8

Nuclear reactions

In a nuclear reaction two nuclei, or a nucleon and a nucleus, come together in such close contact that they interact through the strong force. The deuteron-stripping reaction of equation (7.3) is one example. A reaction which contributes to energy generation in stars is

$$^{16}\mathrm{O} + {}^{16}\mathrm{O} \rightarrow {}^{28}\mathrm{Si} + \alpha + 9.6\ \mathrm{MeV}, \tag{8.1}$$

and a nuclear reaction important in power technology is

$$\mathrm{n} + {}^{235}\mathrm{U} \rightarrow \text{fission products}.$$

The latter two are both *exothermic* reactions in which the kinetic energy of the final nuclei is greater than that of the initial nuclei. In an *endothermic* reaction energy must be supplied before the reaction will take place, as in the reaction inverse to (8.1) above.

8.1 The Breit–Wigner formula

The concept of *cross-section* (Appendix A) is important for understanding and classifying nuclear reactions. Figure 8.1 shows the total cross-section for neutrons to interact with the $^{16}_{8}\mathrm{O}$ nucleus as a function of the kinetic energy E (in the centre-of-mass system) up to $E = 2.3$ MeV. The principal features of the cross-section are the high but narrow *resonance peaks*, superposed on a slowly varying background. These peaks are due to the formation of excited states of $^{17}\mathrm{O}$ from the neutron and $^{16}\mathrm{O}$ at the reso-

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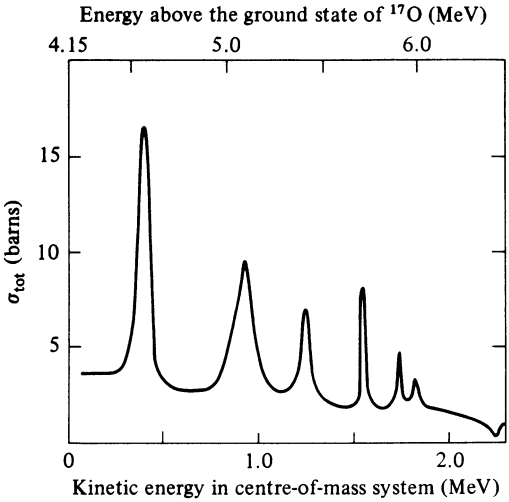


Fig. 8.1 The total cross-section for neutrons interacting with ^{16}O as a function of centre-of-mass energy, showing resonances that correspond to the formation of excited states of ^{17}O (top scale: see also Fig. 7.3). (Data from Garber, D. I. & Kinsey, R. R. (1976), *Neutron Cross Sections*, vol. II, Upton, New York: Brookhaven National Laboratory.)

nance energies. When the energy of the incident neutron is such that the total energy of the system matches, to within the width Γ, one of the excited states energies of ^{17}O, the neutron is readily accepted into the target to form that state. Note that the binding energy of the neutron in the ground state of the so-called *compound nucleus* becomes available as excitation energy. In our example of ^{17}O, if excitation energies are measured from the ground state the neutron binding energy of 4.15 MeV (cf. Table 4.2 and Fig. 7.3) has to be added to the resonance energies to obtain the ^{17}O excitation energies. This displaced energy scale is also given in Fig. 8.1. Thus only those excited states above 4.15 MeV can appear in the data.

The six peaks which appear in Fig. 8.1 correspond to the top six levels of the energy-level diagram, Fig. 7.3. The two lowest of these six correspond to states found in the deuteron-stripping reaction we discussed earlier in §7.1.

It is shown in Appendix D that excited states make a contribution to the total cross-section in the neighbourhood of the resonance energy E_0 of approximately the form

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$$\sigma_{\text{tot}}(E) = \frac{\pi}{k^2} \frac{g\Gamma_i \Gamma}{(E - E_0)^2 + \Gamma^2/4}, \tag{8.2}$$

where $k = |\mathbf{k}|$, and $\mathbf{k}$ is the wave-vector of the incoming neutron in the centre-of-mass frame, Γ_i is the partial width for decay into the incident channel $^{16}\text{O} + \text{n}$, g is a statistical factor (in this case $g = (2j + 1)/2$ where j is the spin of the excited state). The expression (8.2) is known as the *Breit–Wigner formula*. For $\Gamma \ll E_0$ the cross-section is at a maximum when $E = E_0$, and falls to half its maximum value at $E = E_0 \pm \Gamma/2$. Thus Γ is the 'full width at half-maximum' of the peak. The peak width seen experimentally depends also on the energy spread of the incident neutron beam (no particle beam is ever perfectly mono-energetic), on the thermal motion of the nuclei in the target, and on the characteristics of the detectors, so that a careful analysis may be necessary before a true intrinsic width can be obtained from the raw experimental data.

Consider the peak at $E = 0.41$ MeV in Fig. 8.1. The estimated width of the peak, $\Gamma \approx 0.04$ MeV, corresponds to a mean life of $\approx 1.6 \times 10^{-20}$ s. This is short compared with mean lives for γ-emission, but still quite long on the nuclear time scale (§5.2) of the oxygen nucleus of $\sim 10^{-22}$ s. Such a long mean life can be understood as resulting from the nature of the excited state, which is a compound nuclear state in which many nucleons participate. The neutron entering the nucleus loses its energy by collisions with other nucleons, and if it loses more than 0.41 MeV it can no longer escape. The nucleus then stays in the excited state until such time as a single neutron again acquires enough energy to get away, or (in this case with much lower probability) the nucleus decays electromagnetically. In the latter case, if the decay is to the ground state or any other state below the neutron separation energy, the neutron is captured; this process is known as *radiative capture*.

In between resonance peaks, an analysis of the background cross-section suggests that the nucleus resists penetration by the incident neutron. The neutron appears to be repelled from the surface of the ^{16}O nucleus at energies off resonance.

Resonance peaks are a feature of all cross-sections for neutron scattering from nuclei with $A \geqslant 4$ and neutron energies up to a few MeV. The 'binding energy of the last neutron' (or separation energy: see equation (5.6)) which is available for excitation energy usually lies in the range 5 MeV–15 MeV. As explained in §7.2, the density of excited states at fixed energy increases rapidly with A. Thus for neutron scattering from heavy nuclei the number of resonances per MeV increases rapidly with A.

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Also, as A increases, the width of the states becomes narrower: the states become more stable since in the compound nucleus the incoming neutron has more nucleons with which to share its energy, and the probability of any one of them acquiring enough energy to escape decreases.

All this is illustrated in Fig. 8.2, which shows the total cross-section at low energies for neutrons interacting with the heavy nucleus $^{238}_{92}\mathrm{U}$. Note that the horizontal energy scale is in electron volts, and the vertical cross-section scale is logarithmic. The resonance peaks are associated with the formation of excited states of $^{239}\mathrm{U}$, and the spacings between the peaks are only ≈ 20 eV. The resonances are very narrow, with an intrinsic width of order 10^{-2} eV. Indeed, the states are here so narrow that γ-decay competes significantly with other decay modes, and roughly half of the decays of the excited states formed at these resonances are electromagnetic and result in radiative capture. The other prominent decay mode is neutron emission. Less-common modes include α-decay and fission.

For neutron energies that are off resonance the cross-section of Fig. 8.2 is dominated by the neutron scattering from the surface of the $^{238}\mathrm{U}$ nucleus. However, other nuclear reactions are energetically possible and may occur. For example, the neutron could pick up two protons and another neutron from the $^{238}\mathrm{U}$ surface to form an α-particle:

$$\mathrm{n} + {}^{238}_{92}\mathrm{U} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{235}_{90}\mathrm{Th}.$$

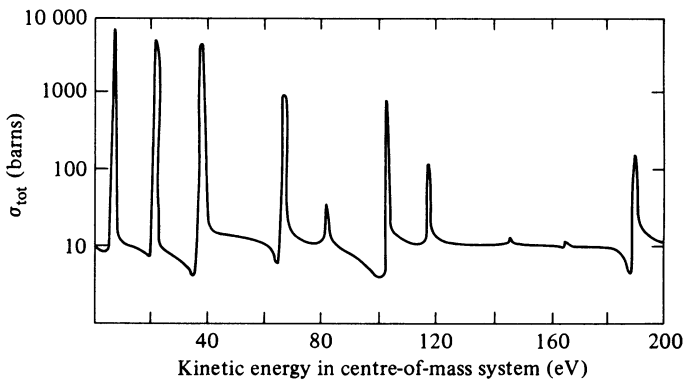


Fig. 8.2 The total cross-section for neutrons interacting with $^{238}\mathrm{U}$, as a function of centre-of-mass energy. Note that the vertical scale for the cross-section is logarithmic and the horizontal energy scale is in electron volts. (Data as in Fig. 8.1.)

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Such a reaction, when the neutron energy is off resonance, does not proceed through the formation of $^{239}_{92}$U, and is known as a *direct nuclear reaction*.

8.2 Neutron reactions at low energies

Since neutrons are uncharged there is no Coulomb barrier to overcome; hence neutrons of very low energy easily penetrate matter and interact with nuclei. In the limit $E \to 0$, only elastic scattering and exothermic nuclear reactions can take place. When a nuclear reaction is possible it can be expected that the reaction rate at sufficiently low energies will be independent of E, and simply proportional to the density of neutrons in the neighbourhood of the nucleus. The cross-section σ_{ex} for exothermic nuclear reactions is given (Appendix A) by

$$(\text{neutron flux}) \times \sigma_{\text{ex}} = \text{reaction rate per nucleus.}$$

The neutron flux is $\rho_{\text{n}} v$ (where ρ_{n} is the neutron number density in the beam and v is the velocity of the neutrons relative to the target nucleus). Since the right-hand side of the equation is also proportional to ρ_{n}, it follows that

$$\sigma_{\text{ex}} = \frac{(\text{constant})}{v} \tag{8.3}$$

at sufficiently low energies. This is the behaviour that is seen experimentally.

If the low-energy region lies in the wing of a resonance, the $(1/v)$ law follows from the Breit–Wigner formula (8.2). In this case, we must take into account the energy dependence of the partial width $\Gamma_i(E)$, found in equation (D.9) of Appendix D. $\Gamma_i(E)$ contains the factor $n_i(E)$, which is proportional to k (from equations (B.6) and (B.8) of Appendix B). For $E \approx 0$, the Breit–Wigner formula then gives

$$\sigma \approx \frac{1}{k} \frac{(\text{constant})}{E_0^2 + \Gamma^2/4}. \tag{8.4}$$

Since $\hbar k = mv$, where m is the reduced mass of the neutron and target nucleus, we recover the $(1/v)$ law. The combination of the $(1/v)$, or,

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equivalently, $(1/E^{\frac{1}{2}})$ law with a low-lying resonance is well illustrated in Fig. 8.3, which shows the low-energy cross-section for cadmium.

In quantum mechanics, non-elastic processes are always accompanied by elastic scattering, just as, in optics, absorption is always accompanied by diffraction. The elastic scattering of neutrons by nuclei takes place through compound nucleus formation and by surface scattering; the two processes are not independent and must be considered together. It may be shown that the elastic-scattering cross-section of slow neutrons does not follow the $(1/v)$ law but tends to a constant value as $E \to 0$. This limiting value depends sensitively on the presence of resonances near threshold. If the target nucleus has spin, the cross-section also depends on the relative orientation of the spins of the neutron and nucleus.

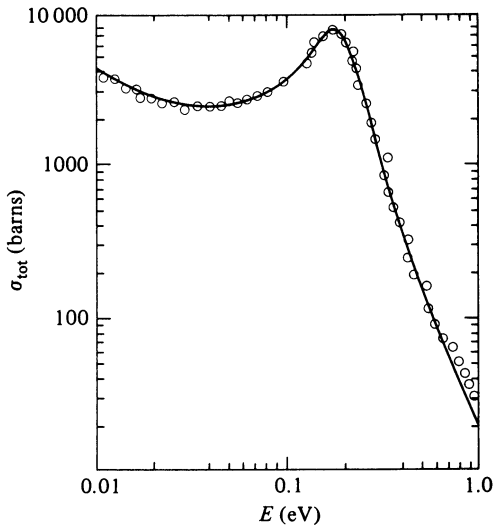


Fig. 8.3 The total cross-section for neutrons interacting with natural cadmium. The open circles ○ are experimental points (data as in Fig. 8.1). The line is a fit with

$$\sigma_{\text{tot}} = \frac{(\text{constant})}{v[(E - E_0)^2 + \Gamma^2/4]},$$

taking $E_0 = 0.18$ eV and $\Gamma = 0.12$ eV. Note that both the scales are logarithmic. On a logarithmic plot the '$1/v$' form at very low energies gives a straight line with a slope of $-\frac{1}{2}$ (cf. equation (8.4), $v = (2mE)^{\frac{1}{2}}$). This is evident for $E < 0.03$ eV. The very large resonance cross-section is due to ^{113}Cd, which constitutes 12.3% of natural cadmium.

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8.3 Coulomb effects in nuclear reactions

Our discussion of nuclear reactions has so far emphasised reactions involving neutrons. In a nuclear reaction involving a proton and a nucleus, or two nuclei, there are seen the same features of resonance scattering with the formation of a compound nuclear state, and direct nuclear reactions off resonance.

However, the effect of the Coulomb repulsion between particles in the initial or final channels of the reaction leads to significant differences in the reaction cross-sections at low energies below the Coulomb barrier height. This effect is illustrated in Fig. 8.4, which shows the low-energy cross-section for the nuclear reaction

$$\alpha + {}^{13}_{6}\mathrm{C} \rightarrow \mathrm{n} + {}^{16}_{8}\mathrm{O},$$

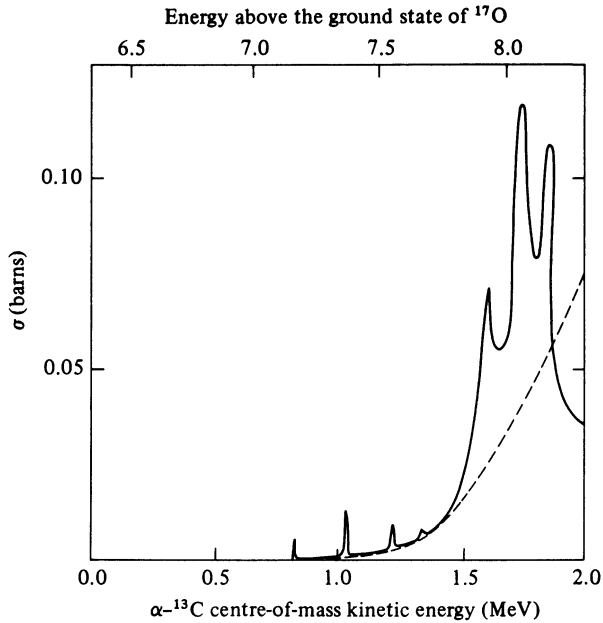


Fig. 8.4 The cross-section for the reaction $\alpha + {}^{13}\mathrm{C} \rightarrow \mathrm{n} + {}^{16}\mathrm{O}$. The dashed curve – – – exhibits the large Coulomb suppression at low energies (see text). Note the resonances at high excited-state energies of ^{17}O (top scale) which are above those shown in Fig. 7.3. (Data from Blair, J. K. & Haas, F. X. (1973), *Phys. Rev.* **C7**, 1356.)

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as a function of the centre-of-mass kinetic energy E of the incident nuclei. The reaction is in fact exothermic, with an energy release of 2.2 MeV (Fig. 7.3) and so it can in principle occur at any energy. However, at low energies the α-particle must tunnel through a Coulomb barrier before it can interact with the ^{13}C nucleus. The barrier is about 4 MeV high. In classical mechanics a nuclear interaction could not occur for an α-particle having lower energy than this. In quantum mechanics the particle can tunnel through, but the low-energy cross-section is much suppressed, as the figure clearly demonstrates. The tunnelling probability for the α-particle to penetrate the barrier from the outside is the same as the probability for tunnelling in the other direction, as in α-decay, and this we estimated in Chapter 6 to be $e^{-G(E)}$, where $G(E)$ is given by equation (6.15) (but with Q replaced by E).

It is usual to parametrise charged-particle reaction cross-sections at low energies by the expression

$$\sigma(E) = \frac{1}{E} S(E) e^{-G(E)}, \tag{8.5}$$

and Fig. 8.4 also shows this curve with $S(E)$ chosen to be a constant 0.3 barn MeV to fit the cross-section at the lowest energies. The background cross-section below the resonances roughly follows this curve, but large resonance peaks due to the formation of excited states of ^{17}O are evident.

The precise form of charged-particle nuclear reaction cross-sections at low energies is of great importance, both in astrophysics and for the prospect of controlled thermonuclear reactions on Earth. It may be shown that, as $E \to 0$, the function $S(E)$ in (8.5) tends to a constant value, which depends on the particular reaction and is very sensitive to the proximity of resonances. We can give a qualitative derivation of this result for the case when the low-energy region lies in the wing of a nearby resonance. The Breit–Wigner formula (8.2) may be written

$$\sigma(E) = \frac{\pi \hbar^3}{2mE} \frac{g\Gamma(\Gamma_i/\hbar)}{(E - E_0)^2 + \Gamma^2/4}, \tag{8.6}$$

where m is the reduced mass of the interacting particles and $\Gamma_i/\hbar$ is the decay rate into the incident channel. For energies close to threshold it is again important to include the energy dependence of Γ_i. Recalling the discussion of α-decay in Chapter 6, we replace the decay rate $(\Gamma_i/\hbar)$ by

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$(1/\tau_0)\mathrm{e}^{-G(E)}$, where τ_0 is a constant nuclear time. For $E \approx 0$ the expression (8.6) then reduces to

$$\sigma(E) = \frac{\pi\hbar^3}{2m\tau_0}\frac{g\Gamma}{E_0^2 + \Gamma^2/4}\frac{\mathrm{e}^{-G(E)}}{E}.$$

This is of the same form as (8.5) with

$$S(E) = \frac{\pi}{2m}\frac{\hbar^3}{\tau_0}\frac{g\Gamma}{E_0^2 + \Gamma^2/4},$$

a constant.

If the nuclei in a reaction have charges Z_1e, Z_2e, the expression (6.15) for $G(E)$ must of course be generalised slightly: $2Z_\mathrm{d}e^2$ is replaced by $Z_1Z_2e^2$, and m becomes the reduced mass of the nuclei involved. At very low energies r_c is large and so $\mathscr{G}(r_\mathrm{s}/r_\mathrm{c}) \to 1$. Thus

$$\begin{aligned} G(E) &\approx \frac{\pi}{\hbar c}\left(\frac{Z_1Z_2e^2}{4\pi\varepsilon_0}\right)\sqrt{\frac{2mc^2}{E}} \\ &= \sqrt{\frac{E_G}{E}}, \quad \text{say,} \end{aligned}$$

where

$$E_G = 2mc^2\left(\frac{\pi Z_1Z_2e^2}{\hbar c(4\pi\varepsilon_0)}\right)^2$$

and

$$\sigma(E) \approx \frac{1}{E}S(0)e^{-\sqrt{(E_G/E)}}. \tag{8.7}$$

8.4 Doppler broadening of resonance peaks

We mentioned in §8.1 that the thermal motion of the nuclei in the target affects the width of a resonance as seen experimentally. Neutrons in a beam incident on a target, and mono-energetic with respect to that target, are not mono-energetic with respect to the individual nuclei in the target,

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since these will be in random thermal motion. The energy that appears in the Breit–Wigner formula is the energy in the centre-of-mass frame of the neutron and the target nucleus. If the neutron has velocity $\mathbf{v}_1$ and the nucleus velocity $\mathbf{v}_2$, the centre-of-mass energy is

$$E = \tfrac{1}{2}m(\mathbf{v}_1 - \mathbf{v}_2)^2 = E_1 + \frac{m}{M}E_2 - 2\sqrt{\left(\frac{m}{M}E_1E_2\right)}\cos\theta,$$

where M is the mass of the nucleus, $m = m_n M/(m_n + M)$ is the reduced mass, E_1 is the centre-of-mass energy when thermal energy is neglected, E_2 is the thermal energy of the nucleus and θ the angle between $\mathbf{v}_1$ and $\mathbf{v}_2$.

The term $(m/M)E_2$ can be neglected if the neutron energy is much greater than thermal or if the target nucleus is heavy. The thermal energy E_2 is of order of magnitude $k_B T$, where k_B is Boltzmann's constant and T is the temperature of the target. Since $\cos\theta$ lies between -1 and $+1$, it can be seen that, when averaged over many nuclei, E will have a spread in energy about E_1 of magnitude

$$\Delta E \approx 2\sqrt{\left(\frac{m}{M}E_1k_BT\right)}.$$

Thus if a cross-section is measured in the laboratory as a function of E_1, in the neighbourhood of a resonance at energy E_0, the Breit–Wigner form is modified and, in particular, the width of the resonance peak will be larger than the natural width by an amount of order

$$\Delta\Gamma \approx 2\sqrt{\left(\frac{m}{M}E_0k_BT\right)}.$$

This is *Doppler broadening*. A more detailed analysis shows that the total area under the resonance peak is independent of temperature, so that the height of the peak is reduced as the width increases. This is illustrated in Fig. 8.5 for a resonance in ^{238}U. (We shall see in Chapter 9 that Doppler broadening is of crucial importance for the thermal stability of nuclear reactors.) In the resonance peak of Fig. 8.3, on the other hand, it is easy to check that the effect of Doppler broadening at room temperature $T_r(k_BT_r \approx (1/40)\text{ eV})$ is small.

Data on neutron scattering cross-sections will be found in McLane, V., Dunford, C. L., and Rose, P. F. (1988), *Neutron Cross Sections* Vol. 2, San Diego: Academic Press.

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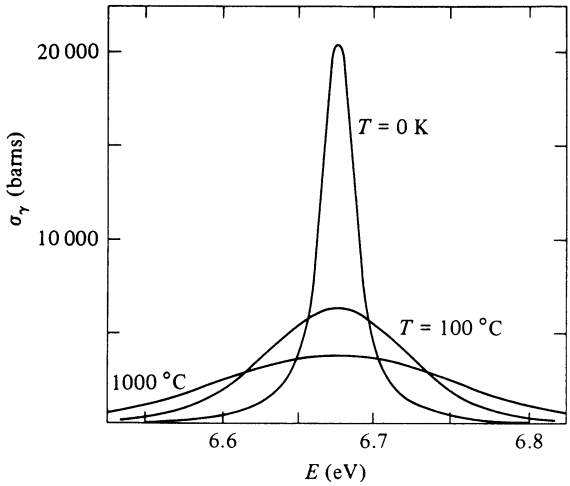


Fig. 8.5 The Doppler broadening of the Breit–Wigner cross-section σ_γ for neutron radiative capture by ^{238}U. The resonance is at 6.67 eV and its natural width is 0.026 eV.

Problems

8.1 Quantum mechanics gives the total cross-section for scattering from an impenetrable sphere of radius R at low energies ($kR \ll 1$) to be $\sigma = 4\pi R^2$. For the cross-section of Fig. 8.2, show that the order of magnitude of the cross-section between resonances is given by this formula with R the radius of the uranium nucleus, and at a resonance the order of magnitude is given by $(\lambda/2\pi)^2$, where λ is the neutron wavelength ($\lambda = 2\pi/k$), as is implied by the Breit–Wigner formula (8.2).

8.2 Neutron detectors register individual neutrons by their production of charged, ionising particles in a nuclear reaction. One method, appropriate to thermal neutrons ($E < 0.1$ eV) uses the reaction

$$\mathrm{n} + {}^3_2\mathrm{He} \rightarrow \mathrm{p} + {}^3_1\mathrm{H} + 0.73 \text{ MeV}.$$

The cross-section for this reaction, which dominates at low energies, follows the $(1/v)$ law,

$$\sigma = 0.039(c/v)\,\mathrm{b}.$$

The mean distance a neutron travels through ^{3}He gas before it interacts is $l = 1/(\rho_{\mathrm{He}}\sigma)$, where ρ_{He} is the number density of helium atoms (Appendix A). What detector thickness is needed, using ^{3}He gas at a pressure of 10 bars (which gives $\rho_{\mathrm{He}} = 2.4 \times 10^{26}$ m^{-3}) in order that at least 90% of incident neutrons with energy 0.1 eV produce ionisation?

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8.3 The nucleus $^{5}_{3}$Li is apparent as a resonance in the elastic scattering of protons from $^{4}_{2}$He at a proton energy ≈ 2 MeV. The resonance has a width of 0.5 MeV and spin $\frac{3}{2}$.

(*a*) What is the lifetime of $^{5}_{3}$Li?

(*b*) Estimate the cross-section at the resonance energy.

8.4 Figure 9.1(*b*) shows the measured total cross-section for neutrons incident on ^{238}U. What conclusion can you draw from the apparent absence of $(1/v)$ behaviour at low neutron energies?

8.5 The zero-temperature radiative capture cross-section illustrated in Fig. 8.5 is the intrinsic cross-section to which the Breit–Wigner formula is immediately applicable. The excited state has spin $\frac{1}{2}$, and there are two significant decay channels; the dominant one is γ-emission and the other is neutron emission. Estimate the relative probability of neutron radiative capture at resonance, and estimate the elastic neutron scattering cross-section at resonance. (Hint: use equation (D.11).)

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