J. Plasma Physics (2002), *vol.* 68, *part* 3, *pp.* 173–190. © 2002 Cambridge University Press
DOI: 10.1017/S0022377802001903 Printed in the United Kingdom

Does neutral dynamics suppress instabilities in partially ionized plasma?

A. A. SHAIKH[1] and A. C. DAS[2]

[1]C.U. Shah Science College, Ashram Road, Ahmedabad 380014, Gujarat, India
[2]Theoretical Physics Division, Physical Research Laboratory, Navrangpura, Ahmedabad 380009, India
(acd@prl.ernet.in)

(Received 2 August 2001 and in revised form 15 April 2002)

Abstract. We present two findings: (i) in the absence of compressibility of (cold) neutrals, the instability triggered by neutral drift due to gradients in plasma density and magnetic field and (ii) the equilibrium pressure-gradient- (PG-) driven instabilities with and without neutral dynamics. The results of detailed analysis suggest that on account of neutral dynamics the growth of these instabilities is suppressed by a neutral perturbational effect. Suppression of PG-driven instabilities is highlighted and the critical wavelength for sustainable growth is found. The physics involved in the mechanism of instabilities is discussed. The endeavour in these investigations is justifiable in its usefulness for understanding the phenomena pertinent to partially ionized plasma (PIP). The stability analysis presented here may be useful for understanding the plasma processes in 'deep' coma (very near to the nucleus of coma), at the edges of a plasma column, near a divertor region and in the case of sublimating gas from a cloud.

1. Introduction

Partially ionized plasmas (PIP, hereafter) are often encountered in nature and in the laboratory environment. Sublimating gas from barium clouds and coma, lower planetary ionospheres, interaction between interstellar-born neutrals and the hileosphere, the interstellar medium, gas during the infant stage of an astrophysical object, etc. are examples of PIP in nature. On the other hand, plasma used in a magnetohydrodynamic (MHD) generator, electromagnetic launchers, magnetic and inertial confinement schemes, at the edge of a plasma column in a tokamak, near the divertor of a tokamak, etc. are some examples of PIP in a laboratory environment. The study of the collective process in PIP with neutral dynamics may help us to understand the phenomena observed both in nature and in laboratory set ups. The collective phenomena in PIP with the inclusion of neutral dynamics have not been studied seriously. Only a few recent publications have dealt with the effects (Zweibel 1989; Dwivedi and Das 1992; Huba et al. 1994; Huba 1995 and references therein; Shaikh and Das 1997, 1998 and references therein; Doughton et al. 1998; Shaikh et al. 1999) of neutral dynamics on instabilities in PIP. A recent publication by Doughton et al. (1998) has claimed the first ever consideration of neutral dynamics in PIP. However, we would like to mention that work in this direction started a decade ago (Zweibel 1989; Dwivedi and Das 1992 and references therein) and later

https://doi.org/10.1017/S0022377802001903 Published online by Cambridge University Press

further developed (Huba et al. 1994; Huba 1995; Shaikh and Das 1997, 1998; Shaikh et al. 1999) over the course of time. It has been substantiated that the consideration of neutral dynamics in the study of collective phenomena in PIP is the benchmark for a complete and coherent study of collective processes. We cite a few examples in support of the importance of neutral dynamics:

1. the inclusion of neutral dynamics has offered modifications and new results which are consistent with those of earlier published works (Huba 1990, 1995; Huba et al. 1994; Shaikh and Das 1997, 1998, 1999; Dwivedi and Das 1992);
2. the inclusion of neutral dynamics in the calculation of instability of the heliopause by Liewer et al. (1996) has, almost, removed the disagreement between theory and simulation in the estimation of growth rate;
3. the consideration of the collision effect in the neutral-dominated astrophysical fluid (i.e. PIP) has provided a better understanding of the alteration of conversion of magnetic field energy to plasma kinetic energy (Zweibel 1989).

With a view to maintaining consistency with other published works, the following principal assumptions are made: (i) there is a stationary equilibrium during the time scale involved in the physical processes considered for the investigation; (ii) we have retained a stable population of neutrals on a given time scale; (iii) the paper uses the background of local theory with a short mean free path compared with the scale length; (iv) $m_e\nu_{ei} = m_i\nu_{ie}$; (v) fluids are inviscid; (vi) $m_i = m_n$; (vii) there is no magnetic tension force $(B \cdot \nabla B)/4\pi$ (parallel shear); and (viii) $T_i = T_e = T_n$.

In PIP, e.g. during the early stage of formation of an astrophysical object, there has always been either finite neutral, plasma or magnetic pressure gradients which drive the neutrals or give rise to a relative velocity between ions and neutrals. Either case is subjected to pressure gradient (PG) instabilities. In Sec. 2, we discuss the fundamental equations and the equilibrium and derive the general dispersion relation in Sec. 3. We found that for a positive growth rate there is a threshold in relative streaming between the plasma and neutral fluids. The role of $\epsilon = \nu_{in}/\nu_{ni}$ is highlighted. We believe, from our results, that neutral dynamics is probably responsible for the stability of lower planetary atmospheres against classical and collisional Rayleigh–Taylor (RT) instability. In Sec. 3, we discuss how a partially ionized plasma is prone to pressure gradient instability with a mechanism that is similar to classical RT. On introducing the neutral perturbation in the problem, we found that neutral dynamics is responsible for damping of PG instabilities similar to RT and collisional RT (Appendix B, B5 and B6). The physics of this mode is described with physically feasible assumptions. Finally, we conclude with a summary in Sec. 4. Detailed algebraic derivations are given in Appendices A and B.

2. Fundamental equations and equilibrium

We write the equations of a fluid for a three-component plasma made up of ions, electrons and neutrals,

$$\partial_t n_\alpha + \nabla \cdot (n_\alpha \mathbf{V}_\alpha) = 0, \tag{1}$$

where n_α is the number density, V_α is the velocity of the fluid and α represents the charged species and neutral particles. Adding electron (inertialess) and ion

https://doi.org/10.1017/S0022377802001903 Published online by Cambridge University Press

momentum equations and using the law of conservation of momentum, quasi-neutrality, vector identity and Maxwell's equations with a definition of current density **J**, we obtain

$$m_i n_i D_t \mathbf{V}_i = -\nabla\left(\frac{B^2}{8\pi} + P\right) - n\nu_{eff}\left(\mathbf{V}_i - \mathbf{V}_n\right) + \frac{cm_e\nu_{en}}{4\pi e}\nabla \times \mathbf{B} + m_i n_i \mathbf{g}, \quad (2)$$

where $D_t = \partial_t + V \cdot \nabla$, $\nu_{eff} = m_i\nu_{in} + m_e\nu_{en}$, $m_e\nu_{ei} = m_i\nu_{ie}$, $n_i = n_e = n$, $m_i = m_n$ and $P = P_e + P_i$ are used. The momentum equation for neutrals is

$$D_t \mathbf{V}_n = -\frac{\nabla P_n}{m_n n_n} - (\nu)\left(\mathbf{V}_n - \mathbf{V}_i\right) + \frac{\nu_{ne}c}{4\pi n_n e}\left(\nabla \times \mathbf{B}\right) + \mathbf{g}, \quad (3)$$

where $\nu = \nu_{ni} + \nu_{ne}$ and we have used Maxwell's equations with a definition of current density **J**. The system of equations is closed by an isothermal equation of state. We consider that the magnetic field $\mathbf{B} = B_0(x)\hat{e}_z$, plasma and neutral density scale lengths are in the positive X-direction, while gravity and the magnetic field gradient are in the negative X-direction. For equilibrium, setting $\frac{\partial}{\partial t} = 0$ in the governing equations we obtain

$$\nabla \cdot (n_\alpha \mathbf{V}_\alpha) = 0 \quad (4)$$

$$0 = -\nabla\left(\frac{B^2}{8\pi} + P\right) + n_i m_i \mathbf{g} - \nu_{eff} n_i \left(\mathbf{V}_i - \mathbf{V}_n\right) + \frac{\nu_{en} m_e c}{4\pi e}\left(\nabla \times \mathbf{B}\right), \quad (5)$$

where we have neglected the dynamic pressure and the magnetic tension force. The X-component of (5) gives

$$-\partial_x\left(\frac{B_0^2}{8\pi} + P\right) - n_{i0} m_i g - \nu_{eff} n_{i0}(V_{i0x} - V_{n0x}) + \frac{\nu_{en} c m_e}{4\pi e}\left(\nabla \times B_0\right)_x = 0.$$

In the absence of a frictional force,

$$\partial_x\left(\frac{B_0^2}{8\pi} + P\right) = -n_{i0} m_i g \quad (6)$$

$$V_A^2\left[L_B^{-1} + \beta L_P^{-1}\right] = -g \quad (7)$$

where $L_P^{-1} = \frac{\partial_x n_{i0}(x)}{n_{i0}}$, $L_B^{-1} = \frac{\partial_x B_{i0}(x)}{B_0}$, $V_A^2 = \frac{B_0^2}{4\pi n_{i0} m_i}$, $C_s^2 = \frac{2T}{m_i}$ and $\beta = \frac{C_s^2}{V_A^2}$.

In the absence of gravity, its role is played by a frictional term $\nu_{in} V_{n0x}$, therefore we obtain

$$V_{n0x} = \frac{V_A^2}{\nu_{in}}\left[L_B^{-1} + \beta L_P^{-1}\right], \quad (8)$$

where we have assumed that $V_{i0x} \sim 0$, $m_e\nu_{en} \ll m_i\nu_{in}$. For a uniform magnetic field the momentum equation of ions yields

$$V_{i0y} = \frac{C_s^2}{\Omega_i}\left[L_p^{-1} + \epsilon L_n^{-1}\right], \quad (9)$$

where $\Omega_i = eB_0/m_i c$ and we have used $V_{e0y} = -(cT_e/eB_0)L_p^{-1}$ from the electron momentum equation and $T_i = T_e = T_n$. Also, we note that the above expression for V_{i0y} is obtained from zeroth-order momentum equation for the ions.

https://doi.org/10.1017/S0022377802001903 Published online by Cambridge University Press

In the absence of gravity (6) reduces to the usual plasma equilibrium,

$$\partial_x\left(\frac{B_0^2}{8\pi}+P\right)=0 \tag{10}$$

i.e. $B_0^2/8\pi + P$ = constant and $\beta = -L_P/L_B$.

The Venus ionopause provides an example of magnetohydrodynamic static balance ($g = 0$).

The y-component of (5) is

$$V_{i0y}-V_{n0y}=-\frac{\nu_{en}m_e c}{4\pi e\nu_{eff}n_{i0}}\partial_x B_0. \tag{11}$$

In the absence of gravity, in (3), it gives

$$V_{n0x}=\frac{V_{nt}^2}{\nu_{ni}L_n} \tag{12}$$

provided that $V_{i0x} = 0$, where $V_{nt}^2 = T_n/m_n$. Note that this is consistent with (8) for a uniform magnetic field. Now, the y-component of (3) gives

$$V_{i0y}-V_{n0y}=-\frac{\nu_{ne}c}{4\pi n_{i0}e}\frac{\partial_x B_0}{(\nu_{ni}+\nu_{ne})}. \tag{13}$$

It is to be noted that (11) and (13) must be consistent and as a result, we must have

$$\frac{\nu_{ne}}{\nu_{ne}+\nu_{ni}}=\frac{m_e\nu_{en}}{m_i\nu_{in}+m_e\nu_{en}}.$$

This can be verified by substituting for $\nu_{ne} = \frac{m_e}{m_i}\frac{n_i}{n_n}\nu_{en}$ (Rishbeth and Garriott 1969) and $\nu_{ni} = \frac{n_i}{n_n}\nu_{in}$ with $m_i = m_n \gg m_e$, which is valid for the F region (Kelley 1989).

We conclude this section with a word of caution that the presence of instability growth has to be examined in the context of the possible dynamic nature of the equilibrium considered. Otherwise the non-stationary equilibrium behaviour may change the conclusion of the investigation.

3. Dispersion relation and stability analysis

Assuming that the perturbed physical quantities vary as $\exp[i(ky-\omega t)]$ and linearizing the basic equations (1)–(3) under a local approximation, we arrive at the following dispersion relation for a cold neutral (see Appendix A, A3):

$$\omega_c\omega_{m1}\omega_{mx}-k^2C_s^2\omega_{m1}-i\frac{\omega_A^2}{kL_B}\frac{\nu_{en}}{\Omega_e}D_1\omega_{m1}+\frac{\omega_{mx}}{kL_P}\left(\frac{k^2C_s^2}{kL_P}+gk\right)=0, \tag{14}$$

where $D_1 = \omega_{cn}/\omega_{ny}$,

$$\Omega_i=\frac{eB_0}{m_ic},\quad C_s^2=\frac{T_i+T_e}{m_i},\quad \omega_c=\omega-kV_{i0y},\quad \omega_{cn}=\omega-kV_{n0y},$$

$$\omega_{my}=\omega_c+i\nu_{in}\frac{\omega_{cn}}{\omega_{nx}},\quad \omega_{mx}=\omega_{m1}=\omega_c+i\nu_{in}\left(\frac{\omega_{cn}}{\omega_{nx}}\right)$$

$$\omega_{nx}=\omega_{cn}+i\,(\nu_{ni}+\nu_{ne}),$$

$$\eta_{en}=\frac{\nu_{en}c^2}{\omega_{pe}^2},\quad \omega_{pe}=\left(\frac{4\pi ne^2}{m_e}\right)^{1/2}.$$

https://doi.org/10.1017/S0022377802001903 Published online by Cambridge University Press

3.1. Low-frequency mode for cold neutrals

For low frequencies ($\omega, \omega_{cn} \ll \Omega_i,\ kV_A,\ kC_s$), tightly coupled ($\omega \ll \nu_{ni}$) and cold neutral $\omega\,(\omega_{cn}) \gg \omega_t$ conditions, relation (14) support two modes.

Mode I

$$\begin{aligned}
\omega_{m1} = \omega_{mx} = 0 &= \omega_c + i\nu_{in}\frac{\omega_{cn}}{\omega_{nx}} \\
\omega_c\omega_{nx} + i\nu_{in}\omega_{cn} &= 0 \\
\omega_{cn} &= \frac{k\Delta V}{1+\epsilon} \\
\omega &= \frac{kV_{i0y} - kV_{n0y} + (1+\epsilon)kV_{n0y}}{1+\epsilon} \\
\omega &= \frac{kV_{i0y}}{\epsilon+1} + \frac{\epsilon kV_{n0y}}{\epsilon+1}.
\end{aligned} \tag{15}$$

This is a real oscillating part, where $\epsilon = \nu_{in}/\nu_{ni}$.

Mode II

$$\begin{aligned}
&\omega_c\omega_{mx} - k^2C_s^2 - \frac{\omega_A^2}{kL_B}\frac{\nu_{en}}{\Omega_e}\frac{\omega_{cn}}{\nu_{ni}} + \frac{1}{kL_P}gk = 0 \\
&\omega_{cn}^2(\epsilon+1) - \omega_{cn}(2k\Delta V) - \left[k^2C_s^2 - \frac{k^2C_s^2}{k^2L^2} - (k\Delta V)^2 + \frac{gk}{kL_P}\right] = 0,
\end{aligned} \tag{16}$$

where

$$\begin{aligned}
\Delta V = V_{i0y} - V_{n0y} &= -\frac{c}{4\pi n_{i0}e}\frac{m_e\nu_{en}}{m_i\nu_{in}}\partial_x B_0 \\
&= -\frac{V_A^2}{\Omega_e}\frac{\nu_{en}}{\nu_{in}}\frac{1}{L_B} \\
&= -\eta_{en}\Omega_i\frac{1}{L_B}\left(\text{since } \eta_{en}\Omega_i = \frac{V_A^2}{\Omega_e}\nu_{en}\right).
\end{aligned}$$

The solution of the dispersion relation (17) is

$$\omega_{cn} = \frac{k\Delta V}{(\epsilon+1)} \pm \frac{1}{(\epsilon+1)}\left[k^2\Delta V^2 - (\epsilon+1)\left(\frac{gk}{kL_P} + k^2\Delta V^2 - k^2C_s^2 + \frac{k^2C_s^2}{k^2L^2}\right)\right]^{1/2}. \tag{17}$$

The solution (17) can now be analysed for various feasible conditions such as

1. For warm PIP we obtain following condition for growth:

$$k^2\Delta V^2\frac{\epsilon}{(\epsilon+1)} + \frac{gk}{kL_P} > k^2C_s^2\left(1 - \frac{1}{k^2L^2}\right).$$

2. For cold PIP, without gravity, the growth rate and real frequency, respectively, are

$$\gamma = \pm\frac{k\Delta V}{(\epsilon+1)}\epsilon^{1/2} \tag{18}$$

https://doi.org/10.1017/S0022377802001903 Published online by Cambridge University Press

$$\omega_0 = \frac{k\Delta V}{(\epsilon + 1)} + kV_{n0y}. \tag{19}$$

Furthermore, the growth rate (18) can also be expressed as

$$\gamma = \frac{kV_A^2}{\nu_{in}L_B}\frac{(\epsilon + 1)}{\epsilon^{1/2}}\frac{\nu_{en}}{\Omega_e} \tag{20}$$

or

$$\gamma = \frac{(\epsilon + 1)}{\epsilon^{1/2}}\sqrt{\frac{m_e}{m_i}}kV_d, \tag{21}$$

where $\mathbf{V}_d = -\nabla P \times \mathbf{B}/nqB^2$.

This instability can be treated as being similar to collisional RT. However, unlike collisional RT (which is a purely growing mode), it moves with a finite velocity that depends on the coupling between the magnetic field gradient and the collision frequencies. The underlying physics for inferring this instability is that the relative streaming causes charge separation at the interface separating the magnetic field density from the plasma density. Charge separation produces an electric field in the direction opposite to the relative streaming, i.e. in a direction that gives rise to the growth of the wave. Thus, the presence of a magnetic field (energy) gradient causes the generation of the required electric field, triggering the RT-type instability in PIP. We could call it a manifestation of the conversion of magnetic energy into electric energy. We note that the result of this case reduces to that of earlier work (Huba 1990) for $V_{n0y} = 0$.

3. In the presence of both relative streaming and gravity in cold PIP we obtain

$$\omega_{cn} = \frac{k\Delta V}{(\epsilon + 1)} \pm \sqrt{-\epsilon k^2\Delta V^2 - \frac{gk}{kL_p}}. \tag{22}$$

Thus, it is clear that growth is enhanced.

Note that $gk/kL_p = \gamma_{RT}^2$, i.e. the expression of the RT growth rate. The above three cases demand conditions for the onset of instabilities in partially magnetized and inhomogeneous plasma for the strongly coupled case in the low-frequency regime. A closer look at any of the above conditions reveals that the relative streaming between ions and neutrals is of paramount importance. Also, optimum conditions can be obtained for $\epsilon \sim 1$ that bring out the importance of the neutral response/dynamics.

The following interesting facts can be derived from the above analytical study.

1. In the absence of gravity, the relation (17) shows that there is a competition between the thermal velocity and the relative streaming for a positive growth rate. However, it should be noted that this condition for positive growth is strictly controlled by the neutral dynamics via the factor ϵ.

2. In addition to the above, under a given approximation we have growth attributed to the parameter ϵ. This instability is mainly induced due to neutral dynamics with some dynamo-like effect, i.e. the collisional effect of the neutrals gives rise to a favourable electric field to start with. Hence, we call it neutral-induced collisional RT (NICRT) (Shaikh and Das 1997). The growth

https://doi.org/10.1017/S0022377802001903 Published online by Cambridge University Press

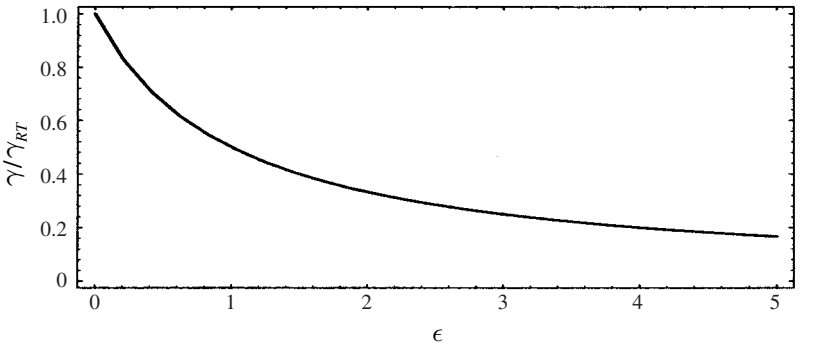


Figure 1. Plot of $\gamma/\gamma_{RT} = gamma \rightarrow \epsilon$ based on (24).

rate γ is given by

$$\gamma = \left(\frac{gk}{kL_P}\right)^{1/2} \frac{1}{(\epsilon + 1)}, \tag{23}$$

where $\epsilon = \nu_{in}/\nu_{ni}$:

$$\gamma = \gamma_{RT} \frac{1}{(\epsilon + 1)}. \tag{24}$$

For $\epsilon < 1$

$$\gamma = \gamma_{RT}(1 - \epsilon). \tag{25}$$

Thus RT is damped because of a neutral response, which is obvious from Fig. 1. It can be seen that in the absence of ϵ, NICRT reduces to conventional RT. In plotting Fig. 1 we have taken a normalized growth rate and the parameter ϵ on the Y- and X-axes, respectively. The above result seems to be interesting for phenomena near the 'coma' of a comet, where the intrinsic gravity of the coma is negligible and it is dressed up by IMF/draped by Solar wind.

3.2. Low-frequency electrostatic mode for warm neutrals

The low-frequency dispersion relation for warm neutrals (Appendix B, B5 and B6) is given by

$$\begin{aligned}
&-\omega_c\left(-\omega_c + \frac{\omega_t^2}{\omega_c} - \frac{\nu_{in}\nu_{ni}}{\omega_{ny}D_2}\right) + \frac{kg\nu_{in}\nu_{ni}}{\omega k L_P \omega_{ny} D_2} \\
&+ \left(-i\nu_{in} - \frac{\nu_{in}\nu_{ni}}{\omega_{nx}D_2}\right)\left(-\frac{\nu_{in}\nu_{ni}}{\omega_{ny}D_2} + \frac{\omega_t^2}{\omega_c}\right) \\
&+ i\omega_c\nu_{in} + \left(-i\nu_{in} - \frac{\nu_{in}\nu_{ni}}{\omega_{nx}D_2}\right)(-i\nu_{in}) + \frac{ikg\nu_{in}}{\omega_c k L_P} \\
&- \frac{gk\omega_t^2\nu_{in}\nu_{ni}}{\omega^2 k L_n \omega_{nx}\omega_{ny}D_2} + \frac{\omega_t^2\nu_{ni}}{\omega_{nx}\omega_{ny}D_2 k L_n}\left(\frac{-i\omega_t^2}{kL_P\omega_c} + \frac{i\omega_t^2\nu_{in}\nu_{ni}D_2}{kL_n\omega_{cn}\omega_{nx}\omega_{ny}}\right) \\
&- \frac{\nu_{in}^2\nu_{ni}f_x}{\omega_{ny}D_2 kL_P\omega_c} - \frac{if_x\nu_{in}^2}{kL_P\omega_c} + \frac{\omega_t^2\nu_{in}^2\nu_{ni}f_x}{\omega_c kL_n\omega_{cn}\omega_{nx}\omega_{ny}D_2} = 0
\end{aligned} \tag{26}$$

with $\omega \ll kC_s$, kV_{nt}, ν_{ni}, $T_i \sim T_n$.

https://doi.org/10.1017/S0022377802001903 Published online by Cambridge University Press

This is the dispersion relation with warm neutrals. It is to be noted that this relation is the most general one because it has incorporated all the effects and therefore it is not hard to retrieve the results for a cold neutral. We investigate for the case of a non-magnetized PIP with the following ordering scheme:

$$\frac{\omega}{\nu_{ni}} \sim \frac{\omega}{\omega_t} \sim \frac{\nu_{in}}{\nu_{ni}}(\epsilon) \sim \delta \ll 1, \quad kL_n, kL_p \gg 1.$$

Here, we note that effective gravity 'g' has come from $\nu_{ni}(V_{n0x} - V_{i0x})$ in the equilibrium equation.

Under a given ordering scheme, we are left with the following sleek dispersion relation:

$$a\omega^2 + ib\omega - c = 0 \tag{27}$$

where

$$a = \frac{\omega_t^2}{kL_n kL_p \nu_{in}} + \frac{\omega_t^2}{(kL_n)^2 \nu_{ni}} + \nu_{ni}$$

$$b = \frac{\omega_t^4}{(kL_n)^2 \nu_{in}\nu_{ni}} + \omega_t^2$$

$$c = \frac{\omega_t^4}{(kL_n)^2 \nu_{ni}}.$$

The solution of (27) yields the following mode:

$$\omega = \frac{-ib}{2a} \pm \frac{(-b^2 + 4ac)^{1/2}}{2a} \tag{28}$$

for $b^2 > 4ac$, which is consistent with local theory, we obtain

$$\gamma_- = \gamma_p - k_y^2 D_{in} \tag{29}$$

where $D_{in} = V_{nt}^2/\nu_{in}$ provided that $\epsilon\gamma_p < \nu_{in}$, which leads to $l_{in}l_{ni} \ll L^2$, where l is the mean free path, L is the scale length involved and $\gamma_p = \frac{\omega_t^2}{(kL_n)^2\nu_{in}}$, with $L_p = L_n$.

The sleek form of the growth rate comprises two terms: growth rate (γ_p) and diffusion. The former is attributed to zeroth-order neutral pressure, which drives the neutral fluid, while the later term is associated with diffusion phenomena that arise due to neutral perturbation. The following comments are in order.

1. The equilibrium neutral pressure/total plasma pressure-driven instability is analogous to the classical Rayleigh–Taylor instability. This could be seen as follows. From the basic equations we derive the dispersion relation for 'pressure-gradient'-driven instabilities for the case of an unperturbed neutral population in PIP as

$$\omega^2 L_p + iL_p\nu_{in}\omega + V_p\Omega_{ci} = 0, \tag{30}$$

where $V_p = \frac{\nu_{in}}{\Omega_{ci}}V_{n0x}$. Here consider the fact that V_p is not $\mathbf{g} \times \mathbf{B}$.
The above simple dispersion relation, identical to the dispersion relation for conventional RT and collisional RT, leads to two novel PG-driven instabilities

https://doi.org/10.1017/S0022377802001903 Published online by Cambridge University Press

similar to RT. The prime features of these instabilities are their presence in PIP (until the neutrals are unperturbed).
For the collisionless limit

$$\gamma = \sqrt{\frac{V_A^2/L_B}{L_p} + \frac{C_s^2/L_p}{L_p}}, \tag{31}$$

while in the collisional limit

$$\gamma_p = \frac{V_A^2\left(L_B^{-1} + \beta L_p^{-1}\right)}{L_p \nu_{in}}. \tag{32}$$

2. These novel modes are seeded by the neutral equilibrium flow that arises due to the finite neutral temperature in the presence of a neutral density gradient. Like RT, the PG mode is capable of triggering a wide spectrum of irregularities/instabilities (independent of k). Note that the instabilities have very noticeable effects on the medium because it exhibits the coupling of plasma and neutral scale lengths and collision frequencies between ions and neutrals and vice versa. For very sharp gradients, the effects of these instabilities have to be reckoned with. This result may find a connection with the discovery of a region called the ionopause and the collisionopause, very near to the nucleus of a comet (towards the Sun, e.g. Mendis 1988). It was observed that this region is identified as a magnetic-field-free cavity (FFC), where the ion temperature is high enough and the inbuilt neutral pressure causes the higher thermal velocity of the neutral gas to overcome the grip of the weak intrinsic gravity of the comet. Of course in the FFC region $L_B^{-1} \rightarrow 0$ and the second term of the above growth rate expression, i.e. (32), come to the fore. The mechanism of this instability is that the inbuilt pressure gradient drives the sublimating gas so that the neutral velocity/relative velocity gives rise to a frictional force. Here the equilibrium is between the PG and the frictional force.

3. We find, from (29), that the perturbation in the neutral population is manifested as the diffusion of neutrals from the region. Hence we may write that this growth rate is arrested by neutral perturbation via diffusion. Here the question is, 'does the classical diffusion phenomenon fully suppress this instability in PIP*?*'. The answer can be obtained by equating relation (29) to zero, and *ergo* we obtain the critical wavelength for the suppression of growth. This could be tested by equating relation (29) with zero,

$$\gamma_p = k_y^2 D_{in}. \tag{33}$$

This leads to the critical wavelength λ_c. However, we admit that $\lambda_c > L_n$, which violates the local approximation. Therefore, we conclude that there must be a very high magnitude of D_{in} in order to suppress the instability shown in (29).

4. We note that neutral perturbation provides a damping effect via diffusion. The underlying physics, in the context of the driving mechanism of instability growth, could be understood as follows. When neutral density perturbation is strong enough then the pressure gradient along the x-direction is flattened, which, in turn, depends on the perturbed velocity of the neutrals. Thus, the perturbed neutral pressure is reduced. Now, because of the reduced perturbed neutral pressure, neutrals are driven with less velocity compared with that of

https://doi.org/10.1017/S0022377802001903 Published online by Cambridge University Press

the unperturbed neutral pressure. Hence the very cause of the instability is fading away, i.e. the neutral frictional force is reduced and hence the growth of the instability is arrested/damped. Another line of thought for this mechanism is the reduction in velocity of the neutrals, i.e. the perturbed neutral velocity slows down the plasma drift velocity, which creates a polarized electric field. Thus, fluctuation in the neutral velocity reduces the very cause of the instability. Also, we can point out that PIP tends to acquire a constant velocity (a reduction in plasma acceleration) in the state of perturbed neutrals. In other words, the reduction in plasma acceleration causes damping of the instability. Here one could envisage (Doughton et al. 1998), in our study, the role of the electrons (as a single fluid) and viscosities of ions and neutrals as damping agents for the growth rate shown by (29). However, we do not expect the same analogy of this mode with that of RT because of the perturbed state of the neutrals. This could be well justified because here the free energy for instability is coming from the frictional force, i.e. the internal force of the system. On the other hand, gravity 'g' appears in the law of conservation of momentum, i.e the force of gravity is an external force. This is a conspicuous difference between RT- and PG-driven instability.

5. It is interesting to note that the neutral perturbation is not even allowing conventional collisional RT to grow. This fact can be accounted for by keeping gravity and frictional terms in the perturbed linearized momentum equations for the plasma and neutrals, *ergo* the very source of free energy for collisional RT is cancelled out with the terms coming from neutral perturbation in the final dispersion relation (Appendix B, B5 and B6). Hence we note that the neutral dynamics damp the growth of classical collision RT and offer a damping effect via a diffusion term to the PG instability, in PIP. This is the reason, perhaps, for the absence of RT and similar instabilities in lower planetary ionospheric plasmas where the neutrals are dominant. We ought to note that we have a two-fluid approximation, i.e. the electron and ions together form one fluid, while the neutrals form another fluid. Therefore, we have neither an equilibrium electric field nor electron dynamics explicitly. Also both fluids are assumed to be inviscid and therefore our model is not similar to that appearing in recent publications (Doughton et al. 1998). Doughton et al. (1998) have shown that the neutral dynamics can suppress PG. However, they have considered three fluid models with viscosity. We should note that viscosity is temperature dependent and for a fluid it decreases with increasing temperature, so we assume that for a warm plasma and neutral fluids, the viscosity could not be as effective at damping conventional collisional RT and PG instabilities compared with the mechanism that we have proposed on the basis of neutral dynamics/perturbation without a viscous term in the momentum equation.

6. We have considered $T_i + T_e \sim T_n$. Even if, we assume that $T_i + T_e \neq T_n$ we would obtain a similar result under the given ordering scheme. We note that if $T_e \sim T_i \sim T_n$ then the acoustic velocity has a factor of 2 in the numerator.

A few further results are a natural outcome of the converse set of assumptions taken in above sections. However, all of these results are analytical and can be easily seen. Nevertheless all of these results are hidden in our monster-like dispersion relation, which could be visualized as a kaleidoscope with a zoo of various modes.

https://doi.org/10.1017/S0022377802001903 Published online by Cambridge University Press

4. Summary

In this paper we have developed a mathematical model for a three-component plasma. Attention has been paid to the collisional effect and its consequences on neutral dynamics in PIP. We have exhaustively discussed the role of neutral dynamics in collective modes. We list the following results:

1. we have reported the conditions under which the onset of instability is expected for the case of cold neutrals (in the absence of compressibility) in PIP;
2. neutral-induced collisional RT (NICRT) with a neutral perturbational effect – we note that the growth rate is controlled by the parameter ϵ;
3. introducing neutral perturbation in the warm neutral case we realized that the growth of PG instability could be controlled by the terms originating from neutral perturbational effects. We find that neutral diffusion could arrest the instability triggered by either plasma, neutral or magnetic PG. This result seems to be suggestive enough that the deliberate incorporation of a neutral population even in a fully ionized plasma could pacify macroscopic instabilities that have serious consequences in the field of plasma technology, such as a change in configuration of a system and mass motion. In a fully ionized plasma, RT instability has to be pacified by other means, e.g. by tailoring the desired density profile (Velikovich et al. 1996). Hence we may suggest that a small population of neutrals may serve as a novel remedy to damping macroscopic instabilities even in FIP.

A final piece of information the authors would like to share is that if we could have included viscosity in the problem one could have seen its consequences (Doughton et al. 1998). We admit that viscosity is important, no matter how small it is, because its presence may change the dynamics of the fluid. Nevertheless, concerning this paper, viscosity decreases with temperature and hence for a warm PIP its absence is justifiable. Moreover, it is well documented (Chia-Shun Yih 1980) that the presence of viscosity provides a stabilizing effect to the growth rate. Also it is known that conventional RT gives rise to a convection phenomenon which smooths out the density/temperature gradient. We admit that the analytical work carried out, here, is within the framework of MHD, which only takes care of changes at a macroscopic level, e.g. the physical displacement of magnetized PIP. However, changes in the velocity function are not included in this model and hence the model excludes this information, which may be vital in some laboratory set-up and important cometary process (Frederick et al. 1986). Also the analytical work presented here is beset with assumptions (although all the assumptions are reasonable in the context of the work presented here), which may lead to missing part of the physics involved in the problem. We conclude with a note that our endeavour to understand the role of neutral dynamics in PIP, is only a few points on the canvas of the physics of PIP and hence more work is needed with a relaxation of assumptions. However, numerical work may give a better insight into the problem, which could be the course for future work.

Acknowledgements

A.A.S. is indebted to Physical Research Laboratory for providing facilities to carry out this work and Professor P. K. Shukla for his financial support and

https://doi.org/10.1017/S0022377802001903 Published online by Cambridge University Press

encouragement to present part of this work in the meeting held during 2000 July 1–5, at the AICTP, Trieste. Also A.A.S. acknowledges Professor D. A. Mendis for his valuable suggestion regarding the probable application of this work during the meeting (2000 July) at the AICTP, Trieste.

Appendix A

A.1. Continuity equations

The linearized continuity equation for neutrals and ions are:

$$\frac{\tilde{n}_n}{n_{n0}} = -i\left(\frac{L_n^{-1}}{\omega_{cn}}\right)\tilde{V}_{nx} + \left(\frac{k}{\omega_{cn}}\right)\tilde{V}_{ny},$$

where $\omega_{cn} = \omega - kV_{n0y}$, $L_n^{-1} = \frac{1}{n_{n0}}\frac{\partial n_{n0}}{\partial x}$ and

$$\frac{\tilde{n}_i}{n_{i0}} = -i\left(\frac{L_p^{-1}}{\omega_c}\right)\tilde{V}_{ix} + \left(\frac{k}{\omega_c}\right)\tilde{V}_{iy},$$

where $\omega_c = \omega - kV_{i0y}$, $L_p^{-1} = \frac{1}{n_{i0}}\frac{\partial n_{i0}}{\partial x}$.

A.2. Plasma momentum transfer equations

X-component

$$-in_{i0}m_i\omega_c\tilde{V}_{ix} = -T\partial_x\tilde{n}_i - \tilde{n}_i m_i g - \nu_{eff}n_{i0}(\tilde{V}_{ix} - \tilde{V}_{nx})$$

where

$$\nu_{eff} = m_e\nu_{en} + m_i\nu_{in} \sim m_i\nu_{in}$$

$$\omega_c = \omega - kV_{i0y}$$

$$\omega_c\tilde{V}_{ix} = -\frac{T}{-in_{i0}m_i}\partial_x\tilde{n}_i - \frac{m_i g}{-im_i}\left(\frac{\tilde{n}_i}{n_{i0}}\right) - \frac{m_i\nu_{in}n_{i0}(\tilde{V}_{ix} - \tilde{V}_{nx})}{-in_{i0}m_i}$$

$$\omega_c\tilde{V}_{ix} = -iC_s^2\frac{1}{n_{i0}}\partial_x\tilde{n}_i - i\,g\left(\frac{\tilde{n}_i}{n_{i0}}\right) - i\nu_{in}(\tilde{V}_{ix} - \tilde{V}_{nx}),$$

where $C_s^2 = T/m_i$; $T = T_i + T_e$. For $T_i = T_e = T$, $C_s^2 = 2T/m_i$. Substituting $\tilde{n}_i/n_{i0}$, $\tilde{V}_{nx}$ and $\partial_x\tilde{n}_i$,

$$\omega_c\tilde{V}_{ix} = C_s^2\left(-\frac{in_{i0}}{\omega_c}\frac{1}{L_p^2}\tilde{V}_{ix} + \frac{k}{L_p\omega_c}\tilde{V}_{iy}\right)$$

$$-ig\left(\frac{-i}{\omega_c}\frac{1}{n_{i0}}\partial_x n_{i0}\tilde{V}_{ix} + \frac{k}{\omega_c}\tilde{V}_{iy}\right)$$

$$-i\nu_{in}\tilde{V}_{ix} + i\nu_{in}\left(i\frac{\nu_{ni}}{\omega_{nx}}\tilde{V}_{ix}\right),$$

where we have assumed $\nu_{ni} \gg \nu_{ne}$.

$$\left(\omega_c + \frac{k^2C_s^2}{\omega_c k^2L_p^2} + \frac{g}{\omega_c}\frac{k}{kL_p} + i\nu_{in} + \frac{\nu_{in}\nu_{ni}}{\omega_{nx}}\right)\tilde{V}_{ix} + \left(\frac{ik^2C_s^2}{kL_p\omega_c} + \frac{igk}{\omega_c}\right)\tilde{V}_{iy} = 0$$

$$\left(\omega_c\omega_{m1} + \frac{k^2C_s^2}{k^2L_p^2} + \frac{gk}{kL_p}\right)\tilde{V}_{ix} + \left(\frac{ik^2C_s^2}{kL_p} + igk\right)\tilde{V}_{iy} = 0, \quad \text{(A 1)}$$

https://doi.org/10.1017/S0022377802001903 Published online by Cambridge University Press

where we have simplified the first parentheses:

$$\omega_{m1} = \omega_c + \frac{i\nu_{in}}{\omega_{nx}}\omega_{cn}$$

$$\omega_{nx} = \omega_{cn} + i\nu_{ni} + i\frac{\omega_t^2}{\omega_{cn}kL_n}.$$

A.3. Y-component

$$-in_{i0}m_i\omega_c\tilde{V}_{iy} = -ik\tilde{n}_iT - \nu_{eff}n_{i0}\tilde{V}_{iy} + \nu_{eff}n_{i0}(\tilde{V}_{ny}) - (V_{i0y} - V_{n0y})(\nu_{in}m_i + \nu_{en}m_e)\tilde{n}_i,$$

where we have taken $\nu_{in}m_i \gg m_e\nu_{en}$. Now using ΔV from equilibrium, i.e.

$$\Delta V = V_{i0y} - V_{n0y} = -\frac{c}{4\pi n_{i0}e}\frac{m_e\nu_{en}}{m_i\nu_{in}}\partial_x B_0$$

$$= -\frac{V_A^2}{\Omega_e}\frac{\nu_{en}}{\nu_{in}}\frac{1}{L_B}$$

$$= -\eta_{en}\Omega_i\frac{1}{L_B}\left(\text{since } \eta_{en}\Omega_i = \frac{V_A^2}{\Omega_e}\nu_{en}\right)$$

$$(\omega_c + i\nu_{in})\tilde{V}_{iy} = i\nu_{in}\tilde{V}_{ny} + \left(kC_s^2 + i\frac{m_ec\nu_{en}}{4\pi n_{i0}em_i}\partial_x B_0\right)\left(\frac{\tilde{n}_i}{n_{i0}}\right).$$

Substituting $\tilde{V}_{ny}$ from (B4)

$$(\omega_c + i\nu_{in})\tilde{V}_{iy} = i\nu_{in}\frac{i\nu_{ni}}{\omega_{ny}}\tilde{V}_{iy} - \frac{i\nu_{ne}c}{4\pi n_{i0}e}\frac{\partial_x B_0}{\omega_{ny}}\left(\frac{\tilde{n}_i}{n_{i0}}\right) + \left(kC_s^2 + i\frac{m_ec\nu_{en}}{4\pi n_{i0}em_i}\partial_x B_0\right)\left(\frac{\tilde{n}_i}{n_{i0}}\right)$$

$$\left(\omega_c + i\nu_{in} + \nu_{in}\frac{\nu_{ni}}{\omega_{ny}}\right)\tilde{V}_{iy} = \left(\frac{\nu_{in}\nu_{ne}c}{4\pi n_{i0}e}\frac{\partial_x B_0}{\omega_{ny}} + kC_s^2 + i\frac{m_ec\nu_{en}}{4\pi n_{i0}em_i}\partial_x B_0\right)\frac{\tilde{n}_i}{n_{i0}}$$

$$\left\{\omega_c + \left[\frac{i\nu_{in}\left(\omega_{cn} - \omega_t^2/\omega_{cn}\right)}{\omega_{ny}}\right]\right\}\tilde{V}_{iy} = \left[kC_s^2 + \frac{\partial_x B_0 m_ec\nu_{en}}{4\pi n_{i0}em_i}\left(\frac{\nu_{in}}{\omega_{ny}} + i\right)\right]\left(\frac{\tilde{n}_i}{n_{i0}}\right),$$

where we have used $\nu_{en} = \nu_{ne}m_i/m_e$

$$(\omega_{mx})\tilde{V}_{iy} = \left\{kC_s^2 + \frac{V_A^2}{L_B}\frac{\nu_{en}}{\Omega_e}\left[\frac{\nu_{in} + i\left(\omega_n + i\nu_{in} - \omega_t^2/\omega_{cn}\right)}{\omega_{ny}}\right]\right\}\left(\frac{\tilde{n}_i}{n_{i0}}\right)$$

$$\omega_{mx}\tilde{V}_{iy} = \left[kC_s^2 + i\frac{kV_A^2}{kL_B}\frac{\nu_{en}}{\Omega_e}\left(\frac{\omega_{cn} - \omega_t^2/\omega_{cn}}{\omega_{ny}}\right)\right]\left(-i\frac{L_P^{-1}}{\omega_c}\tilde{V}_{ix} + \frac{k}{\omega_c}\tilde{V}_{iy}\right) - i\left(kC_s^2 + \frac{ikV_A^2}{kL_B}\frac{\nu_{en}}{\Omega_e}D_1\right)\left(\frac{L_P^{-1}}{\omega_c}\right)\tilde{V}_{ix} + \left[\frac{k}{\omega_c}\left(kC_s^2 + i\frac{kV_A^2}{kL_B}\frac{\nu_{en}}{\Omega_e}D_1\right) - \omega_{mx}\right]\tilde{V}_{iy} = 0,$$

https://doi.org/10.1017/S0022377802001903 Published online by Cambridge University Press

where $D_1 = \frac{\omega_{cn} - \omega_i^2/\omega_{cn}}{\omega_{ny}}$, $\omega_{ny} = \omega_{cn} + i\nu_{ni} - \left(\omega_t^2/\omega_{cn}\right)$

$$\left(-i\frac{k^2C_s^2}{kL_P} + \frac{k^2V_A^2}{kL_P}\frac{D_1}{kL_B}\frac{\nu_{en}}{\Omega_e}\right)\tilde{V}_{ix} + \left(k^2C_s^2 + i\frac{k^2V_A^2}{kL_B}\frac{\nu_{en}}{\Omega_e}D_1 - \omega_c\omega_{mx}\right)\tilde{V}_{iy} = 0. \quad \text{(A 2)}$$

From (A1) and (A2)

$$\det\begin{pmatrix} D_{xx} & D_{xy} \\ D_{yx} & D_{yy} \end{pmatrix} = 0$$

$$\left(-i\frac{k^2C_s^2}{kL_p} + \frac{\omega_A^2}{kL_p}\frac{D_1}{kL_B}\frac{\nu_{en}}{\Omega_e}\right)\left(i\frac{k^2C_s^2}{kL_p} + igk\right)$$

$$-\left(k^2C_s^2 + i\frac{\omega_A^2}{kL_B}\frac{\nu_{en}}{\Omega_e}D_1 - \omega_c\omega_{mx}\right)\left(\omega_c\omega_{m1} + \frac{k^2C_s^2}{k^2L_p^2} + \frac{gk}{kL_p}\right) = 0$$

$$-i\frac{\omega_A^2}{kL_B}\frac{\nu_{en}}{\Omega_e}D_1\omega_c\omega_{m1} - k^2C_s^2\omega_c\omega_{m1} + \omega_c^2\omega_{m1}\omega_{mx} + \frac{\omega_c\omega_{mx}}{kL_P}\left(\frac{k^2C_s^2}{kL_P} + gk\right) = 0$$

$$-k^2C_s^2\omega_{m1} - i\frac{\omega_A^2}{kL_B}\frac{\nu_{en}}{\Omega_e}D_1\omega_{m1} + \omega_c\omega_{m1}\omega_{mx} + \frac{\omega_{mx}}{kL_p}\left(\frac{k^2C_s^2}{kL_p} + gk\right) = 0. \quad \text{(A 3)}$$

This is the general dispersion relation for the electrostatic mode in a partially ionized, inhomogeneous plasma. Here we consider inhomogeneities in a plasma, neutral and magnetic field with neutral dynamics.

Appendix B

B.1. Neutral momentum transfer equation

$$D_t\mathbf{V}_n = -\nu_{in}(\mathbf{V}_n - \mathbf{V}_i) - \nu_{ne}(\mathbf{V}_n - \mathbf{V}_e) - \frac{\nabla P_n}{n_n m_n} + \mathbf{g}$$

$$= -\nu_{ni}(\mathbf{V}_n - \mathbf{V}_i) - \nu_{ne}\left(\mathbf{V}_n - \mathbf{V}_i + \frac{\mathbf{J}}{n_i e}\right) - \frac{\nabla P_n}{n_n m_n} + \mathbf{g}$$

$$D_t\mathbf{V}_n = -(\mathbf{V}_n - \mathbf{V}_i)(\nu_{ni} + \nu_{ne}) - \frac{\nu_{ne}\mathbf{J}}{n_{ie}} - \frac{\nabla P_n}{n_n m_n} + \mathbf{g}$$

$$= -(\mathbf{V}_n - \mathbf{V}_i)(\nu_{ni} + \nu_{ne}) - \frac{\nu_{ne}c}{4\pi n_{ie}}\nabla \times \mathbf{B} - \frac{\nabla P_n}{n_n m_n} + \mathbf{g},$$

where we have used $\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J}$, $\mathbf{J} = n_i e(\mathbf{V}_i - \mathbf{V}_e)$. It is clear that the introduction of the magnetic field in the neutral momentum is due to the frictional forces of the neutral with either species. The neutral momentum equation can be linearized with a perturbation $\propto\ i(ky - \omega t)$.

X-component

$$\frac{\partial \mathbf{V}_n}{\partial t} + [(\mathbf{V}_n \cdot \nabla)\mathbf{V}_n]_x = -(\mathbf{V}_n - \mathbf{V}_i)_x(\nu_{ni} + \nu_{ne}) - \frac{\nu_{ne}c(\nabla \times \mathbf{B})_x}{4\pi n_i e} - \frac{\nabla_x P_n}{m_n n_n} - \mathbf{g},$$

https://doi.org/10.1017/S0022377802001903 Published online by Cambridge University Press

$$-i\omega\tilde{V}_{nx} + V_{n0y}ik\tilde{V}_{nx} = -(\nu_{ni}+\nu_{ne})\,\tilde{V}_{nx} + (\nu_{ni}+\nu_{ne})\,\tilde{V}_{ix}$$

$$-\frac{\nu_{ne}c}{4\pi n_{i0}e}\left(1-\frac{\tilde{n}_i}{n_{i0}}\right)\left[\partial_y(B_{0z}+\tilde{B}_z)\right] - \frac{\nabla_x\tilde{P}_n}{m_n n_n},$$

$$-i\omega\tilde{V}_{nx} + V_{n0y}ik\tilde{V}_{nx} + (\nu_{ni}+\nu_{ne})\,\tilde{V}_{nx}$$

$$= (\nu_{ni}+\nu_{ne})\,\tilde{V}_{ix} - \frac{\nu_{ne}c}{4\pi n_{i0}e}\partial_y\tilde{B}_z - v_{nt}^2\frac{\nabla_x n_{n0}}{n_{n0}}\left(\frac{\tilde{n}_n}{n_{n0}}\right),$$

$$-i\tilde{V}_{nx}\left[\omega_{cn} + i(\nu_{ni}+\nu_{ne})\right]$$

$$= (\nu_{ni}+\nu_{ne})\,\tilde{V}_{ix} - \frac{ik\nu_{ne}c}{4\pi n_{i0}e}\tilde{B} - v_{nt}^2\frac{iL_n^{-2}}{\omega_{cn}}\tilde{V}_{nx} + \frac{k^2v_{nt}^2}{kL_n\omega_{cn}}\tilde{V}_{ny},$$

$$-i\left[\omega_{cn} + i\,(\nu_{ni}+\nu_{ne}) - \frac{k^2V_{nt}^2}{k^2L_n^2}\frac{1}{\omega_{cn}}\right]\tilde{V}_{nx} = (\nu_{ni}+\nu_{ne})\,\tilde{V}_{ix} - \frac{ik\nu_{ne}c}{4\pi n_{i0}e}\tilde{B} + \left(\frac{\omega_t^2}{kL_n}\right)\tilde{V}_{ny}$$

$$\tilde{V}_{nx} = \frac{i\nu_{ni}}{\omega_{nx}}\tilde{V}_{ix} + \frac{k\nu_{ne}c}{4\pi n_{i0}e}\frac{\tilde{B}}{\omega_{nx}}\hat{e}_x + \left(\frac{i\omega_t^2}{kL_n\omega_{cn}}\right)\frac{\tilde{V}_{ny}}{\omega_{nx}}\hat{e}_x, \tag{B1}$$

where $\nu_{ni} \gg \nu_{ne}$ and $\omega_{nx} = \omega_{cn} + i\nu_{ni} - \frac{\omega_t^2}{(kL_n)^2}\frac{1}{\omega_{cn}}$.

Y-component of the neutral momentum equation

$$\partial_t\tilde{V}_{ny} + V_{n0y}\partial_y\tilde{V}_{ny} = -\nu_{ni}(\tilde{V}_{ny}) + \tilde{V}_{iy}\nu_{ni} - \frac{\nu_{ne}c}{4\pi n_{i0}e}\left(\frac{\tilde{n}_i}{n_{i0}}\partial_x B_{0z}\hat{e}_y\right)$$

$$-\frac{k^2v_{nt}^2}{kL_n\omega_{cn}}\tilde{V}_{nx} - \frac{ik^2v_{nt}^2}{\omega_{cn}}\tilde{V}_{ny},$$

where we have assumed $\nu_{ni} > \nu_{ne}$

$$-i\left(\omega_{cn} + i\nu_{ni} - \frac{k^2v_{nt}^2}{\omega_{cn}}\right)\tilde{V}_{ny} = \nu_{ni}\tilde{V}_{iy} - \frac{\nu_{ne}c}{4\pi n_{i0}e}\partial_x B_0\left(\frac{\tilde{n}_i}{n_{i0}}\right) - \frac{k^2v_{nt}^2}{kL_n\omega_{cn}}\tilde{V}_{nx}$$

$$\tilde{V}_{ny} = i\frac{\nu_{ni}}{\omega_{ny}}\tilde{V}_{iy} - \frac{i\nu_{ne}c}{4\pi n_{i0}e}\frac{\partial_x B_0}{\omega_{ny}}\left(\frac{\tilde{n}_i}{n_{i0}}\right) - i\frac{k^2v_{nt}^2}{\omega_{ny}kL_n\omega_{cn}}\tilde{V}_{nx}. \tag{B2}$$

Substituting (B2) in (B1) we obtain

$$\tilde{V}_{nx} = \left(\frac{i\nu_{ni}}{\omega_{nx}D_2} - i\frac{f_x}{\omega_c}\frac{1}{kL_p}\right)\tilde{V}_{ix} - \left(\frac{\omega_t^2\nu_{ni}}{\omega_{nx}\omega_{ny}\omega_{cn}D_2kL_n} + \frac{f_x}{\omega_c}\tilde{V}_{iy}\right), \tag{B3}$$

where $f_x = k\Delta V\nu_{in}\frac{kL_n}{\omega_t^2}\omega_{cn}$, $\omega_{ny} = \omega_{cn} + i\nu_{ni} - \frac{k^2v_{nt}^2}{\omega_{cn}}$.

Now substituting (B3) in (B2) we have,

$$\tilde{V}_{ny} = \left(\frac{i\nu_{ni}}{\omega_{ny}D_2} - i\frac{f_y}{\omega_c}\right)\tilde{V}_{iy} + \left(\frac{\omega_t^2\nu_{ni}}{kL_n\omega_{cn}\omega_{nx}\omega_{ny}D_2} - \frac{f_y}{kL_p\omega_c}\right)\tilde{V}_{ix}, \tag{B4}$$

where

$$f_y = \frac{\nu_{in}k\Delta V(kL_n)^2\,\omega_{nx}\omega_{cn}^2}{\omega_t^4}$$

https://doi.org/10.1017/S0022377802001903 Published online by Cambridge University Press

$$D_2 = 1 - \frac{\omega_t^4}{(kL_n)^2 \omega_{cn}^2 \omega_{nx} \omega_{ny}}.$$

B.2. Plasma momentum equation

X-component

$$f_y = \frac{\nu_{in} k \Delta V (kL_n)^2 \omega_{nx} \omega_{cn}^2}{\omega_t^4}$$

$$D_2 = 1 - \frac{\omega_t^4}{(kL_n)^2 \omega_{cn}^2 \omega_{nx} \omega_{ny}}.$$

B.3. Plasma momentum equation

X-component

$$\omega_c \tilde{V}_{ix} = -ig\left(-i\frac{1}{\omega}\frac{1}{L_p}\tilde{V}_{ix} + \frac{k}{\omega}\tilde{V}_{iy}\right) - i\nu_{in}\tilde{V}_{ix} + i\nu_{in}\left(\frac{i\nu_{ni}}{\omega_{nx}D_2} - \frac{if_x}{\omega_c k L_p}\right)\tilde{V}_{ix}$$
$$+ i\nu_{in}\left(-\frac{\omega_t^2\nu_{ni}}{\omega_{nx}\omega_{ny}D_2 k L_n} + \frac{f_x}{\omega_c}\right)\tilde{V}_{iy} + \frac{i\nu_{in}k\nu_{ne}c}{4\pi n_{i0}e\omega_{nx}}\frac{B_0}{D_2} + \frac{\tilde{B}}{B_0}\hat{e}_x.$$

For the electrostatic case with warm neutrals we obtain

$$\left(-\omega_c - \frac{kg}{\omega_c k L_p} - i\nu_{in} - f\nu_{in}\nu_{ni}\omega_{nx}D_2 + \frac{f_x}{kL_p\omega_c}\nu_{in}\right)\tilde{V}_{ix}$$
$$+ \left(-\frac{igk}{\omega_c} - \frac{\omega_t^2\nu_{ni}}{\omega_{nx}\omega_{ny}D_2 k L_n} + \frac{if_x\nu_{in}}{\omega_c}\right)\tilde{V}_{iy} = 0. \tag{B5}$$

Y-component

$$\omega_c \tilde{V}_{iy} = kC_s^2\left(-i\frac{L_p^{-1}}{\omega_c}\tilde{V}_{ix} + \frac{k}{\omega_c}\tilde{V}_{iy}\right) - i\nu_{in}\tilde{V}_{iy}$$
$$+ i\nu_{in}\left(\frac{\omega_t^2\nu_{ni}}{kL_n\omega_{cn}\omega_{nx}\omega_{ny}D_2}\tilde{V}_{ix} - \frac{f_y}{kL_p\omega_c}\tilde{V}_{ix} + \frac{i\nu_{ni}}{\omega_{ny}D_2}\tilde{V}_{iy} + \frac{if_y}{\omega_c}\tilde{V}_{iy}\right)$$
$$+ i\nu_{ni}\left[-\frac{i\omega_t^2 k\nu_{ne}cB_0}{kL_n(4\pi n_{i0}e)\omega_{cn}\omega_{nx}\omega_{ny}D_2}\right]\frac{\tilde{B}}{B_0}.$$

For the electrostatic case with warm neutrals, we obtain

$$\left(-\frac{ik^2C_s^2}{kL_p\omega_c} + \frac{i\omega_t^2\nu_{in}\nu_{ni}}{kL_n\omega_{cn}\omega_{nx}\omega_{ny}D_2} - \frac{if_y\nu_{in}}{kL_p\omega_c}\right)\tilde{V}_{ix}$$
$$+ \left[-\omega_c + \frac{k^2C_s^2}{\omega_c} - \frac{\nu_{in}\nu_{ni}}{\omega_{ny}D_2} - i\nu_{in} - i\frac{f_y}{\omega_c}(i\nu_{in})\right]\tilde{V}_{iy} = 0. \tag{B6}$$

https://doi.org/10.1017/S0022377802001903 Published online by Cambridge University Press

B.4. Magnetic field evolution

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{V}_i \times \mathbf{B}) - \frac{m_i c}{e}\nabla \times \frac{d\mathbf{V}_i}{dt} - \frac{m_i c}{e}\nu_{in}\nabla \times (\mathbf{V}_i - \mathbf{V}_n)$$
$$+ \eta_{ei}\nabla^2 B + \frac{m_i c}{e}(\nabla \times \mathbf{g})$$
$$-i\omega\tilde{B}_z = -V_{i0y}\partial_y \tilde{B}_z - (\partial_x B_{0z})\tilde{V}_{ix} - B_{0z}\partial_y \tilde{V}_{iy}$$
$$+ \frac{m_i c}{e}\partial_y \frac{d}{dt}\tilde{V}_{ix} + \frac{m_i c}{e}\nu_{in}\partial_y(\tilde{V}_{ix} - \tilde{V}_{nx}) + \eta_{ei}\nabla^2 \tilde{B}_z,$$

where we have assumed that there is no velocity gradient (shear).

$$\tilde{B}_z\left(-i\omega + ikV_{i0y} + \eta_{ei}k_y^2\right) = -\tilde{V}_{ix}\partial_x B_{0z} - ik\tilde{V}_{iy}B_{0z} + \frac{m_i c}{e}ik(-i\emptyset)\tilde{V}_{ix}$$
$$+ i\frac{m_i c}{e}\nu_{in}k(\tilde{V}_{ix} - \tilde{V}_{nx})$$
$$\tilde{B}_z\left[-i\left(\omega_m\right)\right] = -\tilde{V}_{ix}\partial_x B_{0z} - ikyB_{0z}\tilde{V}_{iy} + \frac{m_i c}{e}k(\omega - kV_{i0y} + i\nu_{in})\tilde{V}_{ix}$$
$$- i\frac{m_i c}{e}\nu_{in}k\tilde{V}_{nx}.$$

Substituting (B3) we have

$$\frac{\tilde{B}_z}{B_0} = \frac{1}{\omega_m}\left[-iL_B^{-1} + \frac{ik}{\boldsymbol{\Omega}_i}(\omega_c + i\nu_{in}) - \frac{\nu_{in}}{\boldsymbol{\Omega}_i}k\left(\frac{i\nu_{ni}}{\omega_{nx}D_2} - \frac{f_x}{\omega_c k L_p}\right)\right]\tilde{V}_{ix}$$
$$+ \frac{1}{\omega_m}\left[k - \frac{\nu_{in}}{\boldsymbol{\Omega}_i}k\left(\frac{\omega_t^2}{\omega_{nx}\omega_{ny}\omega_{cn}D_2 k L_n} + \frac{f_x}{\omega_c}\right)\right]\tilde{V}_{iy}, \tag{B7}$$

where

$$\omega_m = \omega_c + ik_y^2\eta_{ei} - \frac{c^2k^2\nu_{in}\nu_{ne}}{\omega_{pe}^2\omega_{nx}D_2}.$$

References

Chia-Shun, Yih. 1980 *Stratified Flows*. New York: Academic Press, Ch. 4.

Doughton, W., Peter, J., Coppi, B. and Kraheninnikov, S. 1998 Instabilities in a partially ionized plasma. *Phys. Plasma* **5**, 2217.

Dwivedi, C. B. and Das, A. C. 1992 Neutral induced low frequency instability. *Planet. Space Sci.* **40**, 1197.

Frederick, L. et al. 1986 Plasma wave observations at Comet Giacobini–Zinner. *Science* **232**, 377.

Huba, J. D. 1990 Universal interchange instabilities in partially ionized gases. *Phys. Fluids* **B2**, 2547.

Huba, J. D. 1991 Theory and simulation of high frequency magnetic drift wave. *Phys. Fluids* **B3**, 3217.

Huba, J. D., Grossmann, J. M. and Ottinger, P. F. 1994 Hall MHD modeling of long conduction time plasma opening switch. *Phys. Plasma* **1**, 3444.

Huba, J. D. 1995 Hall MHD in space and laboratory plasmas. *Phys. Plasma* **2**, 2504.

Kelley, M. C. 1989 *The Earth's Ionosphere*. New York: Academic Press.

Mendis, D. A. 1988 A post-encounter view of comet. *Ann. Rev. Astron. Astrophys.* **26**, 11–49.

https://doi.org/10.1017/S0022377802001903 Published online by Cambridge University Press

Liewer, P. C., Karmesin, S. R. and Brackbill, J. W. 1996 Hydrodynamic instability of the heliopause driven by plasma-current charge exchange interactions. *J. Geophys. Res.* **101**, 17 119–17 127.

Rishbeth, H. and Garriott, O. 1969 *Introduction to Ionospheric Physics*. New York: Academic Press, p. 130.

Shaikh, A. A. and Das, A. C. 1997 Effects of neutral dynamics in a partially ionized plasma. *Adv. Space Res.* **20**, 493.

Shaikh, A. A. and Das, A. C. 1998 Effects of neutral dynamics on electrodynamics modes in partially ionized plasma. *Physica Scripta* **T75**, 249.

Shaikh, A. A., Das, A. C. and Agrawal, B. P. 1999 Neutral dynamics and high frequency electromagnetic modes in partially ionized plasma. *Physica Scripta* **59**, 458.

Velikovich, A. L., Cochran, F. L. and Davis, J. 1996 Suppression of Rayleigh–Taylor instability in Z-pinch loads with tailored density profiles. *Phys. Rev. Lett.* **77**, 853–856.

Zweibel, E. 1989 Magnetic reconnection in partially ionized gases. *Astrophys. J.* **340**, 550.

https://doi.org/10.1017/S0022377802001903 Published online by Cambridge University Press