambda^4)}.$$

There are formally small, but qualitatively very important, differences between expression (a) of our Theorem 1 and Perron's expression (a′), in that our asymptotic $T(\hat{\rho} - 1)$ value indicates that it is sometimes possible to correctly reject the unit root null hypothesis. For example, this hypothesis could be asymptotically rejected whenever λ takes a value lower than 0.064, this is to say, whenever the break occurs in the first 6% of the sample.

Let us now study the behavior of the pseudo *t*-ratio. It was shown in Perron (1989) that $\hat{\rho}$ goes asymptotically to 1 under the breaking trend alternative. Therefore, it could be expected that the τ_τ statistic would go to zero. However, part (b) of our Theorem 1 shows that this statistic does not vanish asymptotically but rather diverges. Moreover, it leads us to the asymptotic rejection of the unit root

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null hypothesis against a stationary alternative whenever the break point occurs in the first half of the sample. By contrast, it goes to $+\infty$ if the break occurs in the second half of the sample ($\lambda > 0.5$) and tends to 0 when the break occurs in the center. It is also worth pointing out that this statistic, as well as the normalized bias, goes toward $-\infty$ when $\lambda = 1$. This fact implies that both statistics present a discontinuity at the end of the sample, and, therefore, we can expect a poor approximation of their limiting distribution in finite samples for values of λ near 1.

To assess the relevance of these asymptotic results in finite samples, we carry out a simple Monte Carlo simulation exercise. Let us consider that y_t is generated by model (1). The parameters of the model can take the following values: $\mu_1 = \beta_1 = 1$, $\beta_2 - \beta_1 = \{0.5, 1, 2, 3, 4, 5\}$, $\lambda = \{0.05, 0.3, 0.4, 0.5, 0.6, 0.7\}$, and u_t is an independent and identically distributed (i.i.d.) $N(0,1)$ distributed innovation. Table 1 reports the power of the two Dickey–Fuller statistics under these combinations of parameters. From Table 1, we can note that $T(\hat{\rho} - 1)$ behaves according to the results of Theorem 1 whenever $(\beta_2 - \beta_1) \geq 1$. Some alterations to this pattern appear for a lower value of the break magnitude. The situation is quite different for the pseudo *t*-ratio. Here, we can observe serious deviations from its limiting values, which are especially important when $\lambda < 0.5$. Thus, we can conclude that, although the asymptotic results allow for the rejection of the unit

TABLE 1. Power of the Dickey–Fuller statistics under the changing growth alternative[a]

	λ					
$\beta_2 - \beta_1$	0.05	0.3	0.4	0.5	0.6	0.7
			$T(\hat{\rho} - 1)$			
0.5	100	0.7	0	0	0.1	0.1
1	100	0	0	0	99.7	99.4
2	100	0	0	0	100	100
3	100	0	0	0	100	100
4	100	0	0	0	100	100
5	100	0	0	0	100	100
			τ_τ			
0.5	100	7.8	0	0	0.1	0.2
1	100	0	0	0	97.7	99.7
2	100	1.3	0	0	100	100
3	100	69.5	0	0	100	100
4	100	99.9	0	0	100	100
5	100	100	0	0	100	100

[a]The variable y_t is generated under model (1) with the innovation following an i.i.d. $N(0,1)$ distribution. The sample size is 100, and the number of replications is 10,000. Columns with $\lambda \leq 0.5$ report the percentage of rejections of the unit root null hypothesis against a stationary alternative. The other columns report the percentage of rejections of the unit root null hypothesis against an explosive alternative.

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root null hypothesis when the break appears in the first half of the sample, this statistic will nevertheless lead us to accept the unit root hypothesis for sample sizes and break magnitudes of practical interest. Consequently, our asymptotic results do not alter the main message of Perron (1989).

We now turn to a consideration of the second of our alternative models. Once again, according to the seminal work of Perron (1989), we can see that no differences exist in the behavior of the Dickey–Fuller tests when the variable y_t is generated by either model (1) or (2). However, in the following theorem we shall verify that the behavior of both Dickey–Fuller unit root tests is different in the two models.

THEOREM 2. *Let* $\{y_t\}_0^T$ *be a sample of size* $T+1$ *generated under model* (2), *with the innovations* $\{u_t\}$ *satisfying Assumption* 1. *Let "*$\rightarrow$*" denote convergence in probability and let* $T_B = \lambda T$, *for all* T *and* $0 < \lambda < 1$. *Thus, as* T *goes to infinity*:

(a)

$$T(\hat{\rho} - 1) \rightarrow -\frac{3(1 + \lambda + 4\lambda^2)}{2(1 - 2\lambda + 4\lambda^2)\lambda},$$

(b)

$$\tau_\tau \rightarrow -\frac{(1 + \lambda + 4\lambda^2)\sqrt{3(\lambda - 3\lambda^2 + 6\lambda^3 - 4\lambda^4)}}{2(1 - 2\lambda + 4\lambda^2)\lambda}.$$

Proof. See Appendix

On comparing Theorems 1 and 2, it is worthy of note that the two Dickey–Fuller unit root tests do not behave in a similar way. These discrepancies can be explained by taking into account that the sample moments of the variable y_t are different in the two lemmas of the Appendix.

While the estimator of the autoregressive parameter goes asymptotically to 1, just as it does in the shock model, we can see that the pseudo *t*-ratio does not diverge; rather, it converges to negative values. Similarly, the normalized autoregressive parameter estimator also tends to a limit value that is different from the result obtained in Theorem 1. Both limit values allow for the correct rejection of the null hypothesis, but only when the break appears in the earliest sample observations. For example, for the 5% significance level, the pseudo *t*-ratio rejects the null hypothesis when $\lambda < 0.08$, whereas the normalized estimator of the autoregressive parameter rejects the null hypothesis when $\lambda < 0.09$. Given that these restrictions are quite infrequent in empirical analysis, it can be concluded that these statistics are not capable of rejecting the unit root null hypothesis for cases of empirical interest, and, once again, our asymptotic results do not modify the main message of Perron (1989).

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3. CONCLUSIONS

In this paper we have studied the effect of a shift in the trend function on the Dickey–Fuller unit root tests. We have shown that when the variable is generated under the changing growth model, the limit value of the normalized estimator of the autoregressive parameter is different from that calculated by Perron (1989). The correct limit value allows for the asymptotic rejection of the unit root null hypothesis. However, the probability of rejecting this hypothesis is quite low, in that it is necessary for the break to appear in the earliest observations, a situation that is not usual in the variables that are habitually employed in empirical studies. Similarly, the pseudo t-ratio diverges to $-\infty$ if the break appears in the first half of the sample and to $+\infty$ if it occurs in the second half, and it goes to 0 if the break time coincides with the center of the sample.

When the variable is generated according to the mixed model, we show that the asymptotic behavior of the Dickey–Fuller statistics does not coincide with that of the changing growth model, as is assumed in Perron (1989). On the contrary, both statistics converge to values that only permit the rejection of the unit root null hypothesis if the break time is very close to the beginning of the sample.

In spite of these results, we should also point out that the limit distributions that have been calculated in this paper do not provide a good approximation for the behavior of the statistics in finite samples. Some simulation exercises for the case of the changing growth model have helped us to verify this point. Thus, the message of Perron (1989) remains valid for purposes of empirical application, and we can consider that the Dickey–Fuller tests are biased in favor of the nonrejection of the unit root hypothesis for cases of empirical interest.

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APPENDIX

To prove the two theorems described earlier, we will adopt the same procedure as in Perron (1989). In what follows, all the summations go from 1 to T, with T_B, t, DU_t, and u_t having been defined earlier. We will begin by expressing the autoregressive parameter estimator and the normalized estimator of the autoregressive parameter as follows:

$$\hat{\rho} = \frac{A}{D}, \tag{A.1}$$

$$T(\hat{\rho} - 1) = T\frac{A - D}{D}, \tag{A.2}$$

where A and D are defined in Perron (1989, p. 1391). Similarly, the sum of the squared residuals is expressed as

$$\begin{aligned} SSR = \Big[& -\Big(\sum ty_{t-1} \sum y_t\Big)^2 - 2T \sum ty_t \sum ty_{t-1} \sum y_t y_{t-1} \\ & + 2 \sum t \sum ty_{t-1} \sum y_t \sum y_t y_{t-1} - \Big(\sum t \sum y_t y_{t-1}\Big)^2 \\ & + T \sum t^2 \Big(\sum y_t y_{t-1}\Big)^2 + 2 \sum ty_t \sum ty_{t-1} \sum y_t \sum y_{t-1} \\ & + 2 \sum t \sum ty_t \sum y_t y_{t-1} \sum y_{t-1} - 2 \sum t^2 \sum y_t \sum y_t y_{t-1} \sum y_{t-1} \\ & - \Big(\sum ty_t \sum y_{t-1}\Big)^2 + T\Big(\sum ty_t\Big)^2 \sum y_{t-1}^2 - 2 \sum t \sum ty_t \sum y_t \sum y_{t-1}^2 \\ & + \sum t^2 \Big(\sum y_t\Big)^2 \sum y_{t-1}^2 + T\Big(\sum ty_{t-1}\Big)^2 \sum y_t^2 - 2 \sum t \sum ty_{t-1} \sum y_{t-1} \sum y_t^2 \\ & + \sum t^2 \Big(\sum y_{t-1}\Big)^2 \sum y_t^2 + \Big(\sum t\Big)^2 \sum y_{t-1}^2 \sum y_t^2 - T \sum t^2 \sum y_{t-1}^2 \sum y_t^2 \Big] \Big/ D, \end{aligned} \tag{A.3}$$

with the third element of the main diagonal of the matrix $(X'X)^{-1}$ being

$$(X'X)^{-1}_{\hat{\rho}\hat{\rho}} = \frac{\Big(\sum t\Big)^2 - T \sum t^2}{D}. \tag{A.4}$$

Proof of Theorem 1. Thus, the solution of equations (A.1)–(A.4) depends only on the calculations of several sample moments of the variables of the model. For Theorem 1, we obtain these values in the following lemma.

LEMMA A.1. *Assume that y_t is generated according to* (1), *with the innovation satisfying Perron* (1989) *Assumption* 1. *Thus, it holds that*

(a)

$$\begin{aligned} \sum y_{t-1} = & \tfrac{1}{2}[\beta_1 \lambda(2 - \lambda) + \beta_2(1 - \lambda^2)]T^2 \\ & + [\mu_1 - \tfrac{1}{2}\beta_1 \lambda + \tfrac{1}{2}\beta_2(\lambda - 1)]T + \sum u_{t-1}, \end{aligned}$$

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(b)

$$\sum ty_{t-1} = \tfrac{1}{6}[\beta_1\lambda(3-\lambda^2) + \beta_2(2-3\lambda+\lambda^3)]T^3 + \tfrac{1}{2}[(\beta_1-\beta_2)\lambda(1-\lambda)+\mu_1]T^2 + [\tfrac{1}{2}\mu_1 - \tfrac{1}{3}\beta_1\lambda + \tfrac{1}{3}\beta_2(\lambda-1)]T + \sum tu_{t-1},$$

(c)

$$\sum y_t y_{t-1} = \tfrac{1}{3}[\beta_1^2\lambda^2(3-2\lambda) + \beta_2^2(1-\lambda)^3 + 3\beta_1\beta_2\lambda(1-\lambda)^2]T^3 + [\beta_1\mu_1\lambda(2-\lambda) + \beta_2\mu_1(1-\lambda)^2]T^2 + o_p(T^2),$$

(d)

$$\sum y_{t-1}^2 = \tfrac{1}{3}[\beta_1^2\lambda^2(3-2\lambda) + \beta_2^2(1-\lambda)^3 + 3\beta_1\beta_2\lambda(1-\lambda)^2]T^3 + \{\beta_1[\mu_1\lambda(2-\lambda) - \beta_2\lambda(1-\lambda)] + \beta_2\mu_1(1-\lambda)^2 - \tfrac{1}{2}[\beta_1^2\lambda^2 + \beta_2^2(1-\lambda)^2]\}T^2 + o_p(T^2).$$

On substituting these values in equations (A.1)–(A.4) we obtain

$$T^{-7}A \to \frac{(\lambda-1)^3\lambda^3(\beta_2-\beta_1)^2}{36}, \tag{A.5}$$

$$T^{-7}D \to \frac{(\lambda-1)^3\lambda^3(\beta_2-\beta_1)^2}{36}, \tag{A.6}$$

$$T^{-6}(A-D) \to \frac{(1-2\lambda)(\lambda-1)^2\lambda^2(\beta_2-\beta_1)^2}{24}. \tag{A.7}$$

Thus, it is clear that the asymptotic limit of $\hat{\rho}$ is 1 and that $T(\hat{\rho}-1)$ goes to the value stated in part (a).

To prove part (b) of Theorem 1, we have first to take into account that

$$\hat{\sigma}^2 \to \frac{8(\sigma_u^2-\gamma) + (\beta_2-\beta_1)^2(1-\lambda)\lambda}{4}, \tag{A.8}$$

$$T^3(X'X)^{-1}_{\hat{\rho}\hat{\rho}} \to \frac{3}{(\beta_2-\beta_1)^2(1-\lambda)^3\lambda^3}. \tag{A.9}$$

If the earlier results are substituted in the pseudo *t*-ratio expression, we obtain the limit value reported in part (b). ■

Proof of Theorem 2. To prove Theorem 2 we should start by obtaining the new sample moments of y_t.

LEMMA A.2. *Assume that y_t is generated according to* (2), *with the innovation satisfying Perron* (1989) *Assumption* 1. *Thus, it holds that*

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(a)

$$\sum y_{t-1} = \tfrac{1}{2}[\beta_1\lambda^2 + \beta_2(1-\lambda^2)]T^2 + [\mu_1\lambda + (1-\lambda)\mu_2 + \tfrac{1}{2}\beta_1\lambda - \tfrac{1}{2}\beta_2(1+\lambda)]T + \sum u_{t-1} - (\mu_2 - \mu_1),$$

(b)

$$\sum ty_{t-1} = \tfrac{1}{3}[\beta_1\lambda^3 + \beta_2(1-\lambda^3)]T^3 + \tfrac{1}{2}[2(\beta_1-\beta_2)\lambda^2 + \mu_1\lambda^2 + \mu_2(1-\lambda^2)]T^2 + [\tfrac{3}{2}\lambda\mu_1 + \tfrac{1}{2}\mu_2(1-3\lambda) + \tfrac{2}{3}\beta_1\lambda - \tfrac{1}{3}\beta_2(1+2\lambda)]T + \sum tu_{t-1} - (\mu_2 - \mu_1),$$

(c)

$$\sum y_t y_{t-1} = \tfrac{1}{3}[\beta_1^2\lambda^3 + \beta_2^2(1-\lambda^3)]T^3 + \{\beta_1\lambda^2[\beta_2 + \mu_1] + \beta_2\mu_2(1-\lambda^2) - \beta_2^2\lambda^2\}T^2 + o_p(T^2),$$

(d)

$$\sum y_{t-1}^2 = \tfrac{1}{3}[\beta_1^2\lambda^3 + \beta_2^2(1-\lambda^3)]T^3 + [\beta_1\mu_1\lambda^2 + \tfrac{1}{2}\beta_1^2\lambda^2 - \tfrac{1}{2}\beta_2^2(1+\lambda^2) + \beta_2\mu_2(1-\lambda^2)]T^2 + o_p(T^2).$$

On taking these new values into account, we obtain

$$T^{-7}A \to \frac{(\beta_2-\beta_1)^2\lambda^3(-1+3\lambda-6\lambda^2+4\lambda^3)}{36}, \tag{A.10}$$

$$T^{-7}D \to \frac{(\beta_2-\beta_1)^2\lambda^3(-1+3\lambda-6\lambda^2+4\lambda^3)}{36}, \tag{A.11}$$

$$T^{-6}(A-D) \to \frac{(\beta_2-\beta_1)^2\lambda^2(1+3\lambda^2-4\lambda^3)}{24}. \tag{A.12}$$

On dividing (A.12) by (A.11), we find that the normalized autoregressive parameter goes to the value reported in part (a).

Finally, to show part (b), we have to bear in mind that

$$T^{-1}\hat{\sigma}^2 \to (\beta_2-\beta_1)^2\lambda^2, \tag{A.13}$$

$$T^3(X'X)^{-1}_{\hat{\rho}\hat{\rho}} \to \frac{3}{(\beta_2-\beta_1)^2\lambda^3(1-3\lambda+6\lambda^2-4\lambda^3)}. \tag{A.14}$$

On substituting these values in the expression of the pseudo *t*-ratio, we obtain the asymptotic limit of this statistic, as reported in part (b). ■

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