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APPLICATION AND CASE STUDIES - ORIGINAL

A Simple Analytic Approach to Regression with or without Interaction Terms: Integrating the Covariance Structure Model and the Delta Method

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Abstract

Several textbooks give the incorrect formula to calculate the standard error estimates for standardized regression coefficients. As a remedy, two analytic methods have been developed: (1) the delta method and (2) the covariance structure modeling method. However, neither method is applicable to compute the standard error estimates for unstandardized regression coefficients of products of Z-scores. In the literature, a nonparametric bootstrap procedure is advocated to test the significance of unstandardized regression coefficients of products of Z-scores. In this article, we propose a simple analytic approach that can produce the standard error estimates not only for standardized regression coefficients when interaction terms are not included, but also for unstandardized regression coefficients when interaction terms of Z-scores are included. Two numeric examples are used to compare our analytic approach with the existing methods, and simulation studies are conducted to further evaluate the performances of regular regression, our analytic approach, and the nonparametric bootstrap procedure at finite sample sizes. It is found that (1) regular regression performs well only when the variances of predictor variables are small, (2) our analytic approach performs well at the sample size of 200 or larger, and (3) the nonparametric bootstrap procedure performs (almost) perfectly in all conditions.

Keywords: covariance structure model; delta method; product of Z-scores; regression coefficient; standard error

1. Introduction

Multiple regression is a commonly used tool to study the linear relationship between a criterion variable and a set of predictor variables. To report the results of multiple regression, researchers always provide not only R^2 (i.e., the proportion of variance of the criterion variable explained by the set of predictor variables) but also each of the estimated regression coefficients. To test the significance of a regression coefficient, the estimated regression coefficient and the associated standard error estimate can be used to either compute the t statistic or construct the confidence interval (CI).

In social and behavioral sciences, many variables are measured on arbitrary scales (e.g., Likert scales) and thus lack substantive meanings. With the motivation to compare the relative importance of predictor variables in relation to the criterion variable, researchers often focus on the standardized

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regression coefficients, rather than the unstandardized regression coefficients. For example, if we want to study whether income and perceived social support have a similar impact in predicting life satisfaction, it is better to compare the magnitude of standardized regression coefficients, whereas it is not meaningful to compare the unstandardized regression coefficients because the magnitude of unstandardized regression coefficients can be arbitrarily manipulated with the scales that measure income and perceived social support, separately. Consequently, the results typically reported for regression analysis in social sciences include the point estimates of standardized regression coefficients, the associated standard error estimates, the t values, and the p-values. Nonetheless, such a reporting practice is often problematic. Specifically, several textbooks (e.g., Cohen et al., 2003, p. 86; Harris, 2001, p. 80; Hays, 1994, p. 709) show the formula (referred to as the textbook formula hereafter) that the standard error estimate for a standardized regression coefficient can be obtained by transforming the corresponding standard error estimate for the unstandardized regression coefficient using the sample standard deviations of the criterion variable and the corresponding predictor variable. Howell (2013) did not show the textbook formula but asserted that the same test statistic (i.e., t value) would be obtained no matter whether we test the unstandardized regression coefficient or the standardized regression coefficient (see the last sentence of the second last paragraph on page 521). Nevertheless, Yuan and Chan (2011) and Jones and Waller (2013, 2015) pointed out that the textbook formula (and Howell's assertion based on it) is incorrect and should not be used to compute the standard error estimates for standardized regression coefficients. In the literature, different methods have been developed to solve this problem. However, no analytic method has been proposed when interaction terms of standardized variables are involved in the regression models. As interaction terms of standardized variables are frequently examined in social and behavioral sciences (Aiken & West, 1991), it is imperative to adopt the correct method for standard error estimation for accurate statistical inferences. Therefore, the present article aims to introduce an analytic approach to regression with or without interaction terms. In the rest of this section, we first review the problem of the textbook formula as well as the methods that have been developed to solve this problem. Next, we point out the gap in the literature when the regression model is extended to involve interaction terms of standardized variables, followed by the aim and advantages of our analytic approach.

1.1. The problem of textbook formula

Let the regression model be

$$y_i = b_0 + b_1 x_{i1} + \cdots + b_p x_{ip} + \varepsilon_i,\ \ i = 1,2,\cdots,n,\ \ \mathrm{Var}\left(\varepsilon_i\right) = \sigma_\varepsilon^2,$$

where y_i is the unstandardized criterion variable, x_{ij} ($j = 1, \ldots, p$) is the jth unstandardized predictor variable, ε_i is the error, b_0 is the intercept, and b_j is the jth unstandardized regression coefficient. Let $\mathbf{b} = \left(b_1\ \ b_2\ \ \cdots\ \ b_p\right)'$ be a column vector of unstandardized regression coefficients. The least squares estimate of $\mathbf{b}$ is $\widehat{\mathbf{b}} = \left(\widehat{b}_1\ \widehat{b}_2\ \cdots\ \widehat{b}_p\right)' = \mathbf{S}_{xx}^{-1}\mathbf{s}_{xy}$, where $\mathbf{S}_{xx} = \begin{pmatrix} s_1^2 & & & \\ s_{21} & s_2^2 & & \\ \vdots & \vdots & \ddots & \\ s_{p1} & s_{p2} & \cdots & s_p^2 \end{pmatrix}$ is the sample covariance matrix of $x_{i1}, \ldots, x_{ip}$ and $\mathbf{s}_{xy}$ is a column vector of covariances between $x_{i1}, \ldots, x_{ip}$ and y_i. Then, the estimate of the jth standardized regression coefficient, denoted by β_j, is given by

$$\widehat{\beta}_j = \frac{s_j}{s_y}\widehat{b}_j, \tag{1}$$

where $s_j = \sqrt{s_j^2}$ is the sample standard deviation of x_{ij}, and s_y is the sample standard deviation of y_i.

Let $\mathrm{SE}\left(\widehat{b}_j\right)$ be the standard error estimate for $\widehat{b}_j$. According to the textbooks mentioned before (Cohen et al., 2003, p. 86; Harris, 2001, p. 80; Hays, 1994, p. 709), the standard error estimate for $\widehat{\beta}_j$ is

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given by

$$\mathrm{SE}\left(\widehat{\beta}_j\right) = \frac{s_j}{s_y}\mathrm{SE}\left(\widehat{b}_j\right), \tag{2}$$

where $\mathrm{SE}\left(\widehat{\beta}_j\right)$ is the standard error estimate for $\widehat{\beta}_j$. Obviously, (Equations 1) and (2) have the same form of transformation. If (Equation 2) were true, we would have $\frac{\widehat{\beta}_j}{\mathrm{SE}\left(\widehat{\beta}_j\right)} = \frac{\widehat{b}_j}{\mathrm{SE}\left(\widehat{b}_j\right)}$, which would lead to the same t statistic as asserted by Howell (2013, p. 521). However, s_y is always subject to sampling variability in regression analysis, and s_j is also subject to sampling variability in most applications of regression analysis in social sciences (Yuan & Chan, 2011, p. 673). Formally, when the predictor variables are fixed, the textbook formula fails to consider the sampling variability of s_y; whereas when the predictor variables are random, the textbook formula fails to consider the sampling variability of both s_y and s_j. Simulation studies demonstrated that the textbook formula can either under- or over-estimate the standard errors for standardized regression coefficients (Yuan & Chan, 2011). Therefore, the significance of a standardized regression coefficient should not be assumed to be identical to that of the corresponding unstandardized regression coefficient. Furthermore, the standard error estimates, the t values, and the p-values generated from popular software packages for regression analysis (e.g., SAS PROC REG, SPSS) should not be interpreted for the standardized regression coefficients. In fact, when the input data are the standardized variables (i.e., the Z-scores of the criterion and predictor variables), the standard error estimates from software packages for regression analysis (e.g., SAS PROC REG, SPSS) are in general incorrect.[1] It is only when the input data are unstandardized variables that one can test the significance of unstandardized regression coefficients. In sum, the textbook formula is incorrect and should not be used to compute the standard error estimates for standardized regression coefficients.

1.2. Solutions to the problem of textbook formula

In the literature, several methods have been developed to compute $\mathrm{SE}\left(\widehat{\beta}_j\right)$, including two analytic methods and the bootstrap procedure. The two analytic methods are the delta method (Jones & Waller, 2015; Yuan & Chan, 2011) and the covariance structure modeling method (Bentler & Lee, 1983; Cheung, 2009; Dudgeon, 2017; Kwan & Chan, 2011).

The delta method is outlined as follows. Let $\boldsymbol{\sigma} = \begin{pmatrix} \mathrm{vech}\left(\boldsymbol{\Sigma}_{xx}\right) \\ \boldsymbol{\sigma}_{xy} \\ \sigma_y^2 \end{pmatrix}$ be a column vector of order $(p + 1)(p + 2)/2$ that collects the population covariances of all unstandardized variables, where the vech$(\cdot)$ operator stacks the columns of the lower triangular part of a symmetric matrix. The sample counterpart of $\boldsymbol{\sigma}$ is $\mathbf{s} = \begin{pmatrix} \mathrm{vech}\left(\mathbf{S}_{xx}\right) \\ \mathbf{s}_{xy} \\ s_y^2 \end{pmatrix}$. Using the delta method, Yuan and Chan (2011) derived the asymptotic covariance matrix of $\sqrt{n}\widehat{\boldsymbol{\beta}}$, denoted by $\mathbf{h}\boldsymbol{\Gamma}\mathbf{h}'$, where $\widehat{\boldsymbol{\beta}} = \left(\widehat{\beta}_1 \ \widehat{\beta}_2 \ \cdots \ \widehat{\beta}_p\right)'$, $\boldsymbol{\Gamma}$ is the asymptotic covariance matrix of $\sqrt{n}\mathbf{s}$ and $\mathbf{h}$ is a $p \times (p + 1)(p + 2)/2$ Jacobian matrix of partial derivatives of $\boldsymbol{\beta} = \left(\beta_1 \ \beta_2 \ \cdots \ \beta_p\right)'$ with respect to $\boldsymbol{\sigma}'$. The formulas of $\boldsymbol{\Gamma}$ and $\mathbf{h}$ can be found in Yuan and Chan (2011, pp. 674–675). Because Yuan and Chan's delta method is based on the asymptotic covariance matrix of covariances, it is referred to as a covariance-based delta method. Following the same idea, Jones and Waller (2015) extended the delta method in two directions. First, Jones and Waller (2015, Section 3) provided a correlation-based delta method. Second, Jones and Waller (2015, Section 4) modified Yuan and Chan's covariance-based delta method by replacing $\boldsymbol{\Gamma}$ in $\mathbf{h}\boldsymbol{\Gamma}\mathbf{h}'$ with $\boldsymbol{\Gamma}^{\mathrm{ADF}}$, which is the asymptotic distribution-free (ADF; Browne, 1984) estimator of acov$\left(\sqrt{n}\mathbf{s}\right)$. The formulas of $\boldsymbol{\Gamma}^{\mathrm{ADF}}$ can be found in Jones and Waller (2015,

[1] Cudeck (1989) reviewed the consequences of analyzing the standardized variables (or correlation matrices) in the general framework of covariance structure analysis and demonstrated that the standard error estimates obtained from such analyses are in general incorrect.

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p. 370). Despite the theoretical correctness, the formulas derived by Yuan and Chan (2011) and Jones and Waller (2015) are complicated. To provide an easy implementation, Pesigan, Sun, and Cheung (2023) recently developed the *betaDelta* package in R for the covariance-based delta method developed by Yuan and Chan (2011) and Jones and Waller (2015, Section 4).

As for the covariance structure modeling method, the basic idea is to subsume $\boldsymbol{\beta} = \begin{pmatrix} \beta_1 & \beta_2 & \cdots & \beta_p \end{pmatrix}'$ as a subset of parameters so that $\text{SE}\left(\widehat{\boldsymbol{\beta}}\right)$ can be obtained as a byproduct of parameter estimation. Specifically, Bentler and Lee (1983, Eq. (14)) specified a constrained model as

$$\boldsymbol{\Sigma}\left(\mathbf{D}_0, \mathbf{R}, \boldsymbol{\beta}, \sigma_\varepsilon^2\right) = \mathbf{D}_0 \begin{pmatrix} \boldsymbol{\beta}'\mathbf{R}\boldsymbol{\beta} + \sigma_\varepsilon^2 & \\ \mathbf{R}\boldsymbol{\beta} & \mathbf{R} \end{pmatrix} \mathbf{D}_0, \text{ subject to } \boldsymbol{\beta}'\mathbf{R}\boldsymbol{\beta} + \sigma_\varepsilon^2 = 1, \tag{3}$$

where $\mathbf{D}_0 = \text{diag}\begin{pmatrix} d_y & d_1 & \cdots & d_p \end{pmatrix}$ is a diagonal matrix of standard deviations and $\mathbf{R} = \begin{pmatrix} 1 & & & \\ \rho_{21} & 1 & & \\ \vdots & \vdots & \ddots & \\ \rho_{p1} & \rho_{p2} & \cdots & 1 \end{pmatrix}$ is a $p \times p$ correlation matrix of $x_{i1}, \ldots, x_{ip}$. Dudgeon (2017, Eqs. (24) and (25)) used the same specification as Equation (3)) and adapted various heteroscedastic-consistent estimators to compute $\text{SE}\left(\widehat{\boldsymbol{\beta}}\right)$. Alternatively, Kwan and Chan (2011) specified an unconstrained model through a reparameterization approach that involves the phantom variable and image structure. Fundamentally, the unconstrained model specified by Kwan and Chan (2011) is equivalent to the constrained model by Bentler and Lee (1983, Eq. (14)) and Dudgeon (2017, Eqs. (24) and (25)). Thus, no matter which model (constrained or unconstrained) is estimated, one should obtain the same $\text{SE}\left(\widehat{\boldsymbol{\beta}}\right)$. However, the constrained model expressed in matrix form (Equation 3) must be translated to scalar functions manually, which is prone to errors and tedious, for some popular software packages (see the *lavaan* implementations in R for Example 1 in the Supplementary Material), while it is a laborious task to specify the unconstrained model using Kwan and Chan's reparameterization approach. No matter whether the constrained or unconstrained model is used, it is not a convenient task for practical implementations, especially when more than a few predictor variables are involved.

Finally, it is well known that the bootstrap procedure is a general method for standard error estimation and can be used in almost all situations. However, the bootstrap procedure requires repeated sampling, repeated standardization, and repeated model fitting, and thus lacks computational efficiency.

1.3. Regression models with interaction terms

Mathematically, an interaction term is a product of two or more predictor variables, and the interaction terms should be included in a regression model when the effect of one predictor variable on the criterion variable varies across different levels of another predictor variable (or different combinations of the levels of other predictor variables). In short, an interaction effect (also referred to as a moderation effect) can exist between two or more predictor variables. The most commonly used interaction terms are those created with continuous predictor variables. In this article, we restrict our focus to this type of interaction term.

Depending on the interest of the researcher, there are at least two ways[2] to create the interaction terms. In the first way, the interaction terms are created with unstandardized predictor variables. In the second way, the interaction terms are created with standardized predictor variables (i.e., Z-scores of predictor variables), and such interaction terms are the products of Z-scores. To study the interaction effects in unstandardized metric, one should regress the unstandardized criterion variable on the unstandardized predictor variables and the interaction terms created with the unstandardized predictor variables, and test the significance of the unstandardized regression coefficients of the interaction

[2]The third way to create the interaction terms is to use both unstandardized and standardized predictor variables, but this is not commonly used in practice.

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terms. To study the interaction effects in standardized metric, one should regress the standardized criterion variable on the standardized predictor variables and the products of *Z*-scores and *still* test the significance of the *unstandardized* regression coefficients of products of *Z*-scores (Aiken & West, 1991, p. 44; Friedrich, 1982). It should be remembered that it is the unstandardized regression coefficients of the products of *Z*-scores that represent the interaction effects in standardized metric because the variances of products of *Z*-scores are not unity. However, as discussed above, software packages for regression analysis (e.g., SAS PROC REG, SPSS) do not produce the correct standard error estimates for unstandardized regression coefficients when *Z*-scores are used as the input data.

To test the significance of the unstandardized regression coefficients of products of *Z*-scores, a nonparametric bootstrap procedure is advocated to construct the CIs based on percentiles (Cheung et al., 2022). The implementation of this nonparametric bootstrap procedure is composed of the following steps. First, a random sample (referred to as the bootstrapped sample) of the same size as the original sample is drawn with replacement from the original sample. Second, the criterion and predictor variables in the bootstrapped sample are standardized.[3] Third, the product terms of *Z*-scores are created. Fourth, the standardized criterion variable is regressed on the standardized predictor variables and the products of *Z*-scores, and the unstandardized regression coefficients are recorded. Finally, repeat the above steps a large number of times (i.e., the bootstrap replications). After completing the nonparametric bootstrap procedure, one can use the percentiles to construct the percentile-based CIs for each of the unstandardized regression coefficients. Cheung et al. (2022) developed the *stdmod* package in R to implement this nonparametric bootstrap procedure.

1.4. Aim and advantages of our analytic approach

Despite the analytic methods developed for regression models without interaction terms (Bentler & Lee, 1983; Cheung, 2009; Dudgeon, 2017; Jones & Waller, 2015; Kwan & Chan, 2011; Yuan & Chan, 2011), no analytic method has been developed to compute the standard error estimates for unstandardized regression coefficients of products of *Z*-scores. To fill this gap, we propose an analytic approach to regression that can produce the standard error estimates not only for standardized regression coefficients when interaction terms are not involved but also for unstandardized regression coefficients when interaction terms of *Z*-scores are involved. Technically, our analytic approach integrates the covariance structure modeling method and the delta method. That is, a simple covariance structure model is estimated in the first step to produce the point estimates and the asymptotic covariance matrix of the estimates, and then the delta method is applied in the second step to compute the standard error estimates for relevant regression coefficients.

Compared to the existing methods, our analytic approach has several advantages. Compared to the delta method (Jones & Waller, 2015; Yuan & Chan, 2011), our analytic approach does not require any derivations of new formulas for the asymptotic covariance matrix of the estimates. In the literature of covariance structure analysis, relevant formulas have been derived for the asymptotic covariance matrix of the estimates, and the results can be obtained directly from most software packages designed for covariance structure analysis. In contrast, the formulas derived by Yuan and Chan (2011) and Jones and Waller (2015) are complicated. If the same idea of the covariance-based delta method is to be followed, one has to derive more formulas that involve both *Z*-scores and products of *Z*-scores, which is a very challenging, if not impossible, task. Compared to the covariance structure modeling method (Bentler & Lee, 1983; Cheung, 2009; Dudgeon, 2017; Kwan & Chan, 2011), the covariance structure model specified in the first step of our analytic approach has a very simple form such that several software packages designed for covariance structure analysis (e.g., LISREL, the *lavaan* and *OpenMx* packages in R, SAS PROC CALIS) can be readily used to estimate such a model. Compared to the nonparametric

[3]Some categorical predictor variables (such as gender, ethnicity) should not be standardized in this step and could still be included in the regression model in the fourth step. As a result, regression models may include both unstandardized and standardized predictor variables. Such a situation is illustrated in Example 2 of this article.

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bootstrap procedure, our analytic approach is computationally more efficient.[4] In addition to these three advantages, our analytic approach can easily accommodate the interaction terms created with three or more predictor variables (i.e., higher-order interaction terms). Finally, our analytic approach is flexible enough to subsume different regression models. That is, the standardized criterion variable can be regressed on either unstandardized predictor variables, standardized predictor variables, or both, as well as the interaction terms created with either unstandardized predictor variables, standardized predictor variables, or combinations of both.

1.5. The organization of the remainder sections

The organization of the remainder sections is as follows. In Section 2, we first describe our analytic approach to computing the standard error estimates for standardized regression coefficients when the interaction terms are not involved. Then, we introduce our analytic approach to computing the standard error estimates for unstandardized regression coefficients when the interaction terms are involved. In Section 3, we analyze two examples. In the first example, the criterion variable is regressed on three predictor variables without any interaction terms, and this example is used to compare our analytic approach with (1) the covariance-based delta method, (2) Bentler and Lee's (or Dudgeon's) constrained covariance structure modeling method, and (3) Kwan and Chan's unconstrained covariance structure modeling method. In the second example, the standardized criterion variable is regressed on the unstandardized and standardized predictor variables, as well as one interaction term created with two standardized predictor variables, and this example is used to compare our analytic approach with the nonparametric bootstrap procedure advocated by Cheung et al. (2022). In Section 4, we conduct two simulation studies to further evaluate the performances of regular regression, our analytic approach, and the nonparametric bootstrap procedure at finite sample sizes. In the first simulation study, all variables are generated from multivariate normal distributions. In the second simulation study, all variables are generated from multivariate non-normal distributions. As Micceri (1989) demonstrated, psychological data are seldom normally distributed, so assessing how the proposed method performs under non-normal data is important. Finally, we summarize the main findings from the two simulation studies and discuss the future directions in Section 5.

2. A simple analytic approach

In this section, we first describe our analytic approach to computing the standard error estimates for standardized regression coefficients when the interaction terms are not involved. Then, we introduce our analytic approach to computing the standard error estimates for unstandardized regression coefficients when the interaction terms are involved.

2.1. Standard error estimates for standardized regression coefficients of Z-scores

Let z_{iy} be the standardized criterion variable and z_{ij} ($j = 1, \ldots, p$) be the jth standardized predictor variable. To analyze the data of unstandardized variables (i.e., $y_i, x_{i1}, \ldots, x_{ip}$) or standardized variables (i.e., $z_{iy}, z_{i1}, \ldots, z_{ip}$), we define the first covariance structure model as

$$\boldsymbol{\Sigma}\left(\mathbf{D}_1, \mathbf{P}_1\right) = \mathbf{D}_1\mathbf{P}_1\mathbf{D}_1, \tag{4}$$

[4]The nonparametric bootstrap procedure is a general method for standard error estimation and can be used in almost all situations. In principle, the bootstrap standard errors should be at least as good as the standard error estimates obtained from any analytic methods. However, the nonparametric bootstrap procedure is computationally intensive. Thus, all analytic methods have the advantage of computational efficiency, because no repeated sampling, repeated standardization, or repeated model fitting are required. After all, the availability of the nonparametric bootstrap procedure does not diminish the benefits provided by the analytic methods including the delta method developed by Yuan and Chan (2011) and Jones and Waller (2015) as well as the analytic approach proposed in this paper.

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where $\mathbf{D}_1 = \text{diag}\left(d_y \;\; d_1 \;\; \cdots \;\; d_p \right)$ is a diagonal matrix of standard deviations, $\mathbf{P}_1 = \begin{pmatrix} 1 & \\ \boldsymbol{\rho} & \mathbf{R} \end{pmatrix}$ is a symmetric matrix, $\boldsymbol{\rho} = \left(\rho_{1y} \;\; \cdots \;\; \rho_{py} \right)'$ is a $p \times 1$ vector of correlations between $x_{i1}, \ldots, x_{ip}$ and y_i or between $z_{i1}, \ldots, z_{ip}$ and z_{iy}, and $\mathbf{R}$ is the same as that in (Equation 3). When the data of $y_i, x_{i1}, \ldots, x_{ip}$ are analyzed, the estimates of the diagonal elements of $\mathbf{D}_1$ are equal to sample standard deviations, whereas when the data of $z_{iy}, z_{i1}, \ldots, z_{ip}$ are analyzed, the estimates of the diagonal elements of $\mathbf{D}_1$ are equal to 1. In contrast, the estimates of the elements of $\boldsymbol{\rho}$ and those of the off-diagonal elements of $\mathbf{R}$ are correlations, which are invariant to the scales of the input data.

In some situations, only the criterion variable and a subset of the predictor variables should be standardized. Let q be a positive integer that is smaller than p, indicating the number of predictor variables that are standardized. To analyze the data of $z_{iy}, z_{i1}, \ldots, z_{iq}, x_{i,q+1}, \ldots, x_{ip}$, we define the second covariance structure model as

$$\boldsymbol{\Sigma}\left(\mathbf{D}_2, \mathbf{P}_2\right) = \mathbf{D}_2\mathbf{P}_2\mathbf{D}_2, \tag{5}$$

where $\mathbf{D}_2 = \text{diag}\left(d_y \;\; d_1 \;\; \cdots \;\; d_q \;\; 1_{q+1} \;\; \cdots \;\; 1_p \right)$ is a diagonal matrix with the first $q + 1$ diagonal elements being standard deviations and the last $p - q$ elements fixed to 1, $\mathbf{P}_2 = \begin{pmatrix} 1 & & \\ \boldsymbol{\rho} & \mathbf{R} & \\ \boldsymbol{\sigma}_{xy} & \boldsymbol{\Phi}_{xz} & \boldsymbol{\Phi}_{xx} \end{pmatrix}$ is a symmetric matrix, $\boldsymbol{\rho} = \left(\rho_{1y} \;\; \cdots \;\; \rho_{qy} \right)'$ is a $q \times 1$ vector of correlations between $z_{i1}, \ldots, z_{iq}$ and z_{iy}, $\mathbf{R} = \begin{pmatrix} 1 & & & \\ \rho_{21} & 1 & & \\ \vdots & \vdots & \ddots & \\ \rho_{q1} & \rho_{q2} & \cdots & 1 \end{pmatrix}$ is a $q \times q$ correlation matrix of $z_{i1}, \ldots, z_{iq}$, $\boldsymbol{\sigma}_{xy} = \left(\sigma_{q+1,y} \;\; \cdots \;\; \sigma_{py} \right)'$ is a $(p - q) \times 1$ vector of covariances between $x_{i,q+1}, \ldots, x_{ip}$ and z_{iy}, $\boldsymbol{\Phi}_{xz}$ is a $(p - q) \times q$ matrix of covariances between $x_{i,q+1}, \ldots, x_{ip}$ and $z_{i1}, \ldots, z_{iq}$, and $\boldsymbol{\Phi}_{xx}$ is a $(p - q) \times (p - q)$ covariance matrix of $x_{i,q+1}, \ldots, x_{ip}$. The estimates of the first $q + 1$ diagonal elements of $\mathbf{D}_2$ should be equal to 1, because the standard deviations of Z-scores always equal to 1; the estimates of the elements of $\boldsymbol{\rho}$ and those of the off-diagonal elements of $\mathbf{R}$ are correlations, which are invariant to the scales of the input data; and finally, the estimates of $\boldsymbol{\sigma}_{xy}$, $\boldsymbol{\Phi}_{xz}$, and $\boldsymbol{\Phi}_{xx}$ are influenced by the scales of the last $p - q$ predictor variables.

In terms of the estimation method, generalized least squares (GLS) and maximum likelihood (ML) can be used with the normality assumption, while ML with the Satorra–Bentler correction (referred to as MLSB hereafter; Satorra & Bentler, 1994), the sandwich estimator (known as MLR in the covariance structure modeling literature; Huber, 1967; White, 1982), and the ADF method can be used without the normality assumption. All these estimation methods provide not only the point estimates but also the asymptotic covariance matrix of the estimates. Because the parameters to be estimated in (Equations 4) and (5) can also be estimated without specifying any models, one can calculate the sample statistics first and use them as the starting values. As a result, the optimization algorithm can always reach convergence and avoid numerical problems. Essentially, the purpose of estimating the models in (Equations 4) and (5) is to obtain the asymptotic covariance matrix of the estimates. At the same time, there is no need to derive any new formulas.

Given the estimates from the first two models, one can compute the standardized regression coefficients. Let $\boldsymbol{\beta}_1 = \left(\beta_1 \;\; \cdots \;\; \beta_p \right)'$ be a $p \times 1$ vector of standardized regression coefficients of $z_{i1}, \ldots, z_{ip}$. An estimate of $\boldsymbol{\beta}_1$ can be obtained from the estimates of the first model. That is, $\widehat{\boldsymbol{\beta}}_1 = \widehat{\mathbf{R}}^{-1}\widehat{\boldsymbol{\rho}}$. Let $\boldsymbol{\beta}_2 = \left(\beta_1 \;\; \cdots \;\; \beta_q \;\; b_{q+1} \;\; \cdots \;\; b_p \right)'$ be a $p \times 1$ vector of both standardized regression coefficients of $z_{i1}, \ldots, z_{iq}$ and unstandardized regression coefficients of $x_{i,q+1}, \ldots, x_{ip}$. An estimate of $\boldsymbol{\beta}_2$ can be obtained from the estimates of the second model. That is, $\widehat{\boldsymbol{\beta}}_2 = \begin{pmatrix} \widehat{\mathbf{R}} & \\ \widehat{\boldsymbol{\Phi}}_{xz} & \widehat{\boldsymbol{\Phi}}_{xx} \end{pmatrix}^{-1} \begin{pmatrix} \widehat{\boldsymbol{\rho}} \\ \widehat{\boldsymbol{\sigma}}_{xy} \end{pmatrix}$.

Based on the formulas of $\widehat{\boldsymbol{\beta}}_1$ and $\widehat{\boldsymbol{\beta}}_2$, one can apply the delta method to compute $\text{SE}\left(\widehat{\boldsymbol{\beta}}_1\right)$ and $\text{SE}\left(\widehat{\boldsymbol{\beta}}_2\right)$. Specifically, given the asymptotic covariance matrix of the estimates obtained from the first two models,

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one only needs the Jacobian matrices of partial derivatives of $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$ with respect to the parameters of the first two models, separately. Jones and Waller (2015, Eqs. (25) and (26)) provided the matrix formulas of the partial derivatives of $\boldsymbol{\beta}_1$ with respect to correlations. In contrast, the partial derivatives of $\boldsymbol{\beta}_2$ involve both covariances and correlations, but no formula has been derived for such a problem. Following the recommendations by Lord (1975) and Browne and Du Toit (1992), we compute the partial derivatives numerically for the purpose of automated implementation of the delta method. Admittedly, computational efficiency can be further improved with the use of analytic derivatives, but any changes in the delta method will require extra efforts to derive analytic derivatives and correspondingly modify the software code.

Finally, it should be noted that our approach to compute $\mathrm{SE}\left(\widehat{\boldsymbol{\beta}}_1\right)$ is essentially the same as the correlation-based delta method as Jones and Waller (2015, Section 3) with the advantage that the asymptotic covariance matrix of the estimated correlations (that is a subset of the estimates) can be obtained automatically as a byproduct of parameter estimation, whereas Jones and Waller (2015, Section 3) referred to the formulas derived elsewhere (e.g., Nel, 1985). Additionally, if a robust method (e.g., MLSB, MLR, ADF) is used to estimate the model in (Equation 4), we can easily get the robust estimator of the asymptotic covariance matrix of the estimated correlations. In principle, given a particular estimation method, the correlation-based delta method and the covariance-based delta method should produce the same $\mathrm{SE}\left(\widehat{\boldsymbol{\beta}}_1\right)$.

2.2. Standard error estimates for unstandardized regression coefficients of Z-scores and products of Z-scores

In some situations, quadratic or higher-order interaction terms are involved in research questions (LeBreton et al., 2013). We show that our analytic approach can accommodate the interaction terms of any orders. With p predictor variables, one can create ${}_2C_p$ second-order interaction terms, ${}_3C_p$ third-order interaction terms, …, ${}_{p-1}C_p$ $(p-1)$th-order interaction terms, and ${}_pC_p$ pth-order interaction term. In total, there are up to ${}_2C_p + {}_3C_p + \cdots + {}_{p-1}C_p + {}_pC_p$ interaction terms that can be included in the regression model. Let k be an integer between 1 and ${}_2C_p + {}_3C_p + \cdots + {}_{p-1}C_p + {}_pC_p$ and $\omega_{i1}, \ldots, \omega_{ik}$ be the interaction terms created from the Z-scores of predictor variables. To analyze the data of $z_{iy}, z_{i1}, \ldots, z_{ip}, \omega_{i1}, \ldots, \omega_{ik}$, we define the third covariance structure model as

$$\boldsymbol{\Sigma}\left(\mathbf{D}_3, \mathbf{P}_3\right) = \mathbf{D}_3\mathbf{P}_3\mathbf{D}_3, \tag{6}$$

where $\mathbf{D}_3 = \mathrm{diag}\left(\begin{array}{cccccc} d_y & d_1 & \cdots & d_p & 1_1 & \cdots & 1_k \end{array}\right)$ is a diagonal matrix with the first $p+1$ diagonal elements being standard deviations and the last k elements fixed to 1, $\mathbf{P}_3 = \begin{pmatrix} 1 & & \\ \boldsymbol{\rho} & \mathbf{R} & \\ \boldsymbol{\sigma}_{\omega y} & \boldsymbol{\Phi}_{\omega z} & \boldsymbol{\Phi}_{\omega\omega} \end{pmatrix}$ is a symmetric matrix, $\boldsymbol{\rho} = \left(\begin{array}{ccc} \rho_{1y} & \cdots & \rho_{py} \end{array}\right)'$ is a $p \times 1$ vector of correlations between $z_{i1}, \ldots, z_{ip}$ and z_{iy}, $\mathbf{R} = \begin{pmatrix} 1 & & & \\ \rho_{21} & 1 & & \\ \vdots & \vdots & \ddots & \\ \rho_{p1} & \rho_{p2} & \cdots & 1 \end{pmatrix}$ is a $p \times p$ correlation matrix of $z_{i1}, \ldots, z_{ip}$, $\boldsymbol{\sigma}_{\omega y} = \left(\begin{array}{ccc} \sigma_{\omega_1, y} & \cdots & \sigma_{\omega_k, y} \end{array}\right)'$ is a $k \times 1$ vector of covariances between $\omega_{i1}, \ldots, \omega_{ik}$ and z_{iy}, $\boldsymbol{\Phi}_{\omega z}$ is a $k \times p$ matrix of covariances between $\omega_{i1}, \ldots, \omega_{ik}$ and $z_{i1}, \ldots, z_{ip}$, and $\boldsymbol{\Phi}_{\omega\omega}$ is a $k \times k$ covariance matrix of $\omega_{i1}, \ldots, \omega_{ik}$.

When only q $(< p)$ predictor variables are standardized and $p - q$ predictor variables remain unstandardized, one can create ${}_2C_q + {}_3C_q + \cdots + {}_{q-1}C_q + {}_qC_q$ interaction terms with the Z-scores of predictor variables, ${}_2C_{p-q} + {}_3C_{p-q} + \cdots + {}_{p-q-1}C_{p-q} + {}_{p-q}C_{p-q}$ interaction terms with the unstandardized predictor variables, and $\left({}_2C_p + {}_3C_p + \cdots + {}_{p-1}C_p + {}_pC_p\right) - \left({}_2C_q + {}_3C_q + \cdots + {}_{q-1}C_q + {}_qC_q\right) - \left({}_2C_{p-q} + {}_3C_{p-q} + \cdots + {}_{p-q-1}C_{p-q} + {}_{p-q}C_{p-q}\right)$ interaction terms with the combinations of unstandardized and standardized predictor variables. No matter how the interaction terms are created, we still use ω_{i1},

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$\ldots, \omega_{ik}$ to denote the interaction terms. To analyze the data of $z_{iy}, z_{i1}, \ldots, z_{iq}, x_{i,q+1}, \ldots, x_{ip}, \omega_{i1}, \ldots, \omega_{ik}$, we define the fourth covariance structure model as

$$\boldsymbol{\Sigma}(\mathbf{D}_4, \mathbf{P}_4) = \mathbf{D}_4\mathbf{P}_4\mathbf{D}_4, \tag{7}$$

where $\mathbf{D}_4 = \text{diag}\begin{pmatrix} d_y & d_1 & \cdots & d_q & 1_{q+1} & \cdots & 1_p & 1_1 & \cdots & 1_k \end{pmatrix}$ is a diagonal matrix with the first $q + 1$ diagonal elements being standard deviations and the last $p - q + k$ elements fixed to 1, $\mathbf{P}_4 = \begin{pmatrix} 1 & & \\ \boldsymbol{\rho} & \mathbf{R} & \\ \boldsymbol{\sigma}_{(x\ \omega),y} & \boldsymbol{\Phi}_{(x\ \omega),z} & \boldsymbol{\Phi}_{(x\ \omega),(x\ \omega)} \end{pmatrix}$ is a symmetric matrix, $\boldsymbol{\rho} = \begin{pmatrix} \rho_{1y} & \cdots & \rho_{qy} \end{pmatrix}'$ is a $q \times 1$ vector of correlations between $z_{i1}, \ldots, z_{iq}$ and z_{iy}, $\mathbf{R} = \begin{pmatrix} 1 & & & \\ \rho_{21} & 1 & & \\ \vdots & \vdots & \ddots & \\ \rho_{q1} & \rho_{q2} & \cdots & 1 \end{pmatrix}$ is a $q \times q$ correlation matrix of $z_{i1}, \ldots, z_{iq}$, $\boldsymbol{\sigma}_{(x\ \omega),y} = \begin{pmatrix} \sigma_{q+1,y} & \cdots & \sigma_{py} & \sigma_{\omega_1,y} & \cdots & \sigma_{\omega_k,y} \end{pmatrix}'$ is a $(p - q + k) \times 1$ vector of covariances between $x_{i,q+1}, \ldots, x_{ip}, \omega_{i1}, \ldots, \omega_{ik}$ and z_{iy}, $\boldsymbol{\Phi}_{(x\ \omega),z}$ is a $(p - q + k) \times q$ matrix of covariances between $x_{i,q+1}, \ldots, x_{ip}, \omega_{i1}, \ldots, \omega_{ik}$ and $z_{i1}, \ldots, z_{iq}$, and $\boldsymbol{\Phi}_{(x\ \omega),(x\ \omega)}$ is a $(p - q + k) \times (p - q + k)$ covariance matrix of $x_{i,q+1}, \ldots, x_{ip}, \omega_{i1}, \ldots, \omega_{ik}$.

To estimate the third and fourth models, one should always use the robust methods (e.g., MLSB, MLR, and ADF), because the interaction terms are not normally distributed regardless of the distributions of the predictor variables. With an appropriate estimation method, the asymptotic covariance matrix of the estimates can be easily obtained from the software packages. Because the point estimates of the third and fourth models can also be obtained from the corresponding sample statistics, these point estimates can be used as the starting values to facilitate the optimization algorithm to reach convergence immediately. Essentially, the purpose of estimating the models in (Equations 6) and (7) is to obtain the asymptotic covariance matrix of the estimates. Again, there is no need to derive any new formulas.

Given the estimates from the third and fourth models, one can compute the relevant regression coefficients. Let $\boldsymbol{\beta}_3 = \begin{pmatrix} \beta_1 & \cdots & \beta_p & b_1 & \cdots & b_k \end{pmatrix}'$ be a $(p + k) \times 1$ vector of regression coefficients of $z_{i1}, \ldots, z_{ip}, \omega_{i1}, \ldots, \omega_{ik}$. An estimate of $\boldsymbol{\beta}_3$ can be computed from the estimates of the third model. That is, $\widehat{\boldsymbol{\beta}}_3 = \begin{pmatrix} \widehat{\mathbf{R}} & \\ \widehat{\boldsymbol{\Phi}}_{\omega,z} & \widehat{\boldsymbol{\Phi}}_{\omega,\omega} \end{pmatrix}^{-1} \begin{pmatrix} \widehat{\boldsymbol{\rho}} \\ \widehat{\boldsymbol{\sigma}}_{\omega,y} \end{pmatrix}$. Let $\boldsymbol{\beta}_4 = \begin{pmatrix} \beta_1 & \cdots & \beta_q & b_{q+1} & \cdots & b_p & b_{p+1} & \cdots & b_{p+k} \end{pmatrix}'$ be a vector of regression coefficients of $z_{i1}, \ldots, z_{iq}, x_{i,q+1}, \ldots, x_{ip}, \omega_{i1}, \ldots, \omega_{ik}$. An estimate of $\boldsymbol{\beta}_4$ can be computed from the estimates of the fourth model. That is, $\widehat{\boldsymbol{\beta}}_4 = \begin{pmatrix} \widehat{\mathbf{R}} & \\ \widehat{\boldsymbol{\Phi}}_{(x\ \omega),z} & \widehat{\boldsymbol{\Phi}}_{(x\ \omega),(x\ \omega)} \end{pmatrix}^{-1} \begin{pmatrix} \widehat{\boldsymbol{\rho}} \\ \widehat{\boldsymbol{\sigma}}_{(x\ \omega),y} \end{pmatrix}$.

Based on the formulas of $\widehat{\boldsymbol{\beta}}_3$ and $\widehat{\boldsymbol{\beta}}_4$, one can apply the delta method to compute $\text{SE}(\widehat{\boldsymbol{\beta}}_3)$ and $\text{SE}(\widehat{\boldsymbol{\beta}}_4)$. Specifically, given the asymptotic covariance matrix of the estimates obtained from the third and fourth models, one only needs the Jacobian matrices of partial derivatives of $\boldsymbol{\beta}_3$ and $\boldsymbol{\beta}_4$ with respect to the parameters of the third and fourth models, separately. For automated implementation of the delta method, we compute the partial derivatives numerically.

3. Two numeric examples

In this section, two examples are analyzed. In the first example, the criterion variable is regressed on three predictor variables without any interaction terms, and we use this example to compare our analytic approach with (1) the covariance-based delta method, (2) Bentler and Lee's constrained covariance structure modeling method, and (3) Kwan and Chan's unconstrained covariance structure modeling method. In the second example, the standardized criterion variable is regressed on two standardized predictor variables, one unstandardized predictor variable, and one interaction term created with the

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two standardized predictor variables, and we use this example to compare our analytic approach with the nonparametric bootstrap procedure advocated by Cheung et al. (2022). Additionally, as requested by one of the reviewers, the standard error estimates from regular regression are also added to both examples.

3.1. Software implementations

To implement our analytic approach, the first step is to estimate one of the covariance structure models defined in this article, and the second step is to apply the delta method. In the first step, we use the *lavaan* (Rosseel, 2012) and *OpenMx* (Neale et al., 2016) packages in R and SAS PROC CALIS to estimate the covariance structure models. The model specifications in the *OpenMx* package and SAS PROC CALIS are easy and straightforward, because the *OpenMx* package and SAS PROC CALIS allow the user to define and specify matrices directly. In contrast, the model specifications in the *lavaan* package rely on the use of latent variables, each latent variable corresponding to a manifest variable, so that the standard deviations and correlations can be estimated properly. Additionally, all the predictor variables are modeled (rather than treated as fixed variables) in the first step. Because no interaction term is involved in the first example, we can use five estimation methods: ML, GLS, MLSB, MLR, and ADF. In contrast, we can only use MLSB, MLR, and ADF due to the interaction term in the second example. Among the software packages, the *lavaan* package provides all five estimation methods, SAS PROC CALIS provides ML, GLS, MLSB, and ADF, and the *OpenMx* package only provides ML and ADF. In the second step, it is much more convenient to use the *OpenMx* package than the *lavaan* package to implement the delta method, because the *OpenMx* package allows users to define additional parameters with matrix formulas, whereas matrix formulas must be translated to scalar functions for the *lavaan* package. For the second example, the *lavaan* package is only used in the first step of our analytic approach due to the inconvenience of working out the scalar functions.

To implement the existing methods, we also use several software packages. Specifically, we use the *lavaan* package to estimate Bentler and Lee's constrained model, the *betaDelta* package to implement the covariance-based delta method, and the *stdmod* package (Cheung, 2024) to construct the percentile-based CIs. Additionally, we take the bootstrap results generated from the *stdmod* package to calculate the bootstrap standard errors, which are the standard deviations across bootstrap replications. All the programs used in this section can be found in the Supplementary Material of this article.

3.2. Example 1

The data of the first example were retrieved from the electronic supplementary material of Kwan and Chan (2011). In this example, the criterion variable is child's reading ability (y_i), and the predictor variables are parental occupational status (x_{i1}), parental educational level (x_{i2}), and child's home possession (x_{i3}). The sample size is 200.

To compute the standard error estimates for the three standardized regression coefficients, we estimate the covariance structure model defined in (Equation 4) and apply the correlation-based delta method. Table 1 shows the point estimates of $\mathbf{D}_1$ and $\mathbf{P}_1$ and the associated standard error estimates from the first step of our analytic approach, and Table 2 shows the standard error estimates for the three standardized regression coefficients from the second step of our analytic approach, two existing methods,[5] and regular regression. Several observations can be made. First, the standard error estimates for the estimates of $\mathbf{D}_1$ and $\mathbf{P}_1$ in Table 1 are the same between GLS and ML, and this is also the case between MLSB, MLR, and ADF. Second, given the first observation, our analytic approach produces the same standard error estimates for the three standardized regression coefficients in Table 2 between GLS and ML and between MLSB, MLR, and ADF. Third, given a particular estimation method, our

[5]The results from Kwan and Chan's reparameterization approach (as implemented by EQS 6.1 using ML only) can be found from Kwan and Chan (2011, Table 3).

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Table 1. Point estimates of $\mathbf{D}_1$ and $\mathbf{P}_1$ and associated standard error estimates from the first step of our analytic approach in Example 1

	Point estimate				SE from SAS PROC CALIS, *lavaan*, and *OpenMx* using ML or GLS				SE from SAS PROC CALIS, *lavaan*, and *OpenMx* using MLSB, MLR, or ADF			
$\mathbf{D}_1$	78.0309				3.9113				4.1423			
	15.0419				.7540				.7845			
	3.0115				.1510				.1480			
	.9149				.0459				.0294			
$\mathbf{P}_1$	1*				N.A.				N.A.			
	.2310	1*			.0671	N.A.			.0628	N.A.		
	.2110	.6606	1*		.0677	.0400	N.A.		.0660	.0387	N.A.	
	.2806	.3547	.3743	1*	.0653	.0620	.0610	N.A.	.0687	.0561	.0580	N.A.

Note: * indicates a fixed value.

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Table 2. Standardized regression coefficients and associated standard error estimates from the second step of our analytic approach, two existing methods, and regular regression in Example 1

		Our analytic approach		Covariance-based delta method		Constrained covariance structure model			
Variable	Standardized regression coefficient	SE from SAS PROC CALIS, *lavaan*, and *OpenMx* using ML or GLS	SE from SAS PROC CALIS, *lavaan*, and *OpenMx* using MLSB, MLR, or ADF	SE from *betaDelta* with `type = "mvn"`	SE from *betaDelta* with `type = "adf"`	SE from *lavaan* using ML or GLS	SE from *lavaan* using MLSB, MLR, or ADF	Bootstrap SE from *stdmod* with 5,000 replications	SE from regular regression
x_{i1}: parental occupational status	.1207	.0903	.0813	.0901	.0816	.0903	.0813	.0821	.0914
x_{i2}: parental educational level	.0491	.0914	.0873	.0911	.0878	.0914	.0873	.0876	.0921
x_{i3}: child's home possession	.2194	.0719	.0732	.0718	.0736	.0719	.0732	.0744	.0740

analytic approach produces the same standard error estimates as those from the existing methods: (1) the covariance-based delta method, (2) Bentler and Lee's constrained covariance structure modeling method, and (3) Kwan and Chan's unconstrained covariance structure modeling method (using ML only; see Kwan & Chan, 2011, Table 3). Fourth, the standard error estimates produced by our analytic approach using MLSB, MLR, and ADF are very close to the bootstrap standard errors based on 5,000 bootstrap replications. Note that the bootstrap standard errors obtained from the *stdmod* package are also close to the bootstrap standard errors obtained from EQS based on 1,000 bootstrap replications (see Kwan & Chan, 2011, Table 3). Finally, the standard error estimates from regular regression are inflated. For parental occupational status (x_{i1}), the standard error estimate from regular regression is larger than the corresponding bootstrap standard error and the one from our analytic approach using MLSB, MLR, and ADF by more than 10%. For parental educational level (x_{i2}), the standard error estimate from regular regression is larger than the corresponding bootstrap standard error and the one from our analytic approach using MLSB, MLR, and ADF by more than 5%.

3.3. Example 2

The data of the second example were retrieved from the *stdmod* package in R (Cheung et al., 2022). In this example, the criterion variable is sleep duration (y_i), and the predictor variables are emotional stability (x_{i1}), conscientiousness (x_{i2}), age (x_{i3}), and gender (x_{i4}, coded as female and male). The sample size is 500.

To study the interaction effect of emotional stability and conscientiousness in standardized metric, Cheung et al. (2022) standardized all the variables to Z-scores, except the categorical variable gender (x_{i4}), and used the standardized emotional stability (z_{i1}) and the standardized conscientiousness (z_{i2}) to create the interaction term (ω_{i1}). To match the results produced by the *stdmod* package, we recode gender as 0 (for female) and 1 (for male). Given these settings, we analyze the data of z_{iy}, z_{i1}, z_{i2}, z_{i3}, x_{i4}, ω_{i1}.

To compute the standard error estimates for the five unstandardized regression coefficients, we estimate the covariance structure model defined in (Equation 7) and apply the delta method. Table 3 shows the point estimates of $\mathbf{D}_4$ and $\mathbf{P}_4$ and the associated standard error estimates from the first step of our analytic approach, and Table 4 shows the standard error estimates for the five unstandardized regression coefficients and the symmetric 95% CIs from the second step of our analytic approach, the bootstrap standard errors and percentile-based CIs obtained from the *stdmod* package based on 5,000 bootstrap replications, and the standard error estimates from regular regression. Several observations can be made. First, MLSB, MLR, and ADF produce the same standard error estimates for the estimates of $\mathbf{D}_4$ and $\mathbf{P}_4$ in Table 3, and this is also the case for the five unstandardized regression coefficients in Table 4. Second, the delta method produces the standard error estimates that are close to the bootstrap standard errors. Third, the symmetric 95% CIs from our analytic approach are similar to the percentile-based 95% CIs from the nonparametric bootstrap procedure. Finally, the standard error estimate for standardized age (z_{i3}) from regular regression is larger than the corresponding bootstrap standard error by 24% and the one from our analytic approach by 28%, whereas the standard error estimates for standardized conscientiousness (z_{i2}), gender (x_{i4}), and the interaction term (ω_{i1}) from regular regression are smaller than the corresponding bootstrap standard errors and those from our analytic approach by 9%–12%.

4. Simulations

In this section, we conduct two simulation studies to further evaluate the performances of regular regression, our analytic approach, and the nonparametric bootstrap procedure at finite sample sizes. In the first simulation study, all variables are generated from multivariate normal distributions. In the second simulation study, all variables are generated from multivariate non-normal distributions.

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Table 3. Point estimates of $\mathbf{D}_4$ and $\mathbf{P}_4$ and associated standard error estimates from the first step of our analytic approach in Example 2

	Point estimate						SE from SAS PROC CALIS, *lavaan*, and *OpenMx* using MLSB, MLR or ADF					
$\mathbf{D}_4$	1						.0425					
		1						.0265				
			1						.0311			
				1						.1220		
					1*						N.A.	
						1*						N.A.
$\mathbf{P}_4$	1*						N.A.					
	.1440	1*					.0447	N.A.				
	.1564	.2181	1*				.0480	.0437	N.A.			
	.0442	.0757	.0930	1*			.0351	.0448	.0365	N.A.		
	−.0195	.0212	.0361	.0114	.1414		.0191	.0165	.0180	.0155	.0111	
	−.1161	−.0147	−.0374	.1074	.0097	1.0318	.0538	.0717	.0753	.0415	.0187	.1075

Note: * indicates a fixed value.

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Table 4. Unstandardized regression coefficients, standard error estimates and symmetric confidence intervals from the second step of our analytic approach, standard error estimates and percentile-based confidence intervals from the nonparametric bootstrap procedure, and standard error estimates from regular regression in Example 2

		Our analytic approach			Nonparametric bootstrap procedure			
Variable	Unstandardized regression coefficient	SE from SAS PROC CALIS and *OpenMx* using MLSB or ADF	Lower limit of symmetric 95% CI	Upper limit of symmetric 95% CI	Bootstrap SE from *stdmod* with 5,000 replications	Lower limit of percentile-based 95% CI	Upper limit of percentile-based 95% CI	SE from regular regression
z_{i1}: emotion	.1150	.0448	.0270	.2031	.0443	.0291	.2012	.0449
z_{i2}: conscientiousness	.1305	.0497	.0328	.2282	.0503	.0288	.2265	.0452
z_{i3}: age	.0371	.0345	−.0306	.1048	.0357	−.0347	.1072	.0443
x_{i4}: gender	−.1844	.1328	−.4454	.0766	.1312	−.4392	.0783	.1170
ω_{j1}: emotion × conscientiousness	−.1083	.0488	−.2042	−.0124	.0498	−.2043	−.0090	.0434

Note: Symmetric 95% CI = point estimate ± $t^* \times$SE, where $t^* = 1.9648$ with $df = 494$.

4.1. First simulation study: Data generation, analysis, and evaluation criteria

In the first simulation study, we generate random data from multivariate normal distributions to mimic the second example in the previous section. First, a random dataset of size n is generated from $MVN(\mathbf{0}_5, \boldsymbol{\Sigma})$ for $x_{i1}, x_{i2}, x_{i3}, x_{i4}$, and ε_i, where $\boldsymbol{\Sigma} = \mathbf{HKH}$, $\mathbf{H} = \begin{pmatrix} \sigma_x & & & & \\ 0 & \sigma_x & & & \\ 0 & 0 & \sqrt{2} & & \\ 0 & 0 & 0 & \sqrt{2} & \\ 0 & 0 & 0 & 0 & \sigma_\varepsilon \end{pmatrix}$, $\mathbf{K} = \begin{pmatrix} \mathbf{R} & \\ \mathbf{0}_4' & 1 \end{pmatrix} = \begin{pmatrix} 1 & & & & \\ r & 1 & & & \\ .3 & .5 & 1 & & \\ .6 & .4 & .2 & 1 & \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$, r is the correlation between x_{i1} and x_{i2}, σ_x^2 is the variance of x_{i1} and x_{i2}, and σ_ε^2 is the variance of ε_i. Four factors are manipulated: (1) n, (2) σ_x^2, (3) r, and (4) σ_ε^2. The levels of these four factors are as follows: n = 100, 200, 300; σ_x^2 = 1, 6, 12; r = .2, .4, .8; and σ_ε^2 = 10, 20, 40. Second, we create the criterion variable from $y_i = x_{i1} + x_{i2} + 0.7x_{i3} + 0.5x_{i4} + 0.3x_{i1}x_{i2} + \varepsilon_i$, where $i = 1, \ldots, n$, and gather the data of y_i, x_{i1}, x_{i2}, x_{i3}, and x_{i4}. Third, we standardize y_i, x_{i1}, x_{i2}, and x_{i3} to z_{iy}, z_{i1}, z_{i2}, and z_{i3} while leaving x_{i4} unstandardized. Finally, we create a new variable $\omega_{i1} = z_{i1}\, z_{i2}$ and gather the data of z_{iy}, z_{i1}, z_{i2}, z_{i3}, x_{i4}, and ω_{i1}. We repeat the above steps 1,000 times at each combination of n, σ_x^2, r, and σ_ε^2. Thus, a total of 81,000 (= 3 × 3 × 3 × 3 × 1,000) random datasets are generated.

For data analysis, we use regular regression and our analytic approach to analyze each random data of z_{iy}, z_{i1}, z_{i2}, z_{i3}, x_{i4}, and ω_{i1}, while the *stdmod* package in R can take the data of either y_i, x_{i1}, x_{i2}, x_{i3}, x_{i4} or z_{iy}, z_{i1}, z_{i2}, z_{i3}, x_{i4} as the input dataset to implement the nonparametric bootstrap procedure. The bootstrap replications are set to 5,000 in our simulations.

Upon completion of data analysis, we have 1,000 point estimates, from which we calculate the mean and standard deviation, for each unstandardized regression coefficient at each combination of n, σ_x^2, r, and σ_ε^2. To evaluate the point estimate of each unstandardized regression coefficient, we calculate the relative bias of mean (relative to the true value in the population, which will be discussed in the next paragraph). As for the standard deviation, it will be used as the true standard error to calculate the relative bias of the standard error estimate. In addition, we have 1,000 standard error estimates from regular regression and our analytic approach and 1,000 bootstrap standard errors for each unstandardized regression coefficient at each combination of n, σ_x^2, r, and σ_ε^2, from which we calculate the average standard error estimate from regular regression and our analytic approach and the average bootstrap standard error. Finally, we calculate the relative bias of the average standard error estimate from regular regression and our analytic approach, as well as the relative bias of average bootstrap standard error. The general formula of relative bias is

$$\text{relative bias} = \frac{\overline{\theta} - \theta_0}{\theta_0},$$

where $\overline{\theta}$ is the mean of an unstandardized regression coefficient or the average standard error estimate across replications, and θ_0 is the corresponding true value of the unstandardized regression coefficient or the true standard error.

One challenge in our simulations is to figure out the true values of the unstandardized regression coefficients of z_{i1}, z_{i2}, z_{i3}, x_{i4}, and ω_{i1}. Even though the regression model is not too complicated, it is still difficult to obtain the true values, because the moments of Z-scores and products of Z-scores must be derived analytically. As an alternative, we approximate the true values by analyzing the simulated data of huge sample sizes. That is, at each combination of σ_x^2, r, and σ_ε^2, we generate a random dataset of size 1,000,000 and regress z_{iy} on z_{i1}, z_{i2}, z_{i3}, x_{i4}, and ω_{i1}. The unstandardized regression coefficients of z_{i1}, z_{i2}, z_{i3}, x_{i4}, and ω_{i1} obtained at huge sample sizes are used to approximate the true values. Additionally, we record the associated R^2. Table 5 shows the approximate true values of regression coefficients and R^2 at each combination of σ_x^2, r, and σ_ε^2. The values of R^2 range from .1530 to .8848, covering a wide range of scenarios in social and behavioral research.

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Table 5. Approximate true values of the unstandardized regression coefficients and R^2 at each combination of σ_x^2, r and σ_ε^2 in the first simulation study

σ_x^2	r	σ_ε^2	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	R^2
1	.2	10	.2412	.2420	.2389	.1194	.0721	.4207
		20	.1910	.1912	.1885	.0960	.0565	.2647
		40	.1437	.1463	.1445	.0727	.0425	.1530
	.4	10	.2387	.2380	.2364	.1179	.0718	.4339
		20	.1885	.1917	.1863	.0959	.0562	.2766
		40	.1460	.1444	.1443	.0723	.0441	.1615
	.8	10	.2331	.2316	.2297	.1166	.0696	.4591
		20	.1864	.1895	.1839	.0940	.0550	.2990
		40	.1410	.1457	.1427	.0724	.0441	.1757
6	.2	10	.4047	.4032	.1617	.0815	.2975	.7275
		20	.3584	.3569	.1451	.0726	.2628	.5735
		40	.3012	.2995	.1217	.0604	.2199	.4023
	.4	10	.3887	.3889	.1574	.0799	.2861	.7472
		20	.3477	.3461	.1403	.0721	.2550	.5977
		40	.2937	.2926	.1197	.0595	.2151	.4247
	.8	10	.3617	.3609	.1454	.0726	.2658	.7829
		20	.3277	.3259	.1335	.0668	.2401	.6429
		40	.2828	.2806	.1129	.0569	.2063	.4739
12	.2	10	.4318	.4319	.1236	.0623	.4480	.8448
		20	.4021	.4008	.1145	.0575	.4171	.7317
		40	.3575	.3567	.1014	.0512	.3705	.5763
	.4	10	.4118	.4119	.1177	.0592	.4275	.8595
		20	.3854	.3849	.1099	.0559	.4009	.7527
		40	.3452	.3459	.0980	.0490	.3575	.6032
	.8	10	.3724	.3728	.1058	.0540	.3870	.8848
		20	.3540	.3514	.1017	.0500	.3657	.7933
		40	.3229	.3195	.0914	.0455	.3334	.6569

4.2. First simulation study: Results

In the first simulation study, even though the data of x_{i1}, x_{i2}, x_{i3}, x_{i4}, and ε_i are generated from multivariate normal distributions, the interaction term (i.e., $\omega_{i1} = z_{i1} z_{i2}$, the product of Z-scores) is, in general, not normally distributed. According to the reporting practice recommended by Cain, Zhang, and Yuan (2017), we first report a variety of percentiles of the sample univariate skewness and the sample univariate kurtosis of ω_{i1} at each combination of n, σ_x^2, r, and σ_ε^2 (see Univariate_skew_kurt_MVN.docx in the Supplementary Material). These percentiles cover a wide range of values that are commonly encountered in practical data analysis (see Cain et al., 2017, Table 1).

The relative bias of the mean is included in Relative_bias_of_mean_MVN.docx in the Supplementary Material, and the main results of the first simulation study are included in Tables 6–9. Specifically, Table 6

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Table 6. Means and standard deviations of the unstandardized regression coefficients in the first simulation study

n	σ_x^2	r	σ_ε^2	Mean					Standard deviation				
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}
100	1	.2	10	.2372	.2328	.2441	.1237	.0738	.1018	.0996	.0878	.0755	.0842
			20	.1885	.1921	.1856	.0986	.0594	.1157	.1071	.1014	.0840	.0865
			40	.1402	.1441	.1445	.0749	.0454	.1203	.1212	.1160	.0909	.0980
		.4	10	.2381	.2418	.2345	.1164	.0718	.1023	.0975	.0916	.0738	.0782
			20	.1903	.1898	.1858	.0943	.0577	.1119	.1061	.1013	.0786	.0847
			40	.1438	.1387	.1495	.0710	.0440	.1191	.1142	.1108	.0819	.0933
		.8	10	.2325	.2284	.2294	.1164	.0700	.1504	.1460	.0911	.0703	.0607
			20	.1838	.1915	.1820	.0922	.0569	.1718	.1663	.1004	.0801	.0700
			40	.1335	.1429	.1427	.0768	.0386	.1858	.1780	.1072	.0849	.0780
	6	.2	10	.4016	.4069	.1605	.0844	.2990	.0774	.0718	.0666	.0517	.0607
			20	.3550	.3523	.1495	.0794	.2624	.0885	.0878	.0822	.0626	.0689
			40	.2946	.2958	.1271	.0644	.2195	.1040	.1020	.0946	.0752	.0772
		.4	10	.3874	.3879	.1607	.0803	.2852	.0706	.0697	.0625	.0491	.0534
			20	.3473	.3472	.1425	.0718	.2522	.0878	.0836	.0743	.0599	.0684
			40	.2928	.2926	.1225	.0586	.2148	.1062	.0963	.0915	.0736	.0764
		.8	10	.3592	.3654	.1471	.0729	.2675	.0980	.0917	.0609	.0445	.0446
			20	.3289	.3252	.1317	.0681	.2383	.1179	.1142	.0726	.0549	.0542
			40	.2837	.2793	.1121	.0569	.2064	.1447	.1375	.0879	.0669	.0629
	12	.2	10	.4353	.4324	.1243	.0645	.4500	.0711	.0691	.0516	.0413	.0540
			20	.4048	.4015	.1158	.0570	.4222	.0790	.0774	.0651	.0525	.0619
			40	.3589	.3606	.0996	.0516	.3711	.0894	.0887	.0825	.0654	.0713
		.4	10	.4141	.4133	.1217	.0603	.4286	.0671	.0637	.0459	.0379	.0520
			20	.3872	.3839	.1122	.0579	.4044	.0749	.0739	.0631	.0462	.0562
			40	.3415	.3486	.0990	.0493	.3579	.0896	.0874	.0779	.0609	.0671
		.8	10	.3735	.3780	.1066	.0553	.3932	.0762	.0776	.0451	.0332	.0492
			20	.3598	.3501	.1060	.0496	.3683	.1003	.0950	.0602	.0443	.0519
			40	.3173	.3242	.0931	.0478	.3344	.1187	.1185	.0754	.0541	.0564
200	1	.2	10	.2372	.2399	.2395	.1221	.0721	.0694	.0686	.0624	.0521	.0534
			20	.1915	.1914	.1895	.0946	.0595	.0805	.0782	.0743	.0594	.0620
			40	.1446	.1452	.1439	.0744	.0449	.0862	.0806	.0769	.0630	.0650
		.4	10	.2408	.2366	.2382	.1182	.0703	.0689	.0658	.0632	.0508	.0511
			20	.1861	.1914	.1886	.0979	.0591	.0776	.0784	.0730	.0553	.0569
			40	.1439	.1472	.1418	.0731	.0427	.0890	.0817	.0771	.0599	.0651
		.8	10	.2338	.2331	.2292	.1146	.0710	.1068	.1021	.0601	.0471	.0422
			20	.1817	.1934	.1866	.0932	.0588	.1200	.1130	.0702	.0535	.0497
			40	.1453	.1433	.1422	.0684	.0429	.1296	.1291	.0799	.0579	.0517

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Table 6. Continued

n	σ_x^2	r	σ_ε^2	Mean					Standard deviation				
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}
	6	.2	10	.4047	.4031	.1635	.0824	.2965	.0507	.0486	.0444	.0350	.0380
			20	.3568	.3594	.1447	.0747	.2626	.0625	.0591	.0582	.0452	.0471
			40	.2987	.2975	.1225	.0619	.2194	.0735	.0716	.0655	.0547	.0535
		.4	10	.3953	.3893	.1569	.0780	.2868	.0498	.0491	.0429	.0339	.0387
			20	.3456	.3501	.1404	.0718	.2539	.0593	.0556	.0534	.0426	.0421
			40	.2928	.2928	.1165	.0614	.2135	.0704	.0660	.0627	.0484	.0517
		.8	10	.3600	.3624	.1464	.0737	.2661	.0686	.0673	.0401	.0310	.0305
			20	.3298	.3241	.1328	.0684	.2415	.0886	.0826	.0518	.0394	.0372
			40	.2848	.2750	.1184	.0571	.2086	.1022	.0960	.0608	.0473	.0436
	12	.2	10	.4339	.4323	.1270	.0631	.4480	.0511	.0466	.0355	.0270	.0380
			20	.4039	.3992	.1144	.0586	.4173	.0550	.0546	.0467	.0353	.0416
			40	.3591	.3542	.1029	.0512	.3699	.0642	.0616	.0572	.0453	.0487
		.4	10	.4137	.4108	.1191	.0589	.4271	.0453	.0466	.0339	.0257	.0369
			20	.3839	.3898	.1096	.0562	.4011	.0528	.0520	.0445	.0329	.0397
			40	.3473	.3454	.0970	.0497	.3591	.0609	.0609	.0540	.0434	.0440
		.8	10	.3766	.3722	.1088	.0537	.3886	.0534	.0515	.0309	.0223	.0325
			20	.3551	.3554	.1024	.0498	.3683	.0682	.0671	.0393	.0306	.0352
			40	.3229	.3193	.0957	.0469	.3336	.0847	.0823	.0494	.0375	.0375
300	1	.2	10	.2386	.2399	.2370	.1219	.0708	.0559	.0542	.0529	.0419	.0450
			20	.1943	.1898	.1897	.0943	.0585	.0643	.0599	.0577	.0471	.0509
			40	.1483	.1456	.1435	.0695	.0458	.0709	.0638	.0625	.0518	.0526
		.4	10	.2384	.2370	.2361	.1197	.0720	.0560	.0553	.0510	.0406	.0400
			20	.1907	.1905	.1893	.0937	.0574	.0625	.0632	.0573	.0470	.0465
			40	.1454	.1483	.1406	.0716	.0449	.0701	.0648	.0614	.0502	.0508
		.8	10	.2334	.2290	.2329	.1156	.0694	.0865	.0832	.0511	.0382	.0344
			20	.1841	.1918	.1842	.0928	.0535	.0945	.0912	.0577	.0431	.0395
			40	.1519	.1335	.1464	.0690	.0423	.1059	.1007	.0623	.0455	.0406
	6	.2	10	.4030	.4029	.1647	.0833	.2970	.0440	.0419	.0380	.0283	.0330
			20	.3590	.3571	.1447	.0730	.2617	.0518	.0477	.0472	.0376	.0383
			40	.3002	.3002	.1199	.0615	.2198	.0598	.0571	.0547	.0442	.0445
		.4	10	.3887	.3869	.1585	.0807	.2865	.0413	.0386	.0355	.0270	.0288
			20	.3503	.3467	.1411	.0699	.2559	.0477	.0496	.0457	.0343	.0356
			40	.2933	.2919	.1198	.0606	.2163	.0549	.0551	.0526	.0403	.0404
		.8	10	.3602	.3613	.1470	.0740	.2646	.0553	.0528	.0333	.0252	.0248
			20	.3226	.3304	.1322	.0684	.2397	.0690	.0666	.0415	.0324	.0288
			40	.2788	.2824	.1142	.0552	.2064	.0867	.0805	.0506	.0389	.0344

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Table 6. Continued

n	σ_x^2	r	σ_ε^2	Mean					Standard deviation				
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}
	12	.2	10	.4330	.4325	.1227	.0632	.4484	.0395	.0389	.0284	.0228	.0307
			20	.4028	.4020	.1140	.0596	.4172	.0452	.0449	.0368	.0297	.0334
			40	.3569	.3563	.1027	.0530	.3709	.0517	.0500	.0451	.0357	.0381
		.4	10	.4123	.4133	.1182	.0600	.4272	.0385	.0369	.0277	.0204	.0292
			20	.3866	.3871	.1087	.0560	.4021	.0447	.0428	.0342	.0280	.0329
			40	.3439	.3476	.0988	.0478	.3601	.0511	.0465	.0431	.0338	.0359
		.8	10	.3723	.3749	.1081	.0550	.3875	.0447	.0429	.0256	.0183	.0260
			20	.3524	.3540	.0999	.0514	.3662	.0560	.0534	.0328	.0234	.0284
			40	.3213	.3188	.0921	.0475	.3350	.0686	.0651	.0405	.0309	.0297

shows the means and standard deviations of the unstandardized regression coefficients; Table 7 shows the average standard error estimate from regular regression, and the associated relative bias; Table 8 shows the average standard error estimate from our analytic approach and the associated relative bias; and Table 9 shows the average bootstrap standard error and the associated relative bias.

Regarding the relative bias of mean, 16 of a total of 405 (= 5 × 3 × 3 × 3 × 3) absolute values are larger than .05, of which eight absolute values are associated with the unstandardized regression coefficient of ω_{i1}. Regarding the relative bias of average standard error estimate, drastically different performances are observed among regular regression, our analytic approach, and the nonparametric bootstrap procedure. Based on the pattern in Table 7, we observe that the relative bias of average standard error estimate from regular regression is mainly determined by σ_x^2. More specifically, 4 absolute values of relative bias of average standard error estimate are larger than .1 at $\sigma_x^2 = 6$ (particularly, 1 within $n = 100$, 2 within $n = 200$, and 1 within $n = 300$; see the last column of Table 7), and 45 absolute values of relative bias of average standard error estimate are larger than .1 at $\sigma_x^2 = 12$ (particularly, 15 within each level of n; see the last column of Table 7). Based on the pattern in Table 8, we observe that the relative bias of average standard error estimate from our analytic approach is mainly determined by n, and most values of relative bias are negative.[6] More specifically, 24 absolute values of relative bias of average standard error estimate are larger than .1 at $n = 100$ (particularly, 6 within $\sigma_x^2 = 1$, 8 within $\sigma_x^2 = 6$, and 10 within $\sigma_x^2 = 12$; see the last column of Table 8), 7 absolute values of relative bias of average standard error estimate are larger than .1 at $n = 200$ (particularly, 1 within $\sigma_x^2 = 1$, 3 within $\sigma_x^2 = 10$ and 20, separately; see the last column of Table 8), and 4 absolute values of relative bias of average standard error estimate are larger than .1 at $n = 300$ (particularly, 4 within $\sigma_x^2 = 12$; see the last column of Table 8). Finally, from the nonparametric bootstrap procedure, all absolute values of relative bias of average standard error estimate in Table 9 are smaller than .1. To better demonstrate the different performances of the three methods, the values of relative bias of average standard error estimate for the unstandardized regression coefficient of ω_{i1} in Tables 7–9 are plotted in Figure 1.

According to Hoogland and Boomsma (1998), the point estimates are acceptable when the relative biases of mean are less than .05 in absolute value, whereas the standard error estimates are acceptable when the relative biases of average standard error estimates are less than .1 in absolute value. Therefore, we can reach the following conclusions. First, more than 95% of the point estimates of unstandardized regression coefficients are acceptable (because 16/405 = 3.95% are unacceptable). Second, all standard

[6]To further investigate the standard error estimates from our analytic approach, a boxplot of standard error estimates is created at each combination of the factors for all the predictors. These boxplots are included in Boxplots.docx in the Supplementary Material.

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Table 7. The average standard error estimate from regular regression and its relative bias in the first simulation study

n	σ_x^2	r	σ_ε^2	Average SE					Relative bias of average SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
100	1	.2	10	.1032	.0985	.0943	.0759	.0794	.0133	−.0117	.0741	.0052	−.0572	0
			20	.1157	.1114	.1064	.0857	.0895	−.0004	.0400	.0496	.0203	.0342	
			40	.1243	.1194	.1142	.0915	.0959	.0330	−.0148	−.0158	.0068	−.0214	
		.4	10	.1002	.0964	.0903	.0703	.0744	−.0199	−.0111	−.0135	−.0470	−.0478	
			20	.1139	.1093	.1021	.0798	.0851	.0182	.0307	.0083	.0164	.0041	
			40	.1226	.1180	.1103	.0864	.0911	.0293	.0333	−.0041	.0551	−.0229	
		.8	10	.1514	.1454	.0899	.0692	.0622	.0067	−.0044	−.0130	−.0159	.0239	
			20	.1718	.1651	.1023	.0785	.0707	.0001	−.0072	.0187	−.0200	.0097	
			40	.1867	.1790	.1109	.0851	.0766	.0047	.0058	.0338	.0022	−.0173	
	6	.2	10	.0706	.0679	.0650	.0519	.0547	−.0873	−.0549	−.0241	.0050	−.0987	1
			20	.0886	.0850	.0815	.0650	.0682	.0014	−.0325	−.0082	.0384	−.0092	
			40	.1044	.1003	.0961	.0769	.0806	.0041	−.0170	.0154	.0223	.0443	
		.4	10	.0677	.0650	.0609	.0477	.0505	−.0405	−.0671	−.0260	−.0301	−.0534	
			20	.0848	.0815	.0763	.0598	.0634	−.0342	−.0253	.0274	−.0018	−.0733	
			40	.1017	.0973	.0910	.0715	.0758	−.0421	.0101	−.0053	−.0283	−.0076	
		.8	10	.0966	.0928	.0573	.0440	.0397	−.0150	.0118	−.0580	−.0112	−.1107	
			20	.1238	.1188	.0735	.0564	.0504	.0502	.0398	.0132	.0270	−.0701	
			40	.1478	.1420	.0882	.0676	.0608	.0210	.0329	.0032	.0110	−.0328	
	12	.2	10	.0537	.0516	.0494	.0398	.0416	−.2453	−.2533	−.0419	−.0371	−.2300	15
			20	.0712	.0685	.0656	.0524	.0551	−.0991	−.1151	.0066	−.0007	−.1101	
			40	.0883	.0846	.0810	.0651	.0683	−.0126	−.0457	−.0192	−.0051	−.0415	

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Table 7. Continued

n	σ_x^2	r	σ_ε^2	Average SE					Relative bias of average SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
		.4	10	.0510	.0489	.0459	.0361	.0380	−.2394	−.2313	−.0010	−.0477	−.2681	
			20	.0670	.0644	.0602	.0470	.0501	−.1054	−.1284	−.0446	.0184	−.1095	
			40	.0847	.0816	.0761	.0598	.0634	−.0549	−.0659	−.0224	−.0185	−.0546	
		.8	10	.0706	.0679	.0421	.0322	.0291	−.0730	−.1248	−.0681	−.0305	−.4091	
			20	.0937	.0900	.0561	.0430	.0387	−.0659	−.0525	−.0682	−.0300	−.2539	
			40	.1216	.1169	.0726	.0556	.0503	.0241	−.0138	−.0370	.0278	−.1073	
200	1	.2	10	.0716	.0688	.0659	.0527	.0546	.0331	.0037	.0566	.0118	.0218	0
			20	.0804	.0772	.0741	.0592	.0614	−.0011	−.0127	−.0036	−.0036	−.0104	
			40	.0862	.0832	.0797	.0636	.0658	−.0010	.0321	.0363	.0097	.0120	
		.4	10	.0697	.0671	.0627	.0491	.0510	.0124	.0198	−.0078	−.0325	−.0018	
			20	.0788	.0760	.0712	.0554	.0578	.0155	−.0308	−.0246	.0032	.0161	
			40	.0851	.0816	.0765	.0596	.0624	−.0441	−.0012	−.0078	−.0049	−.0426	
		.8	10	.1051	.1010	.0625	.0477	.0421	−.0158	−.0107	.0391	.0148	−.0013	
			20	.1195	.1148	.0711	.0543	.0478	−.0042	.0160	.0126	.0143	−.0372	
			40	.1293	.1244	.0772	.0589	.0520	−.0023	−.0366	−.0333	.0169	.0071	
	6	.2	10	.0492	.0472	.0453	.0362	.0375	−.0291	−.0286	.0190	.0341	−.0127	2
			20	.0612	.0589	.0564	.0451	.0467	−.0203	−.0042	−.0306	−.0020	−.0097	
			40	.0726	.0698	.0670	.0534	.0553	−.0121	−.0256	.0233	−.0241	.0327	
		.4	10	.0467	.0450	.0421	.0329	.0342	−.0632	−.0851	−.0181	−.0292	−.1147	
			20	.0591	.0567	.0531	.0415	.0431	−.0036	.0190	−.0060	−.0272	.0251	
			40	.0707	.0681	.0638	.0499	.0520	.0037	.0321	.0188	.0305	.0061	

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Table 7. Continued

n	σ_x^2	r	σ_ε^2	Average SE					Relative bias of average SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
		.8	10	.0665	.0639	.0397	.0304	.0270	−.0297	−.0514	−.0104	−.0190	−.1162	
			20	.0854	.0822	.0510	.0389	.0343	−.0359	−.0047	−.0162	−.0129	−.0778	
			40	.1032	.0992	.0616	.0470	.0415	.0105	.0332	.0123	−.0051	−.0483	
	12	.2	10	.0371	.0357	.0342	.0274	.0284	−.2737	−.2346	−.0364	.0132	−.2529	15
			20	.0489	.0469	.0450	.0360	.0372	−.1112	−.1409	−.0363	.0196	−.1079	
			40	.0614	.0588	.0563	.0452	.0467	−.0446	−.0463	−.0143	−.0013	−.0415	
		.4	10	.0352	.0339	.0317	.0248	.0258	−.2223	−.2739	−.0636	−.0334	−.3018	
			20	.0465	.0447	.0419	.0326	.0342	−.1198	−.1403	−.0581	−.0083	−.1406	
			40	.0589	.0564	.0529	.0414	.0432	−.0321	−.0735	−.0209	−.0470	−.0189	
		.8	10	.0492	.0472	.0292	.0224	.0198	−.0778	−.0833	−.0520	.0045	−.3925	
			20	.0654	.0628	.0390	.0298	.0264	−.0422	−.0633	−.0068	−.0276	−.2494	
			40	.0839	.0807	.0500	.0382	.0337	−.0085	−.0196	.0123	.0174	−.1019	
300	1	.2	10	.0581	.0559	.0536	.0427	.0442	.0397	.0304	.0133	.0196	−.0179	0
			20	.0654	.0628	.0603	.0481	.0496	.0163	.0489	.0442	.0223	−.0268	
			40	.0703	.0675	.0647	.0517	.0533	−.0086	.0576	.0343	−.0017	.0146	
		.4	10	.0568	.0547	.0513	.0401	.0412	.0152	−.0120	.0043	−.0126	.0301	
			20	.0642	.0617	.0578	.0451	.0466	.0272	−.0230	.0086	−.0396	.0021	
			40	.0690	.0665	.0623	.0486	.0503	−.0145	.0262	.0150	−.0329	−.0089	
		.8	10	.0853	.0821	.0508	.0388	.0341	−.0136	−.0131	−.0050	.0165	−.0082	
			20	.0971	.0932	.0578	.0441	.0388	.0272	.0220	.0016	.0244	−.0170	
			40	.1051	.1010	.0627	.0479	.0422	−.0075	.0027	.0079	.0544	.0389	

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Table 7. Continued

n	σ_x^2	r	σ_ϵ^2	Average SE					Relative bias of average SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
	6	.2	10	.0398	.0383	.0367	.0293	.0303	−.0948	−.0858	−.0332	.0336	−.0824	1
			20	.0499	.0480	.0460	.0368	.0379	−.0363	.0054	−.0256	−.0220	−.0105	
			40	.0591	.0566	.0544	.0434	.0447	−.0124	−.0080	−.0060	−.0186	.0049	
		.4	10	.0378	.0364	.0341	.0266	.0275	−.0837	−.0590	−.0386	−.0155	−.0449	
			20	.0480	.0461	.0433	.0337	.0348	.0057	−.0699	−.0534	−.0178	−.0227	
			40	.0572	.0550	.0515	.0402	.0415	.0430	−.0023	−.0212	−.0032	.0281	
		.8	10	.0542	.0521	.0323	.0247	.0217	−.0192	−.0140	−.0304	−.0196	−.1251	
			20	.0692	.0666	.0414	.0316	.0277	.0032	−.0000	−.0035	−.0257	−.0381	
			40	.0843	.0808	.0501	.0383	.0336	−.0282	.0030	−.0096	−.0165	−.0220	
	12	.2	10	.0302	.0289	.0277	.0222	.0228	−.2367	−.2556	−.0238	−.0243	−.2576	15
			20	.0397	.0382	.0367	.0293	.0303	−.1227	−.1500	−.0041	−.0155	−.0923	
			40	.0497	.0477	.0457	.0366	.0376	−.0385	−.0451	.0153	.0249	−.0134	
		.4	10	.0285	.0274	.0256	.0200	.0207	−.2602	−.2576	−.0743	−.0206	−.2910	
			20	.0376	.0363	.0340	.0265	.0275	−.1577	−.1518	−.0069	−.0549	−.1657	
			40	.0475	.0457	.0428	.0333	.0345	−.0693	−.0179	−.0069	−.0129	−.0385	
		.8	10	.0397	.0382	.0237	.0181	.0159	−.1120	−.1096	−.0721	−.0140	−.3888	
			20	.0528	.0507	.0314	.0241	.0210	−.0567	−.0499	−.0428	.0273	−.2597	
			40	.0681	.0654	.0405	.0309	.0272	−.0073	.0043	−.0013	−.0005	−.0849	

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Table 8. The average standard error estimate from our analytic approach and its relative bias in the first simulation study

n	σ_x^2	r	σ_ε^2	Average SE					Relative bias of average SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
100	1	.2	10	.0969	.0922	.0884	.0714	.0717	−.0484	−.0745	.0063	−.0535	−.1489	6
			20	.1085	.1038	.1004	.0804	.0808	−.0627	−.0312	−.0096	−.0421	−.0663	
			40	.1170	.1122	.1072	.0860	.0871	−.0277	−.0743	−.0759	−.0534	−.1106	
		.4	10	.0938	.0901	.0846	.0663	.0669	−.0829	−.0758	−.0761	−.1009	−.1444	
			20	.1063	.1024	.0959	.0748	.0764	−.0503	−.0350	−.0535	−.0483	−.0983	
			40	.1153	.1105	.1036	.0817	.0810	−.0321	−.0328	−.0648	−.0027	−.1311	
		.8	10	.1430	.1372	.0845	.0653	.0554	−.0492	−.0606	−.0725	−.0704	−.0872	
			20	.1616	.1554	.0964	.0735	.0630	−.0597	−.0659	−.0399	−.0816	−.0995	
			40	.1766	.1689	.1042	.0803	.0688	−.0495	−.0511	−.0282	−.0544	−.1178	
	6	.2	10	.0695	.0669	.0626	.0493	.0518	−.1011	−.0680	−.0602	−.0462	−.1467	8
			20	.0839	.0803	.0778	.0621	.0613	−.0515	−.0852	−.0533	−.0084	−.1100	
			40	.0968	.0932	.0912	.0725	.0714	−.0694	−.0863	−.0363	−.0356	−.0755	
		.4	10	.0669	.0643	.0589	.0456	.0484	−.0532	−.0765	−.0576	−.0714	−.0938	
			20	.0801	.0765	.0728	.0570	.0580	−.0879	−.0850	−.0202	−.0490	−.1526	
			40	.0954	.0908	.0861	.0678	.0678	−.1014	−.0567	−.0582	−.0777	−.1128	
		.8	10	.0932	.0897	.0561	.0422	.0407	−.0494	−.0219	−.0778	−.0516	−.0876	
			20	.1173	.1129	.0706	.0537	.0463	−.0045	−.0119	−.0274	−.0227	−.1467	
			40	.1395	.1339	.0833	.0642	.0543	−.0361	−.0264	−.0530	−.0402	−.1362	
	12	.2	10	.0604	.0588	.0485	.0382	.0523	−.1514	−.1493	−.0598	−.0754	−.0317	10
			20	.0718	.0695	.0632	.0500	.0567	−.0914	−.1027	−.0293	−.0464	−.0844	
			40	.0840	.0800	.0768	.0622	.0618	−.0607	−.0976	−.0698	−.0495	−.1336	

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Table 8. Continued

n	σ_x^2	r	σ_ε^2	Average SE					Relative bias of average SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
		.4	10	.0586	.0565	.0454	.0350	.0519	−.1273	−.1118	−.0122	−.0772	−.0006	
			20	.0679	.0656	.0581	.0451	.0540	−.0929	−.1130	−.0779	−.0242	−.0405	
			40	.0809	.0775	.0727	.0568	.0590	−.0969	−.1131	−.0664	−.0679	−.1205	
		.8	10	.0736	.0709	.0421	.0311	.0476	−.0342	−.0858	−.0670	−.0630	−.0328	
			20	.0921	.0886	.0549	.0414	.0472	−.0814	−.0672	−.0868	−.0669	−.0892	
			40	.1156	.1111	.0695	.0532	.0493	−.0261	−.0627	−.0782	−.0164	−.1260	
200	1	.2	10	.0687	.0660	.0635	.0509	.0516	−.0087	−.0371	.0169	−.0225	−.0327	1
			20	.0773	.0743	.0713	.0574	.0579	−.0397	−.0500	−.0404	−.0331	−.0671	
			40	.0829	.0804	.0770	.0616	.0620	−.0390	−.0032	.0017	−.0221	−.0450	
		.4	10	.0671	.0645	.0606	.0475	.0483	−.0257	−.0199	−.0419	−.0642	−.0547	
			20	.0756	.0731	.0690	.0536	.0542	−.0257	−.0674	−.0552	−.0301	−.0471	
			40	.0820	.0791	.0739	.0577	.0589	−.0788	−.0326	−.0415	−.0362	−.0962	
		.8	10	.1022	.0980	.0605	.0461	.0395	−.0431	−.0399	.0053	−.0195	−.0633	
			20	.1165	.1116	.0687	.0527	.0445	−.0288	−.0126	−.0210	−.0158	−.1041	
			40	.1259	.1207	.0746	.0571	.0490	−.0290	−.0648	−.0654	−.0144	−.0520	
	6	.2	10	.0497	.0478	.0444	.0354	.0368	−.0198	−.0164	.0007	.0129	−.0308	3
			20	.0592	.0570	.0551	.0439	.0434	−.0530	−.0364	−.0523	−.0288	−.0797	
			40	.0696	.0668	.0652	.0518	.0512	−.0525	−.0668	−.0039	−.0524	−.0425	
		.4	10	.0474	.0458	.0420	.0323	.0345	−.0494	−.0675	−.0221	−.0480	−.1087	
			20	.0572	.0549	.0519	.0405	.0408	−.0343	−.0133	−.0287	−.0504	−.0300	
			40	.0679	.0649	.0622	.0485	.0482	−.0358	−.0154	−.0082	.0010	−.0683	

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Table 8. Continued

n	σ_x^2	r	σ_ε^2	Average SE					Relative bias of average SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
		.8	10	.0660	.0635	.0398	.0299	.0290	−.0379	−.0574	−.0067	−.0337	−.0494	
			20	.0833	.0800	.0500	.0380	.0330	−.0597	−.0309	−.0344	−.0365	−.1134	
			40	.0999	.0960	.0599	.0457	.0384	−.0217	.0000	−.0156	−.0345	−.1192	
	12	.2	10	.0436	.0423	.0344	.0270	.0381	−.1470	−.0913	−.0306	.0002	.0028	3
			20	.0506	.0488	.0444	.0351	.0400	−.0807	−.1068	−.0484	−.0055	−.0387	
			40	.0600	.0576	.0551	.0442	.0444	−.0665	−.0661	−.0358	−.0232	−.0877	
		.4	10	.0421	.0408	.0322	.0246	.0374	−.0721	−.1243	−.0490	−.0420	.0137	
			20	.0486	.0471	.0415	.0319	.0385	−.0802	−.0950	−.0682	−.0296	−.0311	
			40	.0576	.0554	.0517	.0403	.0415	−.0538	−.0911	−.0422	−.0730	−.0575	
		.8	10	.0531	.0511	.0299	.0223	.0345	−.0060	−.0066	−.0304	−.0021	.0601	
			20	.0660	.0638	.0389	.0293	.0341	−.0327	−.0488	−.0099	−.0438	−.0315	
			40	.0819	.0790	.0491	.0372	.0350	−.0322	−.0410	−.0066	−.0087	−.0675	
300	1	.2	10	.0564	.0541	.0521	.0416	.0426	.0093	−.0028	−.0148	−.0076	−.0525	0
			20	.0637	.0611	.0587	.0471	.0476	−.0095	.0204	.0165	.0001	−.0656	
			40	.0685	.0655	.0631	.0506	.0511	−.0328	.0271	.0087	−.0228	−.0276	
		.4	10	.0552	.0531	.0498	.0390	.0396	−.0135	−.0398	−.0239	−.0392	−.0091	
			20	.0623	.0599	.0560	.0441	.0446	−.0025	−.0518	−.0224	−.0615	−.0412	
			40	.0674	.0650	.0610	.0476	.0481	−.0379	.0024	−.0061	−.0524	−.0528	
		.8	10	.0832	.0801	.0495	.0378	.0325	−.0380	−.0367	−.0315	−.0097	−.0547	
			20	.0949	.0911	.0563	.0431	.0370	.0038	−.0012	−.0230	−.0008	−.0631	
			40	.1027	.0988	.0612	.0469	.0407	−.0298	−.0192	−.0161	.0308	.0011	

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Table 8. Continued

n	σ_x^2	r	σ_ϵ^2	Average SE					Relative bias of average SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
	6	.2	10	.0406	.0392	.0366	.0289	.0303	−.0761	−.0648	−.0375	.0185	−.0809	0
			20	.0488	.0469	.0453	.0362	.0361	−.0586	−.0179	−.0407	−.0382	−.0564	
			40	.0572	.0546	.0533	.0427	.0420	−.0445	−.0445	−.0253	−.0348	−.0568	
		.4	10	.0389	.0374	.0343	.0263	.0281	−.0573	−.0306	−.0326	−.0265	−.0239	
			20	.0468	.0453	.0428	.0330	.0335	−.0176	−.0875	−.0648	−.0373	−.0609	
			40	.0554	.0529	.0507	.0393	.0390	.0087	−.0389	−.0362	−.0248	−.0340	
		.8	10	.0545	.0523	.0326	.0246	.0236	−.0138	−.0095	−.0208	−.0229	−.0471	
			20	.0678	.0652	.0410	.0310	.0272	−.0165	−.0202	−.0133	−.0417	−.0558	
			40	.0822	.0786	.0492	.0374	.0317	−.0521	−.0234	−.0264	−.0389	−.0784	
	12	.2	10	.0360	.0351	.0282	.0221	.0316	−.0886	−.0985	−.0072	−.0281	.0288	4
			20	.0415	.0401	.0365	.0289	.0331	−.0819	−.1067	−.0100	−.0292	−.0097	
			40	.0489	.0469	.0451	.0360	.0362	−.0534	−.0610	.0008	.0073	−.0496	
		.4	10	.0345	.0335	.0262	.0200	.0308	−.1033	−.0910	−.0528	−.0231	.0540	
			20	.0399	.0385	.0339	.0262	.0316	−.1080	−.0988	−.0110	−.0654	−.0413	
			40	.0470	.0453	.0422	.0328	.0339	−.0803	−.0269	−.0196	−.0286	−.0546	
		.8	10	.0432	.0420	.0244	.0181	.0289	−.0323	−.0201	−.0446	−.0117	.1131	
			20	.0541	.0521	.0316	.0239	.0283	−.0335	−.0229	−.0370	.0189	−.0047	
			40	.0672	.0646	.0402	.0305	.0288	−.0205	−.0085	−.0096	−.0145	−.0326	

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Table 9. The average bootstrap standard error and its relative bias in the first simulation study

n	σ_x^2	r	σ_ε^2	Average bootstrap SE					Relative bias of average bootstrap SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
100	1	.2	10	.1026	.0976	.0927	.0760	.0798	.0079	−.0201	.0558	.0066	−.0523	0
			20	.1146	.1098	.1053	.0855	.0899	−.0095	.0252	.0385	.0179	.0393	
			40	.1236	.1185	.1123	.0913	.0969	.0274	−.0223	−.0319	.0044	−.0112	
		.4	10	.0992	.0953	.0888	.0706	.0753	−.0303	−.0226	−.0306	−.0434	−.0371	
			20	.1123	.1082	.1005	.0795	.0856	.0036	.0198	−.0079	.0115	.0106	
			40	.1218	.1167	.1085	.0867	.0909	.0227	.0219	−.0208	.0586	−.0257	
		.8	10	.1508	.1447	.0886	.0695	.0632	.0027	−.0089	−.0274	−.0114	.0412	
			20	.1704	.1640	.1010	.0782	.0716	−.0081	−.0138	.0060	−.0237	.0229	
			40	.1862	.1780	.1091	.0854	.0781	.0022	.0000	.0177	.0059	.0013	
	6	.2	10	.0760	.0733	.0659	.0527	.0582	−.0181	.0209	−.0105	.0193	−.0412	0
			20	.0909	.0870	.0817	.0663	.0700	.0271	−.0091	−.0061	.0591	.0160	
			40	.1040	.1002	.0957	.0772	.0813	.0000	−.0176	.0116	.0266	.0531	
		.4	10	.0724	.0695	.0620	.0488	.0540	.0255	−.0029	−.0080	−.0061	.0112	
			20	.0864	.0827	.0765	.0608	.0660	−.0159	−.0108	.0296	.0150	−.0351	
			40	.1022	.0974	.0904	.0722	.0774	−.0377	.0114	−.0120	−.0190	.0131	
		.8	10	.0989	.0950	.0593	.0453	.0460	.0092	.0360	−.0263	.0180	.0314	
			20	.1246	.1197	.0744	.0575	.0546	.0568	.0482	.0248	.0474	.0074	
			40	.1479	.1419	.0875	.0684	.0638	.0221	.0320	−.0046	.0224	.0143	
	12	.2	10	.0699	.0683	.0513	.0411	.0549	−.0169	−.0116	−.0058	−.0048	.0167	0
			20	.0811	.0791	.0667	.0537	.0628	.0266	.0220	.0246	.0229	.0145	
			40	.0934	.0892	.0809	.0664	.0713	.0447	.0056	−.0194	.0153	.0000	

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Table 9. Continued

n	σ_x^2	r	σ_ε^2	Average bootstrap SE					Relative bias of average bootstrap SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
		.4	10	.0656	.0637	.0481	.0377	.0524	−.0224	.0000	.0479	−.0053	.0077	
			20	.0759	.0735	.0615	.0485	.0588	.0134	−.0054	−.0254	.0498	.0463	
			40	.0894	.0857	.0766	.0608	.0676	−.0022	−.0195	−.0167	−.0016	.0075	
		.8	10	.0787	.0759	.0449	.0338	.0483	.0328	−.0219	−.0044	.0181	−.0183	
			20	.0989	.0950	.0584	.0446	.0513	−.0140	.0000	−.0299	.0068	−.0116	
			40	.1241	.1191	.0735	.0571	.0578	.0455	.0051	−.0252	.0555	.0248	
200	1	.2	10	.0706	.0678	.0648	.0524	.0543	.0173	−.0117	.0385	.0058	.0169	
			20	.0793	.0763	.0728	.0590	.0609	−.0149	−.0243	−.0202	−.0067	−.0177	
			40	.0850	.0824	.0786	.0633	.0652	−.0139	.0223	.0221	.0048	.0031	
		.4	10	.0688	.0662	.0619	.0489	.0511	−.0015	.0061	−.0206	−.0374	.0000	
			20	.0775	.0750	.0704	.0551	.0574	−.0013	−.0434	−.0356	−.0036	.0088	
			40	.0840	.0810	.0754	.0593	.0621	−.0562	−.0086	−.0220	−.0100	−.0461	
		.8	10	.1047	.1004	.0618	.0475	.0423	−.0197	−.0167	.0283	.0085	.0024	
			20	.1194	.1143	.0702	.0541	.0476	−.0050	.0115	.0000	.0112	−.0423	
			40	.1288	.1236	.0762	.0587	.0521	−.0062	−.0426	−.0463	.0138	.0077	
	6	.2	10	.0527	.0509	.0455	.0365	.0394	.0394	.0473	.0248	.0429	.0368	0
			20	.0623	.0600	.0564	.0453	.0468	−.0032	.0152	−.0309	.0022	−.0064	
			40	.0726	.0698	.0667	.0534	.0551	−.0122	−.0251	.0183	−.0238	.0299	
		.4	10	.0496	.0482	.0430	.0333	.0367	−.0040	−.0183	.0023	−.0177	−.0517	
			20	.0599	.0574	.0531	.0417	.0440	.0101	.0324	−.0056	−.0211	.0451	
			40	.0705	.0676	.0635	.0499	.0521	.0014	.0242	.0128	.0310	.0077	

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Table 9. Continued

n	σ_x^2	r	σ_ε^2	Average bootstrap SE					Relative bias of average bootstrap SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
		.8	10	.0678	.0652	.0409	.0309	.0309	−.0117	−.0312	.0200	−.0032	.0131	
			20	.0857	.0825	.0512	.0392	.0367	−.0327	−.0012	−.0116	−.0051	−.0134	
			40	.1029	.0989	.0613	.0471	.0428	.0068	.0302	.0082	−.0042	−.0183	
	12	.2	10	.0488	.0476	.0353	.0280	.0380	−.0450	.0215	−.0056	.0370	.0000	0
			20	.0556	.0539	.0455	.0363	.0422	.0109	−.0128	−.0257	.0283	.0144	
			40	.0646	.0623	.0564	.0456	.0482	.0062	.0114	−.0140	.0066	−.0103	
		.4	10	.0457	.0444	.0331	.0255	.0359	.0088	−.0472	−.0236	−.0078	−.0271	
			20	.0526	.0510	.0426	.0330	.0398	−.0038	−.0192	−.0427	.0030	.0025	
			40	.0616	.0595	.0530	.0416	.0452	.0115	−.0230	−.0185	−.0415	.0273	
		.8	10	.0547	.0527	.0308	.0231	.0334	.0243	.0233	−.0032	.0359	.0277	
			20	.0685	.0662	.0400	.0304	.0349	.0044	−.0134	.0178	−.0065	−.0085	
			40	.0850	.0821	.0504	.0384	.0384	.0035	−.0024	.0202	.0240	.0240	
300	1	.2	10	.0575	.0550	.0528	.0423	.0441	.0286	.0148	−.0019	.0095	−.0200	0
			20	.0648	.0622	.0594	.0479	.0492	.0078	.0384	.0295	.0170	−.0334	
			40	.0696	.0666	.0639	.0514	.0528	−.0183	.0439	.0224	−.0077	.0038	
		.4	10	.0562	.0541	.0505	.0397	.0412	.0036	−.0217	−.0098	−.0222	.0300	
			20	.0634	.0609	.0567	.0448	.0462	.0144	−.0364	−.0105	−.0468	−.0065	
			40	.0685	.0660	.0618	.0484	.0499	−.0228	.0185	.0065	−.0359	−.0177	
		.8	10	.0845	.0813	.0501	.0385	.0340	−.0231	−.0228	−.0196	.0079	−.0116	
			20	.0963	.0925	.0571	.0438	.0386	.0190	.0143	−.0104	.0162	−.0228	
			40	.1042	.1002	.0620	.0477	.0423	−.0161	−.0050	−.0048	.0484	.0419	

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Table 9. Continued

n	σ_x^2	r	σ_e^2	Average bootstrap SE					Relative bias of average bootstrap SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
	6	.2	10	.0428	.0413	.0371	.0294	.0317	−.0273	−.0143	−.0237	.0389	−.0394	0
			20	.0508	.0489	.0459	.0368	.0382	−.0193	.0252	−.0275	−.0213	−.0026	
			40	.0590	.0564	.0541	.0434	.0444	−.0134	−.0123	−.0110	−.0181	−.0022	
		.4	10	.0405	.0390	.0348	.0268	.0292	−.0194	.0104	−.0197	−.0074	.0139	
			20	.0486	.0469	.0434	.0337	.0356	.0189	−.0544	−.0503	−.0175	.0000	
			40	.0570	.0547	.0515	.0401	.0415	.0383	−.0073	−.0209	−.0050	.0272	
		.8	10	.0555	.0532	.0331	.0251	.0248	.0036	.0076	−.0060	−.0040	.0000	
			20	.0693	.0666	.0416	.0317	.0294	.0043	.0000	.0024	−.0216	.0208	
			40	.0839	.0803	.0499	.0381	.0344	−.0323	−.0025	−.0138	−.0206	.0000	
	12	.2	10	.0399	.0388	.0287	.0226	.0309	.0101	−.0026	.0106	−.0088	.0065	0
			20	.0453	.0440	.0370	.0295	.0342	.0022	−.0200	.0054	−.0067	.0240	
			40	.0524	.0504	.0458	.0367	.0389	.0135	.0080	.0155	.0280	.0210	
		.4	10	.0371	.0361	.0267	.0204	.0292	−.0364	−.0217	−.0361	.0000	.0000	
			20	.0427	.0415	.0344	.0268	.0319	−.0447	−.0304	.0058	−.0429	−.0304	
			40	.0498	.0481	.0429	.0335	.0361	−.0254	.0344	−.0046	−.0089	.0056	
		.8	10	.0441	.0428	.0249	.0186	.0271	−.0134	−.0023	−.0273	.0164	.0423	
			20	.0556	.0535	.0322	.0245	.0283	−.0071	.0019	−.0183	.0470	−.0035	
			40	.0691	.0665	.0408	.0312	.0309	.0073	.0215	.0074	.0097	.0404	

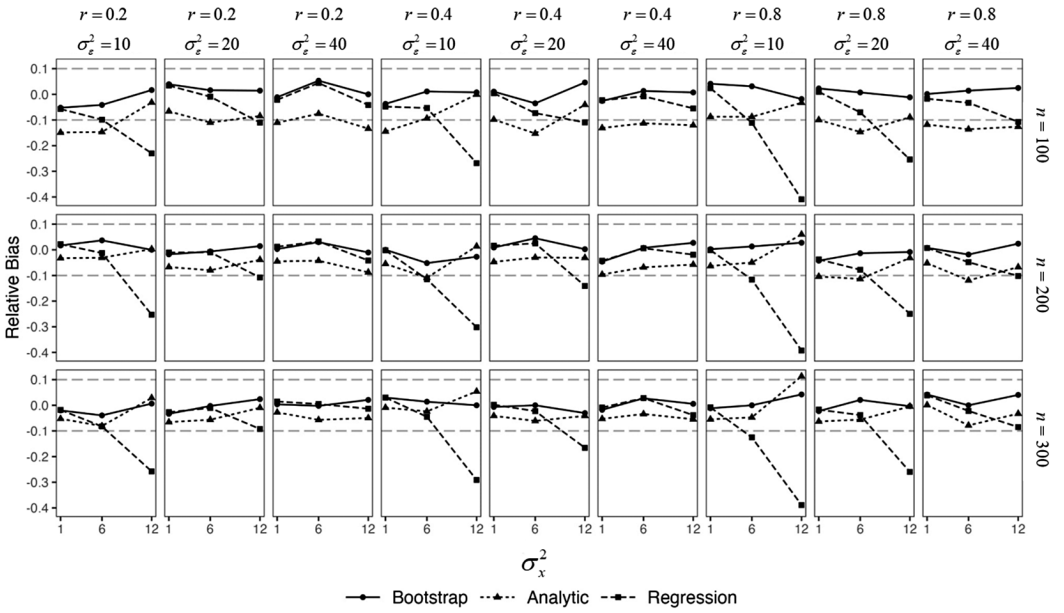


Figure 1. Relative bias in the first simulation study.

error estimates from regular regression are acceptable at $\sigma_x^2 = 1$, more than 95% from regular regression are acceptable at $\sigma_x^2 = 6$ [because $4/(5 \times 3 \times 3 \times 3) = 2.96\%$ are unacceptable], and between 65% and 70% from regular regression are acceptable at $\sigma_x^2 = 12$ [because $45/(5 \times 3 \times 3 \times 3) = 33.33\%$ are unacceptable]. Third, between 80% and 85% of the standard error estimates from our analytic approach are acceptable at $n = 100$ [because $24/(5 \times 3 \times 3 \times 3) = 17.78\%$ are unacceptable], about 95% from our analytic approach are acceptable at $n = 200$ [because $7/(5 \times 3 \times 3 \times 3) = 5.19\%$ are unacceptable], and more than 95% from our analytic approach are acceptable at $n = 300$ [because $4/(5 \times 3 \times 3 \times 3) = 2.96\%$ are unacceptable]. Finally, all bootstrap standard errors are acceptable in all conditions. Remember that all these conclusions are drawn from the specific regression model, the multivariate normal distributions of x_{i1}, x_{i2}, x_{i3}, x_{i4}, and ε_i, and the particular levels of n, σ_x^2, r, and σ_ε^2 used in this first simulation study.

4.3. Second simulation study: Data generation, analysis, and evaluation criteria

In the second simulation study, we use the procedure developed by Qu et al. (2020) to generate the data of x_{i1}, x_{i2}, x_{i3}, x_{i4}, and ε_i from multivariate non-normal distributions. This procedure is implemented by the *mnonr* package (Qu & Zhang, 2020) in R, which requires the user to specify the covariance matrix of the variables and the population values of multivariate skewness and multivariate kurtosis. The covariance matrix $\boldsymbol{\Sigma}$ of x_{i1}, x_{i2}, x_{i3}, x_{i4}, and ε_i used in the second simulation study is the same as the one used in the first simulation study, and the factors manipulated are also the same (i.e., $n = 100$, 200, 300; $\sigma_x^2 = 1$, 6, 12; $r = .2$, .4, .8; and $\sigma_\varepsilon^2 = 10$, 20, 40). The population values of multivariate skewness and multivariate kurtosis are 3 and 35, respectively. For each dataset generated from the *mnonr* package in R, we follow the same steps as in the first simulation study to prepare the dataset for analysis.

To approximate the true values of the unstandardized regression coefficients of z_{i1}, z_{i2}, z_{i3}, x_{i4}, and ω_{i1} at each combination of σ_x^2, r, and σ_ε^2, we generate a random dataset of size 1,000,000 and regress z_{iy} on z_{i1}, z_{i2}, z_{i3}, x_{i4}, and ω_{i1}. Table 10 shows the approximate true values of regression coefficients and R^2 at each combination of σ_x^2, r, and σ_ε^2. The values of R^2 range from .1570 to .9150, covering a wide range of scenarios in real applications.

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Table 10. Approximate true values of the unstandardized regression coefficients and R^2 at each combination of σ_x^2, r and σ_ε^2 in the second simulation study

σ_x^2	r	σ_ε^2	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	R^2
1	.2	10	.2393	.2398	.2373	.1200	.0718	.4263
		20	.1898	.1913	.1891	.0964	.0584	.2716
		40	.1468	.1454	.1449	.0714	.0434	.1570
	.4	10	.2354	.2357	.2342	.1170	.0707	.4462
		20	.1870	.1880	.1877	.0954	.0577	.2870
		40	.1458	.1456	.1418	.0717	.0424	.1684
	.8	10	.2279	.2251	.2247	.1140	.0678	.4853
		20	.1838	.1848	.1816	.0921	.0569	.3220
		40	.1461	.1403	.1405	.0711	.0425	.1919
6	.2	10	.3927	.3925	.1579	.0803	.2886	.7435
		20	.3501	.3496	.1419	.0718	.2585	.5925
		40	.2965	.2952	.1197	.0595	.2159	.4205
	.4	10	.3680	.3690	.1487	.0754	.2707	.7741
		20	.3329	.3333	.1334	.0677	.2437	.6320
		40	.2843	.2851	.1138	.0584	.2091	.4618
	.8	10	.3229	.3259	.1305	.0664	.2388	.8238
		20	.2906	.2887	.1177	.0575	.2127	.7217
		40	.2619	.2627	.1063	.0537	.1934	.5396
12	.2	10	.4145	.4150	.1183	.0595	.4311	.8570
		20	.3870	.3891	.1118	.0559	.4027	.7496
		40	.3459	.3461	.0987	.0507	.3605	.5982
	.4	10	.3794	.3784	.1081	.0548	.3943	.8806
		20	.3583	.3591	.1027	.0514	.3733	.7848
		40	.3270	.3265	.0933	.0472	.3379	.6463
	.8	10	.3202	.3187	.0918	.0461	.3316	.9150
		20	.3047	.3017	.0872	.0437	.3158	.8469
		40	.2851	.2843	.0813	.0416	.2954	.7299

4.4. Second simulation study: Results

In the second simulation study, the data of x_{i1}, x_{i2}, x_{i3}, x_{i4}, and ε_i are generated from multivariate non-normal distributions. Thus, at each combination of n, σ_x^2, r, and σ_ε^2, we first report the percentiles of the sample multivariate skewness and the sample multivariate kurtosis of x_{i1}, x_{i2}, x_{i3}, x_{i4}, and ε_i (see Multivariate_skew_kurt_ms3mk35.docx in the Supplementary Material), followed by the percentiles of the sample univariate skewness and the sample univariate kurtosis of ω_{i1} (see Univariate_skew_kurt_ms3mk35.docx in the Supplementary Material). The reported percentiles cover a wide range of values that are commonly encountered in practical data analysis (see Cain et al., 2017, Tables 1 and 2). Additionally, the percentiles of the sample univariate skewness and the

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sample univariate kurtosis in Univariate_skew_kurt_ms3mk35.docx are larger than those in Univariate_skew_kurt_MVN.docx, indicating more severe nonnormality of ω_{i1} in the second simulation study.

The relative bias of mean is included in Relative_bias_of_mean_ms3mk35.docx in the Supplementary Material, and the main results of the second simulation study are included in Tables 11–14. Specifically, Table 11 shows the mean and standard deviation of the unstandardized regression coefficients; Table 12 shows the average standard error estimate from regular regression, and the associated relative bias; Table 13 shows the average standard error estimate from our analytic approach and the associated relative bias; and Table 14 shows the average bootstrap standard error and the associated relative bias.

Regarding the relative bias of mean, 18 of a total of 405 absolute values of the relative bias of mean are larger than .05, of which 5 absolute values are associated with the unstandardized regression coefficient of ω_{i1}. Regarding the relative bias of average standard error estimate, dramatically different performances are observed among regular regression, our analytic approach, and the nonparametric bootstrap procedure. Based on the pattern in Table 12, we observe that the relative bias of average standard error estimate from regular regression is again mainly determined by σ_x^2. More specifically, there are 43 absolute values of relative bias of average standard error estimate larger than .1 at σ_x^2 = 12 (particularly, 12 within n = 100, 16 within n = 200, and 15 within n = 300; see the last column of Table 12). Based on the pattern in Table 13, we observe that the relative bias of average standard error estimate from our analytic approach is again mainly determined by n, and most values of relative bias are negative. More specifically, 15 absolute values of relative bias of average standard error estimate are larger than .1 at n = 100 (particularly, 4 within σ_x^2 = 1, 2 within σ_x^2 = 6, and 9 within σ_x^2 = 12; see the last column of Table 13), 9 absolute values of relative bias of average standard error estimate are larger than .1 at n = 200 (particularly, 9 within σ_x^2 = 12; see the last column of Table 13), and 7 absolute values of relative bias of average standard error estimate are larger than .1 at n = 300 (particularly, 7 within σ_x^2 = 12; see the last column of Table 13). Finally, 3 absolute values of the relative bias of average bootstrap standard error are larger than .1 (particularly, 1 at the combination of n = 200 and σ_x^2 = 12 and 2 at the combination of n = 300 and σ_x^2 = 12; see the last column of Table 14). To better demonstrate the different performances of the three methods, the values of relative bias of average standard error estimate for the unstandardized regression coefficient of ω_{i1} in Tables 12–14 are plotted in Figure 2.

Given these observations, we have the following conclusions. First, more than 95% of the point estimates of unstandardized regression coefficients are acceptable [because 18/405 = 4.44% are unacceptable]. Second, all standard error estimates from regular regression are acceptable at σ_x^2 = 1 and 6, and between 65% and 70% from regular regression are acceptable at σ_x^2 = 12 [because 43/(5 × 3 × 3 × 3) = 31.85% are unacceptable]. Third, between 85% and 90% of the standard error estimates from our analytic approach are acceptable at n = 100 [because 15/(5 × 3 × 3 × 3) = 11.11% are unacceptable], between 90% and 95% from our analytic approach are acceptable at n = 200 [because 9/(5 × 3 × 3 × 3) = 6.67% are unacceptable], and about 95% from our analytic approach are acceptable at n = 300 [because 7/(5 × 3 × 3 × 3) = 5.19% are unacceptable]. Finally, more than 99% bootstrap standard errors are acceptable [because 3/405 = .74% are unacceptable]. Remember that all these conclusions are drawn from the specific regression model, the particular non-normal distributions of x_{i1}, x_{i2}, x_{i3}, x_{i4}, and ε_i, and the particular levels of n, σ_x^2, r, and σ_ε^2 used in the second simulation study.

5. Discussions

In this article, we propose a simple analytic approach to regression that can produce the standard error estimates not only for standardized regression coefficients when the interaction terms are not involved but also for unstandardized regression coefficients when the interaction terms are involved. Our analytic approach integrates the covariance structure modeling method and the delta method, and several software packages designed for covariance structure analysis can be used to estimate the simple

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Table 11. Means and standard deviations of the unstandardized regression coefficients of the second simulation study

n	σ_x^2	r	σ_ε^2	Mean					Standard deviation				
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}
100	1	.2	10	.2344	.2375	.2364	.1234	.0681	.1000	.0951	.0953	.0747	.0785
			20	.1870	.1879	.1930	.0967	.0626	.1134	.1072	.1054	.0832	.0914
			40	.1440	.1386	.1447	.0784	.0413	.1267	.1156	.1130	.0897	.1008
		.4	10	.2403	.2354	.2332	.1143	.0675	.0991	.0911	.0853	.0678	.0805
			20	.1856	.1851	.1877	.0966	.0578	.1159	.1093	.1008	.0781	.0922
			40	.1398	.1436	.1400	.0723	.0445	.1239	.1187	.1041	.0885	.0966
		.8	10	.2311	.2229	.2228	.1118	.0711	.1529	.1403	.0884	.0673	.0740
			20	.1813	.1799	.1851	.0942	.0589	.1735	.1604	.0988	.0775	.0859
			40	.1353	.1453	.1360	.0729	.0409	.1952	.1757	.1066	.0836	.0934
	6	.2	10	.3983	.3920	.1586	.0808	.2886	.0739	.0713	.0642	.0521	.0553
			20	.3516	.3528	.1406	.0712	.2592	.0898	.0889	.0816	.0652	.0688
			40	.2940	.2914	.1218	.0638	.2109	.1037	.1009	.0950	.0770	.0830
		.4	10	.3699	.3743	.1541	.0759	.2703	.0713	.0653	.0617	.0475	.0525
			20	.3370	.3308	.1325	.0699	.2445	.0841	.0787	.0753	.0578	.0668
			40	.2857	.2903	.1149	.0527	.2102	.1013	.0944	.0893	.0701	.0754
		.8	10	.3242	.3279	.1345	.0681	.2385	.0931	.0877	.0538	.0408	.0435
			20	.3053	.2964	.1267	.0614	.2186	.1192	.1097	.0687	.0513	.0542
			40	.2600	.2676	.1009	.0537	.1944	.1484	.1355	.0837	.0623	.0696
	12	.2	10	.4181	.4173	.1217	.0618	.4342	.0674	.0664	.0515	.0383	.0535
			20	.3965	.3880	.1133	.0574	.4069	.0793	.0747	.0675	.0533	.0597
			40	.3495	.3475	.0984	.0509	.3593	.0897	.0890	.0818	.0652	.0698
		.4	10	.3837	.3855	.1114	.0550	.4042	.0594	.0583	.0457	.0330	.0471
			20	.3655	.3625	.1029	.0540	.3773	.0723	.0677	.0601	.0450	.0535
			40	.3262	.3269	.0970	.0485	.3414	.0863	.0810	.0717	.0578	.0637
		.8	10	.3276	.3274	.0938	.0490	.3406	.0724	.0652	.0398	.0283	.0362
			20	.3144	.3098	.0913	.0441	.3266	.0917	.0846	.0512	.0391	.0453
			40	.2900	.2939	.0802	.0413	.3007	.1163	.1072	.0700	.0510	.0568
200	1	.2	10	.2400	.2427	.2350	.1190	.0722	.0698	.0678	.0619	.0516	.0548
			20	.1867	.1865	.1915	.0993	.0607	.0775	.0758	.0730	.0582	.0606
			40	.1438	.1468	.1432	.0726	.0465	.0884	.0814	.0787	.0648	.0660
		.4	10	.2381	.2369	.2313	.1173	.0698	.0689	.0666	.0607	.0477	.0529
			20	.1944	.1900	.1834	.0935	.0586	.0799	.0767	.0693	.0538	.0635
			40	.1472	.1453	.1403	.0700	.0411	.0865	.0789	.0726	.0605	.0655
		.8	10	.2329	.2232	.2241	.1117	.0670	.1049	.0963	.0591	.0466	.0535
			20	.1780	.1872	.1846	.0966	.0545	.1247	.1108	.0716	.0538	.0592
			40	.1449	.1419	.1364	.0718	.0421	.1284	.1180	.0745	.0572	.0641

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Table 11. Continued

n	σ_x^2	r	σ_ε^2	Mean					Standard deviation				
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}
	6	.2	10	.3949	.3932	.1590	.0819	.2880	.0507	.0486	.0454	.0368	.0391
			20	.3525	.3511	.1431	.0712	.2557	.0607	.0565	.0538	.0448	.0444
			40	.2920	.2920	.1214	.0639	.2149	.0702	.0700	.0686	.0508	.0556
		.4	10	.3692	.3705	.1486	.0761	.2722	.0483	.0455	.0408	.0328	.0359
			20	.3300	.3364	.1363	.0694	.2436	.0594	.0539	.0512	.0409	.0438
			40	.2837	.2854	.1147	.0602	.2097	.0713	.0647	.0620	.0482	.0528
		.8	10	.3288	.3267	.1319	.0664	.2389	.0638	.0601	.0398	.0276	.0314
			20	.3039	.2988	.1200	.0603	.2197	.0837	.0756	.0458	.0365	.0424
			40	.2590	.2665	.1058	.0554	.1924	.0991	.0960	.0607	.0433	.0512
	12	.2	10	.4183	.4172	.1200	.0602	.4337	.0505	.0468	.0346	.0268	.0381
			20	.3902	.3911	.1119	.0561	.4031	.0547	.0515	.0460	.0344	.0419
			40	.3474	.3431	.1036	.0513	.3585	.0629	.0622	.0562	.0447	.0456
		.4	10	.3851	.3807	.1110	.0557	.3983	.0474	.0458	.0328	.0241	.0350
			20	.3625	.3608	.1070	.0520	.3757	.0490	.0494	.0416	.0310	.0375
			40	.3274	.3268	.0925	.0459	.3404	.0612	.0575	.0509	.0398	.0425
		.8	10	.3292	.3246	.0936	.0476	.3384	.0510	.0465	.0262	.0205	.0260
			20	.3095	.3130	.0899	.0448	.3244	.0669	.0589	.0344	.0265	.0334
			40	.2891	.2891	.0842	.0420	.2976	.0792	.0731	.0441	.0351	.0385
300	1	.2	10	.2433	.2361	.2366	.1197	.0711	.0556	.0546	.0517	.0412	.0436
			20	.1871	.1903	.1859	.0972	.0565	.0631	.0598	.0593	.0467	.0500
			40	.1451	.1418	.1460	.0741	.0416	.0724	.0678	.0646	.0554	.0541
		.4	10	.2360	.2361	.2334	.1172	.0694	.0593	.0534	.0492	.0389	.0413
			20	.1915	.1886	.1871	.0929	.0554	.0677	.0599	.0552	.0448	.0513
			40	.1398	.1436	.1438	.0743	.0462	.0697	.0648	.0632	.0479	.0531
		.8	10	.2211	.2263	.2281	.1151	.0703	.0880	.0821	.0510	.0379	.0420
			20	.1882	.1801	.1833	.0932	.0538	.0990	.0901	.0575	.0422	.0477
			40	.1422	.1420	.1440	.0698	.0425	.1089	.0997	.0615	.0462	.0502
	6	.2	10	.3937	.3945	.1608	.0810	.2894	.0431	.0402	.0368	.0298	.0331
			20	.3515	.3514	.1408	.0713	.2569	.0522	.0469	.0471	.0354	.0362
			40	.2925	.2978	.1175	.0607	.2186	.0579	.0545	.0528	.0422	.0452
		.4	10	.3699	.3691	.1499	.0753	.2715	.0401	.0375	.0330	.0264	.0283
			20	.3354	.3336	.1337	.0666	.2430	.0493	.0450	.0393	.0314	.0349
			40	.2895	.2864	.1112	.0565	.2094	.0567	.0536	.0502	.0382	.0433
		.8	10	.3243	.3289	.1328	.0680	.2386	.0514	.0476	.0303	.0230	.0255
			20	.3002	.2985	.1239	.0611	.2210	.0663	.0599	.0385	.0303	.0321
			40	.2633	.2616	.1069	.0543	.1931	.0832	.0771	.0458	.0357	.0385

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Table 11. Continued

n	σ_x^2	r	σ_ε^2	Mean					Standard deviation				
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}
	12	.2	10	.4157	.4166	.1196	.0597	.4329	.0383	.0377	.0285	.0213	.0307
			20	.3908	.3898	.1106	.0550	.4038	.0444	.0402	.0345	.0288	.0329
			40	.3457	.3453	.1013	.0508	.3627	.0505	.0490	.0445	.0345	.0373
		.4	10	.3832	.3823	.1097	.0560	.3966	.0377	.0354	.0257	.0195	.0277
			20	.3633	.3602	.1040	.0515	.3748	.0435	.0401	.0325	.0253	.0308
			40	.3274	.3289	.0930	.0473	.3406	.0489	.0461	.0424	.0322	.0358
		.8	10	.3242	.3244	.0922	.0468	.3371	.0448	.0414	.0218	.0165	.0263
			20	.3096	.3103	.0882	.0457	.3216	.0534	.0502	.0292	.0211	.0294
			40	.2916	.2862	.0829	.0420	.2977	.0663	.0638	.0384	.0292	.0318

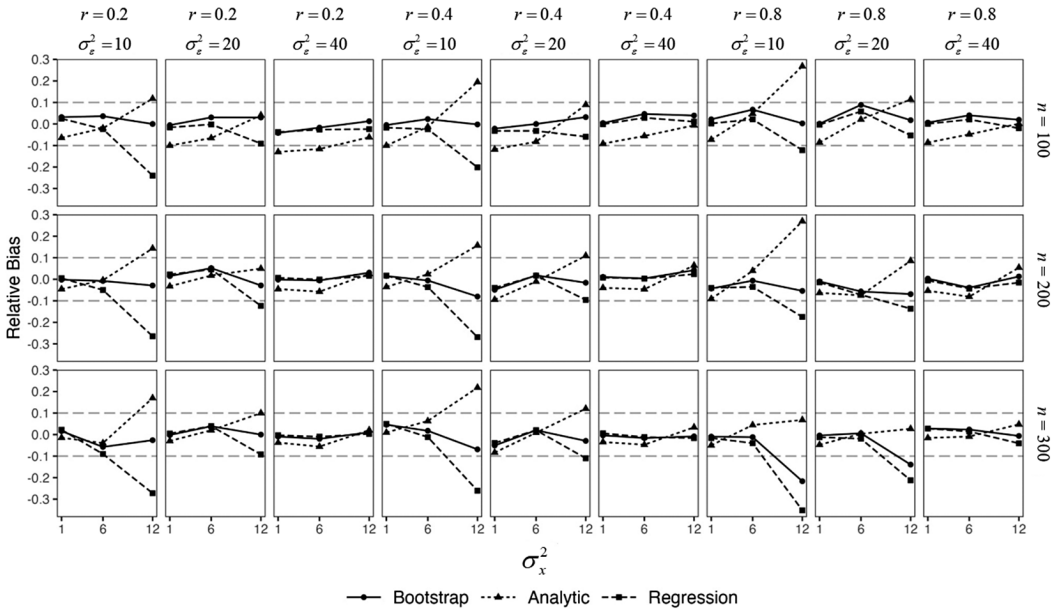


Figure 2. Relative bias in the second simulation study.

models defined in our analytic approach. Two numeric examples are used to compare our analytic approach with the existing methods. In the first example, our analytic approach produces the same standard error estimates as the existing methods. In the second example, our analytic approach produces the standard error estimates that are similar to the bootstrap standard errors. Finally, we conduct two simulation studies to further evaluate the performances of regular regression, our analytic approach, and the nonparametric bootstrap procedure at finite sample sizes. Under both normality and nonnormality, the performance of regular regression is good only at $\sigma_x^2 = 1$ but gets worse at larger values of σ_x^2; the performance of our analytic approach is mostly acceptable at $n = 100$ and gets better at larger sample sizes; and finally, the bootstrap standard errors are (almost) perfect in all conditions. Nevertheless, the implementation of the nonparametric bootstrap procedure is rather time-consuming, especially in simulation studies. For the two simulation studies in this article, it took about 16 seconds for the *stdmod* package to complete 1 replication across all 81 combinations n, σ_x^2, r, and σ_ε^2 (equivalently, about

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Table 12. The average standard error estimate from regular regression and its relative bias in the second simulation study

n	σ_x^2	r	σ_ε^2	Average SE					Relative bias of average SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
100	1	.2	10	.1028	.0987	.0944	.0756	.0805	.0281	.0376	−.0099	.0119	.0250	0
			20	.1153	.1107	.1059	.0850	.0900	.0164	.0323	.0050	.0221	−.0161	
			40	.1244	.1192	.1139	.0914	.0970	−.0180	.0315	.0075	.0190	−.0377	
		.4	10	.1017	.0956	.0894	.0701	.0791	.0259	.0499	.0471	.0342	−.0169	
			20	.1150	.1085	.1017	.0794	.0893	−.0076	−.0070	.0085	.0176	−.0320	
			40	.1247	.1169	.1095	.0858	.0965	.0059	−.0150	.0516	−.0310	−.0011	
		.8	10	.1534	.1413	.0875	.0671	.0741	.0037	.0072	−.0107	−.0026	.0024	
			20	.1768	.1629	.1007	.0769	.0856	.0191	.0156	.0196	−.0078	−.0031	
			40	.1928	.1774	.1098	.0842	.0935	−.0120	.0093	.0304	.0074	.0006	
	6	.2	10	.0690	.0661	.0632	.0508	.0540	−.0662	−.0733	−.0148	−.0245	−.0237	0
			20	.0876	.0838	.0803	.0643	.0687	−.0244	−.0575	−.0163	−.0148	−.0020	
			40	.1031	.0989	.0949	.0763	.0809	−.0056	−.0196	−.0017	−.0091	−.0259	
		.4	10	.0655	.0618	.0578	.0454	.0512	−.0818	−.0537	−.0638	−.0447	−.0243	
			20	.0836	.0783	.0735	.0576	.0646	−.0065	−.0048	−.0236	−.0036	−.0322	
			40	.1000	.0947	.0884	.0690	.0775	−.0126	.0029	−.0096	−.0155	.0288	
		.8	10	.0911	.0838	.0519	.0398	.0444	−.0213	−.0448	−.0350	−.0258	.0213	
			20	.1174	.1083	.0673	.0516	.0573	−.0151	−.0127	−.0204	.0053	.0583	
			40	.1458	.1344	.0833	.0636	.0710	−.0175	−.0081	−.0051	.0217	.0209	
	12	.2	10	.0519	.0494	.0474	.0378	.0407	−.2300	−.2563	−.0789	−.0119	−.2399	12
			20	.0688	.0657	.0631	.0506	.0543	−.1329	−.1203	−.0656	−.0507	−.0909	
			40	.0867	.0830	.0795	.0640	.0682	−.0337	−.0680	−.0287	−.0184	−.0238	

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Table 12. Continued

n	σ_x^2	r	σ_ε^2	Average SE					Relative bias of average SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
		.4	10	.0483	.0455	.0426	.0333	.0376	−.1876	−.2196	−.0670	.0069	−.2016	
			20	.0644	.0607	.0568	.0445	.0503	−.1098	−.1033	−.0549	−.0115	−.0591	
			40	.0822	.0777	.0726	.0567	.0644	−.0471	−.0414	.0123	−.0203	.0109	
		.8	10	.0646	.0594	.0370	.0285	.0318	−.1079	−.0882	−.0705	.0049	−.1216	
			20	.0875	.0804	.0500	.0385	.0428	−.0456	−.0495	−.0250	−.0141	−.0532	
			40	.1139	.1048	.0650	.0499	.0557	−.0205	−.0227	−.0714	−.0218	−.0198	
200	1	.2	10	.0715	.0683	.0655	.0524	.0551	.0250	.0069	.0589	.0154	.0050	0
			20	.0806	.0770	.0738	.0591	.0620	.0399	.0148	.0103	.0160	.0223	
			40	.0866	.0828	.0795	.0631	.0665	−.0202	.0172	.0096	−.0256	.0068	
		.4	10	.0708	.0664	.0622	.0487	.0538	.0279	−.0030	.0234	.0216	.0169	
			20	.0802	.0753	.0706	.0550	.0609	.0036	−.0182	.0184	.0217	−.0399	
			40	.0871	.0814	.0762	.0595	.0660	.0064	.0326	.0504	−.0165	.0076	
		.8	10	.1070	.0984	.0610	.0467	.0513	.0203	.0215	.0317	.0014	−.0402	
			20	.1222	.1128	.0699	.0534	.0584	−.0201	.0181	−.0229	−.0074	−.0143	
			40	.1340	.1235	.0764	.0584	.0638	.0442	.0460	.0253	.0199	−.0056	
	6	.2	10	.0481	.0459	.0441	.0352	.0371	−.0524	−.0560	−.0303	−.0419	−.0498	0
			20	.0604	.0577	.0552	.0442	.0464	−.0053	.0220	.0254	−.0130	.0459	
			40	.0720	.0690	.0660	.0529	.0555	.0249	−.0156	−.0374	.0427	−.0012	
		.4	10	.0454	.0427	.0400	.0313	.0346	−.0592	−.0628	−.0178	−.0482	−.0363	
			20	.0582	.0546	.0512	.0400	.0445	−.0211	.0130	−.0003	−.0210	.0170	
			40	.0700	.0656	.0615	.0480	.0530	−.0177	.0133	−.0089	−.0029	.0033	

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Table 12. Continued

n	σ_x^2	r	σ_ε^2	Average SE					Relative bias of average SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
		.8	10	.0629	.0580	.0360	.0276	.0303	−.0146	−.0347	−.0954	.0002	−.0354	
			20	.0819	.0753	.0467	.0357	.0393	−.0223	−.0041	.0195	−.0228	−.0721	
			40	.1012	.0933	.0579	.0444	.0490	.0213	−.0284	−.0451	.0256	−.0433	
	12	.2	10	.0361	.0346	.0331	.0264	.0280	−.2860	−.2608	−.0417	−.0156	−.2648	16
			20	.0477	.0454	.0436	.0349	.0367	−.1276	−.1172	−.0523	.0142	−.1235	
			40	.0601	.0575	.0551	.0441	.0463	−.0454	−.0755	−.0197	−.0140	.0160	
		.4	10	.0335	.0314	.0295	.0230	.0256	−.2931	−.3137	−.1001	−.0445	−.2690	
			20	.0445	.0417	.0391	.0305	.0339	−.0910	−.1545	−.0609	−.0157	−.0958	
			40	.0571	.0535	.0502	.0391	.0435	−.0669	−.0691	−.0156	−.0156	.0240	
		.8	10	.0445	.0409	.0254	.0195	.0215	−.1272	−.1200	−.0306	−.0504	−.1749	
			20	.0599	.0553	.0343	.0262	.0288	−.1038	−.0611	−.0020	−.0110	−.1366	
			40	.0788	.0726	.0451	.0346	.0379	−.0054	−.0075	.0214	−.0120	−.0153	
300	1	.2	10	.0582	.0556	.0532	.0426	.0446	.0478	.0186	.0300	.0329	.0224	0
			20	.0657	.0627	.0601	.0480	.0502	.0426	.0488	.0147	.0279	.0050	
			40	.0705	.0674	.0646	.0515	.0540	−.0253	−.0061	.0002	−.0712	−.0030	
		.4	10	.0576	.0540	.0506	.0395	.0433	−.0281	.0108	.0281	.0145	.0497	
			20	.0653	.0612	.0574	.0447	.0493	−.0361	.0219	.0402	−.0002	−.0392	
			40	.0706	.0663	.0621	.0485	.0534	.0137	.0234	−.0173	.0112	.0057	
		.8	10	.0867	.0801	.0497	.0379	.0413	−.0142	−.0242	−.0255	−.0008	−.0161	
			20	.0995	.0919	.0569	.0435	.0471	.0058	.0200	−.0116	.0298	−.0121	
			40	.1084	.1001	.0620	.0474	.0516	−.0041	.0042	.0087	.0258	.0278	

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Table 12. Continued

n	σ_x^2	r	σ_ε^2	Average SE					Relative bias of average SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
	6	.2	10	.0391	.0374	.0358	.0288	.0301	−.0929	−.0692	−.0256	−.0336	−.0901	0
			20	.0491	.0470	.0450	.0359	.0376	−.0598	.0037	−.0451	.0160	.0395	
			40	.0585	.0559	.0536	.0428	.0448	.0119	.0256	.0145	.0150	−.0099	
		.4	10	.0370	.0347	.0326	.0254	.0280	−.0776	−.0726	−.0110	−.0365	−.0115	
			20	.0472	.0443	.0416	.0324	.0356	−.0429	−.0140	.0586	.0326	.0204	
			40	.0568	.0533	.0499	.0388	.0428	.0018	−.0069	−.0062	.0153	−.0114	
		.8	10	.0511	.0472	.0293	.0223	.0245	−.0076	−.0085	−.0335	−.0296	−.0399	
			20	.0662	.0610	.0378	.0289	.0315	−.0018	.0182	−.0178	−.0460	−.0193	
			40	.0821	.0756	.0469	.0357	.0391	−.0129	−.0186	.0239	.0005	.0144	
	12	.2	10	.0292	.0279	.0267	.0214	.0224	−.2372	−.2604	−.0610	.0020	−.2723	15
			20	.0388	.0371	.0355	.0285	.0299	−.1266	−.0755	.0297	−.0117	−.0924	
			40	.0487	.0466	.0447	.0356	.0374	−.0355	−.0487	.0026	.0336	.0031	
		.4	10	.0271	.0254	.0238	.0186	.0205	−.2804	−.2814	−.0716	−.0448	−.2605	
			20	.0362	.0340	.0319	.0250	.0275	−.1666	−.1517	−.0205	−.0143	−.1100	
			40	.0464	.0435	.0407	.0319	.0352	−.0526	−.0556	−.0396	−.0110	−.0161	
		.8	10	.0358	.0330	.0205	.0156	.0171	−.2010	−.2028	−.0598	−.0518	−.3520	
			20	.0485	.0447	.0278	.0212	.0231	−.0923	−.1086	−.0479	.0060	−.2124	
			40	.0635	.0586	.0364	.0278	.0305	−.0415	−.0803	−.0517	−.0469	−.0406	

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Table 13. The average standard error estimate from our analytic approach and its relative bias in the second simulation study

n	σ_x^2	r	σ_ε^2	Average SE					Relative bias of average SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
100	1	.2	10	.0963	.0929	.0884	.0715	.0735	−.0367	−.0233	−.0721	−.0434	−.0641	4
			20	.1082	.1035	.0993	.0805	.0823	−.0459	−.0348	−.0581	−.0320	−.1004	
			40	.1171	.1126	.1074	.0865	.0877	−.0757	−.0263	−.0492	−.0357	−.1298	
		.4	10	.0953	.0894	.0843	.0664	.0724	−.0386	−.0192	−.0119	−.0207	−.1004	
			20	.1083	.1014	.0954	.0751	.0813	−.0656	−.0723	−.0538	−.0375	−.1182	
			40	.1176	.1100	.1030	.0813	.0878	−.0513	−.0737	−.0114	−.0821	−.0914	
		.8	10	.1448	.1331	.0826	.0633	.0686	−.0529	−.0516	−.0662	−.0588	−.0725	
			20	.1676	.1539	.0950	.0728	.0786	−.0340	−.0408	−.0384	−.0610	−.0855	
			40	.1833	.1677	.1037	.0797	.0854	−.0610	−.0460	−.0266	−.0464	−.0864	
	6	.2	10	.0689	.0660	.0613	.0489	.0542	−.0682	−.0742	−.0440	−.0619	−.0198	2
			20	.0828	.0792	.0764	.0613	.0643	−.0773	−.1085	−.0633	−.0610	−.0654	
			40	.0962	.0920	.0902	.0728	.0734	−.0715	−.0884	−.0504	−.0546	−.1162	
		.4	10	.0648	.0609	.0565	.0435	.0518	−.0921	−.0675	−.0843	−.0842	−.0133	
			20	.0790	.0741	.0706	.0550	.0613	−.0616	−.0580	−.0617	−.0478	−.0823	
			40	.0933	.0882	.0837	.0655	.0711	−.0791	−.0665	−.0624	−.0665	−.0559	
		.8	10	.0869	.0802	.0509	.0385	.0456	−.0661	−.0861	−.0549	−.0571	.0480	
			20	.1105	.1026	.0650	.0493	.0553	−.0724	−.0648	−.0541	−.0388	.0210	
			40	.1370	.1264	.0789	.0604	.0662	−.0769	−.0668	−.0569	−.0309	−.0482	
	12	.2	10	.0619	.0590	.0473	.0368	.0598	−.0819	−.1113	−.0812	−.0391	.1178	9
			20	.0708	.0683	.0610	.0490	.0622	−.1067	−.0857	−.0973	−.0817	.0407	
			40	.0829	.0795	.0759	.0613	.0655	−.0764	−.1071	−.0721	−.0602	−.0615	

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Table 13. Continued

n	σ_x^2	r	σ_ε^2	Average SE					Relative bias of average SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
		.4	10	.0558	.0529	.0427	.0322	.0563	−.0612	−.0919	−.0645	−.0247	.1947	
			20	.0657	.0617	.0551	.0428	.0582	−.0922	−.0882	−.0830	−.0500	.0892	
			40	.0782	.0739	.0695	.0544	.0633	−.0935	−.0877	−.0299	−.0600	−.0062	
		.8	10	.0642	.0599	.0368	.0277	.0459	−.1129	−.0803	−.0757	−.0216	.2683	
			20	.0845	.0778	.0487	.0372	.0504	−.0789	−.0804	−.0490	−.0476	.1139	
			40	.1086	.1001	.0623	.0480	.0569	−.0663	−.0665	−.1093	−.0594	.0013	
200	1	.2	10	.0687	.0654	.0630	.0505	.0523	−.0162	−.0363	.0193	−.0208	−.0456	0
			20	.0777	.0738	.0712	.0572	.0587	.0022	−.0273	−.0253	−.0177	−.0324	
			40	.0836	.0800	.0769	.0612	.0630	−.0543	−.0169	−.0224	−.0553	−.0455	
		.4	10	.0684	.0636	.0599	.0473	.0511	−.0077	−.0446	−.0142	−.0088	−.0353	
			20	.0775	.0723	.0681	.0533	.0575	−.0302	−.0585	−.0177	−.0099	−.0946	
			40	.0845	.0787	.0739	.0575	.0629	−.0237	−.0027	.0189	−.0495	−.0398	
		.8	10	.1035	.0953	.0590	.0453	.0486	−.0127	−.0106	−.0010	−.0296	−.0908	
			20	.1189	.1094	.0676	.0517	.0555	−.0463	−.0126	−.0557	−.0399	−.0631	
			40	.1302	.1197	.0738	.0566	.0607	.0145	.0143	−.0095	−.0109	−.0532	
	6	.2	10	.0495	.0473	.0437	.0347	.0389	−.0234	−.0275	−.0377	−.0555	−.0044	0
			20	.0587	.0562	.0542	.0431	.0451	−.0323	−.0050	.0081	−.0371	.0175	
			40	.0689	.0658	.0644	.0516	.0524	−.0191	−.0612	−.0613	.0166	−.0575	
		.4	10	.0462	.0437	.0401	.0309	.0368	−.0427	−.0396	−.0155	−.0587	.0235	
			20	.0564	.0526	.0502	.0391	.0433	−.0510	−.0241	−.0184	−.0446	−.0104	
			40	.0672	.0628	.0601	.0468	.0504	−.0581	−.0294	−.0307	−.0283	−.0462	

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Table 13. Continued

n	σ_x^2	r	σ_ε^2	Average SE					Relative bias of average SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
		.8	10	.0615	.0571	.0362	.0272	.0327	−.0373	−.0495	−.0923	−.0136	.0394	
			20	.0794	.0732	.0462	.0350	.0392	−.0522	−.0314	.0085	−.0411	−.0747	
			40	.0980	.0907	.0567	.0433	.0471	−.0115	−.0556	−.0654	.0003	−.0808	
	12	.2	10	.0449	.0434	.0338	.0263	.0435	−.1115	−.0715	−.0223	−.0200	.1434	9
			20	.0506	.0486	.0434	.0343	.0440	−.0741	−.0553	−.0561	−.0041	.0496	
			40	.0592	.0565	.0541	.0432	.0467	−.0594	−.0921	−.0366	−.0336	.0252	
		.4	10	.0406	.0383	.0304	.0231	.0405	−.1444	−.1647	−.0728	−.0428	.1573	
			20	.0468	.0444	.0390	.0301	.0416	−.0431	−.1015	−.0623	−.0311	.1095	
			40	.0564	.0527	.0492	.0384	.0452	−.0786	−.0823	−.0351	−.0347	.0644	
		.8	10	.0459	.0427	.0260	.0194	.0330	−.0985	−.0820	−.0094	−.0534	.2698	
			20	.0597	.0555	.0343	.0258	.0362	−.1073	−.0575	−.0012	−.0244	.0856	
			40	.0768	.0712	.0443	.0339	.0405	−.0308	−.0267	.0039	−.0321	.0543	
300	1	.2	10	.0567	.0538	.0518	.0415	.0430	.0204	−.0138	.0020	.0070	−.0145	0
			20	.0638	.0609	.0586	.0469	.0485	.0122	.0190	−.0120	.0038	−.0297	
			40	.0689	.0658	.0631	.0504	.0521	−.0480	−.0293	−.0228	−.0911	−.0375	
		.4	10	.0559	.0524	.0493	.0385	.0417	−.0574	−.0197	.0010	−.0108	.0099	
			20	.0635	.0593	.0559	.0437	.0470	−.0618	−.0101	.0126	−.0243	−.0834	
			40	.0689	.0647	.0608	.0474	.0512	−.0109	−.0026	−.0388	−.0121	−.0346	
		.8	10	.0848	.0784	.0485	.0370	.0399	−.0364	−.0453	−.0481	−.0242	−.0498	
			20	.0974	.0900	.0555	.0425	.0454	−.0156	−.0002	−.0353	.0070	−.0472	
			40	.1064	.0982	.0606	.0464	.0494	−.0227	−.0149	−.0155	.0040	−.0157	

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Table 13. Continued

n	σ_x^2	r	σ_ϵ^2	Average SE					Relative bias of average SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
	6	.2	10	.0405	.0391	.0360	.0285	.0318	−.0600	−.0277	−.0201	−.0424	−.0394	0
			20	.0481	.0460	.0444	.0354	.0369	−.0780	−.0176	−.0583	.0022	.0201	
			40	.0563	.0537	.0525	.0420	.0427	−.0268	−.0150	−.0051	−.0030	−.0557	
		.4	10	.0381	.0358	.0329	.0253	.0301	−.0517	−.0436	−.0017	−.0409	.0631	
			20	.0462	.0434	.0410	.0319	.0352	−.0627	−.0350	.0454	.0175	.0087	
			40	.0549	.0512	.0488	.0379	.0413	−.0316	−.0456	−.0287	−.0076	−.0470	
		.8	10	.0506	.0471	.0298	.0222	.0266	−.0156	−.0101	−.0183	−.0338	.0444	
			20	.0648	.0600	.0377	.0285	.0323	−.0220	.0024	−.0216	−.0574	.0037	
			40	.0802	.0738	.0463	.0353	.0382	−.0358	−.0430	.0097	−.0126	−.0089	
	12	.2	10	.0369	.0357	.0276	.0214	.0360	−.0360	−.0526	−.0326	.0038	.1697	7
			20	.0417	.0400	.0354	.0281	.0362	−.0610	−.0030	.0258	−.0230	.0999	
			40	.0483	.0463	.0442	.0352	.0381	−.0440	−.0549	−.0073	.0195	.0208	
		.4	10	.0334	.0320	.0246	.0187	.0338	−.1132	−.0944	−.0401	−.0387	.2185	
			20	.0389	.0367	.0321	.0248	.0346	−.1061	−.0830	−.0135	−.0217	.1207	
			40	.0461	.0436	.0404	.0316	.0370	−.0571	−.0540	−.0475	−.0202	.0339	
		.8	10	.0380	.0355	.0212	.0157	.0281	−.1513	−.1442	−.0239	−.0457	.0682	
			20	.0490	.0455	.0281	.0211	.0302	−.0819	−.0934	−.0393	.0023	.0269	
			40	.0629	.0580	.0362	.0275	.0333	−.0510	−.0901	−.0578	−.0593	.0475	

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Table 14. The average bootstrap standard error and its relative bias in the second simulation study

n	σ_x^2	r	σ_ε^2	Average bootstrap SE					Relative bias of average bootstrap SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
100	1	.2	10	.1015	.0981	.0927	.0759	.0810	.0150	.0315	−.0273	.0161	.0318	0
			20	.1139	.1094	.1041	.0854	.0909	.0044	.0205	−.0123	.0264	−.0055	
			40	.1232	.1188	.1125	.0917	.0966	−.0276	.0277	−.0044	.0223	−.0417	
		.4	10	.1003	.0944	.0883	.0706	.0801	.0121	.0362	.0352	.0413	−.0050	
			20	.1137	.1072	.0999	.0797	.0902	−.0190	−.0192	−.0089	.0205	−.0217	
			40	.1233	.1159	.1077	.0861	.0970	−.0048	−.0236	.0346	−.0271	.0041	
		.8	10	.1529	.1404	.0866	.0674	.0756	.0000	.0007	−.0204	.0015	.0216	
			20	.1765	.1622	.0994	.0772	.0860	.0173	.0112	.0061	−.0039	.0012	
			40	.1929	.1767	.1085	.0845	.0940	−.0118	.0057	.0178	.0108	.0064	
	6	.2	10	.0741	.0703	.0648	.0525	.0573	.0027	−.0140	.0093	.0077	.0362	0
			20	.0891	.0850	.0805	.0655	.0709	−.0078	−.0439	−.0135	.0046	.0305	
			40	.1030	.0986	.0947	.0775	.0816	−.0068	−.0228	−.0032	.0065	−.0169	
		.4	10	.0703	.0651	.0598	.0467	.0537	−.0140	−.0031	−.0308	−.0168	.0229	
			20	.0852	.0797	.0744	.0589	.0668	.0131	.0127	−.0120	.0190	.0000	
			40	.0997	.0945	.0880	.0698	.0789	−.0158	.0011	−.0146	−.0043	.0464	
		.8	10	.0939	.0861	.0538	.0414	.0464	.0086	−.0182	.0000	.0147	.0667	
			20	.1187	.1095	.0686	.0528	.0590	−.0042	−.0018	−.0015	.0292	.0886	
			40	.1464	.1346	.0832	.0644	.0724	−.0135	−.0066	−.0060	.0337	.0402	
	12	.2	10	.0685	.0646	.0506	.0400	.0535	.0163	−.0271	−.0175	.0444	.0000	0
			20	.0785	.0752	.0648	.0529	.0615	−.0101	.0067	−.0400	−.0075	.0302	
			40	.0911	.0871	.0803	.0657	.0707	.0156	−.0213	−.0183	.0077	.0129	

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Table 14. Continued

n	σ_x^2	r	σ_ε^2	Average bootstrap SE					Relative bias of average bootstrap SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
		.4	10	.0627	.0581	.0457	.0350	.0470	.0556	−.0034	.0000	.0606	−.0021	
			20	.0733	.0681	.0587	.0463	.0552	.0138	.0059	−.0233	.0289	.0318	
			40	.0865	.0814	.0734	.0583	.0662	.0023	.0049	.0237	.0087	.0392	
		.8	10	.0723	.0665	.0393	.0301	.0363	−.0014	.0199	−.0126	.0636	.0028	
			20	.0929	.0855	.0519	.0402	.0461	.0131	.0106	.0137	.0281	.0177	
			40	.1181	.1086	.0661	.0515	.0579	.0155	.0131	−.0557	.0098	.0194	
200	1	.2	10	.0704	.0670	.0644	.0519	.0547	.0086	−.0118	.0404	.0058	−.0018	
			20	.0796	.0756	.0727	.0587	.0615	.0271	−.0026	−.0041	.0086	.0149	
			40	.0855	.0819	.0785	.0628	.0659	−.0328	.0061	−.0025	−.0309	−.0015	
		.4	10	.0700	.0653	.0612	.0486	.0537	.0160	−.0195	.0082	.0189	.0151	
			20	.0793	.0741	.0694	.0548	.0604	−.0075	−.0339	.0014	.0186	−.0488	
			40	.0863	.0806	.0754	.0590	.0662	−.0023	.0215	.0386	−.0248	.0107	
		.8	10	.1063	.0977	.0603	.0465	.0512	.0133	.0145	.0203	−.0021	−.0430	
			20	.1220	.1121	.0691	.0531	.0586	−.0217	.0117	−.0349	−.0130	−.0101	
			40	.1333	.1226	.0754	.0581	.0643	.0382	.0390	.0121	.0157	.0031	
	6	.2	10	.0517	.0488	.0449	.0359	.0388	.0197	.0041	−.0110	−.0245	−.0077	0
			20	.0613	.0584	.0555	.0445	.0467	.0099	.0336	.0316	−.0067	.0518	
			40	.0715	.0683	.0658	.0531	.0553	.0185	−.0243	−.0408	.0453	−.0054	
		.4	10	.0488	.0452	.0412	.0320	.0357	.0104	−.0066	.0098	−.0244	−.0056	
			20	.0591	.0547	.0515	.0403	.0446	−.0051	.0148	.0059	−.0147	.0183	
			40	.0700	.0652	.0615	.0482	.0530	−.0182	.0077	−.0081	.0000	.0038	

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Table 14. Continued

n	σ_x^2	r	σ_ε^2	Average bootstrap SE					Relative bias of average bootstrap SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
		.8	10	.0646	.0594	.0371	.0282	.0312	.0125	−.0116	−.0678	.0217	−.0064	
			20	.0828	.0761	.0474	.0362	.0400	−.0108	.0066	.0349	−.0082	−.0566	
			40	.1015	.0938	.0580	.0446	.0492	.0242	−.0229	−.0445	.0300	−.0391	
	12	.2	10	.0478	.0457	.0349	.0274	.0370	−.0535	−.0235	.0087	.0224	−.0289	1
			20	.0543	.0518	.0446	.0356	.0407	−.0073	.0058	−.0304	.0349	−.0286	
			40	.0631	.0602	.0556	.0446	.0470	.0032	−.0322	−.0107	−.0022	.0307	
		.4	10	.0442	.0408	.0315	.0241	.0322	−.0675	−.1092	−.0396	.0000	−.0800	
			20	.0506	.0471	.0402	.0312	.0369	.0327	−.0466	−.0337	.0065	−.0160	
			40	.0606	.0562	.0505	.0397	.0443	−.0098	−.0226	−.0079	−.0025	.0424	
		.8	10	.0499	.0458	.0268	.0202	.0246	−.0216	−.0151	.0229	−.0146	−.0538	
			20	.0639	.0590	.0354	.0269	.0311	−.0448	.0017	.0291	.0151	−.0689	
			40	.0809	.0748	.0456	.0352	.0390	.0215	.0233	.0340	.0028	.0130	
300	1	.2	10	.0576	.0547	.0525	.0423	.0443	.0360	.0018	.0155	.0267	.0161	0
			20	.0647	.0619	.0593	.0477	.0499	.0254	.0351	.0000	.0214	−.0020	
			40	.0699	.0668	.0640	.0513	.0536	−.0345	−.0147	−.0093	−.0740	−.0092	
		.4	10	.0568	.0532	.0500	.0392	.0432	−.0422	−.0037	.0163	.0077	.0460	
			20	.0645	.0603	.0566	.0445	.0487	−.0473	.0067	.0254	−.0067	−.0507	
			40	.0699	.0657	.0615	.0482	.0529	.0029	.0139	−.0269	.0063	−.0038	
		.8	10	.0863	.0797	.0492	.0377	.0416	−.0193	−.0292	−.0353	−.0053	−.0095	
			20	.0990	.0915	.0562	.0432	.0475	.0000	.0155	−.0226	.0237	−.0042	
			40	.1081	.0997	.0614	.0472	.0516	−.0073	.0000	−.0016	.0216	.0279	

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Table 14. Continued

n	σ_x^2	r	σ_ϵ^2	Average bootstrap SE					Relative bias of average bootstrap SE					No. of relative bias larger than .1
				z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	z_{i1}	z_{i2}	z_{i3}	x_{i4}	ω_{i1}	
	6	.2	10	.0420	.0400	.0367	.0292	.0312	−.0255	−.0050	−.0027	−.0201	−.0574	0
			20	.0498	.0474	.0450	.0362	.0376	−.0460	.0107	−.0446	.0226	.0387	
			40	.0581	.0552	.0533	.0428	.0443	.0035	.0128	.0095	.0142	−.0199	
		.4	10	.0397	.0366	.0334	.0259	.0288	−.0100	−.0240	.0121	−.0189	.0177	
			20	.0480	.0447	.0417	.0326	.0356	−.0264	−.0067	.0611	.0382	.0201	
			40	.0567	.0527	.0495	.0387	.0426	.0000	−.0168	−.0139	.0131	−.0162	
		.8	10	.0526	.0485	.0303	.0227	.0252	.0233	.0189	.0000	−.0130	−.0118	
			20	.0669	.0618	.0384	.0292	.0323	.0090	.0317	−.0026	−.0363	.0062	
			40	.0824	.0756	.0470	.0360	.0394	−.0096	−.0195	.0262	.0084	.0234	
	12	.2	10	.0389	.0371	.0281	.0220	.0299	.0157	−.0159	−.0140	.0329	−.0261	2
			20	.0443	.0420	.0361	.0288	.0329	−.0023	.0448	.0464	.0000	.0000	
			40	.0511	.0488	.0449	.0359	.0377	.0119	−.0041	.0090	.0406	.0107	
		.4	10	.0358	.0334	.0252	.0192	.0258	−.0504	−.0565	−.0195	−.0154	−.0686	
			20	.0416	.0387	.0327	.0254	.0299	−.0437	−.0349	.0062	.0040	−.0292	
			40	.0490	.0459	.0411	.0323	.0355	.0020	−.0043	−.0307	.0031	−.0084	
		.8	10	.0410	.0379	.0218	.0162	.0206	−.0848	−.0845	.0000	−.0182	−.2167	
			20	.0520	.0480	.0287	.0218	.0253	−.0262	−.0438	−.0171	.0332	−.1395	
			40	.0659	.0606	.0370	.0282	.0316	−.0060	−.0502	−.0365	−.0342	−.0063	

373.3 hours or 15.5 days to complete 1,000 replications across all 81 combinations). In contrast, our analytic approach only needed less than 2 seconds to complete 1 replication across all 81 combinations n, σ_x^2, r, and σ_ε^2 (roughly 1 day or 2 to complete 1,000 replications across all 81 combinations). For methodological researchers, the computational efficiency of our analytic approach is valuable when it is necessary to evaluate a newly developed method or to compare several competing methods.

Our analytic approach can be further extended in multiple ways. First, we can investigate how it performs under different scaling factors. For instance, Gelman (2008) suggested standardizing the variables by dividing by two standard deviations instead of one. He argued that this would make the standardized variables more comparable to the untransformed binary predictors. The proposed approach can be easily adapted to accommodate different scaling factors for standardization. Another avenue for exploration is to assess how the proposed approach performs in summarizing interactions across studies. Hagger et al. (2022) used a similar approach to the one introduced in the present study to meta-analyze interactions. However, they still need to provide empirical evidence on the effectiveness of their approach. Future studies could evaluate how these approaches perform under various simulated conditions. Finally, if the covariance structure model specified in the first step of our approach is re-parameterized purposefully, our analytic approach could be applied to the framework of multivariate component-based regression (Burnham et al., 1996), where multiple criterion variables are regressed on the components that are linear combinations of predictor variables. For example, if $\mathbf{P}_1$ in Equation (4) is re-parameterized by some patterned matrices to conform with canonical correlation analysis (Gu et al., 2019), the standard error estimates for the unstandardized regression coefficients of canonical correlation regression (CCR) could be obtained from the delta method in the second step. Alternatively, if $\mathbf{P}_1$ in Equation (4) is re-parameterized by some patterned matrices to conform with redundancy analysis (Gu et al., 2023), the standard error estimates for the unstandardized regression coefficients of redundancy analysis regression (RAR) can be obtained from the delta method in the second step. More efforts are needed to demonstrate these extensions to CCR and RAR. However, for problems without closed-form solutions in terms of the parameters in the first step, our analytic approach cannot be applied.

In conclusion, our simple analytic approach that integrates the covariance structure modeling method and the delta method can be readily applied to obtain accurate standard error estimates not only for standardized regression coefficients when the interaction terms are not included but also for unstandardized regression coefficients when the interaction terms are included. Both numeric examples and simulation findings demonstrate the superiority of our analytic approach over regular regression, especially in scenarios commonly encountered in realistic settings. Typically, researchers in social and behavioral sciences frequently encounter variables whose variance is (much) larger than 1, and for those exploring interaction terms, their sample size typically exceeds 200 in order to achieve more robust estimates. Therefore, our analytic approach possesses wide applicability in realistic settings. We encourage the wide adoption of this simple and computationally efficient analytic approach in applied research for making accurate statistical inferences.

Supplementary material. To view supplementary material for this article, please visit http://doi.org/10.1017/psy.2026.10107.

Open practices statement. The datasets of all illustrated examples are available from the Supplementary Material.

Data availability statement. The numeric examples used in the manuscript and the computer programs used for the numeric examples are available in the Supplementary Material.

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