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The Boussinesq Debate: Reversibility, Instability, and Free Will

Thomas Michael Mueller

University of Lausanne
E-mail: thomas.mueller.2@unil.ch

Argument

In 1877, a young mathematician named Joseph Boussinesq presented a *mémoire* to the *Académie* des sciences which demonstrated that some differential equations may have more than one solution. Boussinesq linked this fact to indeterminism and to a possible solution to the free will versus determinism debate. Boussinesq's main interest was to reconcile his philosophical and religious views with science by showing that matter and motion do not suffice to explain all there is in the world. His argument received mixed criticism that addressed both his philosophical views and the scientific content of his work, pointing to the physical "realisticness" of multiple solutions. While Boussinesq proved to be able to face the philosophical criticism, the scientific objections became a serious problem, thus slowly moving the focus of the debate from the philosophical plane to the scientific one. This change of perspective implied a wide discussion on topics such as instability, the sensitivity to initial conditions, and the conservation of energy. The Boussinesq debate is an example of a philosophically motivated debate that transforms into a scientific one, an example of the influence of philosophy on the development of science.

In this paper, we will discuss a debate that took place between 1874 and 1880 in France concerning the compatibility of a mechanical and materialistic view of nature, i.e. matter in motion is all that exists and the existence of free will. The main element of this debate is a (re)discovery of Poisson (1806) by Joseph Valentin Boussinesq: some differential equations that describe concrete physical situations may have more than one solution.[1] Boussinesq never claimed priority for such a discovery; nonetheless, he claimed a new understanding of this fact, namely that Newtonian physics is indeterministic in some particular cases. There are cases in which a complete knowledge of the mechanical properties of a physical system (i.e. the entire set of forces and the initial conditions) do not suffice to determine the future state of the

[1] Differential equations may have more than one solution if they fail to satisfy the Lipschitz condition, i.e., that for a function f, $\exists L > 0\ |f(x) - f(y)| \leq L|x - y|$.

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system. Nonetheless, one of the possible future states implied by the dynamics must happen: therefore, matter and motion are not a complete description of nature. They do not exhaust all that exists in the world. Boussinesq's idea was to introduce a new principle, the "*principe directeur*," in order to fill the gap between physical reality and mathematical descriptions. The *principe directeur* was, in Boussinesq's mind, a solution to the incompatibility between the deterministic description of matter in motion and free will. The *principe directeur* restores determinism but introduces a non-mechanical principle which may be compatible with free will.

By the term determinism, Boussinesq refers to the Laplacian demon, i.e. to the potential calculability of every past and future state of the universe from the present state of the universe. He believes that a complete determination of the future state of the universe by the laws of physics is incompatible with the notion of free will. His declination of determinism may be labeled mechanistic, since it implies that the determination depends only on the physical state of the universe and that the laws of mechanics (in the Laplacian view) exhaust all there is to know about the universe.

In contrast, determinism was not a new philosophical commitment. Boussinesq was inspired by a philosophical tradition that we may label "spiritualistic" following the French term *spiritualisme*. *Spiritualisme* was a particular philosophy, well-developed in France at the time which was mainly inspired by Victor Cousin and developed by others such as Etienne Vacherot (1858, 1863), Elme-Marie Caro (1867), and Paul Janet (1865).[2]

Typical, spiritualistic philosophy consisted of confronting materialism and positivism and, of course, their sibling, determinism. The main emphasis of a spiritualistic philosophy of science was to show that matter and motion do not suffice to explain all there is to know about the real world. For example, Vacherot wrote four papers in *La revue de deux mondes* (Vacherot 1869 a,b,c,d) dealing with the sensible question of determinism and trying to escape the fatalist dilemma by denying materialism. Laplace was one of the champions of materialism and determinism and contradicting Laplace was one of the central issues of spiritualistic philosophies.

Spiritualistic philosophies were in general quite sympathetic with a religious view of the world. Defending the existence of more than matter in motion was of course compatible with the idea of God and it was common for spiritualistic philosophers to defend a form of religiosity. This does not imply that spiritualism and religion are one and the same, but there is more than a fortuitous proximity between them. As we will see, Boussinesq's own ideas about religion match quite well with his spiritualistic philosophy: Boussinesq was a faithful Catholic and deeply interested in the question of the compatibility between science and religion.

[2] This choice of authors is not supposed to represent the entire school of spiritualism, nor the majority of it. We have chosen them because they appear at different places in Boussineq's letters and papers. Vacherot published some of his major works on the philosophy of science in *La revue de deus mondes* (Vacherot 1869 a,b,c,d) and we know that Boussinesq used to read it daily (Picard 1934, 55).

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Boussinesq's memoir was followed by quite an important debate in which scientists, philosophers, and scholars from several fields of interest confronted one another on the question of free will. This debate entangled religious and philosophical concerns, especially at the beginning, and more purely scientific concerns later on. This evolution of a religiously- and philosophically-motivated debate into a scientific and mathematical controversy is one of the most interesting aspects of the discussion that followed Boussinesq's *Mémoire*. Few historical works have noticed this debate (Nye 1976; Israel 1992, 1991; Zerner 1994; Van Strien 2014). Van Strien (2014) discusses Boussinesq's *Mémoire* in some depth but as a conclusion to a previous debate. We will consider a different approach: we will analyze Boussinesq's ideas as the starting point of a later debate.

The aim of this paper is to reconstruct this debate, both through the analysis of published answers and the private correspondence of Boussinesq. We will show that the debate started as a defense of a spiritualistic philosophy, anchored in the personal faith of Joseph Boussinesq: religious beliefs were the main motivation for Boussinesq and some of his supporters. This point is extensively discussed in Zerner (1994). The issue of the debate was of a very different nature, what we may call the physico-mathematical rift, i.e. to what extent one should believe mathematical descriptions to be physically true (i.e. the amount of ontological commitment implied by mathematical descriptions). Boussinesq's debate then starts as a private questioning on the matter of free will, i.e. as a metaphysical problem, and ends as a debate on the foundations of physics and mathematics. It is of particular interest to understand how this shift of focus happened, since it is an example of a scientific debate whose origins are not rooted in a scientific concern and, therefore, a debate regarding the influence of philosophy and religion on the development and the advancement of science.

The structure of the paper is as follows: we will discuss Philippe Breton's argument on reversibility as a prologue to Boussinesq. There are two main reasons for linking Breton to Boussinesq, mainly that Boussinesq considered Breton to be a predecessor in his letters, and that other scholars, such as Maxwell, considered Breton's papers to be linked to the Boussinesq's debate (see "Essay for the Eranus Club on Science and Free Will," 11.2.1873, in Maxwell 1995, 814–823). Breton and Boussinesq shared the idea that the laws of physics did not suffice to explain living beings. It will be profitable to compare their views: Breton tries to invalidate the principles of mechanics in a *reductio ad absurdum* argument, while Boussinesq tries to defend them and to add to them some new principle that will be compatible with mechanics.

We will then discuss Boussinesq's *Mémoire* and Bertrand's reaction to it: other scholars have previously discussed this reaction (especially Israel 1991 and Van Strien 2014). We will integrate into this discussion the private correspondence between Boussinesq and Saint-Venant on the topic, a new contribution to the debate.

Finally, we will analyze the debate among several scholars that followed, as a reaction to Boussinesq and Bertrand. This second issue will involve a more tricky one by including a broad range of arguments, such as irreversibility, instability, and similar questions that have a lot do with the relation between physical facts and

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mathematization. The modern reader will recognize topics that characterize the contemporary debate on the instability and chaos theory, on thermodynamics and more generally, on indeterminism and causation. The Boussinesq debate will therefore become an interesting precursor to several, well-developed problems of contemporary philosophies of science.

1. Prologue

Disputes and debates on the compatibility between mechanistic science and free will were a matter of common concern for both philosophers and scientists in the second half of the nineteenth century. One may think of several examples from different possible scientific fields and countries. Let us recall a few: The Belgian statistician Quetelet (1847), the British historian Henri Thomas Buckle (1865), and the French economist Léon Walras were accused of denying free will (Vergé [1874] 1996). James Clerk Maxwell (jointly with Tait and Kelvin) debated against Huxley and Tyndall in England on the compatibility of science and free will (see, for instance, "Essay for the Eranus Club on Science and Free Will, 11.2.1873" in Maxwell 1995, 814–823, mentioned above).[3]

Such a great number of authors did not share a unified view or a common set of worries; nevertheless, all of them were concerned with a similar question, i.e. "Is it possible to reconcile the laws of nature, with freedom of the will?" This question was particularly exacerbated by "mathematical laws," i.e. laws that can be expressed in mathematical form, and, for that reason, may seem to be even more exact and deterministic. This question often hid a second one: "if science is incompatible with free will, then it is incompatible with the religious doctrine of Mercy and Punishment as a consequence of being good or evil?" This was the motivation of many Christian scientists and of some spiritualistic philosophers. But, anticlerical philosophers such as Renouvier also felt uncomfortable with the apparent contradiction of science and free will.

Few of the concerns of that time are still remembered nowadays: There is nonetheless one that is quite well-known by physics students: the reversibility paradox which is generally attributed to Johann Josef Loschmidt. The paradox is generally presented as part of the Kinetic Theory of Gases and a classical objection to Boltzmann's Concept of Entropy.[4] Nevertheless, Loschmidt had no priority on the argument and the paradox was not originally conceived as an objection to Boltzmann's concept of entropy.

[3] Secondary bibliography on the debate in the United Kingdom concerning Maxwell, Buckle, and Tyndall may be found in (Porter 1981 and 1986; Hacking 1983; Smith 2013; Reed 1989; Stanley 2008; Smith and Wise 1989), on Quetelet, one may read Desrosières (2000). On Léon Walras, see Zylberberg (1990)

[4] The reversibility paradox in his contemporary formulation, states that in the case of a physical system whose entropy is increasing, if we reverse the speed of every particle, we will have a decreasing entropy. Therefore, for every physical system with an increasing entropy, there is a physical system with a decreasing entropy. One may then wonder why we always observe physical systems with increasing entropy and not the reverse. The

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As a matter of fact, the first discussions concerning a possible reversibility paradox are mentioned in a French novel, *Lumen*, written by Nicolas Camille Flammarion (1872): It tells the story of an entity called Lumen, who can travel faster than light, and can therefore observe events happening backwards in time. The story seems to have a didactic and entertainment purpose; we know from his correspondence that Boussinesq read it.[5] Of course, Flammarion did not have in mind the Kinetic Theory of Gases, nor the "reversibility paradox" as we (and Loschmidt) understand it. Nevertheless, what is at stake is the idea that depending on our "point of view" things may happen in a time-reversed order, thus suggesting a link between time and causation. Boussinesq's letter credits Flammarion as a precursor of Philippe Breton.

The first "scientific use" of the reversibility argument is disputed: it was used by Thomson (1874) to discuss the Kinetic Theory of Gases, and by Philippe Breton, a French engineer, as a paradoxical argument to show that matter and motion cannot solely explain all that exists in the world. Breton's argument was first published in December 1875 in *Les Mondes*[6] as a series of four articles, therefore after Kelvin and seemingly independently. It was published again in 1876 with a preface by the Abbé Moigno and a short article by the same author (the quotations in this paper are from this last version).

Kelvin's aim was to present the mental experience of Maxwell's demon and some of his troubling consequences. For a short time he considered applying this paradox to living beings, but quickly discards this application because "the real phenomena of living beings infinitely transcend human science" (Thomson 1874, 442). Breton, on the other hand, applies the reversion to living beings. Breton never mentions Kelvin and makes no patent links to the Kinetic Theory of Gases. His aim is to show that the reversal of speeds of a mechanical system implies a patent absurdity when applied to living beings: an old man will become young again, a rotten fruit will become a bud, and so on. Those absurdities are never observed but are a possible consequence of the laws of mechanics. Therefore, states Breton, the hypothesis that all that exists is matter and motion must be wrong. What is at stake for Breton is to show that a mechanistic view of nature does not suffice to explain living beings: it is therefore an attempt to find a place for the "soul" or the *élan vital*, or some other special property that may separate living and non-living beings. Reversibility is therefore not directly

microscopic symmetry of physical systems seems to be in contradiction with the macroscopic asymmetry of irreversible dynamics, thus implying a paradox with the second principle of thermodynamics.

[5] Letter of 23.7.1877. Boussinesq's and Saint-Venant's correspondence is kept in the *Bibliothèque de l'Institut de France* in Paris. Letters to Boussinesq from a great number of authors are alphabetically ordered (cotes MS4228–4229), and sometimes drafts of replies are available. Letters from Boussinesq to Saint-Venant are temporally ordered in Saint-Venant correspondence (cotes MS4226–4227), with some draft answers and hand-written comments by Saint-Venant.

[6] *Cosmos – Les mondes*, often referred as *Les mondes*, was a scientific journal directed by the Abbé Moigno that was published once a week. Moigno's aim was to show that religion and science were not in disagreement and *Les mondes* was his most powerful instrument. On the history of Moigno and his journal, see Redondi (1988).

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linked to free will, but to the more general topic of reducing living organisms to matter in motion, a mechanistic view of nature.

But what is missing in the mechanical picture of the world? Breton is much less explicit on this: one clear point is that mathematical and physical laws lack causality, i.e. the "distinction ... between causes and effects is not a mathematical one, since the formulas do not express it."[7] Breton defends a spiritualistic philosophy, but his spiritualistic affiliation is almost implicit: it is much more explicit in the introduction written by Moigno, an attack against Laplace and his materialism. Moigno was the editor of *Les Mondes*; in the introduction and in his article he attacks Laplace's mechanistic view of nature and explicitly endorses Breton's conclusions on the reversibility paradox.

The paradox is therefore used as a *reductio ad absurdum* argument of the following form:

1. If matter and motion are all that exist, then time reversal is possible.
2. Time reversal is empirically absurd.
3. Therefore, matter and motion are not all that exist.

There is an important difference between Breton's argument and what will later be Loschmidt's paradox. Breton defends reversibility as a meaningful process when applied to non-living beings. Absurdity applies mostly in the presence of life. In the Kinetic Theory of Gases, the paradox applies to mixing gases which are non-living beings. It seems therefore that the reversibility argument was first considered as a way to separate living and non-living things, thus grounding a certain vitalism into physico-mathematical properties: living beings cannot be time reversed, but non-living beings can. Since living-beings may have free will (and Boussinesq will consider free will as a potential property of different living beings other than humans), the Breton-like reversibility paradox shows a possible solution to the free will versus determinism dilemma.

Breton seems to be conscious that irreversibility has much to do with friction and dissipative forces, but his argument is an attack against a mechanistic view of nature. It will be quite the opposite for Loschmidt, therefore embedding a physical paradox into a historical one.

2. Boussinesq on Singular Solutions

2.1 Act I: The Mémoire *of Joseph Boussinesq*

February 1877: The young mathematician Joseph Boussinesq presents a short text to the *Académie des sciences* entitled *Sur la conciliation de la liberté morale avec le déterminisme*

[7] [la] distinction ... des causes et des effets n'est point mathématique, puisque rien ne l'exprime dans les formules (Breton and Moigno 1876, 47).

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scientifique (Boussinesq 1877a). At the time, Boussinesq was a 34-year-old professor at the *Faculté de sciences de Lille*. He was the son of a modest family of cultivators (Picard 1934, 2) who had been trained in mathematics by private tutors to some extent and then he became an autodidact. He was a man of deep Catholic faith who used to meditate over religious questions (ibid., 55) during his free time.

His first paper was followed shortly after by two highly similar papers (Boussinesq 1877b,c), and a more cogent and developed memoir the following year (Boussinesq 1878), then finally, by a fully developed version shortly after (Boussinesq 1879a).[8]

What is at stake is quite surprising: Boussinesq emphasizes that the equation of motion of some mechanical systems under a set of peculiar initial conditions may have more than one solution. Therefore, Boussinesq says, they may behave indeterministically. Boussinesq recognizes the priority of this discovery to Poisson (1806) who first proposed the example $\frac{d^2x}{dt^2} = ax^n$ as a differential equation with more than one solution.[9]

Boussinesq considers that a physical body in a "singular" situation, i.e. in a situation in which more than one solution to his equation of motion is possible, is underdetermined by the equation of motion: the equation of motion does not suffice to determine the future state of the body. Still, something must happen to that body. Therefore, a new principle has to intervene: Boussinesq names it *principe directeur* (hereafter, directing principle).

Boussinesq is quite unclear on the exact nature of this directing principle. It is something that supposedly without disposing of "any mechanical force that could fight those he may find in the world, may profit of their deficiency" (Boussinesq 1879a, 40),[10] i.e. the directing principle doesn't violate or modify any physical law but intervenes when those laws do not completely determine the dynamic of a physical body.

So far, so good: multiple solutions imply indeterminism. But what is really at stake, in Boussinesq's mind, is to reconcile science and free will: a large part of his Mémoire is an effort to show that singular solutions have greater chances of taking place in chemical rather than in physical phenomena, in living beings rather than in inanimate bodies, and so on.[11] His aim is to show that freedom of the will is compatible with the almost deterministic behavior of mechanical bodies, because freedom of the will can employ singular solutions and the directing principle as "elbow room": enough space to escape determinism but without violating any physical law.

[8] Quotations in this article are from this last extended version.

[9] Solutions of this equation for $x'(0) = 0$ are either of the form $x(0) = 0$ or of the form $x(t) \sim t^{\frac{2}{1-n}}$ with $a = \frac{2(1+n)}{(1-n)^2}$, under the condition that $n < 1$.

[10] n'aurait à son service aucune force mécanique qui lui permît de lutter contre celles qu'il trouverait dans le monde; il profiterait seulement de leur insuffisance.

[11] Boussinesq provides a very confusing sketch of the reasons that would give larger chances to singular solutions in living beings. It is a mix of speculation on the interaction between atoms that may be compatible with the existence of singular solutions and of philosophical considerations about the fact that living beings behave differently than non-living beings and must therefore include some non-mechanical principle.

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Unfortunately, there is no such thing as a free lunch and Boussinesq will have to face several problems. He is immediately aware of a set of difficulties that he has to deal with.

First problem: the directing principle applies even to non-living bodies, at times. Clearly non-living bodies are deprived of free will. Boussinesq's answer is that the directing principle "trivially behaves differently, if dealing with fully unconscious acts, almost conscious acts, or conscious deliberation" (ibid., 57).[12] Part of the extended version of his memoir tries to convince the reader that singular solutions have a greater chance of occurring in living bodies and, especially, in human beings. Unfortunately, Boussinesq can provide explicit examples of non-unicity only in very simple cases of mechanical (i.e. non-living) systems. Even if his arguments showing that singular solutions are more common in living beings were convincing – a dubious point – he would have to explain how the directing principle is compatible with unanimated beings. Boussinesq had a long and detailed discussion on singular solutions with Adhémar-Jean-Claude Barré de Saint-Venant: both were deep Catholic believers (Zerner 1994) and their concern with free will was largely an attempt to reconcile their own faith with their scientific commitment.

Saint-Venant, a retired mathematician of great fame at the time, was Boussinesq's mentor and friend. He was 45 years older than Boussinesq and helped him in his professional, personal, and political life; Boussinesq called him "dear master" in their correspondence.[13] Saint-Venant's personal faith occupied him to the point of collecting the sermons of his parson, which have been partially found in his private papers (Verneuil 1992). From a political point of view, he was a conservative. Boussinesq on his end had more liberal views (Zerner 1994) but shared with Saint-Venant's the concern of conciliating science and faith, as well as a penchant for a spiritualistic philosophy.

We know from their letters that a major issue was the interpretation of the directing principle. Saint-Venant wanted to interpret it as a possible explanation of Christian miracles but Boussinesq did not share his opinion. He saw the directing principle as a way to explain ordinary deliberation and not extraordinary events. Boussinesq found it very important to inscribe his memoir into mathematics and physics and not into philosophy or theology.

He repeatedly stated in his letters to Saint-Venant his views on scientific, philosophical, and theological knowledge, insisting on his scientific commitment: "I do not think to have gone beyond the field of science"[14] (letter to Saint-Venant 2.7.1877), he wrote in his letter announcing the first 1877 publication and he insists

[12] se comporte évidemment de différentes manières, suivant qu'il s'agit d'actes complètement inconscients, d'actes plus ou moins conscients, enfin d'actes délibérés [. . .]
[13] *Bien cher maître*
[14] *Je ne crois pas être sorti un instant du champ scientifique*

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on his ambition to reconcile determinism and free will without denying determinism.[15] Boussinesq is particularly fussy on the separation of personal faith and positive science: he tries to avoid theological considerations as much as possible and to restrict himself to mathematical and physical issues.[16]

Second problem: singular solutions exist only in the event of unstable equilibria, i.e. almost never. Boussinesq suggests that this may not be true of living bodies: "an animated body will be . . . one in which the equations of motion admit singular solutions" (Boussinesq 1879a, 40). That singular solutions are not common in inanimate bodies is a strength of Boussinesq's argument: as he repeatedly says, singular solutions do not contradict the laws of physics and inanimate bodies' behavior is well-explained by the laws of physics. Unfortunately, it is also a weakness: as we will see some of the criticisms that will be addressed to him concern exactly this point.

Third problem: Boussinesq follows a kind of ontological commitment to mathematical entities: he believes that any physical or chemical law can be expressed mathematically, and mathematical solutions to differential equations have to be interpreted on physical grounds. "The laws of physics and chemistry are reducible . . . to differential equations"[17] and singular solutions will be more and more common with the increasing complexity of life. Boussinesq does not think that everything that exists physically also exists mathematically: the directing principle would be a counterexample, and his aim is to show that there is more than matter and motion in this world. Nevertheless, Boussinesq thinks that if something is part of the equations (and therefore of the laws of physics) it has to exist. This is a bold claim: Boussinesq thinks that any solution has to be interpreted on physical grounds. In modern philosophical language, Boussinesq has a total ontological commitment to the solutions of the equations of motion. In order to interpret singular solutions as a form of indeterminism, one has to accept both that every physical phenomenon can be reduced to equations and that any consequence (i.e. any solution) of those equations has a physical meaning. Ontology can therefore be deduced from the equations of motion plus something more: but the equations of motion have to be literally understood. This will also be a matter of dispute with his peers. As we will see, Boussinesq is quite conscious of the problem but not always completely coherent.

2.2 Act II: Dangerous Friends

Boussinesq's work is positively received by some mathematicians and philosophers. Paul Janet[18] accepts prefacing his extended version and sings the praises of singular

[15] Letter to Saint-Venant 11.2.1877
[16] Letter to Saint-Venant 3.6.1877
[17] les lois physiques et chimiques sont réductibles, [. . .] à des équations différentielles,
[18] Paul Janet was a French spiritualist philosopher, professor at the Sorbonne and Membre de l'Institut.

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solutions. However, the most important support comes from Saint-Venant. Concerning the singular solutions, Saint-Venant recommends Boussinesq's text to the Académie des sciences (Saint-Venant 1877b).

Nevertheless, Saint-Venant had his own idea on the compatibility between science and free will: In a following note the same year (Saint-Venant 1877a), he presents his own solution to the free will dilemma, the *travail décrochant*. Originally an idea of Cournot, it consists of a very small, but finite, amount of energy that could start new massive phenomena, such as a trigger that shoots the bullet or any other action in which the cause requires almost no work at all and the effect is powerful.

Similar ideas circulated at the time on the relation between science and free will, e.g. James Clerk Maxwell's attempt to reconcile science and free will ("Essay for the Eranus Club on Science and Free Will, 11.2.1873," in Maxwell 1995, 814–823).[19] Saint-Venant and Boussinesq shared the idea of a directing principle that may or may not act and therefore direct conscious action. Nevertheless, Boussinesq's singular solutions didn't require any amount of energy while Saint-Venant invoked small but finite amounts of energy whose origin was unexplained. In this way, Saint-Venant's *travail décrochant* was a violation of physical laws while Boussinesq's singular solutions were not. There were also major differences concerning their understanding of mathematics: Saint-Venant's small forces were something physical; therefore he inscribed his *travail décrochant* into the imperfect match between physics and mathematics. Boussinesq's singular solutions didn't conflict on any ground with mathematics. Even if their ideas were quite close, especially (as we will discuss later) on their spiritualistic ambition, their views on the physico-mathematical rift were different.

Both Saint-Venant and Boussinesq may be considered spiritualists on the philosophical front but one should be cautious in reading Boussinesq as being particularly close to any definite philosophy: he was, after all, not trained in it and did not belong to any given school of thought. At the same time, Boussinesq certainly had great interest and a wide background in philosophy. Quotations and references to philosophers are quite common in his correspondence (not only classics such as Lucretius but also modern ones such as Descartes, Malebranche, Reid, Hobbes, Leibniz). He corresponded with some French philosophers of his time (while writing on singular solutions with Abbé Moigno, Carbonnelle, Alfred Espinas, Renouvier and later on with Pierre Duhem among others). The idea to go beyond materialism is a typical spiritualistic demand but a more explicit endorsement of spiritualistic ideas can be found in his correspondence.[20] Boussinesq even wrote a philosophical essay (Boussinesq 1879b; to be discussed later).

His interest in philosophy and the kind of spiritualism that he defends, without having the sophistication of a professional philosopher, are quite close to Etienne

[19] See also (Porter 1981; Stanley 2008) for a discussion of Maxwell's views on the topic.
[20] See e.g. letter of 10.9.1877, and letter of 2.8.1877

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Vacherot's spiritualism.[21] Boussinesq tries, as Vacherot does, to conciliate science and free will. But he remains as close as possible to a deterministic and materialistic view of nature. He wants to reconcile his Christian faith with his scientific knowledge and, more specifically, a spiritualistic metaphysics that would be compatible with the existence of free will and the deterministic view of nature.

2.3 Act III: Bertrand's Strike

Joseph Louis François Bertrand published, in 1878, a devastating attack against Boussinesq's *mémoire*. Bertrand's fame is nowadays limited to the theory of duopoles in economics and to Bertrand's paradox in probability theory but he was one of the most powerful and well-known mathematicians of his time. He became the *Secrétaire perpetuel de l'Académie des sciences in 1874*, a chief position in the French scientific establishment. Martin Zerner (1991) calls the period 1874–1900, "the reign" of Joseph Bertrand. Boussinesq, a 36-year-old mathematician at the time, was certainly not a (political) equal of the 56-year-old *Secrétaire Perpetuel*.

Bertrand's strike concentrates on one major issue: the translation of physical problems into mathematical language. That the equations of motion may have more than one solution, Bertrand states, is no surprise. They can even have no solution at all and still be of no harm to real physical situations. When a rigid table rests on more than three legs on a perfectly hard ground, the effort on every leg is undetermined. Calculation states this, but neither physicists nor geometers ever believed it for a single moment; they especially avoided the hypothesis that those legs have the power to choose, due to an indispensable will (Bertrand 1878, 520).[22]

Boussinesq's mistake, Bertrand says, is to consider that a mathematical deficiency has to be interpreted on physical grounds. For Bertrand a mathematical deficiency is just a failure of a useful but imperfect instrument: if mathematics shows some weird results, it means that mathematics does not translate physical reality correctly: "the equations of motion may have ... two different solutions ... But the existence of those two solutions shows that the formulae are in that case insufficient and incomplete, and that one should avoid substituting them for real nature with no distinction."[23]

[21] We don't have any direct proof of Boussinesq reading Vacherot; nevertheless, in the correspondence, Saint-Venant mentions a personal friendship with the French philosopher, from whom he could ask some help for publication. Unfortunately, we didn't find any correspondence with Boussinesq nor with Saint-Venant.

[22] Quand une table rigide et pesante repose par plus de trois pieds sur un sol parfaitement dur, l'effort supporté par chaque pied est indéterminé. Le calcul l'affirme, mais ni les physiciens ni les géomètres ne l'ont cru un instant; ils se sont bien gardés surtout de supposer à chaque pied la faculté de choisir, en lui prêtant une volonté devenue indispensable.

[23] les équations différentielles du mouvement peuvent avoir, ... deux solutions différentes. ..., Mais l'existence des deux solutions montre ... que les formules sont dans ce cas insuffisantes et incomplètes, et qu'il faut se garder de les substituer à la nature sans y admettre aucune distinction (Bertrand 1878, 518).

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Bertrand also claims that mathematical entities are translations of abstract entities taken from physical reality: abstract entities simplify the real world. "We believe too easily in the rigor and precision of the products of equations: the certainty of equations is simply that of the principles they translate."[24]

Boussinesq's answer to Bertrand (Boussinesq 1879c) is published in a different journal than Bertrand's critique. We know that Boussinesq would have liked to answer in the *Journal des savants* which denied him the right to reply. After some difficulties, he found a chance to reply in the *Revue philosophique*.

Quite surprisingly, Boussinesq interprets Bertrand's position as the claim that nature is discrete instead of continuous. This is in fact part of Bertrand's arguments, but hardly the most relevant. As we have said, Bertrand's point has more to do with the exactness of mathematical descriptions than with the discrete versus continuous physical ontologies. Therefore, Boussinesq is focusing on what seems to be a minor point. Boussinesq's defense is quite subtle: he claims that there are quantities so small that it would be impossible to measure them. In that case, those quantities that are indistinguishable on physical grounds may be considered identical. In other words, Boussinesq is claiming that epistemologically identical quantities should be considered as ontologically identical. This seems like a strange move at first sight: one would expect Boussinesq to insist on a one-to-one match between mathematics and physics in order to defend his singular solutions.

What Boussinesq defends is that anything that is a (mathematical) law of physics must be true. Mathematical solutions to the equation of motion always possess a physical counterpart but the opposite is not true. Some physical entities may have no mathematical counterpart. This would be the case of very small quantities of energy or force. Mathematical truths constrain the ontology of the physical world but they do not exhaust all that can exist: singular solutions will then be "the only free place outside the realm of the powers of matter" (Boussinesq 1879c, 62).[25]

Concerning these non-measurable quantities, we have two comments to make: First, Boussinesq might have had in mind the (at the time) unclear notion of infinitesimal quantities. An infinitesimal quantity, i.e. a differential in modern mathematical language, may or may not be considered as a physically null quantity. Second, Boussinesq may also have Saint-Venant's *pouvoir décrochant*[26] in mind.

Both questions should be understood in the particular scientific context of the time: questions of instability and sensitivity to initial conditions were a matter of concern for many physicists, a new and unexplored research field.[27] Boussinesq's

[24] On prête trop aisément aux produits es équations une rigueur et une précision absolues; . . . la certitude des équations est simplement celle des principes qu'elles traduisent.

[25] la seule place restée disponible en dehors du domaine incontesté des puissances de la matière brute.

[26] *Pouvoir décrochant* literally means "unhooking power." It is the possibility to give a pulse, a minimal "set in motion" to a given object.

[27] The matter of sensitivity to initial conditions and instability was a concern, e.g. for Maxwell (Maxwell 1995, 814–823), but one may not forget that Poincaré wrote his PhD dissertation in 1879 on the integration of

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singular solutions were exceedingly sensitive to external perturbation. They had much to do with instability.

One may therefore consider Boussinesq's defense to be a strange one: his singular solutions collapse under any infinitesimal perturbation. Therefore, to consider that infinitesimal quantities are indistinguishable from zero quantities may imply that singular solutions do not really exist in the physical world. Let us imagine that small, but non-zero, amounts of energy are always available (zero perturbation is always available, and, if they are indistinguishable from small perturbations, then small perturbations are always available). This would imply a kind of systematic perturbation of every physical system. But in that case, singular solutions would disappear. Therefore, such an indistinguishability between small and zero quantities is a useful one for Saint-Venant's "travail décrochant": one could argue against Saint-Venant that a small amount of energy is not zero, but the assimilation of indistinguishability and identity is a possible escape to this objection. Unfortunately, this is not the case for Boussinesq: even the smallest perturbation will suffice to destroy a singular solution. Therefore, Boussinesq is playing on a dangerous and slippery slope. The physico-mathematical rift is also a matter of concern for Boussinesq. He discusses this topic extensively with Saint-Venant.

Clarifications concerning the question of continuity and the physico-mathematical rift come from a letter (21.6.1877) that Boussinesq wrote to Saint-Venant. Boussinesq discusses the continuity of space and time as they are described by differential equations and their continuous nature in physical reality.

> I will answer an opponent who will refuse the continuity of space and time with a single thing, mainly, that the geometer has fully done his duty when he has adapted as much as our intellectual nature allows us to do, his analysis of reality. Every error small enough to escape abstract analysis does not exist for us; we can summarize what is knowable (by the application of mathematics to things) with this aphorism: "analysis is more severe than nature, it counts insignificant errors that are no more than subtleties for nature; but where she can find no error, neither does nature.[28] (Boussinesq to Saint-Venant, letter of 21.6.1877)

Boussinesq endorses the view that mathematical entities can't be mistaken: all that they "say" is certainly true. Nevertheless, there can be physical entities about whom

differential equations (Chabert and Dahan-Dalmedico 1992, 275), thus beginning his famous contribution to the topic. The work of Boussinesq that we consider in this paper is also an example of this concern.

[28] Je ne me chargerais de répondre à un adversaire qui me refuserait la continuité exacte de l'espace et du temps qu'une seule chose, à savoir, que le géomètre a fait son devoir et tout son devoir quand il a adapté autant que le comporte notre nature intellectuelle, son analyse aux choses. Toute erreur assez petit pour échapper à l'analyse abstraite est pour nous comme si elle n'existait pas; car on peut résumer ce que nous pouvons savoir (en faisant application des mathématiques aux choses) par cet aphorisme: "l'analyse est plus sévère que la nature elle compte des erreurs insignifiants qui ne sont que des subtilités pour la nature ; mais là où elle ne perçoit aucune erreur, la nature n'en voit pas."

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mathematics has nothing to say. It is the case of the directing principle but it may also be the case of Saint-Venant's "pouvoir decrochant."

In a following letter,[29] he clarifies the abstract nature of mathematics. What is mathematically an abstract line may be considered, Boussinesq says, a narrow strip for the same reason that a line with no thickness represents a street on a map. This argument is used by Boussinesq while defending the physicality of singular solutions: "every singular solution revealed by pure analysis, while being mathematically a line, a place of bifurcation, is really a strip, a place of bifurcation, in the real, concrete order."[30]

In this sense, Boussinesq is now defending an imperfect match between mathematical entities and the real world. What remains unclear is to what extent it is acceptable to see mathematics as an idealization of nature: one could use this argument in order to dismiss the physicality of singular solutions. Boussinesq's philosophical writings are not entirely consistent with the views expressed in his correspondence: one may think that while the published papers are an accomplished version of his thought, the correspondence, especially with a close friend such as Saint-Venant, is a suitable place for reasoning, and, therefore, for hesitation and doubts and, eventually, inconsistency and contradictions. It is also a place to openly deliver his own opinions carelessly and, while Boussinesq shows less caution, he also proves to be a more subtle philosophical thinker in his correspondence than in the published writings. Finally, there is a philosophical difference between Boussinesq and Bertrand that may go unnoticed: Bertrand associates mathematical entities with abstraction (Bertrand 1878, 520). Boussinesq associates mathematical entities with abstraction but also with ideality (Boussinesq 1879a, 115). One may speculate on this association between mathematics and ideality as an allegiance to spiritualism: abstraction is a neutral term that refers to some kind of induction, from the particular to the general case, but ideality as a more platonic flavor and suggests the existence of ideal objects in contrast to real objects. Furthermore, to consider mathematical objects as ideal may explain the ontological commitment that Boussinesq shows for mathematical laws.

3. Act IV: When the Dusk Starts to Fall

Bertrand's strike is probably the rudest, but not the only one against Boussinesq's singular solutions. Two others deal specifically with singular solutions, one published by Carbonnelle, and one unpublished by Alfred Espinas. Espinas, a French philosopher and historian, wrote a kind letter to Boussinesq (15.1.1879) mainly complaining about the following three points:

[29] Boussinesq to Saint-Venant 10.9.1877.

[30] chaque solution singulière révélée par l'analyse pure, tout en étant mathématiquement une simple ligne, lieu de bifurcations, est en réalité dans l'ordre concret, l'indice d'une bande étroite, lieu de bifurcations.

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- Boussinesq should prove that singular solutions take place in living beings and do not take place in non-living beings. Otherwise, says Espinas, if a non-living being can be in a singular solution it would either require divine intervention or will remain in an indeterminate state forever.
- There is no evidence to prove that singular solutions were not just mathematical oddities but effectively took place in nature, i.e. Boussinesq provides a proof that these solutions exist mathematically but should also prove their existence empirically.
- The qualitative difference suggested by singular solutions is between living and non-living beings and not between animals and humans, something that Espinas considers to be "contrary to what most spiritualists believe" (ibid.).

Boussinesq's answer to the first question is hazy: in the extended version of his *Mémoire* he explains that singular solutions appear mostly on living beings, but that the exceeding complexity of solving the differential equations will constrain him to the most simple cases. As a matter of fact, he shows only very simple mechanical examples and his arguments on living beings appeal to common sense philosophy and are not very convincing. What he would like to show is that inanimate matter is deterministic while living beings are not. But what he can only prove is that some mechanical systems (i.e. inanimate) present singular solutions. This is a rather frustrating impasse for Boussinesq. Espinas' objection is therefore well-directed even if it is not a novelty for Boussinesq: he previously discussed the difference between living and non-living beings in his correspondence.[31]

The second point is not much developed in Espinas' letter. Nevertheless, it would quickly become the principal objection to Boussinesq's singular solutions. In fact, singular solutions are very special cases of unstable equilibria and zero speed that may have no chance at all of happening in real physical systems. It is quite probable that Espinas did not have a clear idea on this point but, as we will see, other critics of Boussinesq will be much more effective and, as a matter of fact, Boussinesq would never provide any empirical proof for the very good reason that such a proof would be impossible to provide.

Finally, the third argument is also a point that Boussinesq previously discussed in his correspondence with Saint-Venant. Here, Boussinesq is much more explicit and completely endorses the consequence of what his singular solutions suggest.[32] For Boussinesq, animals and even plants "are more than pure chemistry."[33] Despite the fact that the standard religious view on the topic draws the line between humans and non-humans, Boussinesq prefers to separate living from non-living beings. Such a controversial claim (from a religious point of view) is a matter of dispute with Saint-Venant, an occasion for Boussinesq to admit his sympathy for evolutionary theories

[31] E.g. letter to Saint-Venant, 10.9.1877.
[32] Letter of 13.4.1877.
[33] sont plus que de la pure chimie.

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(ibid.). Boussinesq seems to believe that consciousness may increase continuously from simple living beings to complex animals and finally to men, following a kind of evolutionary path. Surprisingly, Boussinesq even talks of animal consciousness, thus admitting that free will may be, to some degree, a property of animals too. Boussinesq's letter is not explicit but we have the impression that he links evolution to the development of a more and more sophisticated consciousness that finally reaches its maximum in human beings. Such a position is surprisingly close to very modern speculations in the philosophy of mind (Dennett 2004). From this last letter, it seems clear that Boussinesq was conscious of the problem about singular solutions in non-living beings. He seems to believe that the problem may be solved by a progressive increase in singular solutions from non-living creatures to vegetative life, animals, and finally humans. This continuous, smooth path from inanimate matter to consciousness and free will is quite a modern one, and, even if it is defended to some degree in the published manuscript, it is entirely endorsed only in the correspondence.

The second similar criticism appears in *Les Mondes* under the pen of Carbonnelle (Carbonnelle 1879b). Carbonnelle's point is the following: singular solutions are unstable and, therefore, any perturbation, no matter how small, suffices to destroy them. Let us suppose that our brain is in a singular solution and that the directing principle wants to act at a certain time in the future in order to direct our will. Unfortunately, there are lots of other objects outside the brain that act as small perturbations: therefore, our actions will not be up to us but up to any surrounding object. One may escape this objection by supposing that the entire universe is in a singular solution but, says Carbonnelle, it does not seem to be a good ground for the problem of free will. In the following publication of the journal, Carbonnelle (1879a) discusses some objections that two unknown readers sent to him, dealing exactly with this last exception, that the entire universe may be in a singular solution. Strictly speaking, it would suffice that one's brain is in a singular solution and that the rest of the universe's total influence on it vanishes in order to allow the free action of the directing principle. But, says Carbonnelle, there are other free agents all around us. Any of them may make free decisions, each one having mechanical consequences corresponding for our brain to small perturbations. To suppose that this is not the case is to suppose that those actions are not free at all. Therefore, concludes Carbonnelle, to say that singular solutions are mathematically unstable is to say that they physically never occur.

This discussion is of a certain interest as an example of a connection between instability and free will, something that will be discussed in several occasions later in time, with a special concern for the extreme sensitivity to initial conditions (see for instance Israel 1992 for a historical overview.). The Boussinesq debate may be seen as an example of an infinite sensitivity to initial conditions and Carbonnelle's objection is a particularly good one.

Boussinesq will answer Carbonnelle in his philosophical treatise one year later (Boussinesq 1879b). He defends that a brain in a singular solution may jump, while perturbed, into another singular solution, an argument that sounds quite desperate.

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More convincing may be a second argument, that some cases of singular solutions are not of the kind of a persisting unstable equilibrium that may be set in motion at any moment but are bifurcating paths that require the directing principle to act in a singular instant. This second argument is not given in Boussinesq's philosophical treatise but in Carbonnelle's first text (Carbonnelle 1879b, 285) as a kind of self-objection, based on the complementary notes of Boussinesq's *Mémoire*. Unfortunately those speculative notes are not developed in deeper detail.

4. Act V: When the Owl of Minerva Spreads Its Wings and Flies

The Boussinesq debate didn't end with those objections and the discussion surrounding them. Slowly, the discussion moved from Boussinesq's *mémoire* on singular solutions to a larger topic, mainly the compatibility between science and free will in a broader sense. We consider here a series of discussions that followed the Boussinesq debate but considerably moved away from the original argument. They still are reactions to Boussinesq, since they explicitly cite Boussinesq and address some criticism to him, but the focus is somehow more general. What is interesting is that the kind of objections that will be given will slowly include such a broad set of topics as instability, sensitivity to initial condition, the arrow of time, and the second principle of thermodynamics, thus resembling more and more the contemporary debate on chaos theory.

Not all those answers are logically related and it is unclear if Boussinesq had the occasion to read them all.

The first one we are going to discuss is the contribution of Charles Renouvier (Renouvier 1882a,b). The link between Boussinesq and Renouvier is grounded on two facts. First, Renouvier explicitly criticizes Boussinesq and discusses his main contributions. Secondly, the two Renouvier articles are published together with the 1922 re-edition of Boussinesq's *mémoire* (Boussinesq 1922). A third article (Renouvier 1880) also discusses Boussinesq's paper but is not published in the 1922 edition.

Renouvier's main argument is that Boussinesq's philosophy goes too far when taking physical laws into account.

> It [the induction from the real case to general laws] can go very far even to the point of including every possible object into a differential equation . . . But do we have to believe that the law of conservation is an absolute one, that it is more than a pure geometrical law, namely an abstraction that the experience confirms as a rule, but that is never exactly concretely achieved?[34] (Renouvier 1882b, 348)

[34] elle [l'induction du cas réel aux lois générales] peut aller extrêmement loin et jusqu'à assujettir (en idée) tous les mobiles imaginables à l'obligation de satisfaire à des équations différentielles du mouvement . . . Mais est-on forcé de croire que la loi de conservation est absolue, qu'elle n'est pas seulement ce que sont les pures lois géométriques, une abstraction que l'expérience vérifie et reçoit comme règle, mais dont l'ordre concret des choses ne souffre jamais une exacte réalisation?

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In his articles, Renouvier questions the ontological commitment to mathematical entities in Boussinesq's work and his understanding of the deep meaning of a scientific law. The difference between Renouvier and Bertrand lies mainly on this point. Renouvier is not questioning the physico-mathematical rift. He's more concerned with the ontological commitment that we should have with respect to scientific laws. Renouvier considers scientific laws as the abstract and idealized form of natural regularities, thus having a theoretical status but not an ontological one. It may sound to some extent like a classical positivistic argument that denies any causal or ontological meaning to scientific laws: science enables one to describe but not to explain. It is quite surprising for Renouvier since he's not a positivist at all. Renouvier's criticism might have to be understood as a declination of the Kantian argument about free will: science is phenomenal while free will is noumenal and nothing can be known about free will from a scientific point of view. Despite his criticism, Renouvier shows an interest in and a good understanding of Boussinesq's argument and recognizes his contribution as a valuable one. He also has some remarks on Saint-Venant's "pouvoir décrochant," mainly concerning the physical difference between a small and a zero amount of energy.

Prior to Renouvier there was a duo of articles by Ernest Naville (1879)[35] and Paul Tannery (1879)[36] in the *Revue Philosophique de la France et de l'Etranger*. Naville will partially endorse Cournot's "pouvoir décrochant" (a line of argument similar to Saint-Venant's and Maxwell's).

Naville's own contribution to the free will compatibility debate is to stress that the conservation of energy allows any direction of movement. For instance, Naville argues that whether a planet moves clockwise or anticlockwise does not imply any violation of the conservation of energy. Naville's point is that moving right or left (or clockwise/anticlockwise) requires the same amount of energy. Therefore, the direction of movement is not determined and is up to the freedom of the will without any violation of (this) physical law. Naville concludes that it is up to human will to decide how to use a certain force.

Tannery is more concerned in his paper with the philosophy of mathematics. Nevertheless, he spends some time criticizing Boussinesq following a similar line to Renouvier (but previously!) and Bertrand: for Tannery, singular solutions are examples of mathematical curiosity with no physical meaning, something that, he says, is quite common. He also pretends that Boussinesq supposes determinism as a hidden hypothesis in his approach. Boussinesq accepts determinism almost everywhere, except in cases of singular solutions. He has a deterministic a priori concerning nature, Tannery says. It is thus not surprising that he finds it in his solutions but, as a matter of

[35] Ernest Naville (1816–1909) was a history and philosophy professor in Geneva, between 1844 and 1848, and professor of apologetics in 1860. He is famous as a public speaker and for his contributions to philosophy and has been an external member of the Institut de France.

[36] Paul Tannery, a French engineer, is known for his contributions to the history of science.

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fact, this does not allow him to conclude that determinism is to some extent proved by science. Boussinesq will spend some lines in his philosophical treatise to answer Tannery but no substantial new arguments are given.

In 1882, the same journal will publish a long article in three parts by Joseph Delboeuf (Delboeuf 1882a,b,c). Delboeuf sharply criticizes both Boussinesq and Saint-Venant. Against Saint-Venant, he argues that a small amount of energy is not zero and, therefore, the "pouvoir décrochant" violates the laws of physics. Against Boussinesq, he uses some of Bertrand's arguments. He seems to confuse to some extent Boussinesq and Saint-Venant.

The interesting contribution of Delboeuf is his own idea of conciliating science and free will. He first considers time as "the measurement of the universal transforming activity" (Delboeuf 1822b, 622), [37] i.e. time is the measurement of irreversible change. This idea of time is, following Delboeuf, in contradiction with the reversible, linear time used in mechanics. This idea sounds quite close to the criticism of Breton but goes a step further in linking the second principle of thermodynamics and irreversibility with free will. Delboeuf considers free agents as those who can suspend their action in time

> It suffices that the individual has the faculty to suspend an action i.e. not to immediately react to the excitation that he receives and thus to delay the moment when he will deploy the force he has stocked in a potential state. By this delay, he obviously does not create any force. (Ibid., 625)[38]

This idea of suspending the action is close to Boussinesq to some extent, but not fully compatible with mechanics. It would imply a discontinuity in the speed of some atoms to suspend their action. Delboeuf seems to be conscious of this, and considers discontinuity as a characteristic of free agents: "There is a discontinuity in the trajectory of an object when its direction and speed are not the immediate consequence of the preceding instant, and," continues Delboeuf, "discontinuity is what characterizes the behavior of free agents" (ibid., 626).[39]

In his reflection on discontinuity, Delboeuf links unpredictability and discontinuity and considers unpredictability as a violation of determinism thus mixing an epistemological level with an ontological one. He also mentions Cournot and the second principle several times: therefore, unpredictability, the arrow of time, causality, and discontinuity, are all linked together with free will in what sounds like a quite confusing but interesting precursor to some modern debates.

[37] mesure de l'activité transformatrice universelle.
[38] il suffit que l'individu ait la faculté de suspendre son action, c'est-à-dire de ne pas répondre immédiatement à l'excitation qui le sollicite, et de retarder le moment où il déploiera la force qui est emmagasinée à l'état de tension. Par ce retard, il n'engendre évidemment pas de forces.
[39] il y a discontinuité dans la trajectoire d'un mobile à l'instant où sa direction et sa vitesse cessent d'être la conséquence immédiate de son mouvement antérieur.

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The *Revue philosophique* will show some interest in Delboeuf's articles and will also publish a critical analysis by Grocler (1882) with a footnote debate between Grocler and Delboeuf.

There will be some quotes and even an extended discussion on the topic in the following years (Ferri 1900; Fargen. 1902; Calò 1907; Freycinet 1896; Couilhac 1897) but mostly on the philosophical front. Scientists will, on their side, ignore the issue after Delboeuf and the Boussinesq debate will slowly be forgotten.

5. Conclusion

The Boussinesq debate is an example of an attempt to reconcile science and free will, which shows multiple interesting aspects, despite having been almost neglected in historical studies.

It originates as an attempt to reconcile religious faith and science despite the fact that one will not notice this aspect without a close look at the private correspondence of Boussinesq (Zerner 1994). The debate entangles spiritualistic concerns with physical and mathematical analysis. Even more interesting is the link that several scholars draw between instability, sensitivity to initial conditions, and free will. Similar arguments have been proposed in the contemporary debate and, despite some new contributions due to quantum mechanics, some of the main arguments have not changed. It seems that there is an historical root that goes from free will and determinism to sensitivity to initial conditions and what we may retrospectively label "chaos." In modern times, problems concerning the sensitivity to initial conditions have been linked to free will and indeterminism. We showed that such a link is a return to sources.

Finally, a recent philosophical debate known as the Norton Dome (Norton 2008; Malament 2007) has rediscovered a particular case of Boussinesq's singular solutions, thus originating a debate on indeterminism and causality that closely resembles part of the Boussinesq debate (see Van Strien 2014 for a comparison of the two debates).

The Boussinesq debate is then an example of moral and religious issues that deploy philosophical arguments in a scientific battlefield. As a consequence, these religiously motivated arguments change their nature and transform into a confrontation on the philosophy of science, more specifically, on the physico-mathematical rift. Under the impulse of this second stake, positive and normative statements entangle in a marvelous epistemological puzzle.

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