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Wine in Your Knapsack?*

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Abstract

We pose several Knapsack Problems (KPs) to select the optimal subset of wines subject to budget and quantity constraints. The first problem seeks the subset of wines, from a single cultivar (zinfandel) that maximizes the sum of ratings subject to a budget constraint. We modify this problem by adding an equality constraint on the number of bottles that must be chosen. The third problem seeks to maximize the sum of ratings from three different cultivars (cabernet sauvignon, pinot noir, and zinfandel) subject to a budget constraint and then a budget and minimum bottle constraints for each cultivar. The rating maximization problem may have multiple solutions. We also pose two expenditure minimization problems, subject to achieving the maximum sum of ratings. We extend the model by defining an integer choice problem, that allows multiple purchases of the same wine, and by accounting for possible non-linearities between wine ratings and utilities. Applications of this model include procurement decisions for tastings, special events or building a personal wine cellar. (JEL Classification: C61 D12)

I. Introduction and Overview

The prototype, 0–1, Knapsack Problem has a hiker contemplating which of $j = 1, 2, \ldots, n$ items to be included in a knapsack of finite capacity, c. Each item has a utility, $u_j > 0$, and a weight (or size), w_j. If the j^{th} item is selected for inclusion in

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the knapsack, $X_j = 1$, if not, $X_j = 0$. The 0–1 Knapsack Problem seeks to

$$\begin{aligned} &\underset{X_j \in \{0,1\}}{\textit{Maximize}} && u = \sum_{j=1}^{n} u_j X_j \\ &\textit{Subject to} && \sum_{j=1}^{n} w_j X_j \leq c \\ & && X_j \in \{0,1\}, j = 1, 2, \ldots, n \end{aligned}$$

There are many variations on the prototype, 0–1, KP including the "Bounded Knapsack," "Subset-Sum," "Change-Making," "0–1 Multiple Knapsack," "Generalized Assignment," and "Bin-Packing" problems. See Martello et al. (1990) or Kellerer et al. (2004).

From the perspective of computer science, all of these problems are NP-hard. NP-hard stands for "non-deterministic polynomial-time hard." NP-hard is a class of problems for which no polynomial-time solution algorithm has been found. Algorithms that run in polynomial-time are viewed as efficient. This means that NP-hard problems cannot, at present, be solved efficiently. This does *not* mean that a particular "instance" of a 0–1 Knapsack Problem cannot be optimally solved. It simply means that there is no polynomial-time algorithm and as n grows, the problem scales exponentially, taking longer to find a "provably optimal solution." Advances in computing and mixed integer programming (MIP) algorithms now allow one to provably solve KPs with $n = 1{,}000{,}000$ on a personal computer. See Srisuwannapa and Charnsethikul (2007).

While the history of KPs in operations research and computer science is interesting, this paper is concerned with the application and solution of KPs for the selection of wines. The wine enthusiast, with a personal computer and an MIP solver, can quickly determine the best selection of wines for a special event (wedding, anniversary, birthday). The purpose of this paper is to illustrate and perhaps inspire a more analytical approach to wine selection.

In the next section we present the KPs of interest to a wine enthusiast planning to host a wine tasting. We start with the prototype, 0–1, KP problem and then add a quantity constraint as might arise if the wine enthusiast is planning a tasting for a given number of guests. This problem has the wine enthusiast selecting a given number of bottles from a set of n different wines based on their rating, price per bottle, and his wine budget for the party. The third problem has the enthusiast selecting a minimum number of bottles from three different cultivars, again subject to a budget constraint. While the items to be chosen in our examples are individual bottles of wine, we could allow X_j to represent a case of a particular wine (with a price including the case discount). Such a formulation may be more appropriate if one is solving a KP to stock a restaurant or wine store with a variety of wines representing different cultivars, wine growing regions, and retail prices.

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In the third and fourth sections, we illustrate the five problems by finding the best selections from 64 North Coast, California zinfandels bottled in 2003. The budget constraint is \$1,000. We then modify this problem by adding a quantity constraint where the wine enthusiast wants precisely 30 bottles for his party and is again willing to spend \$1,000. In the third example (instance), our enthusiast wants to select at least 10 bottles from three cultivars, cabernet sauvignon, pinot noir, and zinfandel, again from the North Coast, California region, all bottled in 2003, to maximize their sum of ranks. We also look at the uniqueness of the solutions found. The fifth section explores non-linearities and extensions to the problem. The sixth section concludes. In an appendix we provide the AMPL code used to solve the numerical KPs presented in this article.

II. The Wine Selection Problems

Our wine enthusiast is planning to host a tasting of California zinfandels with a budget of $B = \$1{,}000$. According to wine experts, 2003 was a good year for the California, North Coast, region. The enthusiast has identified $n = 64$ zinfandels from this region and year which he does not have in his cellar. From the internet he has quickly downloaded the rating of each wine, denoted r_j, and the per bottle price, p_j, as quoted by a dealer he has used in the past. Instead of purchasing multiple bottles of the same wine, he decides to purchase, at most, one bottle of each brand.[1]

These bottles range in their rating from the low 80s to the high 90s. They range in price from \$18 to \$140 per bottle. With a budget of $B = \$1{,}000$, the enthusiast cannot purchase all 64 bottles. The KP in this instance (KP1) seeks to

$$\begin{aligned} &\underset{X_j \in \{0,1\}}{\textit{Maximize}} && r = \sum_{j=1}^{n} r_j X_j \\ &\textit{Subject to} && \sum_{j=1}^{n} p_j X_j \leq B \\ & && X_j \in \{0,1\}, j = 1,\ 2, \ldots, n = 64 \end{aligned} \tag{KP1}$$

Where r is the sum of ratings and X_j equals 1 if wine j is selected, and 0 otherwise. After solving this prototype KP problem, the enthusiast decides to add the quantity

[1] We are using 2003 North Coast Zinfandels as an illustrative example. The analysis could be done for more recent vintages as 2003 Zinfandels are now more scarce. A knowledgeable reviewer also suggested that more recent vintages would be appropriate, given that Zinfandels should be consumed between five and seven years of the vintage.

constraint, $\sum_{j=1}^{30} X_j = 30$ (KP2). The enthusiast now wishes to

$$\begin{aligned} \underset{X_j \in \{0,1\}}{\textit{Maximize}} \quad & r = \sum_{j=1}^{n} r_j X_j \\ \textit{Subject to} \quad & \sum_{j=1}^{n} p_j X_j \leq B \\ & \sum_{j=1}^{30} X_j = 30 \\ & X_j \in \{0,1\},\ j = 1,2,\ldots,n = 64 \end{aligned} \tag{KP2}$$

It turns out that the solution to Problems KP1 and KP2 may not be unique. This can arise if there are wines in the choice set with the same rating and price. There may be several 30-bottle combinations that produce the maximum sum of ratings, $r^* = \sum_{j=1}^{n} r_j X_j^*$.

From the multiple solutions to Problem KP2 the enthusiast then seeks a solution (KP3) which will

$$\begin{aligned} \underset{X_j \in \{0,1\}}{\textit{Minimize}} \quad & E = \sum_{j=1}^{n} p_j X_j \\ \textit{Subject to} \quad & \sum_{j=1}^{n} r_j X_j \geq r* \\ & \sum_{j=1}^{30} X_j = 30 \\ & X_j \in \{0,1\},\ j = 1,2,\ldots,n = 64 \end{aligned} \tag{KP3}$$

where E is the total expenditure for the wine selection problem. As we will see, there may be multiple solutions to KP3 as well. The enthusiast would like to know the number of solutions to both Problems KP2 and KP3.

Finally, the enthusiast is interested in the best wines from $m = 3$ cultivars: cabernet sauvignon ($k = 1$), pinot noir ($k = 2$), and zinfandels ($k = 3$). Let $X_{j,k}$ denote the j^{th} wine in the k^{th} cultivar class and let n_k be the number of wines (bottles) in the k^{th} class. Suppose the enthusiast wants at least $C_k > 0$ bottles from the k^{th} cultivar so that $\sum_{j=1}^{n_k} X_{j,k} \geq C_k$. In our example we will assume that $C_k = 10$ for $k = 1,2,3$. This results in a KP (KP4) seeking to

$$\begin{aligned} \underset{X_{j,k} \in \{0,1\}}{\textit{Maximize}} \quad & r = \sum_{k=1}^{m} \sum_{j=1}^{n_k} r_{j,k} X_{j,k} \\ \textit{Subject to} \quad & \sum_{k=1}^{m} \sum_{j=1}^{n_k} p_{j,k} X_{j,k} \leq B \\ & \sum_{j=1}^{n_k} X_{j,k} \geq C_k = 10 \\ & X_{j,k} \in \{0,1\}, j = 1,2,\ \ldots, n_k, k = 1,2,3 = m \end{aligned} \tag{KP4}$$

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If there are multiple solutions to Problem KP4, the enthusiast may need to solve for the least cost solutions which achieve r^*, KP5. Specifically he would want to

$$\begin{aligned} &\underset{X_{j,k}\in\{0,1\}}{\textit{Minimize}} && E = \sum_{k=1}^{m}\sum_{j=1}^{n_k} p_{j,k}X_{j,k} \\ &\textit{Subject to} && \sum_{k=1}^{m}\sum_{j=1}^{n_k} r_{j,k}X_{j,k} \geq r* \\ & && \sum_{j=1}^{n_k} X_{j,k} \geq C_k = 10 \\ & && X_{j,k} \in \{0,1\}, j = 1,2,\ \ldots,\ n_k, k = 1,2,3 = m \end{aligned} \qquad \text{(KP5)}$$

A. *Finding all Solutions to the Problem*

A relevant question of interest is to find all indifferent solutions to the aforementioned problems KP2–KP5. To answer this question, a new set of dynamic constraints was added, and the problem is solved recursively. These new constraints will identify solutions yielding the same r^* or E^* but which differ in the wines selected. The reason for this is to be able to accommodate the quantity constraints per class. The variables defined are:

Table 1
Sets Defined for Iterative Determination of Solutions

Variable	*Description*
i	Solution Index
P	Set of choices for each class
c	Set of classes (cultivars)
cont	Set of past solutions found
solv	Set of new solutions found

The pseudo-algorithm to identify multiple solutions looks as follows (See appendix for AMPL code).

[1] $i \leftarrow 1$;

[2] Do first run without constraints on past solutions.

[3] $i \leftarrow i + 1$;

[4] **repeat**

[5] Create dynamic set to store solutions (*cont*). This is implemented as a matrix with increasing number of columns in each iteration (dimensions $P.c \times cont$, where $P.c$ is the number of solutions for all classes, *cont* is the number of solutions found -initially 1).

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[6] Store first solution in the first column of the matrix. The implementation places each integer solution per class as a stacked column vector.

[7] **for all** consecutive runs, $i > 1$ **do**

[8] add a constraint such that the squared difference between each element of the new solution and each element of all past solutions is greater than or equal to one. This is equivalent to state that there must be at least one position of the integer solution vector (1's and 0's) that differs.

[9] Store the new solution found in the set of past solutions (*cont*)

[10] $i \leftarrow i + 1$;

[11] **end for**

[12] **until** $i = N_{iter}$, run-time specified by user.

Note that the set *cont* is increasing with each iteration, corresponding to new solutions found.

The overall condition evaluated for each candidate solution is as follows:

$$\forall i \in cont : \sum_{j \in P} \sum_{k \in c} (X[j,k] - solv[j,i,k])^2 >= 1;$$

III. Results

In Table 2 we list the 64 zinfandels comprising the choice set in KP1. The rating and price for each zinfandel may be found at the web site for this article[2] and in Parker (2003). Recall that the budget constraint to KP1 was $B = \$1{,}000$. There are five solutions to this problem, each yielding $r^* = 2,896$. Each solution selects 32 bottles. The solutions differ from one another by one bottle of equivalent price and rating. The bottles selected in each of the five solutions are highlighted and can be identified by moving down a column under a particular solution number. The five solutions share 31 wines in common. Solution 1 includes the zinfandel with Identification Number (IN) 22 (Hartford Vineyard Zinfandel) that is not included in Solutions 2–5. Solution 2 substitutes IN 21 (Hartford's Fanucchi Wood Road Vineyard) for IN 22. IN 19 (Hartford's Dina's Vineyard) is unique to Solution 3, IN 7 (Robert Biale's Monte Rosso) is unique to Solution 4, and IN 3 (Robert Biale's Aldo' Vineyard) is unique to Solution 5. All five solutions completely exhaust the budget and they are the only solutions to the expenditure minimization problem.

Now consider KP2 for the same choice set of 64 zinfandels but with the addition of a quantity constraint, $\sum_{j=1}^{64} X_j = 30$. We know from mathematical programming that the value for r^* in KP1 must equal or exceed the value of r^* in KP2.

[2] The website with the results is http://hortmgt.gomez.dyson.cornell.edu/grad.html

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Table 2
Solution to KP1

				Solution				
Zinfandel (i)	*IN*	*r*	*p*	*1*	*2*	*3*	*4*	*5*
Acorn Zinfandel Heritage Alegria Vineyard	1.00	84.5	32	0	0	0	0	0
Robert Biale Vineyards Zinfandel	2.00	84.5	27	1	1	1	1	1
Robert Biale Vineyards Zinfandel Aldo's Vineyard	3.00	92.5	40	0	0	0	0	1
Robert Biale Vineyards Zinfandel Black Chicken	4.00	92.5	34	1	1	1	1	1
Robert Biale Vineyards Zinfandel Grande	5.00	84.5	40	0	0	0	0	0
Robert Biale Vineyards Zinfandel Old Crane Ranch	6.00	84.5	34	0	0	0	0	0
Robert Biale Vineyards Zinfandel Monte Rosso	7.00	92.5	40	0	0	0	1	0
Carlisle Winery Zinfandel Tom Feeney Ranch	8.00	92.5	33	1	1	1	1	1
Carlisle Winery Zinfandel Riebli Ranch	9.00	92.5	30	1	1	1	1	1
Carlisle Winery Zinfandel Rossi Ranch	10.00	92.5	38	1	1	1	1	1
Carlisle Winery Zinfandel Fava Ranch	11.00	92.5	33	1	1	1	1	1
Carlisle Winery Zinfandel Dry Creek Valley	12.00	92.5	36	1	1	1	1	1
Carlisle Winery Zinfandel	13.00	92.5	25	1	1	1	1	1
Carlisle Winery Zinfandel Carlisle Vineyard	14.00	92.5	38	1	1	1	1	1
Edmeades Estate Zinfandel Ciapusci Vineyard	15.00	92.5	28	1	1	1	1	1
Edmeades Estate Zinfandel	16.00	84.5	18	1	1	1	1	1
Edmeades Estate Zinfandel Piffero Vineyard	17.00	84.5	28	1	1	1	1	1
Franus Zinfandel Brandlin Ranch	18.00	92.5	32	1	1	1	1	1
Hartford Hartford Zinfandel Dina's Vineyard	19.00	92.5	40	0	0	1	0	0
Hartford Zinfandel	20.00	92.5	30	1	1	1	1	1
Hartford Zinfandel Fanucchi Wood Road Vineyard	21.00	92.5	40	0	1	0	0	0
Hartford Zinfandel Hartford Vineyard	22.00	92.5	40	1	0	0	0	0
Hartford Zinfandel Highwire Vineyard	23.00	92.5	35	1	1	1	1	1
J C Cellars Zinfandel Arrowhead Vineyard	24.00	92.5	35	1	1	1	1	1
J C Cellars Zinfandel Iron Hill Vineyard	25.00	92.5	30	1	1	1	1	1
Robert Keenan Zinfandel	26.00	84.5	26	1	1	1	1	1
Mara Zinfandel Reserve Dolinsek Ranch	27.00	84.5	40	0	0	0	0	0
Mara Zinfandel Reserve Luvisi Ranch	28.00	92.5	35	1	1	1	1	1
Martinelli Zinfandel Giuseppe and Luisa	29.00	92.5	36	1	1	1	1	1
Martinelli Zinfandel Jackass Hill Vineyard	30.00	98	75	0	0	0	0	0
Martinelli Zinfandel Jackass Vineyard	31.00	92.5	50	0	0	0	0	0
Louis Martini Zinfandel Monte Rosso Gnarly Vine	32.00	92.5	50	0	0	0	0	0
Murphy-Goode Winery Snake Eyes Elaine Maria	33.00	92.5	35	1	1	1	1	1
Neyers Zinfandel Tofanelli Vineyard	34.00	84.5	29	0	0	0	0	0
Niebaum-Coppola Edizione Pennino Zinfandel Estate	35.00	84.5	25	1	1	1	1	1
Rancho Zabaco Zinfandel Monte Rosso	36.00	92.5	45	0	0	0	0	0
Rancho Zabaco Zinfandel Monte Rosso Toreador	37.00	92.5	60	0	0	0	0	0
Rosenblum Cellars Hendry Vineyard Reserve	38.00	92.5	41	0	0	0	0	0
Rosenblum Cellars Zinfandel Lyons Vineyard Reserve	39.00	92.5	45	0	0	0	0	0
Rosenblum Cellars Zinfandel Maggie's Reserve	40.00	84.5	45	0	0	0	0	0
Saddleback Cellar Venge Family Res. Scouts Honor	41.00	92.5	35	1	1	1	1	1
Saddleback Cellar Zinfandel Old Vines	42.00	92.5	32	1	1	1	1	1
Sausal Zinfandel Old Vine Estate	43.00	84.5	30	0	0	0	0	0
Sbragia Family Vineyards Zinfandel	44.00	84.5	25	1	1	1	1	1
Scherrer Zinfandel Scherrer Vineyard	45.00	84.5	55	0	0	0	0	0

Continued

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Zinfandel (i)	*IN*	*r*	*p*	*Solution* 1	2	3	4	5
Storybook Mountain Zinfandel Atlas Peak	46.00	84.5	25	1	1	1	1	1
Storybook Mountain Zinfandel Eastern Exposure	47.00	92.5	35	1	1	1	1	1
Storybook Mountain Zinfandel Estate Reserve	48.00	92.5	45	0	0	0	0	0
Storybook Mountain Zinfandel Mayacamas Range	49.00	84.5	27	1	1	1	1	1
Summers Ranch Zinfandel Villa Andriana Vineyard	50.00	92.5	24	1	1	1	1	1
Trentadue Zinfandel la Storia	51.00	84.5	30	0	0	0	0	0
Turley Wine Cellars Zinfandel Dragon	52.00	92.5	100	0	0	0	0	0
Turley Wine Cellars Zinfandel Estate	53.00	92.5	70	0	0	0	0	0
Turley Wine Cellars Zinfandel Hayne Vineyard	54.00	92.5	138	0	0	0	0	0
Turley Wine Cellars Mead Ranch Atlas Peak	55.00	92.5	52	0	0	0	0	0
Turley Wine Cellars Moore (Earthquake) Vineyard	56.00	92.5	140	0	0	0	0	0
Turley Wine Cellars Zinfandel Rattlesnake Ridge	57.00	92.5	59	0	0	0	0	0
Turley Wine Cellars Zinfandel Tofanelli Vineyard	58.00	92.5	105	0	0	0	0	0
Turley Wine Grist Vineyard Bradford Mountain	59.00	92.5	80	0	0	0	0	0
Turley Wine Cellars Zinfandel Rancho Burro	60.00	92.5	89	0	0	0	0	0
Vieux-Os Wines Zinfandel Tofanelli Vineyard	61.00	84.5	42	0	0	0	0	0
Vieux-Os Wines Zinfandel Hell Hole Vineyard	62.00	92.5	35	1	1	1	1	1
Vieux-Os Wines Zinfandel Ira Carter Vineyard	63.00	92.5	35	1	1	1	1	1
Williams Selyem Zinfandel Bacigalupi Vineyard	64.00	92.5	42	0	0	0	0	0

There are 50 solutions to KP2 all yielding $r^* = 2,751$. We then solve KP3 where we identify 15 solutions with $r^* = 2,751$ and where the minimized expenditure is $E^* = \sum_{j=1}^{64} p_j X_j^* = 987$, thus saving \$13 from our budget, $B = \$1,000$. The 15 solutions to KP3 are shown in Table 3 where the wines contained in a particular solution are again underlined as one moves down a solution column. The 50 solutions to KP2, containing the 15 solutions to KP3, can be found at the web site for this article.

The Cab-Pinot-Zin selection problem is the most complex and the most interesting. The choice set now includes 164 cabernets, 100 pinots, and the same 64 zinfandels contained in Tables 2 and 3. Table 4 provides descriptive statistics for the overall dataset. Cabernets exhibit the highest average price, \$106.41, which is more than double the average price for zinfandels (\$44.11) and about 50 percent above the average price for pinots (\$73.23). These wines range widely in both price and rating, making a systematic approach to wine selection under a budget constraint all the more important.

Recall that in KP4 and KP5 there was a budget constraint, $\sum_{k=1}^{m} \sum_{j=1}^{n_k} p_{j,k} X_{j,k} \leq B$, and cultivar-quantity constraints, $\sum_{j=1}^{n_k} X_{j,k} \geq C_k = 10$. When we solve KP4 we get four solutions all yielding $r^* = 3,262.5$ and each precisely exhausts the budget of $B = \$1,000$. After running the expenditure minimization problem, we determine that there are no solutions that could achieve $r^* = 3,262.5$ for $\sum_{k=1}^{m} \sum_{j=1}^{n_k} p_{j,k} X_{j,k} < B$. The four solutions to KP4 and KP5 are shown in Table 5.

A careful study of Table 5 provides some interesting insights to the Cab-Pinot-Zin selection problem. First, only 11 cabs, 10 pinots, and 18 zins are involved in the

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four solutions in Table 5. Nine cabs with the INs 2, 20, 38, 87, 88, 121, 126, 129, and 163 appear in all four solutions. Cab IN 125 appears in Solutions 3 and 4 while cab IN 140 substitutes for 125 in Solutions 1 and 2. Of the 11 cabs involved in the four solutions, six were rated at 92.5 and five were rated at 84.5.[3] The same 10 pinots, with INs 15, 17, 28, 41, 48, 49, 51, 52, 86, and 89, appear in all four solutions. Seven out of these 10 pinots have a rating of 84.5 while three were rated at 92.5.

A total of 18 zins appeared in the four solutions. Common to all four solutions were zins with INs 2, 9, 13, 15, 16, 17, 18, 20, 25, 26, 35, 42, 44, 46, 49, and 50. In Solutions 1 and 3 Zin IN 8 appears, but is replaced by Zin IN 11 in Solutions 2 and 4. This is again an example where multiple solutions can arise in a quantity-constrained wine selection problem. From the point of view of the CPLEX Solver, Zin IN8 is identical to Zin IN11.

Of the 18 zins involved in the four solutions, 8 were rated at 84.5 while 10 were rated at 92.5. The fact that the quantity constraint was not binding for the zinfandel selection suggests that it was a cultivar yielding a higher rating per dollar compared to the cabs and pinots. To have more money to spend on the zinfandels, the CPLEX Solver opted to purchase only the minimum number of cabs and pinots and to purchase relatively lower rated, inexpensive wines. Table 6 provides descriptive statistics for KP5.

If we remove the quantity constraints from the Cab-Pinot-Zin problem, the maximized sum of ratings is again $r^* = 3,262.5$ but the composition of the selected cabs, pinots, and zins changes significantly. There are 50 solutions to the KP maximizing the sum of ratings but only one solution to the expenditure minimization problem subject to $r^* = 3,262.5$ (Table 7). In that solution, 8 cabs are chosen, 9 pinots are chosen, and 20 zins are chosen for an expenditure of $995. With no quantity constraints the CPLEX Solver finds higher rating per dollar wines in the zins and limits the number of cabs and pinots to those that can compete on a rating per dollar basis.

Table 7 displays the solution to KP5 after removing all quantity constraints while Table 8 provides descriptive statistics for that solution.

IV. Uniqueness of Solutions for Binding Budget Constraints

Consider the two knapsack problems above defined in KP2 and KP3. In the first problem we wish to maximize the sum of the ratings for selected wines, subject to a budget and quantity constraint. The quantity constraint assumes only one bottle of each wine is chosen. As noted, there may be multiple solutions to KP2. In Problem

[3] These ratings are calculated by taking the midpoint in the range provided by *The Wine Advocate* rating (Parker, 2003).

Table 3
Solution to KP3

		Solution														
Zinfandel (i)	*IN*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
Acorn Zinfandel Heritage Alegria Vineyard	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Robert Biale Vineyards Zinfandel	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Robert Biale Vineyards Zinfandel Aldo's Vineyard	3	1	1	0	1	0	1	1	1	0	1	1	1	1	1	1
Robert Biale Vineyards Zinfandel Black Chicken	4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Robert Biale Vineyards Zinfandel Grande	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Robert Biale Vineyards Zinfandel Old Crane Ranch	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Robert Biale Vineyards Zinfandel Monte Rosso	7	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1
Carlisle Winery Zinfandel Tom Feeney Ranch	8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Carlisle Winery Zinfandel Riebli Ranch	9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Carlisle Winery Zinfandel Rossi Ranch	10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Carlisle Winery Zinfandel Fava Ranch	11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Carlisle Winery Zinfandel Dry Creek Valley	12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Carlisle Winery Zinfandel	13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Carlisle Winery Zinfandel Carlisle Vineyard	14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Edmeades Estate Zinfandel Ciapusci Vineyard	15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Edmeades Estate Zinfandel	16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Edmeades Estate Zinfandel Piffero Vineyard	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Franus Zinfandel Brandlin Ranch	18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Hartford Hartford Zinfandel Dina's Vineyard	19	1	0	1	0	1	1	1	1	1	1	1	1	1	1	0
Hartford Zinfandel	20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Hartford Zinfandel Fanucchi Wood Road Vineyard	21	0	1	1	1	1	1	1	1	1	1	1	0	1	0	1
Hartford Zinfandel Hartford Vineyard	22	1	1	1	1	1	1	1	1	1	0	0	1	0	1	1
Hartford Zinfandel Highwire Vineyard	23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
J C Cellars Zinfandel Arrowhead Vineyard	24	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
J C Cellars Zinfandel Iron Hill Vineyard	25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Robert Keenan Zinfandel	26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Mara Zinfandel Reserve Dolinsek Ranch	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Mara Zinfandel Reserve Luvisi Ranch	28	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Martinelli Zinfandel Giuseppe and Luisa	29	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

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Martinelli Zinfandel Jackass Hill Vineyard	30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Martinelli Zinfandel Jackass Vineyard	31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Louis Martini Zinfandel Monte Rosso Gnarly Vine	32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Murphy-Goode Winery Snake Eyes Elaine Maria	33	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Neyers Zinfandel Tofanelli Vineyard	34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Niebaum-Coppola Edizione Pennino Zinfandel Estate	35	0	0	0	1	1	1	0	1	1	0	1	1	1	1	1	1
Rancho Zabaco Zinfandel Monte Rosso	36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Rancho Zabaco Zinfandel Monte Rosso Toreador	37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Rosenblum Cellars Hendry Vineyard Reserve	38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Rosenblum Cellars Zinfandel Lyons Vineyard Reserve	39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Rosenblum Cellars Zinfandel Maggie's Reserve	40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Saddleback Cellar Venge Family Res. Scouts Honor	41	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Saddleback Cellar Zinfandel Old Vines	42	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Sausal Zinfandel Old Vine Estate	43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Sbragia Family Vineyards Zinfandel	44	1	1	1	1	1	1	1	0	0	1	1	1	0	0	0	
Scherrer Zinfandel Scherrer Vineyard	45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Storybook Mountain Zinfandel Atlas Peak	46	1	1	1	0	0	0	1	1	1	1	0	0	1	1	1	
Storybook Mountain Zinfandel Eastern Exposure	47	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
Storybook Mountain Zinfandel Estate Reserve	48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Storybook Mountain Zinfandel Mayacamas Range	49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Summers Ranch Zinfandel Villa Andriana Vineyard	50	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
Trentadue Zinfandel la Storia	51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Turley Wine Cellars Zinfandel Dragon	52	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Turley Wine Cellars Zinfandel Estate	53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Turley Wine Cellars Zinfandel Hayne Vineyard	54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Turley Wine Cellars Mead Ranch Atlas Peak	55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Turley Wine Cellars Moore (Earthquake) Vineyard	56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Turley Wine Cellars Zinfandel Rattlesnake Ridge	57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Turley Wine Cellars Zinfandel Tofanelli Vineyard	58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Turley Wine Grist Vineyard Bradford Mountain	59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Turley Wine Cellars Zinfandel Rancho Burro	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Vieux-Os Wines Zinfandel Tofanelli Vineyard	61	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Vieux-Os Wines Zinfandel Hell Hole Vineyard	62	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
Vieux-Os Wines Zinfandel Ira Carter Vineyard	63	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
Williams Selyem Zinfandel Bacigalupi Vineyard	64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	

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Table 4
Descriptive Statistics for the Cab-Pinot-Zin Dataset

Rating		*Type*			
		Cabernet	*Pinot*	*Zinfandel*	*Grand Total*
84.50	Average Price ($)	59	47	32	49
	Max Price ($)	220	95	55	220
	Min Price ($)	10	19	18	10
92.50	Average Price ($)	103	86	48	87
	Max Price ($)	485	370	140	485
	Min Price ($)	27	25	24	24
97.50	Average Price ($)	351	411		361
	Max Price ($)	830	490		830
	Min Price ($)	75	333		75
98.00	Average Price ($)			75	75
	Max Price ($)			75	75
	Min Price ($)			75	75
Total Average Price ($)		106	73	44	84
Total Max Price ($)		830	490	140	830
Total Min Price ($)		10	19	18	10

KP3, the set of solutions found in KP2, is searched for the solution or solutions that minimize the expenditure while achieving (or exceeding) r^*.

We claim that the solutions to the consumer problems defined in KP1 to KP5 (maximize utility and minimize expenditure) must lie in the intersection of the solution sets for problems KP2 and KP3. Furthermore, these solutions are the only solutions to the overall problem (i.e. to choose the bundle that minimizes the expenditure by the consumer, subject to a lower bound on the sum of ratings) in cases when the budget constraint is binding.

Claim 1: Denote by $SP2$ the set of solutions to problem KP2 such that the budget is completely exhausted, and $SP3$ the set of solutions to problem KP3. Consider the subset $sp3 = \{\mathbf{X} \in SP3 : \sum_{i=1}^{N} p_i \times X_i = B\}$. $\forall \mathbf{X} \in sp3, \mathbf{X}$ is a solution to the consumer selection problem.

Proof of Claim 1

The first part is to prove that all solutions of KP2 are solutions of KP3.[4]

For the sake of contradiction (FSOC) suppose not $\rightarrow SP2 \neq \{\}, \wedge, \exists \mathbf{X}^j : \mathbf{X}^j \in SP2$ but $\mathbf{X}^j \notin SP3$. Let $\mathbf{X}^k \in SP3$. Therefore $\sum_{i=1}^{N} p_i \times X_i^k < \sum_{i=1}^{N} p_i \times X_i^j, \wedge, u(\mathbf{X}^k) > u(\mathbf{X}^j) = r^j = r^*$, the value function for KP2 from $\mathbf{X}^j$. But then, $\mathbf{X}^j$ cannot be a solution to KP2. Therefore a contradiction is reached ↖.

[4] This is equivalent to saying that If SP2 is a not-empty set (SP2 ≠ { }), SP3 is a not empty set.

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Table 5
Solution to KP4-KP5

					Solution			
Type	*Name*	*IN*	*r*	*p*	*1*	*2*	*3*	*4*
Cabernet	Flora Springs Cabernet Sauvignon	2	85	26	1	1	1	1
Cabernet	Beringer Cabernet Sauvignon	20	85	27	1	1	1	1
Cabernet	Chateau Saint Jean Cabernet Sauvignon	38	93	27	1	1	1	1
Cabernet	Louis Martini Cabernet Sauvignon	87	85	17	1	1	1	1
Cabernet	Louis Martini Cabernet Sauvignon	88	85	24	1	1	1	1
Cabernet	Robert Mondavi Cabernet Sauvignon Napa	121	93	27	1	1	1	1
Cabernet	Ruston Family Vineyards Cabernet Sauvignon Stagecoach Vineyard	125	93	36	0	0	1	1
Cabernet	Rutherford Ranch Cabernet Sauvignon	126	93	10	1	1	1	1
Cabernet	Sbragia Family Vineyards Cabernet Sauvignon Andolsen	129	93	35	1	1	1	1
Cabernet	Seavey Caravina	140	93	36	1	1	0	0
Cabernet	Zahtila Vineyards Cabernet Sauvignon	163	93	33	1	1	1	1
Pinot	Beringer Pinot Noir Stanley Ranch	15	85	30	1	1	1	1
Pinot	Chateau Saint Jean Pinot Noir	17	85	19	1	1	1	1
Pinot	Failla Pinot Noir	28	93	32	1	1	1	1
Pinot	Husch Pinot Noir	41	85	21	1	1	1	1
Pinot	La Crema Pinot Noir	48	85	29	1	1	1	1
Pinot	La Crema Pinot Noir Anderson Valley	49	85	29	1	1	1	1
Pinot	La Crema Pinot Noir Russian River	51	85	29	1	1	1	1
Pinot	Landmark Pinot Noir Grand Detour	52	93	25	1	1	1	1
Pinot	Walter Hansel Winery Pinot Noir Estate	86	85	29	1	1	1	1
Pinot	Walter Hansel Winery Pinot Noir The North Slope	89	93	30	1	1	1	1
Zinfandel	Robert Biale Vineyards Zinfandel	2	85	27	1	1	1	1
Zinfandel	Carlisle Winery Zinfandel Tom Feeney Ranch	8	93	33	1	0	1	0
Zinfandel	Carlisle Winery Zinfandel Riebli Ranch	9	93	30	1	1	1	1
Zinfandel	Carlisle Winery Zinfandel Fava Ranch	11	93	33	0	1	0	1
Zinfandel	Carlisle Winery Zinfandel	13	93	25	1	1	1	1
Zinfandel	Edmeades Estate Zinfandel Ciapusci Vineyard	15	93	28	1	1	1	1
Zinfandel	Edmeades Estate Zinfandel	16	85	18	1	1	1	1
Zinfandel	Edmeades Estate Zinfandel Piffero Vineyard	17	85	28	1	1	1	1
Zinfandel	Franus Zinfandel Brandlin Ranch	18	93	32	1	1	1	1
Zinfandel	Hartford Zinfandel	20	93	30	1	1	1	1
Zinfandel	J C Cellars Zinfandel Iron Hill Vineyard	25	93	30	1	1	1	1
Zinfandel	Robert Keenan Zinfandel	26	85	26	1	1	1	1
Zinfandel	Niebaum-Coppola Edizione Pennino Zinfandel Estate	35	85	25	1	1	1	1
Zinfandel	Saddleback Cellar Zinfandel Old Vines	42	93	32	1	1	1	1
Zinfandel	Sbragia Family Vineyards Zinfandel	44	85	25	1	1	1	1
Zinfandel	Storybook Mountain Zinfandel Atlas Peak	46	85	25	1	1	1	1
Zinfandel	Storybook Mountain Zinfandel Mayacamas Range	49	85	27	1	1	1	1
Zinfandel	Summers Ranch Zinfandel Villa Andriana Vineyard	50	93	24	1	1	1	1

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Table 6
Statistics, Solution Found to KP5

Rating		*Type*			
		Cabernet	*Pinot*	*Zinfandel*	*Grand Total*
84.5	Number	5.00	7.00	8.00	20.00
	Min Price ($)	10.00	19.00	18.00	10.00
	Average Price ($)	20.80	26.57	25.13	24.55
	Max Price ($)	27.00	30.00	28.00	30.00
	Expenditure ($)	104.00	186.00	201.00	491.00
	Exp. Percentage	50.00	70.00	47.06	54.05
92.5	Number	5.00	3.00	9.00	17.00
	Min Price ($)	27.00	25.00	24.00	24.00
	Average Price ($)	31.60	29.00	29.33	29.94
	Max. Price ($)	36.00	32.00	33.00	36.00
	Expenditure ($)	158.00	87.00	264.00	509.00
	Exp. Percentage	50.00	30.00	52.94	45.95
Total Number		10.00	10.00	17.00	37.00
Total Min Price ($)		10.00	19.00	18.00	10.00
Total Average Price ($)		26.20	27.30	27.35	27.03
Total Max Price ($)		36.00	32.00	33.00	36.00
Total Expenditure ($)		262.00	273.00	465.00	1000.00

Note that due to the discrete nature of the choices in this problem, the utility function is not continuous.

The above results are derived from construction and definition of problem KP3, $SP3 = \{\mathbf{X} : \mathbf{X}\ minimizes\ C, \wedge, \sum_{i=1}^{r} N_i \times X_i \geq r^*\}$.

The second part is to prove that the solutions in *sp3* are the unique solutions to the overall consumer problem -and therefore are solutions to both KP2 and KP3.

FSOC Suppose not $\rightarrow \exists \mathbf{X}^m \in sp3 : \mathbf{X}^m \notin SP2$. Let $\mathbf{X}^j \in SP2$. Therefore $r^* = r^j = u(\mathbf{X}^j) > u(\mathbf{X}^m), \wedge, \sum_{i=1}^{N} p_i \times X_i^j < \sum_{i=1}^{N} p_i \times X_i^m = B$. But then, $\mathbf{X}^m$ cannot be a solution to KP3, and hence $\mathbf{X}^m \notin sp3 (sp3 \subseteq SP3)$. Therefore a contradiction is reached $\nwarrow$.

The extension to different classes is straightforward and therefore this proof also applies to the problems defined by KP4 and KP5.

V. Integer Problems and Nonlinearities

The prototype KP problem is a linear-binary problem. However, the wine selection problem might be more appropriately framed as an integer-choice problem where the wine enthusiast is allowed to buy multiple bottles of the same brand and where the utility of a bottle with rating r_j may be nonlinear with respect to the number of

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Table 7
Solution to KP4 and KP5 Without Quantity Constraints

Type	*Name*	*IN*	*r*	*p*
Cabernet	Flora Springs Cabernet Sauvignon	2	84.5	26
Cabernet	Beringer Cabernet Sauvignon	20	84.5	27
Cabernet	Chateau Saint Jean Cabernet Sauvignon	38	92.5	27
Cabernet	Louis Martini Cabernet Sauvignon	87	84.5	17
Cabernet	Louis Martini Cabernet Sauvignon	88	84.5	24
Cabernet	Robert Mondavi Cabernet Sauvignon Napa	121	92.5	27
Cabernet	Rutherford Ranch Cabernet Sauvignon	126	92.5	10
Cabernet	Zahtila Vineyards Cabernet Sauvignon	163	92.5	33
Pinot	Chateau Saint Jean Pinot Noir	17	84.5	19
Pinot	Failla Pinot Noir	28	92.5	32
Pinot	Husch Pinot Noir	41	84.5	21
Pinot	La Crema Pinot Noir	48	84.5	29
Pinot	La Crema Pinot Noir Anderson Valley	49	84.5	29
Pinot	La Crema Pinot Noir Russian River	51	84.5	29
Pinot	Landmark Pinot Noir Grand Detour	52	92.5	25
Pinot	Walter Hansel Winery Pinot Noir Estate	86	84.5	29
Pinot	Walter Hansel Winery Pinot Noir The North Slope	89	92.5	30
Zinfandel	Robert Biale Vineyards Zinfandel	2	84.5	27
Zinfandel	Robert Biale Vineyards Zinfandel Black Chicken	4	92.5	34
Zinfandel	Carlisle Winery Zinfandel Tom Feeney Ranch	8	92.5	33
Zinfandel	Carlisle Winery Zinfandel Riebli Ranch	9	92.5	30
Zinfandel	Carlisle Winery Zinfandel Fava Ranch	11	92.5	33
Zinfandel	Carlisle Winery Zinfandel	13	92.5	25
Zinfandel	Edmeades Estate Zinfandel Ciapusci Vineyard	15	92.5	28
Zinfandel	Edmeades Estate Zinfandel	16	84.5	18
Zinfandel	Edmeades Estate Zinfandel Piffero Vineyard	17	84.5	28
Zinfandel	Franus Zinfandel Brandlin Ranch	18	92.5	32
Zinfandel	Hartford Zinfandel	20	92.5	30
Zinfandel	J C Cellars Zinfandel Iron Hill Vineyard	25	92.5	30
Zinfandel	Robert Keenan Zinfandel	26	84.5	26
Zinfandel	Neyers Zinfandel Tofanelli Vineyard	34	84.5	29
Zinfandel	Niebaum-Coppola Edizione Pennino Zinfandel Estate	35	84.5	25
Zinfandel	Saddleback Cellar Zinfandel Old Vines	42	92.5	32
Zinfandel	Sbragia Family Vineyards Zinfandel	44	84.5	25
Zinfandel	Storybook Mountain Zinfandel Atlas Peak	46	84.5	25
Zinfandel	Storybook Mountain Zinfandel Mayacamas Range	49	84.5	27
Zinfandel	Summers Ranch Zinfandel Villa Andriana Vineyard	50	92.5	24

bottles purchased. It might also be the case that the the utility of a bottle with rating r_j may depend on whether one's wine cellar is near capacity. Taking into account multiple purchases of the same wine and non-linearities in utility extends the KP problems (KP1)–(KP5) to address these relevant questions. We begin with an integer problem that allows our wine enthusiast to select more than one bottle of the same wine subject to an upper bound constraint on the number of bottles for each wine, a budget, and a variety constraint. Next, we solve this integer problem

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Table 8
Statistics, Solution Found to KP5 Without Quantity Constraints

Rating		*Type*			
		Cabernet	*Pinot*	*Zinfandel*	*Grand Total*
84.5	Number	5.00	6.00	9.00	20.00
	Min Price ($)	10.00	19.00	18.00	10.00
	Average Price ($)	20.80	26.00	25.56	24.50
	Max Price ($)	27.00	29.00	29.00	29.00
	Expenditure ($)	104.00	156.00	230.00	490.00
	Exp. Percentage	62.50	66.67	45.00	54.05
92.5	Number	3.00	3.00	11.00	17.00
	Min Price ($)	27.00	25.00	24.00	24.00
	Average Price ($)	29.00	29.00	30.09	29.71
	Max. Price ($)	33.00	32.00	34.00	34.00
	Expenditure ($)	87.00	87.00	331.00	505.00
	Exp. Percentage	37.50	33.33	55.00	45.95
Total Number		8.00	9.00	20.00	37.00
Total Min Price ($)		10.00	19.00	18.00	10.00
Total Average Price ($)		23.88	27.00	28.05	26.89
Total Max Price ($)		33.00	32.00	34.00	34.00
Total Expenditure ($)		191.00	243.00	561.00	995.00

employing two alternative nonlinear utility functions and relaxing the upper bound constraint on the number of bottles purchased. We also discuss the possibility that utility is nonlinear and convex in the rating r_j and variety constraints (i.e. a minimum assortment requirement).

A. *Formulation*

Suppose our tasting host chooses up to $b_{j,max}$ bottles of wine $j = 1, 2, \ldots, n$, where $b_j = 0, 1, \ldots, b_{j,max}$. We assume that utility is linear in wine ratings in this integer problem. With the option of buying multiple bottles of the same wine, utility becomes $u = \sum_{j=1}^{b_{j,\max}} r_j b_j$.

Our multiple-bottle, integer-optimization problem (KP6) may be stated as

$$\begin{aligned} \underset{b_j \in \{0,1,\ldots,b_j,\max\}}{\textit{Maximize}} \quad & u = \sum_{j=1}^{n} r_j b_j \\ \textit{Subject to} \quad & \sum_{j=1}^{n} p_j b_j \leq B \\ & b_j = 0, 1, \ldots, b_{j,\max} \end{aligned} \tag{KP6}$$

Problem KP6 assumes that utility is linear in wine ratings. However, conventional economic theory follows the notion of decreasing marginal utility, in which additional consumption of an identical good provides incrementally

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less utility for subsequent units consumed. In KP7 we will employ two utility functions to reflect decreasing marginal utility for successive bottles of the same wine:

(1) A power function with a given preference parameter:

$$u(b_j) = (r_j * b_j)^{\gamma}, 0 \leq \gamma \leq 1$$

for $\gamma = 1$, utility is linear in rating.

(2) A logarithmic transformation with standardization for repeated purchases of wine j:

$$u(b_j) = \alpha_j \log\left(b_j \times \frac{r_j}{100} + 1\right)$$

where α_j allows the enthusiast to provide a utility weight for each wine. Figure 1 illustrates this transformation.

Employing these two functional forms, the non-linear utility problem (KP7) may be stated as

$$\begin{aligned} \underset{b_j \in \{0,1,\ldots,b_{j,\max}\}}{\textit{Maximize}} \quad & u = \sum_{j=1}^{n} u(b_j) \\ \textit{Subject to} \quad & \sum_{j=1}^{n} p_j b_j \leq B \\ & b_j = 0, 1, \ldots, b_{j,\max} \end{aligned} \tag{KP7}$$

Similar to the previous cases, if problem KP7 yields multiple solutions, our wine enthusiast may need to solve for the least cost solution(s) that achieve at least u^*.

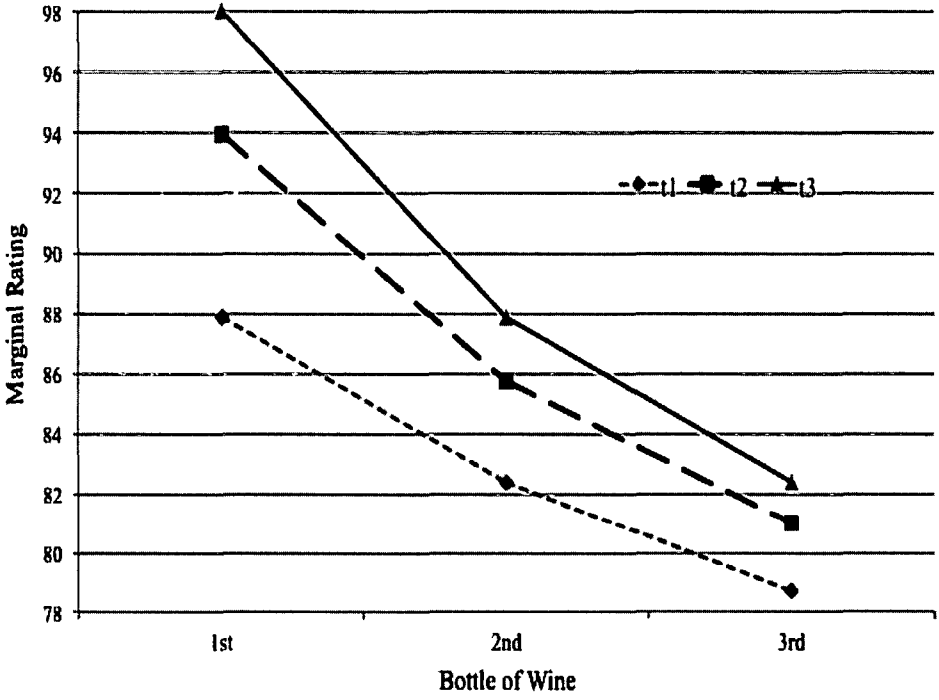
B. *Results for the Integer Problem*

We solve the models in (KP6)–(KP7) for the zinfandel wine set. The maximum number of bottles of any wine $b_{j,max}$ is constrained to three. In the case of the linear utility, problem KP6, there are more than fifty solutions, yielding $r^* = 3363$. Each one of these solutions chooses 38 bottles of wine, with 14 different wines chosen and an expenditure of $998. Allowing multiple purchases of the same wine leads to 11 wines selected three times in all optimal solutions. The rating of the bottles repeatedly selected is divided almost equally between a high rating (92.5, 6 brands) and a low rating (84.5, 5 brands).[5]

[5] The set of 50 solutions is available on the website under 'Linear-Utility'.

Figure 1

Marginal Utility for Multiple Purchases of the Same Wine
logarithmic transformation



α_j scales the rating so that the utility of the second bottle of the highest rated wine will equal the utility of the first bottle of the lowest rated wine. That is, the lowest-rated first bottle of wine is weakly preferred to the second bottle of the highest-rated wine. While this is an arbitrary way of setting a preference for variety, herein is used for illustrative purposes. α_j can be calibrated according to the preferences of the wine selector.

In the two versions of KP7 the solutions depend on the form of the utility function. In the log transformation, there are only two solutions, with 38 bottles of wine chosen and an expense of $1,000. In the two solutions, there are only 7 brands that are repeatedly chosen 3 times, while 4 brands are chosen two times. For brands where three bottles are chosen, the solution indicates a preference for lower rated wines, with only 2 highly rated wines (92.5) chosen repeatedly. The difference between the two solutions is due to an exchange between equivalent bottles of wine (in terms of rating and price), where Zin IN 8 (Carlisle Winery Zinfandel Tom Feeney Ranch) is exchanged for Zin IN 11 (Carlisle Winery Zinfandel Fava Ranch). The results are shown in Table 9.

In the case of the power function, with $\gamma = 0.8$, there are 24 solutions, with 35 bottles of wine chosen and a total expense of $1,000. In the solutions found, only 2

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Table 9
Solution to Second Stage of Equation (7)
logarithmic transformation

Zinfandel (i)	*IN*	*p*	*Solution* 1	2
Acorn Zinfandel Heritage Alegria Vineyard	1	32	0	0
Robert Biale Vineyards Zinfandel	2	27	2	2
Robert Biale Vineyards Zinfandel Aldo's Vineyard	3	40	0	0
Robert Biale Vineyards Zinfandel Black Chicken	4	34	0	0
Robert Biale Vineyards Zinfandel Grande	5	40	0	0
Robert Biale Vineyards Zinfandel Old Crane Ranch	6	34	0	0
Robert Biale Vineyards Zinfandel Monte Rosso	7	40	0	0
Carlisle Winery Zinfandel Tom Feeney Ranch	8	33	0	1
Carlisle Winery Zinfandel Riebli Ranch	9	30	1	1
Carlisle Winery Zinfandel Rossi Ranch	10	38	0	0
Carlisle Winery Zinfandel Fava Ranch	11	33	1	0
Carlisle Winery Zinfandel Dry Creek Valley	12	36	0	0
Carlisle Winery Zinfandel	13	25	3	3
Carlisle Winery Zinfandel Carlisle Vineyard	14	38	0	0
Edmeades Estate Zinfandel Ciapusci Vineyard	15	28	2	2
Edmeades Estate Zinfandel	16	18	3	3
Edmeades Estate Zinfandel Piffero Vineyard	17	28	2	2
Franus Zinfandel Brandlin Ranch	18	32	1	1
Hartford Hartford Zinfandel Dina's Vineyard	19	40	0	0
Hartford Zinfandel	20	30	1	1
Hartford Zinfandel Fanucchi Wood Road Vineyard	21	40	0	0
Hartford Zinfandel Hartford Vineyard	22	40	0	0
Hartford Zinfandel Highwire Vineyard	23	35	0	0
J C Cellars Zinfandel Arrowhead Vineyard	24	35	0	0
J C Cellars Zinfandel Iron Hill Vineyard	25	30	1	1
Robert Keenan Zinfandel	26	26	3	3
Mara Zinfandel Reserve Dolinsek Ranch	27	40	0	0
Mara Zinfandel Reserve Luvisi Ranch	28	35	0	0
Martinelli Zinfandel Giuseppe and Luisa	29	36	0	0
Martinelli Zinfandel Jackass Hill Vineyard	30	75	0	0
Martinelli Zinfandel Jackass Vineyard	31	50	0	0
Louis Martini Zinfandel Monte Rosso Gnarly Vine	32	50	0	0
Murphy-Goode Winery Snake Eyes Elaine Maria	33	35	0	0
Neyers Zinfandel Tofanelli Vineyard	34	29	1	1
Niebaum-Coppola Edizione Pennino Zinfandel Estate	35	25	3	3
Rancho Zabaco Zinfandel Monte Rosso	36	45	0	0
Rancho Zabaco Zinfandel Monte Rosso Toreador	37	60	0	0
Rosenblum Cellars Hendry Vineyard Reserve	38	41	0	0
Rosenblum Cellars Zinfandel Lyons Vineyard Reserve	39	45	0	0
Rosenblum Cellars Zinfandel Maggie's Reserve	40	45	0	0
Saddleback Cellar Venge Family Res. Scouts Honor	41	35	0	0
Saddleback Cellar Zinfandel Old Vines	42	32	1	1
Sausal Zinfandel Old Vine Estate	43	30	1	1
Sbragia Family Vineyards Zinfandel	44	25	3	3
Scherrer Zinfandel Scherrer Vineyard	45	55	0	0

Continued

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Zinfandel (i)	*IN*	*p*	*Solution* 1	2
Storybook Mountain Zinfandel Atlas Peak	46	25	3	3
Storybook Mountain Zinfandel Eastern Exposure	47	35	0	0
Storybook Mountain Zinfandel Estate Reserve	48	45	0	0
Storybook Mountain Zinfandel Mayacamas Range	49	27	2	2
Summers Ranch Zinfandel Villa Andriana Vineyard	50	24	3	3
Trentadue Zinfandel la Storia	51	30	1	1
Turley Wine Cellars Zinfandel Dragon	52	100	0	0
Turley Wine Cellars Zinfandel Estate	53	70	0	0
Turley Wine Cellars Zinfandel Hayne Vineyard	54	138	0	0
Turley Wine Cellars Mead Ranch Atlas Peak	55	52	0	0
Turley Wine Cellars Moore (Earthquake) Vineyard	56	140	0	0
Turley Wine Cellars Zinfandel Rattlesnake Ridge	57	59	0	0
Turley Wine Cellars Zinfandel Tofanelli Vineyard	58	105	0	0
Turley Wine Grist Vineyard Bradford Mountain	59	80	0	0
Turley Wine Cellars Zinfandel Rancho Burro	60	89	0	0
Vieux-Os Wines Zinfandel Tofanelli Vineyard	61	42	0	0
Vieux-Os Wines Zinfandel Hell Hole Vineyard	62	35	0	0
Vieux-Os Wines Zinfandel Ira Carter Vineyard	63	35	0	0
Williams Selyem Zinfandel Bacigalupi Vineyard	64	42	0	0

bottles are repeatedly chosen (3 times), a lower-rated wine, Zin IN 16 (Edmeades Estate Zinfandel), and a higher-rated wine, Zin IN 50 (Summers Ranch Zinfandel Villa Andriana Vineyard). In fact, only one highly rated wine (Carlisle Winery Zinfandel, 92.5) is chosen twice, while 3 different lower rated wines are chosen twice.[6]

Relaxing the three repetitions constraint, and using a power function with $\gamma = 0.8$, there are 40 solutions found, completely using the budget of \$1,000. In this case, the preference for variety is clear. In the final solution set, 37 bottles are selected, but only one bottle is repeatedly chosen: IN 16, rating 84.5, Edmeades Estate Zinfandel. Five INs are chosen twice in different solution combinations: IN 13, Carlisle Winery Zinfandel; IN 35, Niebaum-Coppola Edizione Pennino Zinfandel Estate; IN 44, Sbragia Family Vineyards Zinfandel; IN 46, Storybook Mountain Zinfandel Atlas Peak; and IN 50, Summers Ranch Zinfandel Villa Andriana Vineyard.[7] All the other 22 instances are single bottle selections.

C. *Extensions*

The integer choice problems defined above can be extended to consider other relevant issues in the wine selection problem, including preference for variety,

[6] The set of solutions is available on the website under 'Power-Function-3'.
[7] The set of solutions is available on the website under 'Power-Function-A'.

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possible convexity in the relationship between wine ratings and utility, and cellar capacity constraints.

An additional constraint can be imposed to problem KP6 to reflect the preference for variety of our wine enthusiast. Such constraint ensures that at least a certain number of wine brands are chosen. To do this, we introduce a new binary variable, $v_j \in \{0, 1\}$ where

$$v_j = \left\{ \begin{matrix} 0 & if & b_j = 0 \\ 1 & if & b_j > 0 \end{matrix} \right\}$$

Our multiple-bottle, integer-optimization problem may be stated as

$$\begin{aligned} \underset{b_j \in \{0,1,\ldots,b_{j,\max}\}}{\textit{Maximize}} \quad & u = \sum_{j=1}^{n} r_j b_j \\ \textit{Subject to} \quad & \sum_{j=1}^{n} p_j b_j \leq B \\ & b_j = 0, 1, \ldots, b_{j,\max} \\ & v_j = \left\{ \begin{matrix} 0 & if & b_j = 0 \\ 1 & if & b_j > 0 \end{matrix} \right\} \\ & \sum_{j=1}^{n} v_j \geq V \end{aligned} \qquad \text{(KP8)}$$

where $V \geq 1$ is a variety constraint requiring that the number of different wines selected equals or exceed V. Although this is an interesting variation, given the number of different wines selected in KP6, $\sum_{j=1}^{n} v_j = 14$, the variety constraint would not be binding for $V \leq 14$.

Suppose now that utility is nonlinear in the rating r_j so that $u_j = u(r_j)$. Economist's normally think that utility functions should be quasi-concave or strictly concave, but is this the case with wine consumers? We could imagine a wine consumer where utility is convex in r_j. Consider the utility points for wines rated 90, 92, and 94 (Figure 2).

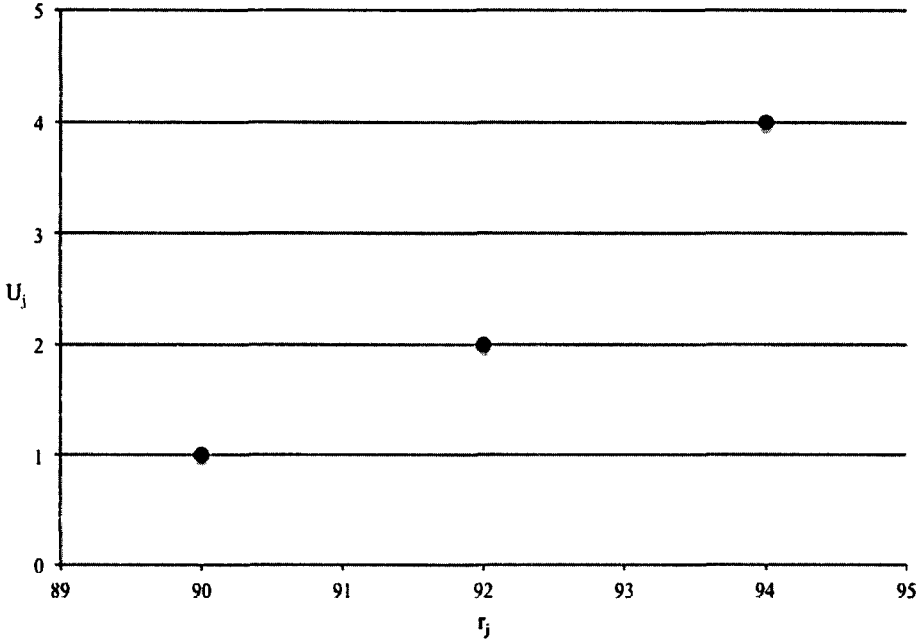
Let p_{90}, p_{92}, and p_{94} denote the per bottle price of wines rated 90, 92, and 94 and assume that $2p_{92} = p_{90} + p_{94}$. The cost of two bottles of the wine with a 92 rating equals the cost of one bottle of wine rated 90 plus one bottle rated 94. If these were the only two-bottle combinations under consideration, based on the utilities in Figure 2, the consumer would prefer purchasing one bottle rated at 90 and one bottle rated at 94 to purchasing two bottles rated at 92. Such a convex utility over ratings might be consistent with a consumer who would prefer a sip of a superior wine to multiple glasses of a lower-rated wine. If we combine convex utility over ratings with the option of purchasing multiple bottles we might easily observe our consumer purchasing a few highly-rated, expensive wines as opposed to a case of lower-rated, inexpensive wine. This variation can be accommodated in the framework established for problem KP7, with utility convex in ratings. The

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Figure 2

Utility Convex in Rating U(90) = 1, U(92) = 2, U(94) = 4 and p is price

if $2p_{92} = p_{90} + p_{94}$, then the preferred choice is [90,94]



solutions would likely display a preference for higher rated wines along with a selection of low-rated, inexpensive wines.

Finally, the selection might become even more rational if we allow for cellar capacity. Let S denote the storage capacity (number of bottles) of the consumer's wine cellar and let $S_t \leq S$ denote the number of bottles currently cellared in period t. It seems plausible that the utility of wines with different ratings might depend on the rating of wines currently cellared and on the remaining capacity to cellar newly acquired wine. To dynamically account for composition effects leads to a very complex problem which we sidestep in this article. We can at least speculate on nonlinear utilities that might also depend on remaining capacity. The utility of a wine with rating r_j is now given by $u_{j,t} = u[r_j, (S - S_t)]$. We can imagine a situation where as $S - S_t$ gets close to zero, higher ranked wines confer even higher utility. When proper storage gets scarce the consumer opt for higher-rated more expensive wines. This is shown in Figure 3.

VI. *Conclusions*

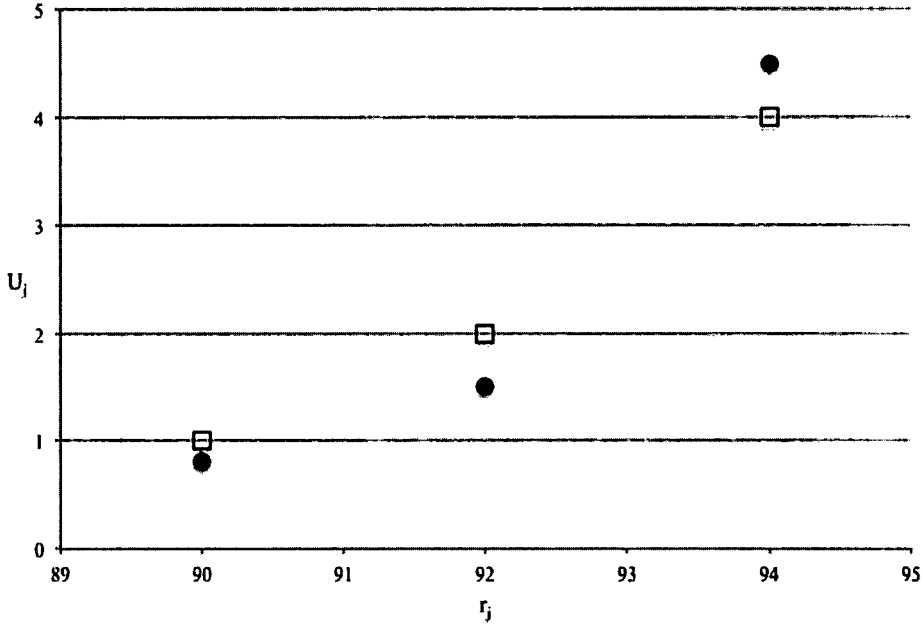
Most wine enthusiasts simply enjoy sharing a good bottle of wine and restrict their inventory to a modest wine cellar. The enjoyment and collection of wine does not

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Figure 3

Utility of Rating Dependent on Remaining Cellar Capacity

$u_{j,t}$ when $(S - S_t) = 100$ (circular shape), and $u_{j,t}$ when $(S - S_t) = 10$ (square shape)



preclude taking an analytical approach, and in fact the wine selection problems in this article were meant to illustrate how one might get the most out of a modest budget for wine. The wine selection problems in this article are examples of Knapsack Problems, an interesting class of problems in the fields of operations research and computer science. The ability to find optimal solutions to this NP-Hard class of problems has been improved by the development of efficient mixed integer programs (MIPs) so that the analytical wine enthusiast may now pose and quickly solve various wine selection problems on a computer. In our most complex problem, with 164 cabs, 100 pinots, and 64 zins from the California North Coast appellation in 2003, the CPLEX Solver found zins to provide higher rank per dollar than cabs or pinots. With a budget of $1,000 and no quantity constraints, it chose 20 zins compared to 8 cabs and 9 pinots. For the log utility transformation, and allowing the purchase of multiple bottles of the same wine, there were 20 different brands chosen, with 11 multiple bottle purchases from the set of 64 zins.

The wine enthusiast with strong personal tastes (such as those held by Paul Giamatti's character in the movie 'Sideways') may trump the selection made by a computer, but given the rapid increase in the number of wines and vintages

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now available at stores, and especially auction sites, the selection of wine from a single cultivar and region (as in our first selection problem) might still benefit from an analytical approach. Extending the model to accommodate repeated purchases of the same wine and possible nonlinearities in preferences further illustrates the benefits of taking an analytical approach to the wine selection problem.

Applications of the KP can be extended to help buyers at restaurants and wine stores make more rational wine procurement decisions. Such buyers often face limited shelf or storage space and need to select a wine assortment (cultivars and regions) to meet customer expectations. Given the proliferation of wine brands, the resulting KPs are likely to be more complex than the applications presented in this study. This "buyer problem" calls for further research.

The fact that wine selection problems are likely to generate multiple solutions (because of similarly ranked and priced wines) is perhaps a good thing. It allows our wine enthusiast to make the ultimate choice based on experience, personal taste, or simply the desire to try a wine not yet tasted.

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Appendix: AMPL Programs

The AMPL code for solving problems KP4 and KP5 is listed below. For an introduction to AMPL, see Fourer et al. (2002). The first section corresponds to the problem formulation for a single selection. The second part corresponds to the cycle for finding all the solutions to the problem, as explained in subsection II. A, 'Finding all solutions to the problem'.

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A. Model for KP4

```
set P;
set c; # classes of wine
param r {j in P, k in c} default 0; # ratings
param p {j in P, k in c} default 0; # prices
param cont;
param solv{j in P, 1..cont, k in c}; # vector with
solutions to problem
param b; # budget
param t;
param num{k in c}; # number of wines
var X {j in P, k in c} binary;
maximize Total_Util: sum {j in P, k in c} r[j, k] *
X[j, k];
subject to budget: sum {j in P, k in c} p[j, k] * X[j, k]
<= b;
subject to numberc {k in c}: sum {j in P} X[j, k] >=
num[k];
subject to diff {i in 1..cont}: sum {j in P, k in c}
(j*X[j, k]/j - j*solv[j, i, k]/j)^2>=1;
```

B. Model for KP5

```
set P;
set c; # classes of wine
param r {j in P, k in c} default 0; # ratings
param p {j in P, k in c} default 0; # prices param cont;
param solv{j in P, 1..cont, k in c}; # vector with
solutions to problem
param u; # utility param t;
param num{k in c}; # number of wines
var X {j in P, k in c} binary;
minimize Expend: sum {j in P, k in c} p[j, k] * X[j, k];
subject to util: sum {j in P, k in c} r[j, k] * X[j, k] >= u;
subject to numberc {k in c}: sum {j in P} X[j, k] >=
num[k];
subject to diff {i in 1..cont}: sum {j in P, k in c}
(j*X[j, k]/j - j*solv[j, i, k]/j)^2>=1;
```

C. Iterative Process for KP4

```
# Iterative cycle for wine problem, 1st stage
reset; # reset all information
cd /Applications/ampl; # change directory file
option solver cplex;
model wineicn.mod; # set model to run
data wineicn.dat; # set data to run, full data, no class
limits
param ncont := 100; # tries fixed
param loadstart;
param loadend;
param loadtime;
let loadstart := time();
let cont := 1; # define variables to be filled
for {i in 1..cont}{
      for {j in P}{
            for {k in c}{
                  let solv[j, i, k] := 0;
                  }
            }
      }
option show_stats 1;
solve;
for {j in P}{
      for {k in c}{
            let solv[j, cont, k] := X[j, k];
      }
}
for {i in 2..ncont}{
      option show_stats 1;
      solve;
      let cont := i;
      for {j in P}{
            for {k in c}{
                  let solv[j, i, k] := X[j, k];
            }
      }
      let loadend := time();
      let loadtime := loadend-loadstart;
```

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```
        if loadtime>ncont then{
                break;
        }
  }
  display solv >> wine1n_out;
```

D. Iterative Process for KP5

```
  # Iterative cycle for wine problem, 2nd stage
  reset; # reset all information
  cd /Applications/ampl; # change directory file
  option solver cplex;
  model wine2icn.mod; # set model to run
  data wine2icn.dat; # set data to run, full data, no class
limits
  param ncont := 100; # tries fixed
  param loadstart;
  param loadend;
  param loadtime;
  let loadstart:= time();
  let cont:= 1; # define variables to be filled
  for {i in 1..cont}{
        for {j in P}{
                for {k in c}{
                        let solv[j, i, k]:= 0;
                }
        }
  }
  option show_stats 1;
  solve;
  for {j in P}{
        for {k in c}{
                let solv[j, cont, k]:= X[j, k];
        }
  }
  for {i in 2..ncont}{
        option show_stats 1;
        solve;
        let cont:= i;
```

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```
        for {j in P}{
                for {k in c}{
                        let solv[j, i, k]:= X[j, k];
                }
        }
                let loadend:= time();
                let loadtime:=loadend-loadstart;
                if loadtime>ncont then{
                        break;
                }
}
display solv >> wine2n_out;
```

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