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JUDGMENT AGGREGATION AND SUBJECTIVE DECISION-MAKING*

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I present an original model in judgment aggregation theory that demonstrates the general impossibility of consistently describing decision-making purely at the group level. Only a type of unanimity rule can guarantee a group decision is consistent with supporting reasons, and even this possibility is limited to a small class of reasoning methods. The key innovation is that this result holds when individuals can reason in different ways, an allowance not previously considered in the literature. This generalizes judgment aggregation to subjective decision situations, implying that the discursive dilemma persists without individual agreement on the logical constraints. Notably, the model mirrors the typical method of choosing political representatives, and thus suggests that no voting procedure other than unanimity rule can guarantee representation that reflects electorate opinion. Finally, I apply the results to a normative argument for unanimity rule in contract theory and juries, as well as to problems posed for deliberative democratic theory and the concept of representation.

1. INTRODUCTION

Can groups make reasoned decisions? Can a group itself have a belief or make an argument? The sentiment is common enough. We think nothing of hearing that a legislature decided on a policy because it believed *A*, *B* and *C*, or that voters re-elected a Senator because of electorate opinion

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on education and defence. Logical deduction at the group level is equally commonplace.

Using the framework of judgment aggregation (JA) theory, I argue that it is generally impossible to guarantee a group will be logically consistent in the domain of decision-making, hence we cannot rationalize decisions purely at the group level. What I take to be special about decision-making, and where I depart from the existing JA literature, is that individuals may disagree on how a decision is logically connected to other considerations. Even allowing for this subjective diversity, I prove it is generally impossible to derive group decisions from group opinions on supporting reasons. We must either abandon the aim of describing group decisions in terms of reasoned arguments or accept the possibility of group inconsistency.

The subjective group-decision model considered here is more flexible and realistic than the fixed logical frame assumed by existing JA theory. In particular, the model applies to political representation. An individual's vote choice may flow logically from independent issue opinions, but not necessarily in an objective and universal manner. Some in the electorate value education more than defence, and others the opposite. A pressing question is whether, given these individual opinions and votes, we can define *group* issue opinions consistent (by some given rule) with a *group* choice of representative. Whereas existing JA theory cannot deal with such subjectivity, this paper's model demonstrates that consistency generally cannot be guaranteed. Not only may a representative disagree on some group opinions, but the set of group opinions may also imply that a different representative should have been chosen.

Section 1 reviews past results in JA theory, then describes my model and how it relates to this literature. Some concrete examples of the model are given. Section 2 rigorously presents the model and several results. I have structured the paper so it is possible to skip Section 2 and still understand the central argument. Section 3 analyses the results and applies them to democratic and political theory. I present a normative argument for unanimity rule in the social contract and jury trials, and illustrate problems for deliberative democratic theory and political representation. Section 4 suggests future work that can expand the model and analysis.

1.1. Past results

The central insight of JA theory is the difficulty of translating sets of logically interconnected individual judgments into a set of logically consistent group judgments. The social representation focus is similar to that of preference aggregation theory, and several papers have reproduced major preference aggregation results within a JA framework (List and Pettit, 2004; Dokow and Holzman, 2005; Dietrich and List, 2007a). However, whereas a narrow preference focus can be questioned because

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	P_1	P_2	$P_1 \wedge P_2$
Individual 1	T	T	T
Individual 2	T	F	F
Individual 3	F	T	F
Majority	T	T	F

TABLE 1. The doctrinal paradox/discursive dilemma. Each individual is consistent, but the majority is not.

of its connection to the rational choice frame, the relevance of JA is not so easy to escape. Since it operates in the far more general domain of logical beliefs instead of preferences, JA can capture a wider range of group relationships and behaviour. This paper concentrates on subjective group decision-making; previous work has had more of a pure judgment focus.

A curious paradox in legal theory called the "doctrinal paradox" provided some initial inspiration (Kornhauser and Sager 1986; Brennan 2001). Suppose a panel of three judges is deciding whether "Mary is guilty of slander". To vote guilty, a judge must find that "Mary lied" (proposition P_1) *and* that "Mary intentionally defamed" (proposition P_2). Each judge applies True or False to each of the three propositions. As Table 1 demonstrates, if we aggregate these judgments to a group judgment by majority vote, the panel's judgments may be logically inconsistent. Pettit (2001) refers to the general problem of group inconsistency under majority voting on interrelated propositions as the "discursive dilemma". List and Pettit (2002) arrived at the first systematic result in JA theory with an impossibility result focused on the doctrinal paradox. They show that no aggregation rule can guarantee *consistency* in the group result under the assumptions of *universal domain* (individuals are allowed to hold any logically consistent set of beliefs), *systematicity* (a proposition's group judgment depends only on the proposition's individual judgments, with the same dependence for different propositions and for and against a proposition), and *anonymity* (equal treatment of different individuals).

From this beginning, JA theory has retained the basic framework of aggregating from individual to group judgments on separate logical propositions, but has rapidly broadened its scope along two dimensions. First, the set of logical propositions being aggregated, called the *agenda,* has been generalized. Instead of referring to a specific agenda, most results now apply to all agendas meeting certain logical conditions, such as *atomic closedness* (Pauly and van Hees, 2006), *strong connectedness* (Dietrich and List, 2007a), and *total blockedness* (Nehring and Puppe, 2007). The

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field has also been extended beyond propositional logic to more general logics (Pauly and van Hees, 2006; Dietrich, 2007; van Hees, 2007). Recent work has relaxed *completeness*, the assumption that each individual judges every proposition in the agenda (Gärdenfors, 2006; Dietrich and List, forthcoming).

Second, the conditions placed on the aggregation rule have varied. For instance, the original List and Pettit (2002) result can replace anonymity with *non-dictatorship* (the group judgments do not always agree with a single individual) and retain impossibility (Pauly and van Hees, 2006). In most impossibility results, the strong systematicity condition used in List and Pettit (2002) is weakened to *independence* (the group judgment on a proposition depends only on the proposition's individual judgments) by dropping the requirement of equal treatment of different propositions. Variations on independence have been particularly diverse (Dietrich, 2006). For general discussions, see Chapman (2002), List (2006), and List and Puppe (forthcoming).

Despite these generalizations, the literature has retained the assumption that the logical connections among propositions are constant across individuals. The individual faces a fixed logical frame and, for each proposition, judges it or chooses not to (if we relax completeness). Although sensible for a purely logical focus, it is a substantial assumption of homogeneity when we think about realistic group decisions. Much of the complexity in group decision-making arises because members disagree about the importance of different considerations. In this study, I broaden the existing JA approach by allowing for distinct individual reasoning.[1] Each individual is allowed her own understanding of how a decision logically relates to other propositions.

1.2. A new model

I now introduce the basic elements of the model. In brief, each individual judges a decision based on her judgments on independent considerations, or premises, and individuals may do so in different ways. If we then aggregate to group judgments on each premise and the decision, it is generally not possible to describe the group decision as derived from group judgments. Expressed in the usual tabular form (e.g. Table 1), decisions are horizontal, and aggregation is vertical. The impossibility of reasoned group

[1] One might surmise this is equivalent to a normal JA agenda that includes all possible biconditionals connecting propositions and requires the choice of exactly one, so that an individual can affirm the connection she likes and satisfy "individual reasoning" in that way. However, under independence, the group's affirmed biconditional is then required to be a proposition-wise function of the individuals' affirmed biconditionals, which is a considerable restriction. I assume no dependence of the group "decision rule" on the individual decision rules. I thank Franz Dietrich for clarity on this point.

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decision-making concerns the potential disagreement between horizontal and vertical implications. I introduce two distinct ways of looking at this impossibility, and conclude with three illustrative examples.

What is the usual method by which decisions are made? Frequently, there is a substantive Yes/No decision under consideration, such as whether to build a bridge across the Hudson River or not, to elect Grover Cleveland or another man, or to declare war or sue for peace. Such central decisions are typically based on ancillary premises that together determine the choice of action. In the last case, they may include judgments on whether the war can be won, whether victory would be worth the cost, and so on. In particular, this structure parallels how legislators decide policy, how voters elect representatives, and how financial experts invest.

In this paper's model, each individual judges a central decision as a function of her judgments on several premises. This premise-decision structure has been investigated in Nehring and Puppe (forthcoming) and Dokow and Holzman (2005), the former of which calls it the "truth-functional model". However, both papers assume there exists a fixed logical frame.[2] The key innovation of this paper's model is that the function, or decision rule, connecting the premises and decision may differ between individuals. In this way, the model can capture both objectively logical and subjective decisions. For example, suppose a legislator is deciding whether to support funding for stem cell research. She may begin by considering whether the research will cure certain diseases, if it involves the taking of potential life, and other relevant questions. But not only are the judgments themselves a matter of opinion, there is also no clear rule for which sets of judgments should lead to support for stem cell research. That, too, is subjective, and the choice of connection is embodied in the individual decision rule. This generalization considerably expands the territory of JA, and the examples below explore some of the new avenues.

Next, we aggregate the individual judgments to a group judgment on each premise and the decision.[3] The central question is: Does the group itself obey a decision rule? In other words, can we describe the group decision as a function of group beliefs? The surprising answer is that, in general, we cannot. Generally, no decision rule exists that will always be followed by the group. Assuming overlap in the premises considered by different individuals, the only exception is if we aggregate by a type

[2] As a warm-up to my main results, Theorem 1 parallels conclusions reached in these papers by assuming a common decision rule.

[3] I put very minimal restrictions on this aggregation rule, with no strong assumptions like anonymity or systematicity. These restrictions are: (1) Universal Domain, (2) Independence, (3) Non-triviality: The group decision is not fixed, and (4) Non-authoritarianism: A single individual does not determine the group judgments.

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of unanimity rule *and* every decision rule is such that each premise can "veto" the decision.[4] A group aggregating by majority rule cannot follow a group decision rule. Hence, assuming all individuals reason in the same way is not necessary for the discursive dilemma to apply.

This general impossibility can be interpreted in two distinct ways:

Interpretation 1: Given any aggregation rule, there generally does not exist any decision rule guaranteed to be consistent with the group judgments. For each decision rule, some set of individual judgments will violate the rule after being aggregated. By this interpretation, we do not need to wonder where the group decision rule comes from, since no such rule can exist. Absent a group decision rule, there is no way to specify how group premise judgments lead to the group decision. We are confronted with the general impossibility of reasoned group decision-making.

Interpretation 2: Given any group decision rule, there generally does not exist any aggregation rule that can guarantee the group judgments will be consistent with the decision rule. This may be the favoured interpretation if a formal or sensible group standard exists or the decision *is* a matter of objective logic. Should such a group standard exist, the individuals will not necessarily follow it. The third example presents such a case.

I now illustrate the model and its implications with three examples. To demonstrate its versatility, I consider in turn a legislature, an electorate and a team of financial experts.

Example: Political decision-making

Consider a legislature deciding on a bill supporting stem cell research. As noted before, individual legislators may disagree about *how* to decide their vote, not just what that vote will be. To capture this situation, the model imagines that each legislator bases her vote (the decision) on her judgments on several premises, with the connection between premises and vote expressed through her decision rule. Suppose we then aggregate to group opinions on each premise and to a group vote, which represents the passage or defeat of the bill. Table 2 displays an example of such a legislature (using majority rule for aggregation). In this example, the bill is defeated.

It is sensible to then ask whether the legislature can provide a reasoned argument for its decision. In other words, is there a group decision rule that can connect the group premise judgments to the legislative outcome? As Interpretation 1 above indicates, the general answer is no. If the aggregation rule is by majority, for example, there exists no group decision

[4] There exists one other normatively dubious combination, the even/odd aggregation rule and even/odd decision rule, but this can be passed over for now.

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Premises	P_1	P_2	P_3	P_4	Decision Rule	Q = Vote
Voter 1	T	T	T	F	$Q \Leftrightarrow \neg(P_1 \wedge P_2)$	F
Voter 2	F	F	T	T	$Q \Leftrightarrow \neg P_1 \wedge P_3 \wedge P_4$	T
Voter 3	T	T	F	T	$Q \Leftrightarrow P_3 \wedge P_4$	F
Legislature	T	T	T	T	$Q \Leftrightarrow ?$	F

TABLE 2. A legislature, deciding on a bill supporting stem cell research. P_1 = "The research (R) will destroy embryos", P_2 = "Embryos should be considered life", P_3 = "R holds promise for curing disease", P_4 = "R is cost-effective", and the vote Q = "R should be supported".

rule guaranteed to be consistent. For the Table 2 example, we can of course devise a group decision rule consistent for *this set* of judgments. However, this rule cannot describe group reasoning since the group would violate this rule for a different set of consistent individual judgments. If some individuals changed their opinions, a group contradiction may result. Hence, we cannot say the group decides for group reasons. The implication of this conclusion for deliberative democratic theory is explored in Section 3.2.

Example: Political representation

This paper's connection to political representation is brought out by a somewhat obscure result called the Ostrogorski Paradox. The paradox concerns a set of voters who each favour one of two parties on different issues. Suppose that each person votes for the party he or she favours on the majority of issues. Suppose further that we define the group opinion on each issue and the vote winner by majority rule. The resulting paradox is the possibility that the group will favour the losing party on a majority of issues, or even *all* issues, meaning that an unpopular platform will win (Rae and Daudt, 1976; Nurmi, 1998).[5]

In a more general Ostrogorski model, any rule could be applied to either the decision of how to vote or the aggregation of group opinion.[6] Despite the absence of truth values, any rule indicating how a person votes based on his issue opinions can be captured by an individual decision rule. It follows that the generalized Ostrogorski model (without the assumption of majority rule for votes or aggregation, nor that voters decide similarly)

[5] Lagerspetz (1996) overviews the result's connection to other voting paradoxes and the challenge it presents for representative democracy. Pigozzi (2005) notes its similarity to JA.

[6] Some work has been done to generalize the Ostrogorski Paradox (Bezembinder and van Acker, 1985; Deb and Kelsey, 1987; Nermuth 1992), but the assumption of a constant decision rule is always retained.

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Issues	Education	Defence	Immigration	Health Care	Honesty	Vote
Voter 1	D (1)	D (1)	R (1)	D (0)	D (2)	D
Voter 2	D (1)	R (1)	R (2)	D (1)	R (0)	R
Voter 3	R (0)	R (1)	R (1)	D (0)	D (1)	R
Electorate	D	R	R	D	D	R

TABLE 3. An electorate. Parentheses indicate weights voters place on issues.

is identical to this paper's model, with issues as the premises and the vote as the decision.

To illustrate, imagine each voter favours either candidate D or R on a set of issues (which may include character judgments like honesty). I write "D" if the voter affirms the statement "I prefer D on this issue" and "R" otherwise. Suppose each voter determines his vote (D or R) from his issue judgments using a decision rule, and that an aggregation rule produces group issue opinions and a group choice of representative. Table 3 displays a case in which voters differently weight each issue and vote by the higher weighted sum, and aggregation is by majority rule.

We may ask: Is there a group decision rule that can justify the winning candidate from group issue opinions? Is the winner representative of what the group believes? Under Interpretation 1, such a validation is generally impossible. There exists no group decision rule guaranteed to be consistent with the group opinions. Interpretation 2 offers a different insight: If there does exist a group decision rule, there is no guarantee the choice of representative will be consistent with the group judgments. In other words, the decision rule may indicate the losing candidate better represents group opinion. These conclusions are explored further in Section 3.3.

Example: Expert prediction

A corporation is deciding if it should invest in the Tiger Mutual Fund (TMF), and consults its team of financial analysts. Suppose that each analyst gives an ↑ or ↓ on stocks in TMF, and recommends investment in TMF if a majority of these judgments are ↑. (I give these symbols a similar treatment as D and R in the last example.) To make it interesting, suppose each analyst is an expert on a different (but overlapping) set of stocks, and derives the TMF judgment from these only. One knows biotech, another knows the Midwest, and so on. Notice that the differing decision rules result from differing information, not subjective distinctions. In this case, there exists a sensible group decision rule we suppose the corporation adopts: Invest in TMF if the team judges ↑ on a majority of

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Stocks:	*A*	*B*	*C*	*D*	*E*	TMF
Analyst 1	↑	↑	↑			↑
Analyst 2		↑	↓	↓		↓
Analyst 3		↓	↓		↑	↓
Team	↑	↑	↓	↓	↑	↓

TABLE 4. A team of financial analysts.

stocks. Unfortunately, as Interpretation 2 above indicates, there generally does not exist any aggregation rule – whether or not it treats each analyst or each stock equally – that will always be consistent with the group decision rule. For some sets of judgments, the team will recommend a majority of stocks, and not recommend TMF. Such a paradox (with majority rule on expert judgments, and non-expert judgments omitted for clarity) is displayed in Table 4.

2. THE MODEL

Section 2.1 defines the basic elements of the model. As a preliminary, Section 2.2 considers the case of a common decision rule (Theorem 1). Section 2.3 is the technical core of the paper, demonstrating in Theorem 2 a strong (im)possibility result even when allowing for differing decision rules. Theorem 3 strengthens the result by adding a monotonicity assumption. In Section 2.4, Theorem 4 fills in a few details on the full catalog of consistent decision and aggregation rules.

2.1. Basic structure

A *simple agenda* is a list of K ($\geq$3) logical propositions ($P_1, P_2, \ldots, P_{K-1}, Q$). Each proposition takes on a value from $\{T, F\}$, the 2-valued Boolean logic. The first $K-1$ propositions of a simple agenda, called *premises*, are logically independent (any assignment of truth values is allowed).

A *decision rule* is a logical rule (or Boolean function) that uniquely determines the value of Q, called the *decision*, from the values of the premises. For the sake of brevity in future definitions, I sometimes denote $Q = P_K$. A premise is *relevant* to a decision rule if Q's entailed value depends in some way on the value of the premise. I assume that a decision rule has at least two relevant premises. Some premises in the simple agenda may not be relevant to a particular decision rule.

Consider N ($\geq$2) agents who each make *judgments*, or assignments from $\{T, F\}$, on the propositions in a simple agenda. Each agent i affirms a decision rule D^i. For example, if i judges Q as true if and only if P_1 and

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P_2 are true, then for D^i I write: $Q \Leftrightarrow P_1 \wedge P_2$. [7] The key innovation of this paper is that each agent may obey a different decision rule.

Define a *decision frame* X to be a simple agenda combined with a list of decision rules $(D^1, D^2, \ldots, D^N, D^S)$, one for each agent and society. I use "society" and "group" interchangeably.

A *judgment set* is a list $J \in \{T, F\}^K$ of judgments on each proposition in a given simple agenda. I assume *completeness* (agents judge every proposition).[8] A judgment set for agent i (or society) is *consistent* if its assignment of truth values obeys i's (or society's) decision rule. A *profile* is an N-tuple $(J^1, J^2, \ldots, J^N)$ of judgment sets, representing the N individuals' judgments on each proposition in the simple agenda.

A judgment aggregation rule (JAR) g (for a decision frame X) maps a profile for the simple agenda in X to a complete social judgment set. If $\mathrm{J} = \{T, F\}^K$ is the space of possible judgment sets, $g : \mathrm{D} \rightarrow \mathrm{J}$, for some $\mathrm{D} \subseteq \mathrm{J}^N$. A JAR thus aggregates a set of individual judgments on each proposition to a group judgment on each proposition.

For any profile R and JAR g, I use the following notation:

P_j^i = Agent i's judgment on P_j (the jth element of the ith element of R)
P_j^S = The social judgment on P_j (the jth element of g(R))
$\propto P_j^i$ = "Agent i's judgment on P_j changes".
$\propto P_j^S$ = "The social judgment on P_j changes".

Thus, Q^i indicates agent i's decision, and Q^S indicates the group's ultimate decision. Using this notation, premise P_j is relevant to i's decision rule (or simply, relevant to i) if there exists a judgment set for i such that $\propto P_j^i \Rightarrow \propto Q^i$.

I now define the conditions I place on each JAR g. Recall that $P_K = Q$.

1. *Universal domain:* g's domain is the set of all profiles consisting of consistent judgment sets. This condition assumes all individuals obey their own decision rules.

[7] I use "$\Leftrightarrow$" for a decision rule (to signify that an agent regards it as a logical equivalence between Q and the premises) and "$\leftrightarrow$" as a logical connector. Since the decision rule is always affirmed, there is no need to distinguish between material and subjunctive bi-implications, as noted by a reviewer.

[8] It may be objected that some agents judge premises they regard as irrelevant. However, I demonstrate below that these judgments will have no effect on the group judgments. As a result, full completeness is not required – agents only need to judge relevant premises, and only socially relevant premises need to be aggregated. The above definitions will stand to avoid further complications, but the extended model should be obvious. For more extensive relaxations of completeness, see Gärdenfors (2006) and Dietrich and List (forthcoming).

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2. *Independence:* A proposition's social judgment depends only on that proposition's individual judgments. Formally, for all profiles R, R', and $j \in (1, 2, \ldots, K)$, the following holds: (For all $i \in (1, 2, \ldots, N)$, P_j^i in profile $R = P_j^i$ in profile R') $\Rightarrow$ (P_j^S in profile $R = P_j^S$ in profile R').

3. *Non-triviality*: Q^S is not pre-determined, so that for each $\theta \in \{T, F\}$, $Q^S = \theta$ for some profile.

4. *Non-authoritarianism:* No agent i exists such that the social judgments depend only on i's judgments. This strengthens non-dictatorship, since a social judgment may be the opposite of an authoritarian's judgment on some proposition. Formally, i is an authoritarian if, for each $j \in (1, 2, \ldots, K)$, $P_j^i = P_j^S$ for all profiles or $P_j^i \neq P_j^S$ for all profiles.

5 *Collective rationality:* For every profile R, $g(R)$ is a consistent social judgment set. This condition guarantees the group obeys the social decision rule.

Call a JAR *democratic* if it satisfies these five conditions. Note the minimal demand of this characteristic, since no assumption of equal treatment of voters or propositions is being made.

Example. Take (P_1, P_2, P_3, Q) as our simple agenda, and consider two agents. A decision frame X combines this simple agenda with a set of decision rules for both agents and society. Say that D^1 is $Q \Leftrightarrow P_1 \vee P_3$, D^2 is $Q \Leftrightarrow P_1 \wedge \neg P_3$, and D^S is $Q \Leftrightarrow P_1 \vee \neg P_2$. Both P_1 and P_3 are relevant to D^1.

Each agent produces a judgment set for (P_1, P_2, P_3, Q). If agent 1's judgment set is (T, F, F, T), it is consistent, as it obeys D^1. A profile is an ordered pair of judgment sets on (P_1, P_2, P_3, Q). A JAR (for X) maps this profile to a social judgment set on (P_1, P_2, P_3, Q). The JAR is independent if P_1^S depends only on P_1^1 and P_1^2 and the equivalent holds for the other propositions. If a potential output of the JAR is (F, T, T, T), then the JAR fails collective rationality, as this social judgment set violates D^S.

2.2. Group decisions with a common decision rule

As a warm-up to the main results, this section swims in familiar waters by assuming all individuals and the group employ a common decision rule, which is equivalent to the truth-functional model. Theorem 1 parallels very similar results in Nehring and Puppe (forthcoming) (their Theorem 1), which adds a monotonicity condition, and Dokow and Holzman (2005) (their Theorem 4.8), which adds a Pareto condition and allows the decision to consist of a multi-proposition set.[9]

[9] Theorem 1 generalizes Nehring and Puppe's Theorem 1, but their later results, which allow interconnections among the premises, are not so generalized.

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Assume all premises are relevant to the common decision rule (since otherwise they can be ignored). I now define two classes of JARs and two types of decision rules:

Oligarchy rule and conjunctivity: Say that a JAR is an *oligarchy rule on* P_j if there exists a group of agents who must unanimously assign $\theta \in \{T, F\}$ to P_j to give P_j^S the value θ, and P_j^S cannot be affected by non-members. As a result, all members of the group have veto power over P_j^S. Formally, there exists a group of agents M ($|M| \geq 2$) and $\theta \in \{T, F\}$ such that for all profiles: $(P_j^i = \theta \text{ for all } i \in M) \Leftrightarrow (P_j^S = \theta)$. Otherwise, P_j^S takes the default value $\neg\theta$.

A decision rule D^i is *conjunctive* if there exists a truth-value for every premise sufficient to determine Q. In other words, every premise has veto power over Q. Formally, for some $\theta \in \{T, F\}$ and set of $Y_j \in \{T, F\}$, the following holds for D^i: $(P_j^i = Y_j \text{ for all } j \in (1, 2, \ldots, \text{K}-1)) \Leftrightarrow (Q^i = \theta)$. These are decision rules of the form $Q \Leftrightarrow (P_1 \wedge \neg P_2) \wedge \cdots \wedge P_{K-1}$, or any variation with added negations, or the equivalent rules with all $\vee$ signs. As a counter-example, the decision rule $Q \Leftrightarrow (P_1 \wedge P_2) \vee P_3$ is not conjunctive.

Even/odd: Say that a JAR is an *even/odd rule on* P_j if there exists a group of agents such that P_j^S is determined by an even/odd count of supporters in the group. Equivalently, in any profile, any member of this group can switch P_j^S, and non-members have no effect. Formally, there exists a group of agents M ($|M| \geq 2$) such that for all profiles: $(i \in M) \Leftrightarrow (\propto P_j^i \Rightarrow \propto P_j^S)$.

Similarly, a decision rule D^i is *even/odd* if Q is affirmed depending on whether an even or odd number of premises are affirmed. For example, the decision rule $Q \Leftrightarrow (P_1 \leftrightarrow P_2)$ is even/odd.

Theorem 1. *Suppose a decision frame contains a common decision rule D for all individuals and society. If D is not conjunctive and not even/odd, no democratic JAR exists. If D is conjunctive, the only democratic JARs are oligarchy rules on all propositions. If D is even/odd, the only democratic JARs are even/odd rules on all propositions.*

All proofs are contained in the Appendix.

Besides even/odd decision rules, only conjunctive decision rules can have a democratic JAR, which must then be an oligarchy rule. This implies the Ostrogorski Paradox and the original doctrinal paradox result (List and Pettit, 2002). If D is even/odd, the JAR can be an even/odd rule on each proposition. This combination implies that *every* judgment switch by an agent in M switches Q^S. Thus, any democratic JAR must force a change in Q^S for any judgment change by an agent in M, or allow a change only to or from a single profile of unanimous agreement.

What is there to make of even/odd rules? Barring an inspired argument, they must be considered a strange artifact of the model. Since an even/odd determination is so intuitively unappealing as an aggregation rule, and a non-even/odd assumption could easily be added

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to the list of JAR constraints, my later analysis in Section 3 ignores this option.

2.3. Group decisions with differing decision rules

I now turn to this paper's key innovation: the possibility of differing decision rules. Agents may rely on premises in different ways, and may even disagree on what premises are relevant.

A few definitions describe the agents' connections through relevant premises. An agent i is *empowered* if there exists a profile in which $\propto Q^i \Rightarrow \propto Q^S$. A premise is *shared* if it is relevant to more than one empowered agent. Although one expects the agents' decisions to be based on many of the same premises, a concept is needed for when that is not the case. A JAR is *partitioned* if there exists a partition of the empowered agents into at least two (non-empty) subgroups such that no premise is shared between two agents in different subgroups.

Assume for simplicity that each socially relevant premise is relevant to at least one empowered agent. This is unrestrictive, since if a premise is not relevant to any empowered agent, there is no way for the premise's social judgment to change and affect Q^S.

Because of the added complexity, slightly broader versions of oligarchy rule and conjunctive and even/odd decision rules are needed. A JAR is a *unitary rule on* P_j if there exists a group of agents who all have veto power over P_j^S, and P_j^S cannot be affected by non-members. Formally, there exist a set of agents M ($|M| \geq 2$), $\theta^S \in \{T, F\}$, and a (possibly distinct) set of $\theta^i \in \{T, F\}$ for each $i \in M$, such that the following holds for all profiles: ($P_j^i = \theta^i$ for all $i \in M$) $\Leftrightarrow$ ($P_j^S = \theta^S$). Otherwise, P_j^S takes the default value $\neg\theta^S$.

For a set of premises P, a decision rule D^i is *semi-conjunctive on* P if there exists a truth-value for every premise in P sufficient to determine Q. Thus, it may be that only *some* premises (namely, all those in P) have veto power over Q. Formally, for $\theta \in \{T, F\}$, non-empty $P \subseteq \{P_1, P_2, \ldots, P_{K-1}\}$, and $Y_j \in \{T, F\}$ for each $P_j \in P$, the following holds for D^i: ($Q^i = \theta$) $\Rightarrow$ ($P_j^i = Y_j$ for all $P_j \in P$). For example, the decision rule $Q \Leftrightarrow P_1 \wedge P_2 \wedge (P_3 \vee P_4)$ is semi-conjunctive on $\{P_1, P_2\}$.

For a set of premises P, a decision rule D^i is *semi-even/odd on* P if, holding all non-P judgments fixed, Q is affirmed depending on whether an even or odd number of premises in P are affirmed. Equivalently, the following holds for D^i: For non-empty $P \subseteq \{P_1, P_2, \ldots, P_{K-1}\}$, $\propto P_j^i \Rightarrow \propto Q^i$ for any $P_j \in P$. For example, the decision rule $Q \Leftrightarrow (P_1 \leftrightarrow P_2) \leftrightarrow (P_3 \vee P_4)$ is semi-even/odd on $\{P_1, P_2\}$.

Theorem 2. *Given a non-partitioned, democratic JAR g (for decision frame X), one of the following holds:*

(a) *g is a unitary rule on all propositions, society's decision rule in X is semi-conjunctive on the set of shared premises, and all empowered agents'*

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decision rules in X are semi-conjunctive on the set of premises shared by the agent.

(b) *g is an even/odd rule on all propositions, society's decision rule in X is semi-even/odd on the set of shared premises, and all empowered agents' decision rules in X are semi-even/odd on the set of premises shared by the agent*.

A proposition P_j is *dominated* if there exists an agent i such that $\propto P_j^i \Rightarrow \propto P_j^S$ in every profile. Say that a JAR *is full-democratic* if it satisfies universal domain, independence, non-triviality, collective rationality, and no proposition is dominated.

Corollary 2.1. *Every non-partitioned, full-democratic JAR (for decision frame X) is a unitary rule on all propositions, and requires the decision rules in X of all empowered agents and society to be semi-conjunctive on the set of relevant premises*.

A stronger result than Theorem 2 obtains with the additional assumption of *monotonicity* on each proposition, meaning that increased support for a particular judgment cannot switch the social judgment in the opposite direction. Formally, a JAR is *monotonic on* P_j if the following holds: For $\theta \in \{T, F\}$, if {For all $i \in (1, 2, \ldots, N)$, ($P_j^i = \theta$ in profile R) $\Rightarrow$ ($P_j^i = \theta$ in profile R')}, then {($P_j^S = \theta$ in profile R) $\Rightarrow$ ($P_j^S = \theta$ in profile R')}. Say that a JAR is *monotone-democratic* if it is democratic and monotone on every proposition.

Theorem 3. *Every non-partitioned, monotone-democratic JAR (for decision frame X) is an oligarchy rule on all propositions, and requires society's decision rule in X to be semi-conjunctive on the set of shared premises and all empowered agents' decision rules in X to be semi-conjunctive on the set of premises shared by the agent*.

2.4. Partitioned subgroups

The previous theorems provide strong limiting results for non-partitioned JARs. The final theorem and corollaries consider several partitioned subgroups, demonstrating that only a little freedom is gained. For a partitioned subgroup G, the subgroup's *profile segment* is the subset of the profile restricted to the judgments of G's members on Q and each premise relevant to a member of G.

Theorem 4. *Suppose a democratic JAR g contains R* ($\geq$2) *partitioned subgroups. For* $r \in (1, 2, \ldots, R)$, *each subgroup* G_r *has a profile segment* A_r *in the space* $\mathbf{A}_r$. *Say that* Q^S *is described by a function h*: $\mathbf{A}_1 \times \mathbf{A}_2 \times \cdots \times \mathbf{A}_R \rightarrow \{T, F\}$. *The following hold*:

1. *We can construct functions* $g_r : \mathbf{A}_r \rightarrow \{T, F\}$ *and a function f*: $\{T, F\}^R \rightarrow \{T, F\}$ *such that* $h(A_1, A_2, \ldots, A_R) = f(g_1(A_1), g_2(A_2), \ldots, g_R(A_R))$.

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2. *There are three possibilities for each g_r: (a) G_r has a single member, (b) g_r is a unitary rule, or (c) g_r is an even/odd rule.*
3. *The space of possible g_r's and all functions $f: \{T, F\}^R \rightarrow \{T, F\}$ delimits all the possible choices for h.*

Corollary 4.1. *Using the formalism of Theorem 4: For any monotone-democratic JAR g, every g_r is an oligarchy rule and f is monotonic.*

Corollary 4.2. *If a democratic JAR aggregates the group decision by majority rule (or any supermajority rule short of unanimity), then no premises are shared and every empowered agent dominates the set of premises relevant to the agent.*

3. ANALYSIS

The theorems of Section 2 share a similar structure: Assuming all individuals *and* the group employ decision rules and the aggregation rule satisfies five minimal constraints, the allowed combinations of decision and aggregation rules are severely restricted.[10] The five constraints are universal domain (all individual judgments consistent with the individuals' decision rules are allowed), independence, non-triviality (non-fixed group decision), non-authoritarianism (a single individual does not determine the group judgments, which slightly strengthens non-dictatorship), and collective rationality (the group judgments are always consistent with the group decision rule). I make no restrictive assumptions like anonymity or the neutrality part of systematicity, according to which all propositions must be aggregated in the same way. With the possible exception of independence (discussed in Section 3.2), the first four constraints appear essential. Thus, we can interpret the theorems in the following way: If the decision and aggregation rules are *not* one of the required combinations and we retain the four necessary constraints, then collective rationality cannot be satisfied. Hence, the group cannot consistently follow the given decision rule.

For Theorem 1, I make the simplifying assumption that individuals and the group employ a common decision rule, which makes the model identical to the truth-functional model. Collective rationality then requires aggregation on every proposition to be made by oligarchy rule (unanimity rule restricted to a fixed group) and the common decision rule to be conjunctive (every premise can veto the decision). Nehring and Puppe (forthcoming) and Dokow and Holzman (2005) obtain very similar results. For example, if a group wants to make a decision based on whether at least

[10] In the succeeding analysis, I ignore even/odd rules, which were discussed at the end of Section 2.2.

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two out of three premises are true, there is no way to guarantee a consistent group result.[11]

Starting with Theorem 2, I allow differing decision rules, the central innovation of this paper. The major complication that arises concerns how premises individuals use to make their decisions overlap, or are shared. An exception to each of the results in Section 2.3 is if there exist partitioned subgroups of individuals such that members of different subgroups have no premises of shared concern. In the general case without such a partition, Theorem 2, collective rationality requires aggregation to be made by an analogue of oligarchy rule (every member of a fixed group can veto the group decision and any shared premise) and each decision rule to be semi-conjunctive (every shared premise can veto the decision). For example, if we aggregate by majority rule, collective rationality cannot be satisfied regardless of the chosen decision rules. Also, if just one individual makes a "two out of three" decision, collective rationality cannot be satisfied.

Theorem 3 adds monotonicity to the aggregation constraints and strengthens the conclusion of Theorem 2 to requiring oligarchy rule for collective rationality. This is probably the most useful result. However, non-monotonic aggregation rules are worth considering, such as when a group finds a particular individual's judgment unreliable.

Theorem 4 considers partitioned subgroups, concluding that only a little freedom is gained. Essentially, partitioning breaks down the group decision into a "vote" by each subgroup, where now the determination of each subgroup vote is limited in the manner of Theorem 2. However, we are freed up somewhat in that we can use any rule to derive the group decision from the subgroup votes. For example, we can satisfy collective rationality if each subgroup votes by unanimity rule and the group decision is determined by the majority of these votes. Ironically, the more the group as a whole agrees on what issues are relevant, the more difficult it becomes to guarantee a consistent decision. Corollary 4.2 notes that assuming collective rationality, if we make the group decision by majority or supermajority rule across all individuals, then every individual must form his own subgroup, meaning that no two voters care about the same premise. Thus, for the most common aggregation method, the group decision can only be linked consistently to group judgments that are determined by single individuals, which hardly rings true as a reasoned *group* decision.

[11] This impossibility for the "two out of three" decision rule is at the root of preference aggregation impossibility theorems. For any three alternatives, x, y, and z, consider the logically independent premises $P_1 =$ "x is preferred to y", $P_2 =$ "y is preferred to z", and $P_3 =$ "If $\neg(P_1 \leftrightarrow P_2)$, then x is preferred to z". $Q =$ "x is preferred to z" is true if and only if at least two out of three of these premises are true, hence no preference aggregation rule can ensure transitivity in the group result.

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The central claim of this paper is that decision-making cannot be understood purely at the group level. If one follows Dahl (1989: 5) in seeing democracy as primarily "a unique process of making collective and binding decisions", then certainly these results are relevant to democratic theory, and may help to illuminate some of the "half-hidden premises, unexplored assumptions, and unacknowledged antecedents [that] form a vaguely perceived shadow theory that forever dogs the footsteps of explicit, public theories of democracy" (Dahl 1989: 3). In that spirit, this section explores this paper's implications for unanimity rule, the deliberative democratic tradition, and political representation.

3.1. Unanimity

When a consistent, reasoned group argument is essential, unanimity rule (or a close parallel) is required. Despite majority rule's ubiquity, unanimity rule does appear in the democratic tradition in two highly important cases this paper may help illuminate. One case is the ultimate basis for democratic government. The other concerns the most consequential of interpersonal decisions.

3.1.1. Contract theory. For both Locke and Rousseau, unanimity is a core requirement for the social contract, from which majority rule is established as a more efficient mode of decision-making. Coleman (2003: 235) sees this founding act as a bargain that requires everyone's consent: "Only unanimity could possibly guarantee both that no person is made worse off by collective policy and that the outcome of collective policy is stable".

Bargaining, in general, accords with this paper's model. Suppose each party to a proposed bargain judges if each aspect of the arrangement is acceptable (or as good as can be expected from further bargaining). If an individual finds some aspect unacceptable, she rejects the bargain; otherwise, she accepts it. It follows that each decision rule is semi-conjunctive, hence unanimity rule enables the group to consistently judge the acceptability of each provision in the bargain *and* decide on the bargain itself.[12] In the context of the social contract, acceptability criteria can be regarded as justice criteria, so that unanimity rule allows a state to have a consistent justice rationale.

In effect, this paper suggests that founding a state may be the only group decision guaranteed to be consistent. Although subsequent majority rule decisions may not be rationally supported, the rule itself may derive

[12] This binary model lacks the flexibility to choose the bargain under debate. Rather, the decision is to accept or reject a bargain proposed by some external process. This process can repeat, however, until an acceptable bargain is reached.

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from a collectively rational decision. In effect, a form of group consistency holds prior to any group decision; it is actuated by the very existence of the group.

3.1.2. Juries. Of course, at least one major institution employs unanimity rule: the jury. Previous formal work has assessed the unanimity requirement for juries mainly in terms of the rule's effect on individuals. Feddersen and Pesendorfer (1998) contend that unanimity rule encourages strategic voting, *increasing* the chance an innocent defendant is convicted. Jurors, they argue, may strategically vote guilty because their vote can only matter if everyone else is already voting to convict, which strongly suggests guilt. Coughlan (2000) responds that non-strategic voting is expected when factoring in mistrials and juror communication. However, the rule may inhibit jurors' incentives to gather and communicate information (Persico, 2004).

This paper provides a strong normative and logical argument in favour of unanimity rule in jury trials. The decision rule structure is analogous to the logical requirement for conviction. Jurors must judge several independent issues, such as motive, intent and opportunity, that together decide guilt or innocence.[13] Reaching a publicly declared, logically consistent argument for the final decision is essential in the deliberative context of a jury. With a few sensible assumptions, this has been shown to only be possible with unanimity rule and semi-conjunctive decision rules. As it happens, legal requirements for guilt tend to have little wiggle room; every condition in the logical chain must be met. Hence, jury decision rules can be well-described as semi-conjunctive, and only unanimity rule will facilitate social consistency. It may not be accidental that the jury employs the only aggregation rule that can guarantee collective rationality.

3.2. Deliberative democratic theory

Inspired by Oakeshott's "politics as conversation" and Habermas's "ideal speech situation", deliberative democratic theory idealizes "decision making by discussion among free and equal citizens" (Elster 1998: 1). In addition, the theory considers it "necessary for democratic legitimacy to supplement collective decisions on actions or policies with supporting reasons" (List 2006: 365).

This paper puts in stark light the fact that deliberative democracy's core ideals, the individual contribution to group opinions and the public display of accepted reasons, are at loggerheads. Even allowing

[13] Hence, I am imagining a richer deliberative environment than the one presented in the standard information aggregation account of juries, such as in the Condorcet Jury Theorem and the papers mentioned above.

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for pluralistic, subjective decision-making, a group cannot consistently present reasons for its decisions (short of unanimity rule). As List and Pettit (2002: 89) put it, JA "highlights a tension between two plausible demands: on the one hand, that a group be responsive to the judgments of individual members in forming collective judgments and, on the other, that it be rational in the judgments it collectively endorses". For Pettit (2001: 268), deliberative democrats falter because they have made "an implicit assumption that it does not matter whether the discipline of reason is imposed at the individual or at the collective level". This assumption fails not only for the exercise of cold logic, but for any manner of subjective decision-making.

A possible response is that this failure indicates the *need* for deliberation. Deliberative democrats stress that "democracy revolves around the transformation rather than simply the aggregation of preferences" (Elster 1998: 1). Deliberation can expand consensus by "revealing private information, lessening bounded rationality, or encouraging more public-spirited proposals" (Fearon 1998: 56). However, unless deliberative democrats restrict themselves to small, homogeneous groups for which unanimity is practical, it remains unclear how they can elucidate group decision-making.

A final try for collective rationality without unanimity rule is to discard one of the four remaining constraints. Since universal domain, non-triviality and non-authoritarianism appear essential, we are stuck with abandoning independence, meaning that group judgments need not be derived solely from individual judgments on the same proposition.[14] Although adding consistency considerations for determining group judgments may ensure collective rationality, this route is especially problematic for reasoned decision-making. As Dietrich and List (2007b) demonstrate, independence is a necessary condition for non-manipulability and strategy-proofness, calling into question the genuineness of the reasoning process under non-independent aggregation. Moreover, what is meant by a group opinion not derived from individual opinions on the same issue? Surely we cannot interpret a group reason as authoritative if the members themselves do not endorse it. Non-independent aggregation is promising when the goal is mediation among conflicting opinions, but much less so if each group judgment can be scrutinized and needs to stand on its own. A similar problem arises if deliberative democrats hope individuals factor in group consistency for their own judgments. What is meant by an individual opinion if it is influenced by group consistency demands? The relationship between judgments at the individual and group level is not as clear as deliberative theory assumes.

[14] Pigozzi (2006) and List and Pettit (2006) recommend this approach.

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3.3. Representation

It is a common lament that political representatives fail to remain faithful to the voters. This assumes they were in accord in the first place. Generally, an elected representative cannot be guaranteed to cohere to any rule that indicates the proper choice of representative based on the list of group issue opinions. Given any rule, group opinions may imply a different representative should have been chosen.

Thus, an elected representative cannot consistently be justified purely based on group opinions. An inherent incommensurability exists between the representative as an embodiment of the electorate and the representative as an embodiment of electorate opinion. Schumpeter (1950: 245 – 6) similarly challenges the coherency of popular representation, with the exception of "small and primitive communities with a simple social structure in which there is not much to disagree on", so that one can speak of the "will or the action of the community".

On the positive side for representation, direct democracy cannot guarantee consistent group opinions, whereas a representative can be consistent. (Of course, inconsistency rematerializes in a representative assembly, so think of an executive.) Brennan (2001: 208) notes this advantage in application to political parties, which must rationalize their policy ideas: "That consistency requirement constrains those parties to consider and to declare their reasons for action. Reasons weigh in a manner that they do not in direct democratic processes where voters simply offer a judgment on the outcome – with no requirement at all that the reasons of different voters need be the same or logically consistent in the aggregate".

What can we make of a representative system that guarantees logically consistent beliefs will stand in for a population, but the beliefs will not necessarily be those supported by popular opinion? More than anything, the dilemma questions the meaning of the representative: Do voters elect a person or a set of beliefs? The answer offers a tentative claim about whether political representatives should rule in accordance with their own beliefs or with their electors'. It also contests the common assumption that these two aspects of representation are not in conflict. Like majority rule, even if representation is the best practicable standard for a modern democracy, it may exist without the solid theoretical basis we might hope for it.

4. FUTURE WORK

This paper has investigated a JA model widely applicable to group decision-making, including decisions that are subjective in nature. A number of avenues exist for generalizing and extending the results. More interrelated agendas can be considered, as well as more general logics. I claim, but cannot prove here, that the results hold if individuals can assign to each premise any of a finite number of arguments. For example, instead

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of judging whether unemployment will go up or down, an individual could give a predicted unemployment rate (say, in steps of 0.1%). A probabilistic analysis can ascertain how common inconsistency is for different aggregation and decision rules. Social choice theory has focused great attention on adjusting universal domain to engineer combinable preference orderings. Similar modifications may help guarantee collective rationality.

Finally, the debate continues as to what JA results like this paper say, if anything, about democratic theory. The parallel debate concerning preference aggregation theory has been impassioned and illuminating. Since JA theory is more general, and avoids the divisive rational choice framework, its contribution ought to be even more informative. Unfortunately, it appears that we cannot expect the academic community to reach any consistent conclusions on the matter.

APPENDIX: PROOFS

I prove Theorems 2 and 4, from which the corollaries and Theorem 3 follow immediately. Theorem 1 follows directly from Corollary 2.1. These further proofs are omitted.

Proof of Theorem 2. Some definitions: An agent i is *pivotal on* P_j in profile R if $\propto P_j^i \Rightarrow \propto P_j^S$. Note that an agent i is empowered if there exists a profile in which i is pivotal on Q. For an agent i, a profile A' is an *i-variant* of A if the judgment sets are identical in both profiles for all agents $l \neq i$. Note that A itself is an i -variant of A. If i's decision rule is semi-conjunctive (respectively, semi-even/odd) on a set P, I abbreviate this as Q^i is semi-conjunctive (semi-even/odd) on P.

Consider a democratic JAR g for decision frame X. For each $i \in (1, 2, \ldots, N, S)$, X^i is the set of premises relevant to i's decision rule. M^* is the set of empowered agents for g.

Lemma 2.1. *If an agent i is pivotal on Q in a profile A, then in any i-variant of A, the agent is pivotal on Q and* P_j *for all* $P_j \in X^i$.

Proof. Suppose $\propto Q^i \Rightarrow \propto Q^S$ in profile A. Since Q^i is uniquely determined by the other judgments in i's judgment set, there exists a k such that $\propto P_k^i \Rightarrow \propto Q^i \Rightarrow \propto Q^S$. As Q^S is similarly determined by the other judgments in the social judgment set, by independence we must have $\propto P_k^S$. Hence, i is pivotal on P_k in profile A.

For any $P_j \in X^i$, there exists a set of i's judgments such that $\propto P_j^i \Rightarrow \propto Q^i$. By independence, i is pivotal on Q in any i-variant of A. As above, this implies i is pivotal on P_j in some i-variant of A. By independence, i is pivotal on P_j in any i-variant of A.

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Lemma 2.2. $\cup^{i \in M^*} X^i = X^S$.

Proof. Consider any $i \in M^*$ and $P_j \in X^i$. There must exist a profile in which i is pivotal on Q and $\propto P_j^i \Rightarrow \propto Q^i$. By independence, $\propto P_j^S \Rightarrow \propto Q^S$. Thus, P_j is relevant to the social decision rule, meaning $P_j \in X^S$. It follows that $\cup^{i \in M^*} X^i \subseteq X^S$. By assumption, $X^S \subseteq \cup^{i \in M^*} X^i$.

Lemma 2.3. *Q^S and P_j^S for each $P_j \in X^S$ depend only on the judgments of empowered agents.*

Proof. If P_j^S (for $P_j \in X^S$) depends on the judgment P_j^i for $i \notin M^*$, then there exists a profile A in which $\propto P_j^i \Rightarrow \propto P_j^S$. There also exists a profile with the same judgments on P_j as in A, and in which $\propto P_j^S \Rightarrow \propto Q^S$, which implies $\propto P_j^i \Rightarrow \propto Q^S$, but $\propto Q^i \not\Rightarrow \propto Q^s$, a contradiction.

Lemma 2.4. *Consider shared $P_j \in X^i$ for $i \in M^*$. If i is non-pivotal on P_j in some profile, then Q^i is semi-conjunctive on a set of premises including P_j.*

Proof. Suppose that agent $i \in M^*$ is pivotal on Q in profile A. Consider any shared $P_j \in X^i$. From Lemma 2.1, i is pivotal on P_j in profile A, but suppose that i is non-pivotal on P_j in some profile.

From profile A, we can change the agents' judgments on P_j one by one. We must reach a profile B in which i is pivotal on P_j, but, for $l \neq i$, $\propto P_j^i \Rightarrow i$ is no longer pivotal on P_j (call this profile B'). By Lemma 2.1, i is not pivotal on Q in B'. By independence, in B', i is pivotal on all $P_{k \neq j}$ such that $P_k \in X^i$. It follows that with the social judgments outside of X_i unchanged from B', the value of P_j^S in B' is sufficient to decide the value of Q^S in B'. Starting at profile A, let i set P_j^S to its value in B' (call this profile A'). In A', $\propto Q^i \Rightarrow \propto Q^S$ (by independence), but P_j^S decides Q^S. Hence, Q^i cannot change without P_j^i changing. It follows that P_j^i in A' is sufficient to decide Q^i in A'.

Lemma 2.5. *If i is pivotal on shared P_j in every profile, then $\propto P_j^i \Rightarrow \propto Q^i$ in every profile.*

Proof. Suppose i is pivotal on some shared P_j in every profile, and consider $k \in M^*$ with $P_j \in X^k$. From Lemma 2.2, $P_j \in X^S$. Using Lemma 2.1, there exists a profile A in which k is pivotal on P_j, and thus both i and k are pivotal on P_j. Starting from A, adjust the non-P_j judgments as necessary so that $\propto P_j^S \Rightarrow \propto Q^S$. Call this profile B and note that (by independence) both i and k are pivotal on P_j in B. It follows that i and k are both pivotal on Q in B.

From B, $\propto P_j^k \Rightarrow \propto P_j^S \Rightarrow \propto Q^S$, which implies $\propto P_j^k \Rightarrow \propto Q^k \Rightarrow \propto Q^S$. Following these changes, both i and k are still pivotal on Q since i is always

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pivotal on P_j. To be consistent, after $\propto P_j^i \Rightarrow \propto Q^i \Rightarrow \propto Q^S$ starting from B, both i and k are still pivotal on Q.

Suppose there exists an i-variant of B (call it profile C) for which $\propto P_j^i \not\Rightarrow \propto Q^i$. Since i is pivotal on P_j in C, but $\propto P_j^i \not\Rightarrow \propto Q^S$ by independence, it follows that $\propto P_j^S \not\Rightarrow \propto Q^S$ in C. However, k is still pivotal on Q (by the last paragraph), so $\propto P_j^k \Rightarrow \propto Q^k \Rightarrow \propto Q^S$ (since k's judgment set in C is the same as in B), requiring $\propto P_j^S \Rightarrow \propto Q^S$, a contradiction. Therefore, $\propto P_j^i \Rightarrow \propto Q^i$ in each of i's judgment sets, and thus in every profile.

Lemma 2.6. *If i is pivotal on P_j in every profile and $P_j \in X^k$ for $k \in M^*$, then k is pivotal on P_j in every profile.*

Proof. For the purpose of contradiction, suppose i is pivotal on P_j in every profile and $P_j \in X^k$ for $k \in M^*$, but there exists a profile such that k is non-pivotal on P_j. As in Lemma 2.5, consider a profile A in which $\propto P_j^S \Rightarrow \propto Q^S$ and both i and k are pivotal on P_j. From A, $\propto P_j^k \Rightarrow \propto Q^k \Rightarrow \propto Q^S$, and then both i and k are still pivotal on Q.

From Lemma 2.4, Q^k is semi-conjunctive on a set of premises containing P_j, so there exist Z_j^k, $\theta \in \{T, F\}$ such that $P_j^k = Z_j^k \Rightarrow Q^k = \theta$. Since more than one premise is relevant to k's decision rule by assumption, there must be two different sets of values for k's non-P_j judgments such that, with $P_j^k \neq Z_j^k$, $Q^k = \theta$ in one and $Q^k \neq \theta$ in the other. It follows that there exists a set of values for k's non-P_j judgments such that $\propto P_j^k \not\Rightarrow \propto Q^k$. Let A' be a k-variant of A such that $\propto P_j^k \not\Rightarrow \propto Q^k$.

From the first paragraph, both i and k are pivotal on Q in A'. By Lemma 2.5, from A', $\propto P_j^i \Rightarrow \propto Q^i \Rightarrow \propto Q^S$, so $\propto P_j^S \Rightarrow \propto Q^S$. However, k is pivotal on P_j by Lemma 2.1, hence $\propto P_j^k \Rightarrow \propto P_j^S \Rightarrow \propto Q^S$, but $\propto P_j^k \not\Rightarrow \propto Q^k$, a contradiction by independence.

Lemma 2.7. *Suppose, for each agent $i \in M^*$, Q^i is semi-conjunctive on the set of shared premises in X^i, and for each shared $P_j \in X^i$, i is non-pivotal on P_j in some profile. If g is not partitioned, then g is a unitary rule on Q and every shared premise, and Q^S is semi-conjunctive on the set of all shared premises.*

Proof. I use the following notation: For each agent $i \in M^*$, there exist $\theta^i \in \{T, F\}$ and $Y_j^i \in \{T, F\}$ for each shared $P_j \in X^i$ such that $(P_j^i = Y_j^i$ for all $P_j \in X^i) \Leftrightarrow (Q^i = \theta^i)$. For each shared $P_j \in X^i$, i is non-pivotal on P_j in some profile. Assume g is not partitioned, so that every empowered agent shares at least one premise.

Sublemma 2.7.1. *Suppose i is pivotal on Q in profile A and $Q^i = \theta^i$. If i shares P_j with $k \in M^*$, then k is pivotal on Q in A and $Q^k = \theta^k$ in A.*

Proof. Suppose i is pivotal on Q in profile A, with $Q^i = \theta^i$, and $P_j \in X^i \cap X^k$. If $k \in M^*$, there exists a profile B in which k is pivotal on Q. For

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each $r \in M^*$, X^r is semi-conjunctive with at least two relevant premises, so setting $P_j^r = Y_j^r$ is not sufficient to decide Q^r. Hence, there exists a profile B' with the same judgments on Q as in B, and in which $P_j^r = Y_j^r$ for all $r \in M^*$. By independence, k is pivotal on Q in profile B' and thus pivotal on P_j by Lemma 2.1. By the same logic, there exists a profile A' with the same judgments on Q as in A, and in which $P_j^r = Y_j^r$ for all $r \in M^*$. Note that i must be pivotal on Q in A' and $Q^i = \theta^i$ in A' by construction.

In profile A', $\propto P_j^i \Rightarrow \propto Q^i \Rightarrow \propto Q^S$, hence $\propto P_j^S \Rightarrow \propto Q^S$. By independence, k is still pivotal on P_j in A', so $\propto P_j^k \Rightarrow \propto P_j^S \Rightarrow \propto Q^S$, and thus k is pivotal on Q in A' and A. By independence, $\propto P_j^k \Rightarrow \propto Q^k$ in A' and $P_j^k = Y_j^k$ in A' by construction. It follows that $Q^k = \theta^k$ in profile A' and A.

Sublemma 2.7.2. *Q^S is semi-conjunctive on the set of all shared premises.*

Proof. Consider $i \in M^*$ and a profile A in which i is pivotal on Q. Take any shared $P_j \in X^i$.

Changing the judgments on P_j one by one, there must exist a profile B in which i is pivotal on P_j, but, for $l \neq i$, $\propto P_j^l \Rightarrow i$ is no longer pivotal on P_j (call this profile B'). By Lemma 2.1, i is not pivotal on Q in B'. By independence, i is pivotal on all $P_{k \neq j}$ for $P_k \in X^i$. Thus, with the social judgments on premises outside of X^i fixed, the value of P_j^S in profile B' is sufficient to decide the value of Q^S in profile B'.

Suppose a set of agents $M \subseteq M^*$ are all pivotal on Q in profile C. Call $X^M = \cup^{r \in M} X^r$. It follows that with the social judgments on premises outside of X^M fixed, then for each shared $P_k \in X^M$, there exists a value for P_k^S sufficient to decide Q^S.

Since g is not partitioned, repeated application of Sublemma 2.7.1 demonstrates that there exists a profile D in which all members of M^* are pivotal on Q. Since $\cup^{r \in M^*} X^i = X^S$ (by Lemma 2.2), it follows that with the social judgments on premises outside of X^S fixed, then for each shared $P_k \in X^S$, there exists a value for P_k^S sufficient to decide Q^S. But there are no premises outside of X^S, so for every profile there exists a value for P_k^S sufficient to decide Q^S. Thus, Q^S is semi-conjunctive on all shared premises.

Proof of Lemma 2.7. From the argument of Sublemma 2.7.2, there exists a profile D in which all members of M^* are pivotal on Q. Moreover, there exist $\theta^S \in \{T, F\}$ and $Y_j^S \in \{T, F\}$ for each shared $P_j \in X^S$ such that $(P_j^S = Y_j^S$ for all $P_j \in X^S) \Leftrightarrow (Q^S = \theta^S)$. From Sublemma 2.7.1, it also follows that $Q^i = \theta^i$ in D for all $i \in M^*$.

For any $r \in M^*$ and shared premise $P_j \in X^r$, from D, $\propto P_j^r \Rightarrow \propto Q^r \Rightarrow \propto Q^S$, so $\propto P_j^S \Rightarrow \propto Q^S$. It follows that $Q^S = \theta^S$ in D. Hence, $(Q^i = \theta^i$ for all $i \in M^*) \Rightarrow (Q^S = \theta^S)$.

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Suppose there exists a profile E in which $Q^S = \theta^S$, but there is an agent $l \in M^*$ such that $Q^l \neq \theta^l$. As in the proof to Lemma 2.7.1, we can move from the Q judgments in D to the Q judgments in E without changing any judgments on shared $P_k \in X^l$ (call this profile E'). In E', $P_k^l = Y_k^l$, $Q^l \neq \theta^l$, $Q^S = \theta^S$, and l is still pivotal on P_k. It follows that $\propto P_k^l \Rightarrow \propto P_k^S \Rightarrow \propto Q^S$ (since Q^S is semi-conjunctive on all shared premises), but $\propto P_k^l \not\Rightarrow \propto Q^l$ (since Q^l is semi-conjunctive on all shared premises in X^l), a contradiction. Thus, $(Q^i = \theta^i \text{ for all } i \in M^*) \Leftrightarrow (Q^S = \theta^S)$.

Proof of Theorem 2. It is not possible that Q^i is semi-conjunctive on a set of premises and another premise always changes Q^i, so Lemmas 2.4 and 2.5 prove that there are two possibilities for each $i \in M^*$:

1. Q^i is semi-conjunctive on the set of shared premises in X^i. Also, for each shared premise in X^i, the agent is non-pivotal on the premise in some profile.
2. All shared premises in X^i can change Q^i in any profile, hence Q^i is semi-even/odd on the set of shared premises in X^i. Also, i is pivotal on all shared premises in X^i in every profile.

From Lemma 2.6, the two types cannot share a premise. If we suppose g is not partitioned, it follows that all empowered agents must be the same type.

If all are the first type, Lemma 2.7 proves that g must be a unitary rule on Q and every shared premise, and Q^S is semi-conjunctive on the set of all shared premises.

Suppose all are the second type. Every shared premise is determined by an even/odd rule since all sharing agents are pivotal on the premise in every profile. If $i \in M^*$ is pivotal on Q in a profile A and shares P_j with k, then $\propto P_j^i \Rightarrow \propto Q^i \Rightarrow \propto Q^S$, so $\propto P_j^S \Rightarrow \propto Q^S$, which implies $\propto P_j^k \Rightarrow \propto P_j^S \Rightarrow \propto Q^S$, demonstrating that k is pivotal on Q in A. Since all empowered agents are connected to one another by shared premises (assuming g is not partitioned), repeated application of this argument proves that all empowered agents are pivotal on Q in A. Moreover, if $\propto Q^i$ from A, then i is still pivotal on Q, hence all empowered agents are still pivotal on Q. It follows that g is an even/odd rule on Q and every shared premise, and that Q^S is semi-even/odd on the set of all shared premises.

Proof of Theorem 4. Define a function $h(A_1, A_2, \ldots, A_R) = Q^S$. We find R functions $g_r : \mathrm{A_r} \rightarrow \{T, F\}$ that each capture all the information in A_r that can affect Q^S. If G_r contains a single member i, then G_r can only affect Q^S by changing Q^i, so setting $g_r(A_r) = Q^i$ retains all the information that can affect Q^S.

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Now suppose G_r has multiple members. Suppose the total profile is such that changing A_r can change Q^S. If we restrict our attention to only the profile segment A_r, the social judgments on the propositions included in A_r, and Q^S, the relationship must be the same as if they represented an entire profile. For this hypothetical (non-partitioned) profile, define a function $g_r(A_r) = Q^S$. As proved in Theorem 2, the empowered agents either (a) all have semi-even/odd decision rules and aggregate with an even/odd rule on Q or (b) all have semi-conjunctive decision rules and aggregate with a unitary rule on Q. Since the aggregation rule must be the same in any total profile (because the decision rules do not change), g_r must also follow the same rule. It follows that g_r captures the entire effect of A_r on Q^S.

This ordered set $(g_1(A_1), g_2(A_2), \ldots, g_R(A_R))$ thus determines $h(A_1, A_2, \ldots, A_R)$, so there is a function $f : \{T, F\}^R \rightarrow \{T, F\}$ such that $f(g_1(A_1), g_2(A_2), \ldots, g_R(A_R)) = h(A_1, A_2, \ldots, A_R)$. Since no premises are shared between subgroups, any choice of g_r for each G_r and any choice of f is possible. This space of functions delimits the possible choices for h.

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