Conclusions

Clearly these methods, incorporating numerical differentiation, are effective in reducing the number of ordinates to be calculated and thereby reducing or eliminating the effects of cumulative rounding errors.

The relative simplicity of R_n^{+2} given by (5) and its superiority over T_n, R_n, T_n^{+2} and $S_{\text{even } n}$ does have some appeal particularly for use on a programmable calculator. On a spreadsheet, however, by writing simple Macros to calculate T_n, R_n and $\delta(\varepsilon)$ the best method involves finding both $T_n'(\varepsilon)$ and $R_n'(\varepsilon)$ before combining them into $S_{2n}'(\varepsilon)$.

The choice of ε depends on the rule employed and the calculating device to be used. Finding Δ_{2p-1} working to m significant figures, in order to balance the mathematical error of the approximate differentiation with the numerical error of inexact arithmetic, we need $\varepsilon = 10^{-k}$ where k satisfies $m \approx (2p + 1)k$.

The ability to calculate the Δ_{2p-1} terms allows for an accurate prediction of the number of intervals needed to achieve a specified error. This means a single calculation of the integral rather than a search for consistency with increasing n. This could be particularly useful in accurate calculation of functions which are only defined as a definite integral.

Finally, it must be stressed, that these methods are dependent on the integrand being 'well behaved'. If not, they are unlikely to offer improvement on the basic methods of T_n or R_n with their combination as described in [2].

References

1. A. Ralston, *A first course in numerical analysis*, McGraw-Hill, (1965) pp.133, 156.
2. Robin Murphy, Numerical integration on a spreadsheet, *Math. Gaz.* **95** (November 2011) pp. 528-532.

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103.46 A four-hundred year calendar

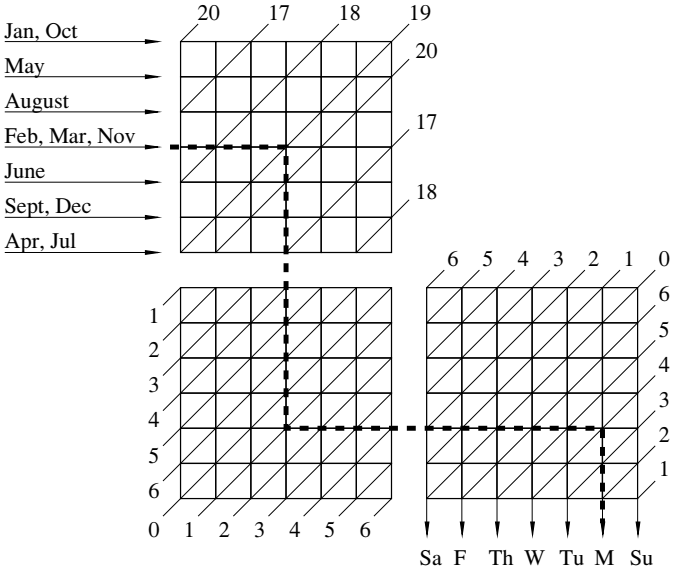
The relationships between days and dates, in the form of calendars, tables, schedules, etc., figure prominently in our management of affairs, both domestically and nationally. A new version of these relationships, valid from the introduction of the Gregorian calendar in 1752, is presented in this Note in the form of a nomogram, the use of which is exemplified.

The objective in constructing the nomogram (Figure 1) was to develop a means whereby, given the date in the conventional form of date, month and year (e.g., 18th June 1815), the corresponding day of the week could be found. The starting point for the writer's interest in the topic was a short paper by Lewis Carroll, published in the journal *Nature* [1].

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The date is considered in four parts, viz. the century number (18 in the example above), the year (15), the month (June) and the date itself (18). (Note: The distinction between the century number (eg 18) and the century itself (the 19th) is to be noted.) The diagonal lines in the first grid of the nomogram are labelled with the century numbers 17, 18, 19, 20 which cover the 400-year day/date repeat cycle. (The century numbers given (17, 18, 19, 20) could be replaced with any similar four-century block eg 21, 22, 23, 24; 25, 26, 27, 28; and so on without affecting the validity of the nomogram.)

The diagonal lines in the second grid are labelled 0 to 6. For a year Y (< 100) these numbers are obtained as $(5Y/4)$ (i.e., the number of 4s in the quantity 5 times Y) less multiples of 7. (These numbers repeat every 28 years. For convenience values of $(5Y/4)$ for Y from 0 to 27 are shown in the inset in Figure 1.)



Y	0	1	2	3	4	5	6	7	8	9	10	11	12	13
$(5Y/4)$	0	1	2	3	5	6	0	1	3	4	5	6	1	2

Y	14	15	16	17	18	19	20	21	22	23	24	25	26	27
$(5Y/4)$	3	4	6	0	1	2	4	5	6	0	2	3	4	5

FIGURE 1: The nomogram and the Y to $(5Y/4)$ conversion table

The diagonal lines in the third grid are also labelled 0 to 6. These numbers are the date, D, less multiples of 7 (or mod (D, 7) i.e. the remainder after dividing D by 7). The vertical lines of this grid are extended to give the days of the week in succession, beginning on the right with Sunday (Su) and continuing though to Saturday (Sa).

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In using the nomogram for the determination of the day of a given date, the entry is via the month, as arranged in the vertical column to the left of the first grid. From the appropriate month, one moves to the right into the first grid of the nomogram onto the diagonal labelled with the appropriate century number (17, 18, 19, 20). From this point one moves vertically downwards into the second grid and onto the diagonal labelled with the appropriate $(5Y/4)$ value. From here one continues horizontally into the third grid onto the appropriate D diagonal and thence downwards to identify the required day.

As an example a broken line in the nomogram traces the determination of 11th November 1918 as a Monday.

The nomogram can be seen as a once-and-for-all calendar, valid for any year of the 400 year repeat cycle of the day/date system, with the proviso that for the months of January and February in a Leap Year the following corrections are made:

(i) If the day of a specified date is required, then the derived day should be moved one to the right.

e.g. What is the day of the 1st of January 2020? The nomogram gives the day as Thursday. The corrected day is Wednesday.

(ii) If the date of a specific day is being sought then 1 should be added to the derived date.

e.g. What is the date of the first Sunday in January 2024? The nomogram yields the date as 6. The necessary correction changes this to 7.

Examples of the use of the nomogram in ways other than the month-to-day/date route follow.

1. In what years of the present century will Christmas Day be a Sunday?

   One enters the nomogram at Sunday and moves up to diagonal 4 $(25 - 3 \times 7 = 4)$, and thence across into the second grid. A line from December is brought onto diagonal 20 in the first grid and down into the second grid. The two lines in the second grid intersect on diagonal 6, corresponding to years 2005, 2011, 2016 and 2022. This block increases by 28 through the century to give finally 2089 and 2095.

2. In what month(s) of 2025 will the 13th be a Friday?

   One enters the nomogram at Friday and moves up to diagonal 6 $(13 - 7 = 6)$, then across to diagonal 3 in the second grid (see inset in Figure 1 for year 25). From there one moves up to diagonal 20 and across from there to give the month of June.

Reference

1. L. Carroll, To find the day of the week for any given date, *Nature* 1887, March 31st, p. 517.

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