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FORWARD GUIDANCE CONTRACTS

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We examine "Forward Guidance Contracts," which penalize central bankers for choosing high interest rates. We integrate those contracts into the New Keynesian Framework and show that they can be used to overcome a liquidity trap. Moreover, although the government takes only a share of the social benefits into account when it has to decide whether to offer the contract, we demonstrate that for plausible parameter values the government will always find it desirable to offer the contract in a liquidity trap but not in normal times. Finally, we show that the optimal duration of such contracts is typically very short.

Keywords: Forward Guidance, Zero Lower Bound, Central Banks, Incentive Contracts, Transparency

1. INTRODUCTION

In the aftermath of the global financial crisis of 2007–2008, interest rates hit the zero lower bound in many countries. Hence, traditional monetary policy that works through changes in short-term nominal interest rates was unable to stimulate the respective economies further. Since then, forward guidance, i.e., announcing the future stance of monetary policy, has become a common component in the toolbox of central banks. It may be effective through two channels. First, it may convey private information of the central bank. Second, it may represent a promise of the central bank to pursue a particular policy in the future. This paper concentrates on the second channel.

In line with theoretical findings that suggest that a central bank in a liquidity trap should commit to keeping nominal interest rates low—even beyond the duration of the recession,[1] the Bank of England and the Federal Reserve, for instance, started to make public statements about their future actions.[2] In particular, they pledged in

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2012 and the following years, to refrain from increasing short-term interest rates until the economic situation has improved.[3] However, as pointed out in Thornton (2012), central banks' verbal guidance of "lower for longer" tends to have little impact due to a lack of credibility since central banks do not incur losses when they deviate from their announcements.[4]

In this paper, we consider a new approach to make forward guidance more effective at the zero lower bound: Forward Guidance Contracts (henceforth FGCs). In particular, we integrate FGC into a standard New Keynesian Framework with the zero lower bound and two possible realizations of a shock to the natural real rate of interest [see Eggertsson and Woodford (2003) and Eggertsson (2003)].

The contracts work as follows. While they are in effect, central bankers are punished when choosing high interest rates. For instance, their pay, pension, or the length of their term could monotonically decrease with the level of interest rates.[5]

The gist of our model is that FGCs create partial commitment. Central bankers will try to keep interest rates low but still deviate to some extent if future developments make such a commitment too costly. We show that repeated short-term FGCs can yield favorable trade-offs between the efficacy of forward guidance in helping to jump-start the economy and a reduction of flexibility in responding to future developments.

At a more specific level, our results are as follows: First, we integrate simple renewable FGCs offered by the government into the New Keynesian Framework and provide a microfoundation of these contracts.

Second, we characterize and analyze optimal FGCs when a government commits to using such contracts in downturns. Under these contracts, the central banker sets interest rates in a downturn at zero and sets interest rates immediately after the downturn at levels lower than the ones he would set in the absence of such contracts. The induced higher levels of inflation and output at the beginning of the future boom feed back into higher current output and inflation.

Third, we analyze the incentives of the government to offer FGCs in a crisis if it cannot commit to offer these contracts beforehand. Although the government takes only part of the social benefits from offering the contract into account when it makes its decision, it will typically find it beneficial to offer the contract in a downturn and to refrain from offering it in normal times.

Fourth, we examine the optimal duration of FGCs. While our stylized model does not allow for an in-depth quantitative analysis, it nevertheless suggests that the optimal duration of FGCs is quite short, i.e., it spans a time period of one or two quarters.[6]

Fifth, in an additional online appendix, we characterize FGCs that yield welfare gains for an entire range of negative natural real interest-rate shocks when the contract is designed under a veil of uncertainty about such shocks. Typically, the optimal intensity of central bankers' incentives to stick to low interest rates is moderate in such circumstances.

The paper is organized as follows. In the next section, in order to further motivate our approach, we describe how forward guidance has been used by the Federal

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Reserve. After a brief literature review in Section 3, the model is presented in Section 4. To assess the potential of and the challenges to FGCs, the standard discretionary solution without FGCs is presented as a benchmark in Section 5. In Section 6, we establish the properties and the welfare implications of optimal FGCs. In Section 7, we investigate an alternative environment for FGCs. Practical issues with regard to the implementation of FGCs are considered in Section 8. A discussion and our conclusions are presented in Section 9.

2. FORWARD GUIDANCE IN PRACTICE

Before discussing the related literature, we briefly review and summarize the Fed's responses to the financial crisis with a particular focus on forward guidance. In 2008, the Federal Open Market Committee (FOMC) lowered its policy rate—the federal funds rate—rapidly to a near-zero level. When the policy rate is in the vicinity of the zero lower bound, the substitutability between money and bonds becomes very high. In such circumstances, further monetary easing becomes ineffective since the opportunity cost of holding money is zero and economic agents start hoarding money—a situation called "liquidity trap."

In the presence of a liquidity trap, it is vital to create inflationary expectations to lower the real interest rate. Thus, the Fed started to make public statements about its future actions.[7] On December 16, 2008, the Fed implemented forward guidance for the first time:

> [...] the Committee anticipates that weak economic conditions are likely to warrant exceptionally low levels of the federal funds rate *for some time*. (emphasis added)

Three months later, the Fed made a statement quite similar except that the term "some time" was replaced by "an extended period."

These two announcements can be categorized as "open-ended forward guidance," as no date to exit from the low levels of the federal funds rate was mentioned explicitly. Open-ended forward guidance gives a central bank more flexibility to react to unforeseen events. However, the efficacy in creating inflationary expectation is then substantially reduced, since the public expects the central bank to raise the interest rate as soon as the economy recovers.

On August 9, 2011, the Fed opted for forward guidance with a more specific outlook:

> The Committee currently anticipates that economic conditions—including low rates of resource utilization and a subdued outlook for inflation over the medium run—are likely to warrant exceptionally low levels for the federal funds rate *at least through mid-2013*. (emphasis added)

In January 2012, the announced expected end date was shifted to late 2014. In September 2012, it was further postponed to mid-2015. We call these

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types of forward guidance with explicit time span "calendar-based forward guidance." Later in 2012, the Fed started forward guidance that was contingent on certain macroeconomic variables—known as "state-contingent forward guidance."

In this paper, we focus on "calendar-based" FGCs—contracts with explicit ending date. The "open-ended" FGCs—contracts with ambiguous time span—leave room for ex-post renegotiation and are thus less effective in creating inflationary expectations. The "state-contingent" FGCs—contracts with duration contingent on the state of the economy—are essentially the same as the simple renewable "calendar-based" FGCs in our setup. For example, we will show in Section 6 that the government and the central bank have incentives to sign FGCs repeatedly in downturns and do not sign FGCs in normal times. That is, FGCs will be valid during the entire downturn and $t = 1$ in normal times. We can thus obtain a "state-contingent" FGC that is valid until one period after the economy reverts to normal times. In this sense, the simple renewable "calendar-based" FGCs are equivalent to the "state-contingent" FGCs.

In all cases discussed above, the Federal Reserve communicated low future levels of nominal interest rates. In line with this observation, we only focus on FGCs that help central banks to commit to low interest rates in the future. This is implicit in our assumption that central bankers' utility decreases with the deviation of adopted interest rates from zero if an FGC is in effect. An alternative would be to consider FGCs where central banker's utility decreases with the deviation of interest rates from previously made interest rate forecasts.[8] However, this increased flexibility appears to be unnecessary in the case of a liquidity trap, where central banks' aim is to increase inflation expectations.

3. LITERATURE

FGCs are a new type of contract for central bankers. They are related to earlier and recent literature. Walsh (1995) proposes incentive contracts for central bankers and shows that such contracts can eliminate the inflation bias and can induce socially desirable shock stabilization when central bankers face a classic time-inconsistency problem.[9] Gersbach and Hahn (2014) show that making deviations from inflation forecast costly for central bankers can improve welfare in a standard new Keynesian framework with a time-inconsistency problem due to the so-called stabilization bias.

In the present paper, we examine FGCs in which the central bankers' utility is contingent on their choices of interest rates. Woodford (2003b) shows that it is socially desirable to assign an interest-rate smoothing objective to the central bank. Our paper differs from Woodford (2003b) in several dimensions. First, we explicitly consider the liquidity trap and zero lower bound problem. Second, in Woodford (2003b) the central bank's objective function depends on lagged interest rates, whereas FGCs penalize high levels of interest rates.

Our contribution to the literature on contracts for central bankers is twofold. First, we examine FGCs in the New Keynesian Framework with the zero lower

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bound. The credibility problem in this framework, which was labeled "deflation bias" by Eggertsson (2006), is different from the inflation bias in Kydland and Prescott (1977) and Barro and Gordon (1983) or the stabilization bias in the new Keynesian model without the zero lower bound [see Clarida et al. (1999)]. Second, we provide a microfoundation of such contracts and analytical solutions for optimal renewable, short-term FGCs. We illustrate that such type of contracts can harvest a large fraction of the possible welfare gains. For plausible shocks optimal renewable, short-term FGCs are preferable over long-term contracts. Repeated short-term FGCs can create favorable trade-offs between the commitment to zero interest rates when the economy is hit by a negative natural real interest-rate shock and the desired flexibility in increasing interest rates when the economy returns to normal levels.

The present paper belongs to a recent strand of the literature on the benefits and costs of forward guidance and the optimal way of implementing it. Woodford (2012), Campbell (2008), Gersbach and Hahn (2011), and Hughes Hallett and Acocella (2018) stress the social value of publishing central bank interest rate projections, and Campbell et al. (2012) and Gurkaynak et al. (2005) find that policy inclinations about the forward path of interest rates reveal information and can affect market expectations.[10] Bodenstein et al. (2012) show that, with imperfect credibility captured by a probability of discarding promises, central banks could achieve considerable welfare gains.[11] However, the credibility of the US Federal Reserve and the Swedish Riksbank has been low in the aftermath of 2008 crisis.[12] Lim and Goodhart (2011) are critical of forward guidance, arguing that it may have little impact on expectations.

We add to this literature by proposing to implement forward guidance in the form of FGCs because these contracts make forward guidance credible and therefore effective in influencing expectations. In addition, we discuss which type of FGCs may help in jump-starting an economy. While we focus on FGCs with a microfoundation on how variation of payments, pension or the length of the term of central banks can be used to motivate central bankers to commit partially, other proposals have been made that work through the central bank's balance sheet. Krippner and Thornton (2012) suggest that large-scale purchases of interest-rate-derivative contracts could significantly raise the credibility (and ultimately the efficacy) of central bank's guidance, as central bank would incur great capital loss from breaking its early promise. Levin et al. (2010) have suggested that large-scale asset purchase programs may increase the commitment power of forward guidance.

4. MODEL

4.1. A General Framework

Our model combines FGCs with the standard new Keynesian framework to examine the zero-bound problem. To model the zero-bound problem, we follow Eggertsson and Woodford (2003) and Eggertsson (2003). Time is discrete and

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indexed by $t = 0, 1, 2, \ldots$. The IS Curve is described by

$$x_t = \mathbb{E}_t[x_{t+1}] - \frac{1}{\sigma}(\tilde{i}_t - \mathbb{E}_t[\pi_{t+1}] - \tilde{r}_t), \tag{1}$$

where x_t denotes the (log) output gap in period t and π_t, $\tilde{i}_t$, and $\tilde{r}_t$ are the deviations of the inflation rate, the nominal interest rate and the natural real rate of interest from their steady-state levels.[13] As will be explained below, $\tilde{r}_t$ is given by an exogenous shock process. Parameter σ satisfies $\sigma > 0$. With the steady-state nominal interest rate $\bar{i}$, it is now convenient to introduce the nominal interest rate $i_t := \bar{i} + \tilde{i}_t$ as well as the transformed shock $r_t := \tilde{r}_t + \bar{i}$. Now the IS curve (1) can be written as

$$x_t = \mathbb{E}_t[x_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] - r_t), \tag{2}$$

and the zero-lower-bound constraint on monetary policy can be formulated as $i_t \geq 0$.[14]

Following Eggertsson (2003), we consider two possible realizations of $\tilde{r}_t$ and thus of r_t, which correspond to two different states $s \in \{L, H\}$. With a slight abuse of notation, we write r_L and r_H for these realizations. We assume $r_H > 0$ and $r_L < 0$, which ensures that the zero lower bound typically binds in state L but not in state H. In the following, we will say that the economy is in a "downturn" if the state is L. Similarly, we will use the term "normal times" to describe an economy in state H.

Like Eggertsson (2003), we consider a situation where the economy is initially in a downturn, i.e., $s = L$. In each period $t = 1, 2, \ldots$, the state will change to $s = H$ with constant probability $1 - \delta$ $(0 < \delta < 1)$ and then remain in this state forever. With probability δ, the economy remains in the downturn. The Phillips curve is

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t[\pi_{t+1}], \tag{3}$$

with $\kappa > 0$ and β $(0 < \beta < 1)$ as the common discount factor.

As is well known, the Phillips curve can be microfounded from a standard model with Calvo pricing [see Woodford (2003a)]. To motivate a positive long-run average inflation rate, we consider a variant of Calvo pricing, where intermediate firms that cannot adjust their prices freely increase their prices mechanically at a constant rate.[15] We explicitly allow for positive long-run levels of inflation, as the level of inflation affects the long-run level of nominal interest rates and thereby their distance from the zero lower bound.

The instantaneous social loss function is

$$l_t = \frac{1}{2}\left(\pi_t^2 + \lambda x_t^2\right), \tag{4}$$

where $\lambda > 0$. Future losses are discounted by the factor β.

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As explained in more detail in Appendix E, we assume that the central banker shares the private agents' objectives and thus faces the loss function (4) in each period. In addition, he may face an FGC characterized by parameter b, which implies that the central banker incurs utility losses bi_t^2 when the interest rate he has chosen, i_t, differs from zero.

In Appendix E, we provide a foundation of the central banker's utility function when the government offers him a wage contract composed of a fixed wage and a variable component decreasing with i_t. The parameter b—chosen by the government acting as contract designer—measures the intensity of incentives provided by the FGC.[16] The base salary should be large enough such that the central banker never declines such a wage contract when the government chooses to offer it.

4.2. Forward Guidance Contracts

In this section, we focus on simple renewable FGCs that may be chosen by the government and affect the central banker's incentives in the subsequent period. More precisely, we assume in the first scenario that the government chooses b at an ex-ante stage. In all periods, it can only offer contracts with this parameter. In a particular period $t = 0, 1, 2, \ldots$, the sequence is as follows: First, the current state $s \in \{H, L\}$ is realized and becomes common knowledge. Second, the government decides whether to sign a new FGC with given parameter b [henceforth FGC(b)], which will be effective in period $t + 1$. Third, the private sector forms its expectations about inflation and output in period $t + 1$. Also, the central banker selects the nominal interest rate i_t to minimize his losses, subject to (2) and (3). The central banker's loss function in period t is influenced by a possible FGC signed in period $t - 1$. More precisely, it is

$$l_t^{CB} = \frac{1}{2}\left(\pi_t^2 + \lambda x_t^2\right) + \frac{1}{2} b i_t^2,$$

if an FGC was signed in period $t - 1$ and

$$l_t^{CB} = \frac{1}{2}\left(\pi_t^2 + \lambda x_t^2\right),$$

otherwise. We assume that in the initial period $t = 0$, an FGC is effective.[17]

Figure 1 shows the sequence of events. While we make the simplifying assumption that the government knows the exact size of r_L when determining the value of b, we study FGCs in a situation with uncertainty about this parameter when contracts are designed in additional online appendix.

5. DISCRETION WITHOUT FORWARD GUIDANCE CONTRACTS

To have a benchmark for assessing the potential and the limitations of FGCs, we briefly summarize in this section the standard discretionary solution in the absence

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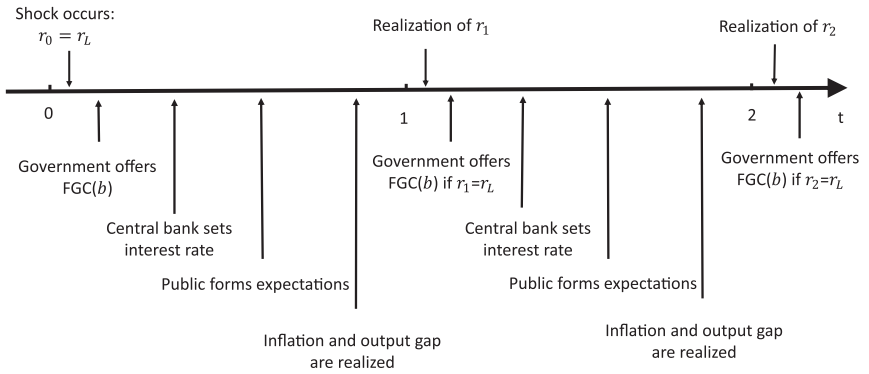


FIGURE 1. The sequence of events.

of FGCs. In the following, we focus on Markov equilibria, i.e., all economic variables depend only on the current state of the economy $s \in \{H, L\}$.

In each period, the central bank discretionarily chooses the nominal interest rate as its policy instrument, taking both its own future behavior and the public's expectations as given. In a Markov equilibrium, there are only two possible realizations for inflation, the output gap, and the nominal interest rate. We use π_L^D, x_L^D, i_L^D for the corresponding values in a downturn and π_H^D, x_H^D, and i_H^D for normal times, where the superscript D stands for "discretionary."

It is easy to compute the values of inflation and the output gap in normal times. When the natural real interest rate has returned to the positive value r_H, i.e., in period t when $s = H$, optimal policy involves $i_H^D = r_H$. Therefore, we obtain $x_H^D = 0$ and $\pi_H^D = 0$. Computing the equilibrium in the downturn is somewhat more involved. During the downturn, the zero lower bound is binding because of $r_L < 0$. Hence, in periods when $s = L$, we obtain $i_L^D = 0$. We note that $\mathbb{E}_t[\pi_{t+1}] = \delta\pi_L^D + (1-\delta)\pi_H^D = \delta\pi_L^D$ in a downturn, where we have used $\pi_H^D = 0$, and we also note that the probability of the state remaining at $s = L$ is δ. Analogously, we observe $\mathbb{E}_t[x_{t+1}] = \delta x_L^D$.

Inserting these expressions into (2) and (3) and solving for π_L^D and x_L^D yields

$$\pi_L^D = \frac{\kappa}{\sigma(1-\delta)(1-\beta\delta) - \delta\kappa} r_L, \tag{5}$$

$$x_L^D = \frac{1-\beta\delta}{\sigma(1-\delta)(1-\beta\delta) - \delta\kappa} r_L. \tag{6}$$

Henceforth we assume that δ is sufficiently small for the denominator in the above equations to be strictly positive.

ASSUMPTION 1. *Parameter δ satisfies*

$$\sigma(1-\delta)(1-\beta\delta) - \delta\kappa > 0. \tag{7}$$

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TABLE 1. Parameter values

$\beta = 0.99$
$\lambda = 0.03$
$\kappa = 0.3$
$\sigma = 2$
$r_H = 0.015$
$\delta = 0.5$

Assumption 1 imposes an upper bound on δ, i.e., the probability of remaining in a downturn cannot be too large. Together with $r_L < 0$, Assumption 1 ensures that π_L^D and x_L^D are negative in a downturn.[18]

In order to get a rough idea about the magnitude of the effects in our model, we calibrate the parameters in the following way. The values $\beta = 0.99$ and $\kappa = 0.3$ are taken from Clarida et al. (2000); $\sigma = 2$ is taken from Eggertsson (2006). With an inflation target of 2% and a discount factor $\beta = 0.99$, and thus a quarterly natural real rate of interest of approximately 1%, $r_H \approx 0.5\% + 1\% = 1.5\%$ appears plausible. $\lambda = 0.03$ is in line with a loss function derived from microeconomic foundations and is also chosen in Gersbach and Hahn (2014). Fernández-Villaverde et al. (2015) find that the average expected duration of zero-lower-bound episodes is approximately two quarters. Hence, we adopt $\delta = 0.5$ for the numerical examples. The parameter values, which can be readily verified to satisfy Assumption 1, are summarized in Table 1.[19]

6. FORWARD GUIDANCE CONTRACTS

In this section, we analyze how the possibility of signing FGCs affects the equilibrium. We assume that both the government and the central bank act under discretion. More specifically, upon observing the current state s, the government decides whether to sign a new contract, taking as given the central bank's decisions both in the current period and in all future periods, together with the possible existence of a contract for the current period. Then the central bank chooses its instrument subject to a possible FGC, taking its own future decisions and the government's future behavior as given. We consider a Markov equilibrium, i.e., an equilibrium where the decision-makers' choices depend solely on payoff-relevant state variables, i.e., on $s \in \{H, L\}$ and the possibility that a contract was signed in the previous period.

For the moment, we assume that the government will always choose an FGC in a downturn and no contract in normal times. Later, we will see that this behavior is indeed optimal for the government. We need to consider $2 \times 2 = 4$ different constellations because there are two different realizations of the shock, $s \in \{H, L\}$, and there may be either an active contract (C) or no active contract (N). The corresponding levels of inflation will be denoted as π_H^C, π_L^C, π_H^N, and

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π_L^N. Analogous notation will be used for the different possible values of the output gap.

In normal times and in the absence of an FGC, it is obvious that $\pi_H^N = x_H^N = 0$ holds. Next, we turn to the constellation, where $s = H$ and a contract was signed in the previous period. Given the fact that in the next period, the output gap will be x_H^N and inflation will amount to π_H^N, π_H^C, and x_H^C can be determined with the help of (2) and (3) as follows:

$$\pi_H^C = \beta\pi_H^N + \kappa x_H^C,$$
$$x_H^C = -\frac{1}{\sigma}(i_H^C - \pi_H^N - r_H) + x_H^N.$$

Using $\pi_H^N = x_H^N = 0$, these equations simplify to

$$\pi_H^C = \kappa x_H^C, \tag{8}$$
$$x_H^C = -\frac{1}{\sigma}(i_H^C - r_H). \tag{9}$$

Minimizing $\frac{1}{2}\left(\pi_t^2 + \lambda x_t^2\right) + \frac{1}{2}bi_t^2$ subject to (8) and (9) yields

$$\pi_H^C = \frac{b\sigma}{\lambda + \kappa^2 + b\sigma^2}\kappa r_H = \kappa f(b), \tag{10}$$
$$x_H^C = \frac{b\sigma}{\lambda + \kappa^2 + b\sigma^2} r_H = f(b), \tag{11}$$
$$i_H^C = \frac{\lambda + \kappa^2}{\lambda + \kappa^2 + b\sigma^2} r_H, \tag{12}$$

where we have introduced

$$f(b) := \frac{b\sigma}{\lambda + \kappa^2 + b\sigma^2} r_H. \tag{13}$$

We note that $f(b)$ is a monotonically increasing function with $f(0) = 0$ and $\lim_{b\to\infty} f(b) = r_H/\sigma$.

Equations (10) and (12) are useful for understanding why FGCs are potentially welfare-improving. With the help of (12), we observe that the nominal interest rate in the first period after the downturn is a decreasing function of b and that it is lower than the level that would prevail in the absence of an FGC, r_H. Hence, FGCs enable the central bank to commit to expansionary monetary policy once the economy has left the downturn. Note that inflation will be higher if an FGC is present in state H, which is shown by the fact that π_H^C is an increasing function of b.[20]

Figure 2 shows how, under an FGC, the nominal interest rate in state H decreases when the incentive intensity b increases.In turn, the inflation and output gap

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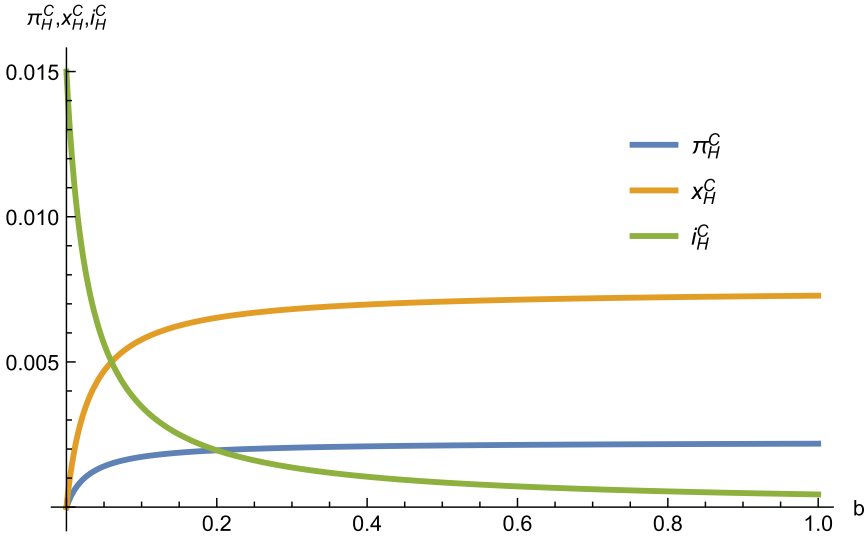


FIGURE 2. Inflation, output gap, and interest rate as a function of the value of b in state H with an active contract.

increase with b in state H. This is plausible because increases in b raise the relative importance of achieving small interest rates compared to the other objectives in the central bank's loss function, namely inflation and output stabilization.

Finally, we examine inflation and output in a downturn. Because there is a constant probability δ of remaining in state L, expectations of inflation and output, measured as deviations from steady-state levels, are

$$\mathbb{E}_t[\pi_{t+1}] = \delta\pi_L^C + (1-\delta)\pi_H^C, \tag{14}$$

$$\mathbb{E}_t[x_{t+1}] = \delta x_L^C + (1-\delta)x_H^C. \tag{15}$$

We are now in a position to compute inflation and the output gap in the downturn. For the moment, we assume that the zero lower bound is binding in the downturn, i.e., $i_L^C = 0$. Later, we will identify the range of values of b for which this is actually the case. Further, we will show that the government's optimal choice of b always lies within this range.

Using (2), (3), (10), (11), (13)–(15), and $i_L^C = 0$ yields

$$\pi_L^C = Af(b) + \pi_L^D, \tag{16}$$

$$x_L^C = Bf(b) + x_L^D, \tag{17}$$

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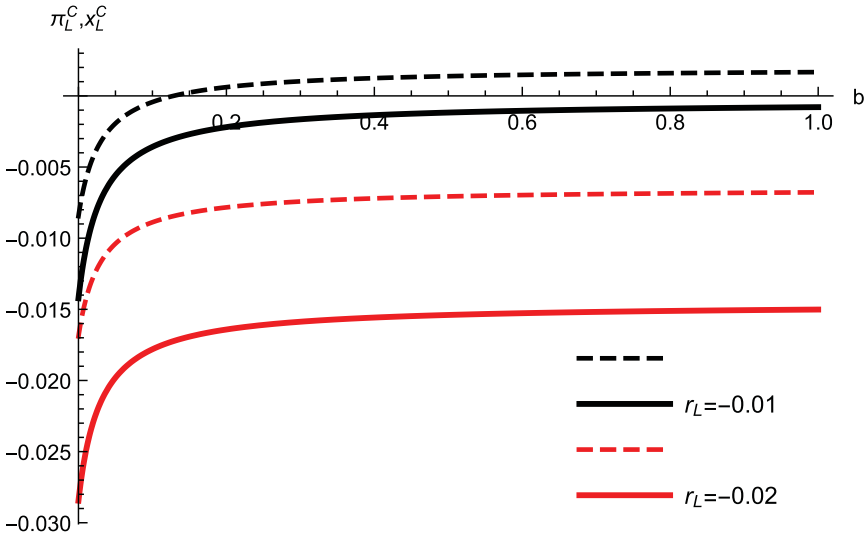


FIGURE 3. Inflation (dashed curves) and output gap (solid curves) as a function of the value of b for different shock sizes.

where

$$A := \frac{\kappa(1-\delta)\{\sigma[1+\beta(1-\delta)]+\kappa\}}{\sigma(1-\delta)(1-\beta\delta)-\delta\kappa}, \tag{18}$$

$$B := \frac{(1-\delta)[\sigma(1-\beta\delta)+\kappa]}{\sigma(1-\delta)(1-\beta\delta)-\delta\kappa}, \tag{19}$$

and π_L^D and x_L^D are defined in (5) and (6), respectively. Recall that, in line with Assumption 1, the values π_L^D and x_L^D, which would prevail without FGCs, are strictly negative. Moreover, Assumption 1 entails $A > 0$ and $B > 0$. Together with $f'(b) > 0\ \forall b \geq 0$, this implies that π_L^C and x_L^C strictly increase with b. Hence, for small b, FGCs can cushion the harmful consequences of a downturn on the output gap and also mitigate the ensuing deflation. These beneficial effects are possible because FGCs enable the central bank to commit to loose monetary policy after the downturn [see (10)–(12)]. This commitment to expansionary policy after the downturn raises inflation expectations during the downturn [see (14)] and thereby enables the central bank to implement a lower real interest rate $i_t - \overline{\pi} - \mathbb{E}_t[\pi_{t+1}] = -\overline{\pi} - \mathbb{E}_t[\pi_{t+1}]$ when the nominal interest rate is constrained by the zero lower bound.

Figure 3 illustrates how, in state L, inflation and the output gap increase with incentive intensity b under an FGC.

Finally, we need to examine the circumstances in which our assumption that the zero lower bound is binding in a downturn is fulfilled under FGCs. The following

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lemma, which is proved in Appendix A, establishes a sufficient condition for the zero lower bound to be binding:

LEMMA 1. *If*

$$f(b) \leq \frac{\kappa^2 + \lambda(1-\beta\delta)}{\kappa(\kappa A + \lambda B)}|\pi_L^D| =: \hat{f}, \tag{20}$$

then the zero lower bound is binding in a downturn with FGCs.

As (i) $f(b)$ monotonically increases with b, (ii) $f(0) = 0$, and (iii) the right-hand side of the condition in the lemma is positive and does not depend on b, the lemma defines a critical value of b, henceforth denoted by $\hat{b}$, such that the zero lower bound is binding in state L for all values of b below this critical value. Note that this value will be infinite if the right-hand side of (20) is at least as large as $\lim_{b\to\infty} f(b) = r_H/\sigma$.

We thus obtain the following corollary:

COROLLARY 1. *The zero lower bound is binding regardless of the value of b when* $r_L \leq \hat{r}_L$*, where*

$$\hat{r}_L := -\frac{(\kappa A + \lambda B)[\sigma(1-\delta)(1-\beta\delta) - \delta\kappa]}{\sigma[\kappa^2 + \lambda(1-\beta\delta)]} r_H.$$

We note that when the shock is severe, i.e., $r_L \leq \hat{r}_L$, the inflationary expectation induced by the FGC is not large enough to lift the optimal nominal interest rate above zero, even if the value of b is set at an extremely high level.

Figure 4 depicts the left-hand side and the right-hand side of (20) from Lemma 1 for different values of r_L. In the case $r_L = -0.01$, a large value for b induces positive inflation and output gap, as already shown in Figure 3. Hence, (20) is only satisfied for low values of b. In the case $r_L = -0.02$, inflation and output gap in downturns are negative regardless of the value of b, as shown in Figure 3. Correspondingly, (20) is always satisfied. Thus, in such a case, the zero lower bound is binding for all values of b.

In the following, we restrict our attention to values of b that satisfy the condition in the lemma. The justification for this assumption is that the government would never find it optimal to select a value of b that would violate (20), which will be demonstrated in the following section.

6.1. Optimal Contracts with Commitment to Contracting

In this section, we derive optimal FGCs and thus determine the socially optimal value of b. In doing so, we continue to assume that the government always offers an FGC in downturns and no contract in normal times. We call this scenario "commitment to contracting" or "CC" for short. Later, we will consider FGCs in the absence of such commitments. We will use the label "NCC" (no commitment to contracting) in this case.

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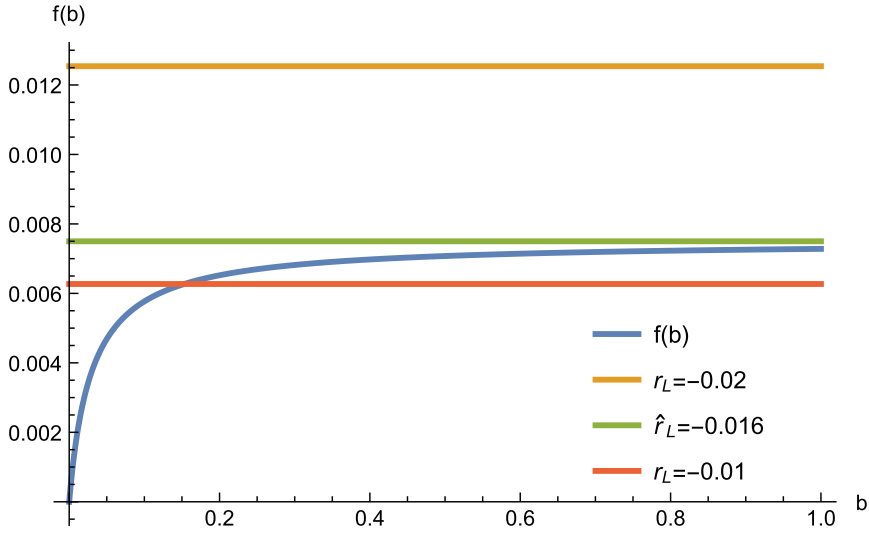


FIGURE 4. The function $f(b)$ and three horizontal curves representing the right-hand side of (20) for different shock sizes.

First, we observe that in equilibrium per-period social losses can take only three different values:

$$l_L^C = \frac{1}{2}\left(\pi_L^C\right)^2 + \frac{1}{2}\lambda\left(x_L^C\right)^2 = \frac{1}{2}\left[Af(b) + \pi_L^D\right]^2 + \frac{1}{2}\lambda\left[Bf(b) + x_L^D\right]^2, \quad (21)$$

$$l_H^C = \frac{1}{2}\left(\pi_H^C\right)^2 + \frac{1}{2}\lambda\left(x_H^C\right)^2 = \frac{1}{2}(\kappa^2+\lambda)\left(x_H^C\right)^2 = \frac{1}{2}(\kappa^2+\lambda)\left[f(b)\right]^2, \quad (22)$$

$$l_H^N = \frac{1}{2}\left(\pi_H^N\right)^2 + \frac{1}{2}\lambda\left(x_H^N\right)^2 = 0, \quad (23)$$

where we have used that $\pi_H^N = x_H^N = 0$, (10), (11), (16), and (17). Social losses expected in period 0 are given by

$$V_L(C) = \sum_{t=0}^{\infty}\beta^t\delta^t l_L^C + \sum_{t=1}^{\infty}\beta^t\delta^{t-1}(1-\delta)l_H^C, \quad (24)$$

where the subscript L stands for the current state of the economy and C stands for the fact that a (C)ontract was signed in the previous period. In (24), we have utilized $l_H^N = 0$ as well as the fact that (a) the probability of the economy being in a downturn is δ^t in all periods t with $t \geq 0$ and (b) the probability that the economy has just left the downturn and hence an FGC is still effective is $\delta^{t-1}(1-\delta)$ in all periods t with $t \geq 1$. It is straightforward to rewrite (24) as

$$V_L(C) = \frac{1}{1-\beta\delta}\left[l_L^C + \beta(1-\delta)l_H^C\right]. \quad (25)$$

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Together with (21) and (22), (25) can be used to explain the trade-off involved with FGCs. First, l_H^C is an increasing function of b, which is a consequence of the facts that $\pi_H^C = \kappa f(b)$ [see (10)], $x_H^C = f(b)$ [see (11)], $f(0) = 0$, and $f'(b) > 0\ \forall b \geq 0$. The interpretation of this observation is that FGCs induce expansionary policy for one period once the downturn has ended. This is socially costly ex-post. Second, l_L^C is a monotonically decreasing function for small b. Hence, FGCs induce welfare gains in the downturn. This follows from the observation that the commitment to expansionary policy after the downturn increases inflation expectations during the downturn and thereby enables the central bank to implement lower real interest rates when the economy is stuck at the zero lower bound.

The socially optimal value of b balances these costs and benefits. In Appendix B, we prove the following lemma:

LEMMA 2. *Suppose the government always offers an FGC in state L and never offers a contract in state H. Then, the optimal value of b, b*, can be determined in the following way:*

1. *If $r_H/\sigma > f^*$, b^* is given by $f(b^*) = f^*$, where*

$$f^* := \frac{A + \lambda B \frac{1-\beta\delta}{\kappa}}{A^2 + \lambda B^2 + \beta(1-\delta)(\lambda + \kappa^2)} |\pi_L^D|. \tag{26}$$

   At the optimal value of b, the zero lower bound is binding in equilibrium.
2. *If $r_H/\sigma \leq f^*$, social losses decrease strictly with $b\ \forall b \geq 0$. In this case, the zero lower bound is binding $\forall b \geq 0$.*

It is instructive to conduct comparative statics with respect to $|r_L|$. For this purpose, observe that f^* is a monotonically increasing function of $|r_L|$ because $|\pi_L^D|$ is a monotonically increasing function of $|r_L|$ [see (5)]. As a result, the optimal value of b, b^*, which is given by $f(b^*) = f^*$ for $r_H/\sigma > f^*$, increases with $|r_L|$. This is plausible, as a higher value of $|r_L|$ corresponds to a larger shock and thus calls for higher inflation expectations to stimulate aggregate demand through a lower real interest rate. Such higher inflation expectations can be induced by a larger value of b [see equations (10) and (14)]. For $|r_L| \to 0$, the optimal value of b converges to zero. By comparing f^* [equation (26)] and $\hat{f}$ [equation (20)] and rearranging terms, it turns out that $f^* < \hat{f}$, where $\hat{f}$ is the critical value, given in Lemma 1, below which the zero lower bound is binding. Thus, the zero lower bound is binding with an optimal FGC(b^*).

Lemma 2 also defines a critical value of r_L^c below which it is optimal to apply extremely harsh FGCs.

COROLLARY 2. *The optimal value of b is infinite when $r_L \leq r_L^c$, where*

$$r_L^c := -\frac{[A^2 + \lambda B^2 + \beta(1-\delta)(\lambda + \kappa^2)][\sigma(1-\delta)(1-\beta\delta) - \delta\kappa]}{\sigma[\kappa A + \lambda B(1-\beta\delta)]} r_H.$$

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This corollary implies that in the presence of severe shocks, it is optimal to sign an FGC with $b = \infty$. In such circumstances, equation (12) implies that the central bank would set the interest rate at the lowest possible value, i.e., $i_H^C = 0$, which leads to the highest possible inflation rate $\pi_H^C = \frac{\kappa}{\sigma} r_H$ and output gap $x_H^C = \frac{1}{\sigma} r_H$ in the initial period of stage H. Obviously, this finding has to be taken with a grain of salt. Infinitely large costs for deviations from very low interest rates would not be optimal in a model with a richer specification of shocks, e.g., a model that would also include markup shocks.

6.2. Optimal Contracts without Commitment to Contracting

Up to now we have considered scenario CC, where we have simply assumed that the government designs the contract at the beginning of the downturn and commits to always signing it in the downturn. It remains to show that this behavior is indeed optimal when the government decides in each period whether to offer the FGC or not, i.e., if there is NCC. That is, we will examine whether it is optimal for the government to always sign FGC(b^*) in the downturn.[21] The next lemma, which is proved in Appendix C, identifies a respective condition.

LEMMA 3. *The assumed behavior of the government, i.e., always signing an FGC with $b = b^*$ in a downturn and refraining from signing a contract in normal times, is optimal under NCC if $f^* \leq 2\tilde{f}$, where*

$$\tilde{f} := \frac{A - P + \lambda(B - Q)\frac{1-\beta\delta}{\kappa}}{A^2 - P^2 + \lambda B^2 - \lambda Q^2 + \beta(1-\delta)(\kappa^2 + \lambda)} |\pi_L^D|. \tag{27}$$

P and Q are constants and given in Appendix C.

The lemma reveals that it is conceivable that for the optimal value of b, b^* identified in Lemma 2, the government would not find it optimal to offer the contract under NCC. This may occur because the government takes its own future behavior and the behavior of private agents as given. This distinguishes the government's decision problem in a particular period under NCC from the problem in the ex-ante stage, where the government can choose b for all future periods under CC.

More specifically, when weighing up the costs and benefits of signing an FGC in a particular period, the government under NCC will fully take the costs into account that would materialize in the next period, provided that the state were H. However, the government under NCC only considers a fraction of the benefits. This can be seen when we look at the well-known representation of the new Keynesian Phillips Curve, where current inflation is proportional to the expected discounted sum of all future output gaps

$$\pi_t = \kappa \mathbb{E}_t \left[\sum_{i=0}^{\infty} \beta^i x_{t+i} \right], \tag{28}$$

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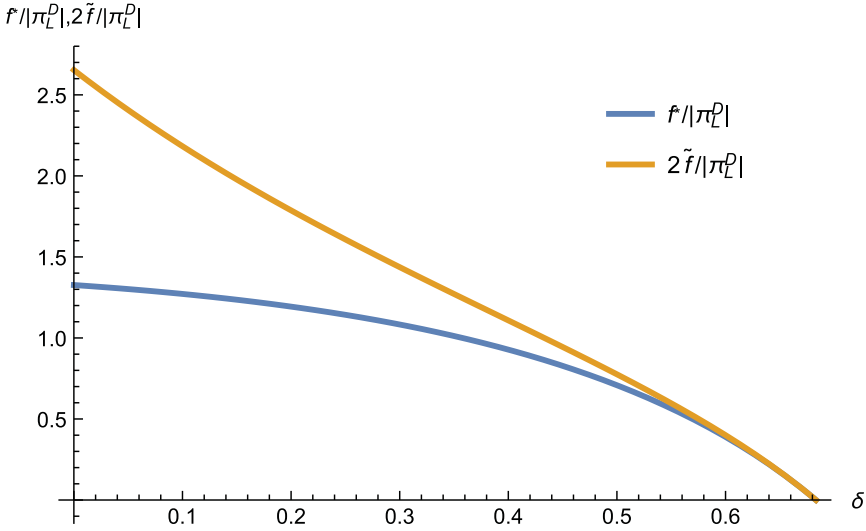


FIGURE 5. The values of f^* and $2\tilde{f}$, scaled by $|\pi_L^D|$, for the range of δ satisfying Assumption 1.

which directly follows from iterating (3). When the government signs an FGC in period t under NCC, it only takes into account the effect this contract has for $\mathbb{E}_t[x_{t+1}]$.

Despite the difficulty that a contract with b^* may not be offered in equilibrium by the government under NCC, it is straightforward to determine the optimal value of b for the case where the government optimally decides in each period whether to sign a contract for the next period or not.

PROPOSITION 1. *Under NCC, the optimal level of b, b^{**}, can be determined in the following way:*

1. *For $2\tilde{f} \geq f^*$ and $f^* < r_H/\sigma$, the optimal level of b is given by $f(b^{**}) = f^*$.*
2. *For $2\tilde{f} < f^*$ and $2\tilde{f} < r_H/\sigma$, the optimal level of b is given by $f(b^{**}) = 2\tilde{f}$.*
3. *For $2\tilde{f} \geq r_H/\sigma$ and $f^* \geq r_H/\sigma$, we obtain $b^{**} = \infty$.*

Figure 5 shows that $f^* \leq 2\tilde{f}$ is fulfilled in the range of δ that satisfies Assumption 1 for the parameter values specified in Table 1. That is to say, the government under NCC would behave in the same way as under CC when designing the FGC, i.e., when selecting the optimal value of b.[22]

To close this section, we present in Figure 6 the discounted social losses with the use of an FGC expected in period 0 [see (25)]. The expected social loss with an optimal FGC(b^*) stays below the one in the discretionary case for all values of r_L. When we compare the ratio of social losses under FGC with the social losses under discretion, we observe that a considerable welfare gain can be achieved with such contracts, as this ratio attains nearly 0.03 for $r_L \in (r_L^c, 0)$

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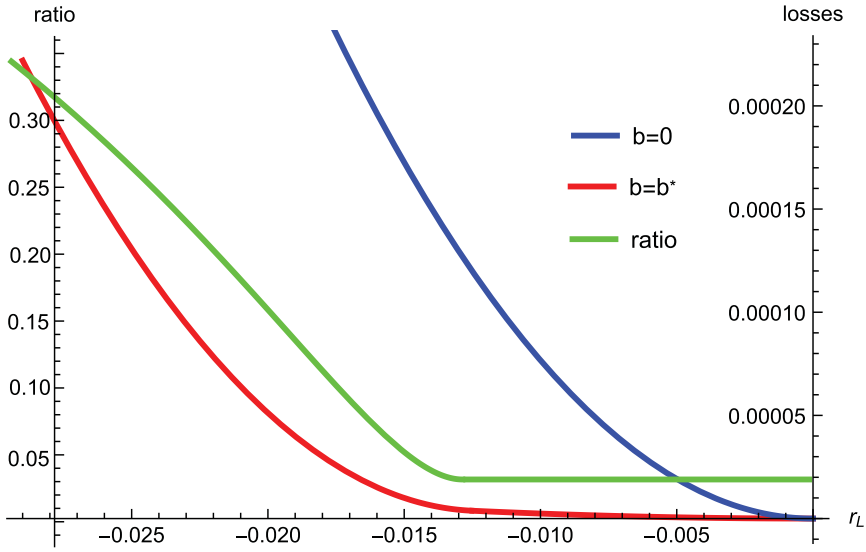


FIGURE 6. Discounted social losses with optimal FGC(b^*), in the discretionary case (right axis), and the ratio of these two discounted social losses (left axis), as functions of r_L (dashed curve represents $r_L = r_L^c$).

and is still around 0.25 for a large negative natural real interest-rate shock. Hence, we obtain quite favorable trade-offs between the efficacy of FGCs at the zero lower bound and the reduced flexibility in reacting to future events. Figure 6 also shows that the social loss with FGC($b^* = \infty$) starts to increase considerably when $r_L < r_L^c$.[23] As a robustness check, we show the discounted social welfare gains with optimal FGC(b^*) in Figure 7 for a range of shock sizes and the probabilities of being trapped in the downturn δ. It is clear that FGCs are more socially desirable for larger shock sizes and for longer expected durations of downturns.

7. ALTERNATIVE CONTRACTS WITH LONGER TERMS

7.1. Two-Period Contracts

So far, we have focused on simple renewable FGCs signed in one period and becoming effective in the next. In this section, we explore a simple alternative: contracts that become effective immediately after signing and remain effective for one more period.

First, however, we need to stress that both types of contract studied in Section 6 and in this section are the shortest (and simplest) contracts under which forward guidance can have an impact.[24] Consider a simpler contract, i.e., one that only applies to the period in which it is written. Such a contract would never be signed

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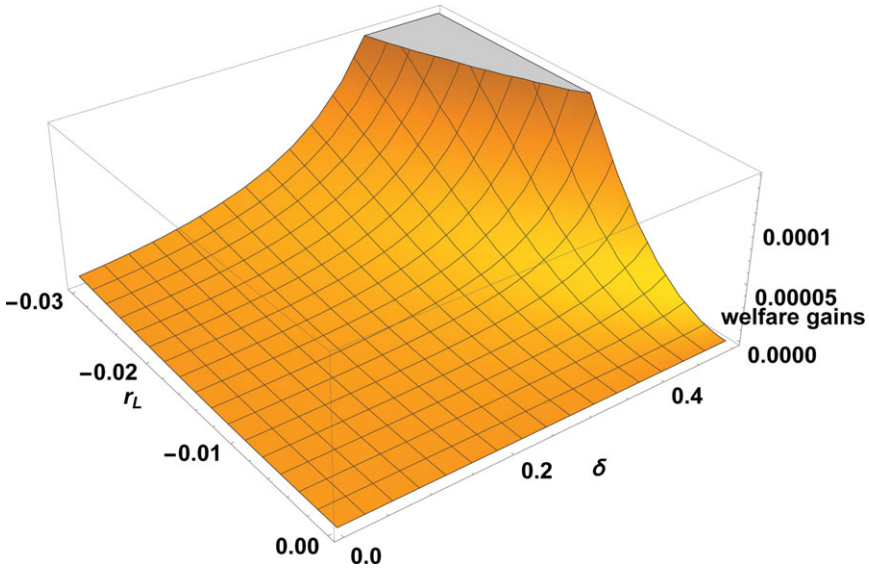


FIGURE 7. Discounted social welfare gains with optimal FGC(b^*) as functions of r_L and δ.

in normal times. In the downturn, the central banker would also set zero interest rate in the absence of an FGC. Hence, such a contract would only replicate the discretionary solution.

We use $\widehat{\text{FGC}}(b)$ to denote FGCs that are signed at the beginning of a period t and provide incentives for central banks to stick to low interest rates in periods t and $t+1$. As before, these contracts are renewed and repeatedly applied as long as the economy is in the downturn. While FGC(b) and $\widehat{\text{FGC}}(b)$ can achieve similar welfare gains,[25] $\widehat{\text{FGC}}(b)$ differs from FGC(b) as they become effective in the period where they are signed and in one or more additional periods. This involves the advantage that $\widehat{\text{FGC}}(b)$'s allow the construction of analytically tractable, long-term contracts that might be needed when the natural real interest-rate shock is extremely severe. We focus on a situation in which a negative shock to the natural real interest rate happens at the beginning of period 0 and the $\widehat{\text{FGC}}(b)$ is signed immediately.

The detailed sequence of events is shown in Figure 8. In period 0, after the realization of r_L, the government signs an $\widehat{\text{FGC}}(b)$ that makes the central banker's remuneration contingent on the level of nominal interest rates. Then, the interest rate in period 0, i^C_{L0}, is set, and the realized inflation and output gap are π^C_{L0} and x^C_{L0}.

In period $t = 1$, the central banker sets i^C_H if the economy has recovered and the corresponding inflation and output gap are π^C_H and x^C_H, respectively. If the economy is still in a downturn, the central banker sets i^C_{L1}, and the corresponding inflation and output gap are π^C_{L1} and x^C_{L1}, respectively.

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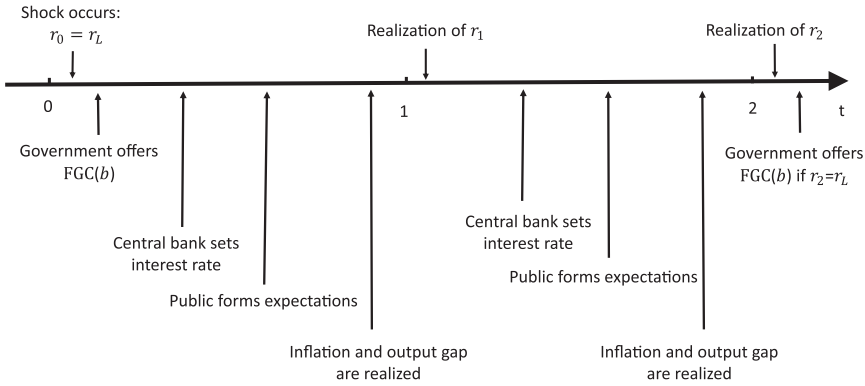


FIGURE 8. The sequence of events.

In period 2, the economy has either recovered and the central bank chooses the discretionary solution leading to zero inflation and zero output gap, or the natural real interest rate is still negative. In the latter case, the $\widehat{\text{FGC}}(b)$ is re-signed and thus the central banker continues to select interest rates under FGCs.[26]

As in Section 6, we derive the economic outcomes in periods 0 and 1:

$$\pi_{L0}^C = A_0 f(b) + \pi_L^D, \tag{29}$$

$$x_{L0}^C = B_0 f(b) + x_L^D, \tag{30}$$

$$\pi_{L1}^C = A_1 f(b) + \pi_L^D, \tag{31}$$

$$x_{L1}^C = B_1 f(b) + x_L^D, \tag{32}$$

where A_0, A_1, B_0, and B_1 are given in Appendix D.

The expected discounted intertemporal social loss is

$$V_{LA}(C) = \frac{l_{L0}^C + \beta[(1-\delta)l_H^C + \delta l_{L1}^C]}{1-\beta^2\delta^2}, \tag{33}$$

where $l_{L0}^C = 0.5[(\pi_{L0}^C)^2 + \lambda(x_{L0}^C)^2]$, $l_{L1}^C = 0.5[(\pi_{L1}^C)^2 + \lambda(x_{L1}^C)^2]$, and $l_H^C = 0.5[(\pi_H^C)^2 + \lambda(x_H^C)^2]$.

7.2. Optimal Two-Period Contracts

We next establish that the zero interest rate is still a binding constraint when $\widehat{\text{FGC}}(b)$ is used.[27]

PROPOSITION 2. *1. There exists a threshold value $\hat{r}_{LA} < 0$ such that the central bank will set $i_{L0}^C = i_{L1}^C = 0$ for any value of b in an $\widehat{FGC}(b)$ if $r_L \leq \hat{r}_{LA}$. $\hat{r}_{LA} < 0$ is given in Appendix D.*[28]

2. For $\delta \leq 0.54$, the central bank sets $i_{L0}^C = i_{L1}^C = 0$ under an optimal $\widehat{FGC}(b)$ for any $r_L < 0$.

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The intuition for Proposition 2 is straightforward. If the natural real interest-rate shock is sufficiently severe, the zero bound will bind for any $\widehat{\text{FGC}}(b)$. No boom or inflation that can be generated by an $\widehat{\text{FGC}}(b)$ after the shock has died out will suffice to lift the economy in the downturn to the level at which the central bank will optimally start to move away from the zero interest rate. By contrast, if the natural real interest-rate shock is moderate or small, optimal levels of b are set at a sufficiently low level, such that the induced boom and inflation after the return to normal times do not cause the central bank to start moderating the economy in the downturn already. Too high values of b would cause inefficiently large booms and inflation.

Finally, we derive the optimal value of b in $\widehat{\text{FGC}}(b)$.

PROPOSITION 3. *There exists a unique optimal $\widehat{FGC}(b)$ characterized by*

$$b = \begin{cases} \frac{\lambda+\kappa^2}{\sigma^2}\frac{r_L}{r_{LA}^c - r_L}, & \text{if } r_L \in (r_{LA}^c, 0) \\ \infty, & \text{if } r_L \leq r_{LA}^c \end{cases}, \tag{34}$$

where $r_{LA}^c < 0$ is a critical value of the natural real interest-rate shock given in Appendix D.[29]

Proposition 3 shows that it is optimal to use $\widehat{\text{FGC}}(b)$ with small values of b if the shock is small and large values if the shock is large.

7.3. Longer-Term Contracts

We have limited ourselves to simple renewable $\widehat{\text{FGC}}$s. It is instructive to examine contracts with longer durations as this enables us to determine the optimal duration of $\widehat{\text{FGC}}$s. For example, one might expect longer durations to be desirable when negative natural real interest-rate shocks are severe and recovery probability is low.

In Figure 9, we show the optimal value of b^*, depending on the size of shock, for $\widehat{\text{FGC}}$s with different lengths. Dashed vertical curves represent the critical values below which $b^* = \infty$ for 2-, 3-, 4-period $\widehat{\text{FGC}}$s (from the right to the left, the same for Figure 11). The optimal value of b^* increases with the size of shock, which indicates that the intensity of incentives should go hand in hand with the severity of the recession. A large deflation and output collapse, caused by a large natural real interest-rate shock, can be mitigated by a large expected inflation and output boom created by either a contract of longer term or a short-term contract with larger values of b. Thus, compared to long-term $\widehat{\text{FGC}}$s, short-term $\widehat{\text{FGC}}$s require larger value of b^*.

Figure 10 shows that for $r_L \geq -0.085$ a 2-period $\widehat{\text{FGC}}$ is more desirable as the inflationary expectation raised by 2-period $\widehat{\text{FGC}}$ with a finite value of b^* is sufficient to compensate the deflation and output collapse caused by the natural real interest-rate shock r_L, while 3- and 4-period $\widehat{\text{FGC}}$s constrain the central bank for too long.

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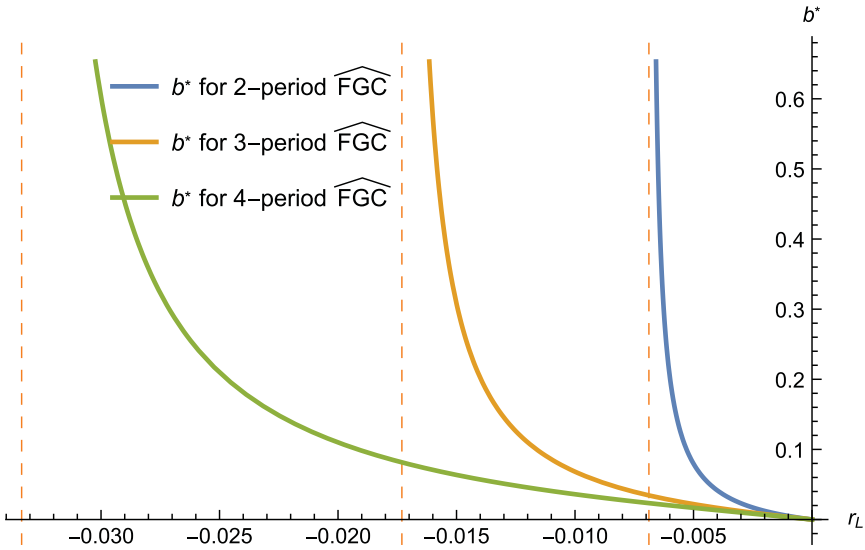


FIGURE 9. The optimal value of b^* for $\widehat{\text{FGC}}$s with different durations.

We would like to note that, given our assumption of $r_H = 0.015$, a value of $r_L = -0.0085$ implies that the quarterly natural real rate of interest drops by more than 2.3 percentage points, which corresponds to a drop by approximately 9 percentage points for the annualized natural real rate of interest rate. It appears unlikely that a shock would exceed this value, which implies that our model tends to indicate that FGCs should have a short duration.

For completeness, we also examine more sizeable shocks r_L. Figure 11 illustrates that longer term $\widehat{\text{FGC}}$s could lower social losses further compared to short-term contracts for extremely large shocks.

8. IMPLEMENTATION

In this section, we discuss two issues related to the implementation of FGC. In particular, we focus on possible renegotiations and on how FGCs could be implemented in a committee setting.

8.1. Credibility of the Contract

A potential problem of FGCs is the credibility of the contract itself, namely in the initial period of stage H, both the government and the central bank may have incentives to renege on the contract—jointly abandon the contract, for instance.[30]

First, we would like to note that the optimal duration of FGCs is quite short. Optimal contracts signed in a particular quarter end in the subsequent quarter. This observation may imply that contracts are hard to renegotiate, as renegotiations take

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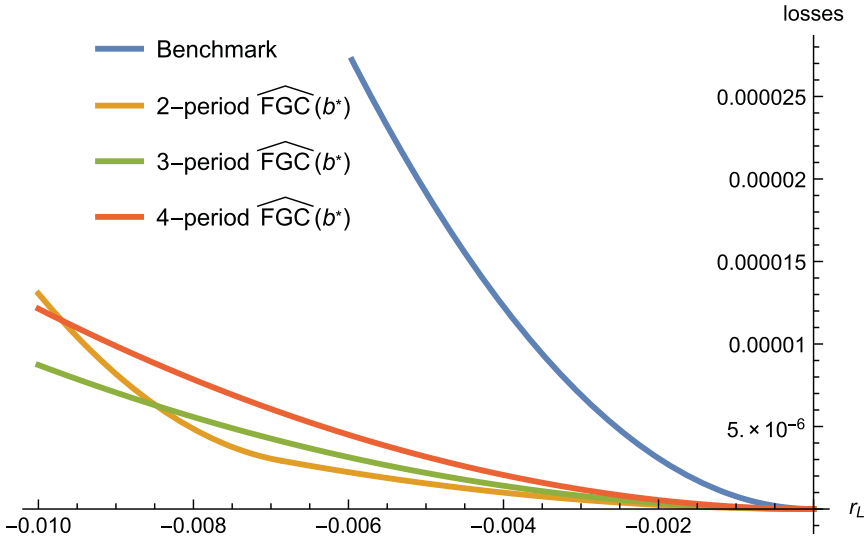


FIGURE 10. The expected social losses for $\widehat{\text{FGC}}$s with different durations, as functions of r_L ($r_L \in [-0.01, 0]$).

time. Second, there could be legal or other institutional constraints such that the government can commit to fulfilling the contract. Third, a related potential remedy for this problem is to stipulate explicitly in the contract that either of the parties or both would have to sacrifice a lot to renegotiate on the contract.

Jensen (1997) finds that with renegotiation costs, the inefficiency associated with the time-inconsistency problem could be alleviated to some degree. One example is to assign a central banker who is reluctant to deviate from his promise to keep interest rates low. In such circumstances, renegotiation implies the appointment of a new central banker, which is costly in terms of reputation losses and loss of independence of monetary policy. Walsh (2016) considers a situation where downturns, instead of one-off occurrence, occur with a certain probability in normal times, leaving the policy maker less incentive to renege on his promise to keep interest rates low, as renegotiation in the current period damages the credibility of FGCs in future downturns. Thus, the key to enhancing the credibility of the contract is to make it costly to either of the parties or to both to renege on it.

8.2. Implementation in a Committee Setting

In this paper, we focus on individual central bankers rather than on monetary policy committees. It would be straightforward to apply FGCs to a committee setting. Two approaches are conceivable. First, one could make individual central bankers' pay dependent on their individual actions in committee meetings, i.e., their individual votes on monetary policy. However, this would create incentives for central bankers who are not pivotal to vote for very low interest rates in the

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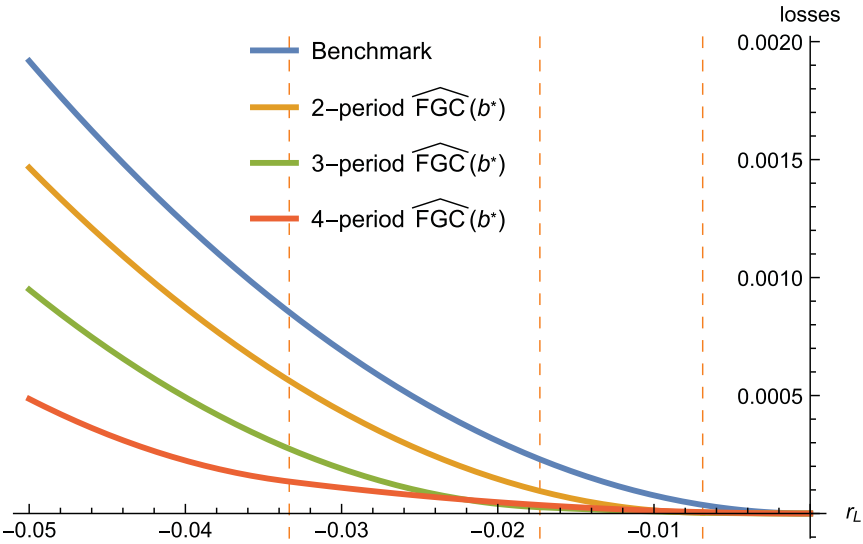


FIGURE 11. The expected social losses for $\widehat{\text{FGC}}$s with different durations, as functions of r_L ($r_L \in [-0.05, 0]$).

presence of a FGC. This would maximize their salaries, but would not influence monetary policy. Such incentives thus might lead to split votes among committee members, which would be only driven by central bankers' incentives to increase their wages. Such artificially induced disagreement is likely to be confusing for the public, which makes this form of individual accountability for central bankers undesirable.

As a consequence, we prefer a second approach. Central bankers would, in addition to deciding on the current interest rate, also vote on whether they would jointly accept a FGC, provided that the government has offered one. Every central banker's salary would then depend on the level of interest rates chosen by the committee.[31]

9. DISCUSSION AND CONCLUSION

Forward guidance aims at influencing the public's expectations, an objective that has a long tradition in monetary policy. Today the reliance on forward guidance has become a central aspect of monetary policy, often associated with the belief that forward guidance can provide a stimulus when economies are mired in longer downturns. We have explored a simple contractual tool that makes forward guidance more effective when the economy is at the zero bound.

Such FGCs strike a balance between Odyssian policy commitments and the need to react to new developments. We have confined ourselves to very simple contracts, written either after a downturn or in normal times. Numerous extensions

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of our research could be pursued. For instance, one could take into account the fact that it takes time to learn the magnitude of the shock, so the government necessarily has to sign such contracts under a veil of ignorance, which, in turn, may call for moderate intensity of incentives.

Also, one could allow for more complex contracts such as state-contingent FGCs. In such contracts, the central bankers' remuneration loss, which would occur if they deviated from low interest rates, would itself depend on macroeconomic variables such as the output gap. Compared to these approaches, simple contracts based only on interest rates appear advantageous for the following reasons. First, the central bank's instrument, a short-term nominal interest rate, can be controlled effectively by the central bank, whereas the central bank only has imperfect control over inflation and output. Second, current interest rates can be observed in a straightforward manner. By contrast, the output gap is difficult to measure because gross domestic product data are often revised and because there is no universally agreed way to compute the natural level of output.[32] Thus, FGCs that draw on alternative macroeconomic variables appear to have little potential over and above the simple contracts examined in this paper.

NOTES

1. See Krugman (1998), Eggertsson and Woodford (2003), Adam and Billi (2006), Werning (2011), and Belgibayeva and Horvath (2017).

2. A more detailed exposition of forward guidance in practice is presented in Section 2.

3. The initial pledges in December 2012 and August 2013, respectively, were conditioned on unemployment rates. Since then a variety of other approaches have been used [see Economist (2014)].

4. One example is the Bank of Japan. After lowering its policy rate to 0.15%, the Bank of Japan announced in April 1999 that the rate would be maintained at this level "until deflationary concerns are dispelled." However, the rate was soon raised against the backdrop of a weak economy (inflation at around -0.5% and the output gap still in the negative territory). Cf. Sayuri Shirai's speech "Monetary policy and forward guidance in Japan" at the IMF on 09/19/2013.

5. Utility losses could also occur when scrupulous bankers are appointed who are intrinsically reluctant to deviate from their promises of low interest rates and thus suffer utility losses when they do indeed deviate.

6. This finding is related to Schaumburg and Tambalotti (2007) and Bodenstein et al. (2012), who show that even a low probability of pursuing previously chosen plans is sufficient to reap most of the gains from commitment.

7. Apart from lowering the policy rate rapidly to the near-zero level and signaling its intention to keep it low for a longer period, the Fed has launched several Large Scale Asset Purchasing (LSAP) programs.

8. This avenue was pursued in a previous version of the paper.

9. The theory of incentive contracts was further developed in the influential papers by Persson and Tabellini (1993), Beetsma and Jensen (1998), Beetsma and Jensen (1999), Jensen (1997), Lockwood (1997), Svensson (1997), and Huang and Tian (2011).

10. Mirkov and Natvik (2013) find that central banks may be unwilling to deviate from previous interest-rate projections.

11. See Schaumburg and Tambalotti (2007) for a related analysis without the zero lower bound.

12. In a recent contribution Walsh (2016) examines whether forward guidance can be made credible through a reputational mechanism.

13. A complete list of the notation can be found in Tables F.1 and F.2.

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14. Alternatively, it could be formulated as $\tilde{i}_t \geq -\bar{i}$.

15. This assumption is adopted by Yun (1996), among others.

16. For the framework of Krippner and Thornton (2012), the parameter b could be related to the number of overnight indexed swap (OIS) contracts purchased by central bank, as the capital losses the central bank would incur if the interest rate is raised before OIS contracts expire are linear in the number of contracts.

17. This assumption is immaterial to our findings.

18. For a discussion of sign reversals in the initial responses of inflation and output when the duration of an interest-rate peg is extended, see Carlstrom et al. (2012).

19. All values of δ with $\delta < 0.68$ satisfy Assumption 1 for given values of the other parameters.

20. We observe that for $b = 0$, (10)–(12) entail the values of inflation, the output gap, and the nominal interest rate from the standard discretionary solution examined in Section 5.

21. In Section 8.1, we discuss whether the government would renege on the contract in $t = 1$ in normal times.

22. We will show in Section 8.1 that what matters is whether the government reneges on the contract once the economy reverts to normal times.

23. In such circumstances, welfare could be further improved by longer term FGCs, which we discuss in Section 7.3.

24. Renewable short-term contracts are attractive as they can reap most—or even all—possible welfare gains from FGCs for small and moderate negative natural real interest-rate shocks. They constrain the central bank as little as possible and thus involve the lowest risk in case of unforeseen events.

25. Details on this comparison are available on request.

26. In this section, we only explore the $\widehat{\text{FGC}}(b)$ when the government commits to contracting. A similar exercise as in Section 6.2 can also be performed for this type of contract.

27. The proof is available upon request.

28. For small values of δ, $\hat{r}_{LA}$ is small (e.g., $\hat{r}_{LA} = -0.029$ for $\delta = 0.1$). For large feasible values of δ, $\hat{r}_{LA}$ becomes larger (e.g., $\hat{r}_{LA} = -0.009$ for $\delta = 0.5$).

29. Again, for small values of δ, r^c_{LA} is small (e.g., $r^c_{LA} = -0.037$ for $\delta = 0.1$). For large feasible values of δ, r^c_{LA} becomes larger (e.g., $r^c_{LA} = -0.009$ for $\delta = 0.5$).

30. Similar to the main argument in McCallum (1995), getting the government involved and signing FGCs relocates the central bank's time-inconsistency problem to the government.

31. In the presence of turnover, a new member would be affected by a FGCs that the committee signed with the government in the past.

32. See Orphanides and van Norden (2002) for an analysis of the difficulties of measuring the output gap in real time.

33. It is not difficult to verify that, for $b = \hat{b}$, $\pi^C_L = \hat{\pi}^C_L = -\frac{\lambda\beta\kappa r_L}{\kappa^3+\sigma(1+\beta(1-\delta))\kappa^2+\lambda\kappa+\sigma\lambda(1-\beta\delta)}$ and $x^C_L = \hat{x}^C_L = \frac{\beta\kappa^2 r_L}{\kappa^3+\sigma(1+\beta(1-\delta))\kappa^2+\lambda\kappa+\sigma\lambda(1-\beta\delta)}$, which implies the continuity of social losses, interpreted as a function of b, at $b = \hat{b}$.

34. This is in line with actual practices, as central bankers have to adhere to procedures for the management of their personal assets that avoid a conflict of interest [see Swiss National Bank, Bankrat (2012) and European Central Bank, Banking Supervision (2014)]. With FGCs, prohibiting the use of hedging instruments would be particularly important.

35. We note that the extreme case $b = \infty$ implies $\zeta''(0) = -\infty$.

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APPENDIX A: PROOF OF LEMMA 1

In this appendix, we examine the circumstances under which our previous assumption holds that the central bank will select an interest rate of zero in a downturn. For this purpose, we use (2) and (3) to replace π_t and x_t, respectively, in the central bank's instantaneous loss function in period t in the presence of a contract. The derivative of the resulting expression with respect to i_t has to be weakly positive at $i_L^C = 0$. Otherwise, it would be profitable to raise interest rates. Formally, this condition can be stated as

$$\kappa \pi_L^C + \lambda x_L^C \leq 0. \tag{**A.1**}$$

As a next step, we evaluate (A.1) at π_L^C and x_L^C, where the latter two variables are specified in (16) and (17):

$$\kappa \left[A f(b) + \pi_L^D \right] + \lambda \left[B f(b) + x_L^D \right] \leq 0.$$

Solving for $f(b)$ yields

$$f(b) \leq -\frac{\kappa \pi_L^D + \lambda x_L^D}{\kappa A + \lambda B} = \frac{\kappa^2 + \lambda(1 - \beta\delta)}{\kappa(\kappa A + \lambda B)} |\pi_L^D|.$$

■

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APPENDIX B: PROOF OF LEMMA 2

To prove the lemma, we proceed in several steps. First, we determine the value of b that minimizes the social losses represented by (25), assuming that the resulting value of b satisfies (20), i.e., is small enough to ensure that the zero lower bound is binding in the downturn. Second, we show that, for this value of b, (20) is actually satisfied. Third, we examine optimal central bank policy if condition (20) fails to hold, i.e., in the case where b is such that the zero lower bound is not binding. Fourth, we show that the government would never find it optimal to select such a value for b.

Step #1 Inserting (21) and (22) into (25) and computing the derivative with respect to b reveals that this derivative is proportional to $f(b) - f^*$, where f^* has been defined in (26). We note that f^* is positive because $\pi_L^D < 0$, $x_L^D < 0$, $A > 0$, and $B > 0$.

Recall that $f(b)$ is a strictly monotonically increasing function with $f(0) = 0$. Hence, for $\lim_{b\to\infty} f(b) = r_H/\sigma > f^*$, there is a unique value of b, b^*, that satisfies $f(b) = f^*$. This value minimizes expected social losses. By contrast, for $\lim_{b\to\infty} f(b) = r_H/\sigma \leq f^*$, social losses are a strictly monotonically decreasing function of b $\forall b \geq 0$. Loosely speaking, the optimal value of b is infinitely high in this case. For the remainder of the proof, we focus on the case where values of b exist for which the zero lower bound would not bind in equilibrium under optimal central bank policy, i.e., we focus on $\lim_{b\to\infty} f(b) = r_H/\sigma > f^*$.

Step #2 It is unclear as yet whether for the value of b, b^*, identified in the previous step, the zero lower bound is actually binding in equilibrium. To show this, we prove that with respect to i_t, the derivative of the central bank's loss function, with the Phillips Curve and the IS Curve used to substitute for inflation and output, is weakly positive at $i_t = 0$ in a downturn. Formally, this condition can be stated as

$$\left.\frac{\partial l_t^{CB}}{\partial i_t}\right|_{i_L^C=0} \geq 0. \tag{B.1}$$

Since

$$l_t^{CB} = l_L^{CB} = \frac{1}{2}\left[\left(\pi_L^C\right)^2 + \lambda\left(x_L^C\right)^2\right] + \frac{1}{2}bi_t^2,$$

with $i_t = i_L^C$ and

$$\frac{\partial l_t^{CB}}{\partial i_t} = \pi_t\frac{\partial \pi_t}{\partial i_t} + \lambda x_t\frac{\partial x_t}{\partial i_t} + bi_t = \pi_L^C\left(-\frac{\kappa}{\sigma}\right) + \lambda x_L^C\left(-\frac{1}{\sigma}\right) + bi_L^C,$$

(B.1) can be rewritten as

$$\kappa\pi_L^C + \lambda x_L^C \leq 0. \tag{B.2}$$

Using $\pi_L^C = Af(b^*) + \pi_L^D$ and $x_L^C = Bf(b^*) + x_L^D$, replacing $f(b^*)$ by f^*, and using the definition of f^* in (26), it is straightforward, though tedious, to show that (B.2) is satisfied for $b = b^*$.

Step #3 So far, we have determined the optimal choice of b from all values for which the zero lower bound binds in equilibrium. However, it is conceivable that the government would select a value of b such that this would not be the case, i.e., a value for which (20) is

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violated. Hence, we consider the equilibrium for this range of b in this step. In the fourth step, we demonstrate that the government would never select such a value for b^*.

If the zero lower bound does not bind, the following first-order condition holds in state L under an FGC, as can easily be shown:

$$\kappa\hat{\pi}_L^C + \lambda\hat{x}_L^C - b\sigma i_L^C = 0. \tag{B.3}$$

The Phillips Curve (3), $\hat{\pi}_L^C = \beta\left[\delta\hat{\pi}_L^C + (1-\delta)\pi_H^C\right] + \kappa\hat{x}_L^C$, the IS Curve (2), $\hat{x}_L^C = -\sigma^{-1}\left\{i_L^C - \left[\delta\hat{\pi}_L^C + (1-\delta)\pi_H^C\right] - r_L\right\} + \left[\delta\hat{x}_L^C + (1-\delta)x_H^C\right]$, (10), (11), and (B.3) can be used to compute

$$\hat{\pi}_L^C = z(b)\frac{\kappa(1-\delta)\left(\beta\lambda + b\sigma\left\{\kappa + \sigma\left[1+\beta(1-\delta)\right]\right\}\right)}{\kappa^2+\lambda+b\sigma^2}r_H + z(b)\kappa r_L, \tag{B.4}$$

$$\hat{x}_L^C = z(b)\frac{(1-\delta)\left\{-\beta\kappa^2 + b\sigma\left[\kappa + \sigma\left(1-\beta\delta\right)\right]\right\}}{\kappa^2+\lambda+b\sigma^2}r_H + z(b)(1-\beta\delta)r_L, \tag{B.5}$$

where

$$z(b) := \frac{b\sigma}{\kappa^2 + \lambda(1-\beta\delta) + b\sigma\left[\sigma(1-\delta)(1-\beta\delta) - \delta\kappa\right]}. \tag{B.6}$$

We observe that $z(b)$ is a monotonically increasing function of b (recall our previous assumption $\sigma(1-\delta)(1-\beta\delta) - \delta\kappa > 0$).

Step #4 Suppose that b is sufficiently high for the zero lower bound not to be binding ($b \geq \hat{b}$). Then, (B.4) and (B.5) can be used to write per-period losses in a downturn as

$$\begin{aligned}\hat{l}_L^C &= \left(\hat{\pi}_L^C\right)^2 + \lambda\left(\hat{x}_L^C\right)^2 \\ &= z(b)^2\left[\left(\frac{\kappa(1-\delta)\left(\beta\lambda + b\sigma\left\{\kappa + \sigma\left[1+\beta(1-\delta)\right]\right\}\right)}{\kappa^2+\lambda+b\sigma^2}r_H + \kappa r_L\right)^2\right. \\ &\quad \left. + \lambda\left(\frac{(1-\delta)\left\{-\beta\kappa^2 + b\sigma\left[\kappa + \sigma\left(1-\beta\delta\right)\right]\right\}}{\kappa^2+\lambda+b\sigma^2}r_H + (1-\beta\delta)r_L\right)^2\right].\end{aligned} \tag{B.7}$$

Using $f(b) = \frac{b\sigma}{\lambda+\kappa^2+b\sigma^2}r_H$ and $\frac{1}{\lambda+\kappa^2+b\sigma^2}r_H = \frac{r_H - \sigma f(b)}{\kappa^2+\lambda}$ [see (13)], we can restate (B.7) as follows:

$$\begin{aligned}\hat{l}_L^C = z(b)^2\left\{\left[\kappa(1-\delta)\left(\frac{r_H - \sigma f(b)}{\kappa^2+\lambda}\beta\lambda + f(b)\left\{\kappa + \sigma\left[1+\beta(1-\delta)\right]\right\}\right) + \kappa r_L\right]^2\right. \\ \left. + \lambda\left[(1-\delta)\left(-\frac{r_H - \sigma f(b)}{\kappa^2+\lambda}\beta\kappa^2 + f(b)\left[\kappa + \sigma\left(1-\beta\delta\right)\right]\right) + (1-\beta\delta)r_L\right]^2\right\}.\end{aligned} \tag{B.8}$$

We will now explain that (B.8) is a monotonically increasing function of b for $b \geq \hat{b}$, where $\hat{b}$ is implicitly defined by (20). This follows from two observations. First, we have already noted that $z(b)$ monotonically increases with b $\forall b \geq 0$. Second, the term in brackets in (B.8) is a quadratic function of $f(b)$. It is straightforward, though tedious, to show that the minimum of this term, interpreted as a function of $f(b)$, is at $f(b) = f(\hat{b})$, where $f(\hat{b})$

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is given by the right-hand side of (20). Hence, (B.8) monotonically increases with b for $b \geq \hat{b}$.

Because at $b = \hat{b}$, $l_L^C = \hat{l}_L^C$ holds[33] and l_L^C, evaluated at $\hat{b}$, has to be larger than at $b = b^*$ as b^* is the value of b minimizing l_L^C, we can conclude that the government would not choose a value of b with $b \geq \hat{b}$. ■

APPENDIX C: PROOF OF LEMMA 3

C.1. Preliminary Steps

We need to define the strategy of the government in the candidate equilibrium precisely. We assume that the government will always sign a new contract for the next period in state L, independently of whether a contract has been signed for the current period. Moreover, we consider the case where the government never signs a contract for the next period if the economy is in state H, irrespective of whether a contract exists for the current period.

Next, we examine whether, for the government, profitable deviations exist in a particular period, when the government takes its own future behavior, the behaviors of the central bank and of the private sector as given. There are four potential deviations. First, in a situation where a contract is present in the current period and where the current economic state is L, the government chooses not to sign a contract for the next period. Second, the government refuses to offer a contract in state L, given that no contract is present in the current period. Third, in state H the government offers a contract for the next period if a contract is active in the current period. Fourth, in state H without a contract in the current period, the government introduces a contract for the next period.

It is comparably straightforward to show that the third and fourth deviation cannot be profitable. Showing that the other deviations are undesirable for the government is more cumbersome and requires a few preliminary steps and some additional notation.

Let $V_s(C)$ be the discounted future social losses for optimal central-bank and private-sector behaviors, given the current state $s \in \{L, H\}$, the fact that the government pursues the strategy described above, and that a contract has been signed for the current period. $V_s(N)$ is the analogous expression for the case where no contract is present in the current period. Moreover, let l_s^{XY} with $s \in \{H, L\}$ and $X, Y \in \{C, N\}$ be the per-period losses in state s if currently there is a contract ($X = C$) or no contract ($X = N$) and if in the current period a contract is signed for the next period ($Y = C$) or not ($Y = N$).

We obtain the following equations:

$$V_L(C) = l_L^{CC} + \beta\left[\delta V_L(C) + (1-\delta)V_H(C)\right], \tag{C.1}$$

$$V_H(C) = l_H^{CN} + \beta V_H(N), \tag{C.2}$$

$$V_L(N) = l_L^{NC} + \beta\left[\delta V_L(C) + (1-\delta)V_H(C)\right], \tag{C.3}$$

$$V_H(N) = l_H^{NN} + \beta V_H(N). \tag{C.4}$$

We note that $l_H^{NN} = 0$, $l_L^{CN} = l_L^{NN}$, and $l_L^{CC} = l_L^{NC}$, where the latter two conditions follow from the observation that the zero lower bound always binds in state $s = L$, irrespective of whether a contract was signed in the previous period. This observation will be shown formally later.

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As a result, we obtain

$$V_L(C) = \frac{1}{1-\beta\delta} l_L^{CC} + \frac{\beta(1-\delta)}{1-\beta\delta} l_H^{CN}, \tag{C.5}$$

$$V_H(C) = l_H^{CN}, \tag{C.6}$$

$$V_L(N) = \frac{1}{1-\beta\delta} l_L^{CC} + \frac{\beta(1-\delta)}{1-\beta\delta} l_H^{CN}, \tag{C.7}$$

$$V_H(N) = 0. \tag{C.8}$$

C.2. Deviation in State L when a Contract was Signed in the Previous Period

We are now in a position to specify the condition that ensures that the government does not find it optimal to refuse to offer a contract for the next period, given that the current state is L and that a contract was signed in the previous period:

$$l_L^{CN} + \beta\left[\delta V_L(N) + (1-\delta) V_H(N)\right] \geq V_L(C). \tag{C.9}$$

The right-hand side of the inequality states the losses incurred if the government does not deviate. The expression on the left-hand side represents social losses if the government does not offer a contract in the period under consideration but pursues its equilibrium strategy in all future periods. With the help of (C.5)–(C.8), (C.9) can be simplified to

$$l_L^{CN} \geq l_L^{CC} + \beta(1-\delta) l_H^{CN}. \tag{C.10}$$

This condition will be analyzed in more detail later.

C.3. Deviation in State L when a Contract was not Signed in the Previous Period

In state L, the government will find it optimal to offer a contract for the next period, provided that no contract was signed in the previous period, if

$$l_L^{NN} + \beta\left[\delta V_L(N) + (1-\delta) V_H(N)\right] \geq V_L(N). \tag{C.11}$$

Because $l_L^{NN} = l_L^{CN}$ and $V_L(N) = V_L(C)$, this condition is equivalent to (C.9) and thus to (C.10).

C.4. Evaluating Condition (C.10)

To evaluate condition (C.10), we have to determine l_L^{CN}, l_L^{CC}, and l_H^{CN}. For this purpose, we observe that l_L^{CC} and l_H^{CN} are per-period losses that also occur in the candidate equilibrium. Hence, we obtain

$$l_L^{CC} = 0.5\left[(\pi_L^C)^2 + \lambda(x_L^C)^2\right], \tag{C.12}$$

$$l_H^{CN} = 0.5\left[(\pi_H^C)^2 + \lambda(x_H^C)^2\right]. \tag{C.13}$$

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To determine l_L^{CN}, we have to compute inflation and the output gap, π_L^{CN} and x_L^{CN}, for the case where the government does not offer a contract in state L for the next period but reverts to its putative equilibrium strategy in all future periods, i.e., it will offer a contract in state L and no contract in state H. In such a situation, expectations of inflation and the output gap are

$$\mathbb{E}_t[\pi_{t+1}] = \delta\pi_L^{NC} + (1-\delta)\pi_H^{NN} = \delta\pi_L^{C}, \tag{C.14}$$

$$\mathbb{E}_t[x_{t+1}] = \delta x_L^{NC} + (1-\delta)x_H^{NN} = \delta x_L^{C}. \tag{C.15}$$

It is tedious but straightforward to show that inserting these two expressions into (2) and (3), evaluated at $i_t = 0$, yields

$$\pi_L^{CN} = Pf(b) + \pi_L^{D}, \tag{C.16}$$

$$x_L^{CN} = Qf(b) + x_L^{D}, \tag{C.17}$$

where π_L^D, x_L^D, and $f(b)$ have been introduced in (6), (13), and (5) respectively, and P and Q are given by

$$P := \delta\left[\left(\frac{\kappa}{\sigma} + \beta\right)A + \kappa B\right], \tag{C.18}$$

$$Q := \delta\left(\frac{A}{\sigma} + B\right). \tag{C.19}$$

Recall that A and B have been defined in (18) and (19).

C.5. Verifying that the Zero Lower Bound Binds for the Deviations

It remains to be verified that the zero lower bound is also binding for the deviations analyzed above if (20) is satisfied, which ensures that it is binding in state L in equilibrium when a contract is present in the current period. We note that this is the case for

$$\kappa\pi_L^{CN} + \lambda x_L^{CN} \leq 0. \tag{C.20}$$

Using (20), (C.16), (C.17), and $x_L^D = \frac{1-\beta\delta}{\kappa}\pi_L^D$, which follows from (5) and (6), and rearranging results in the condition yields

$$\frac{\kappa P + \lambda Q}{\kappa A + \lambda B} \leq 1. \tag{C.21}$$

It is straightforward to show $A - P = \frac{\kappa(1-\delta)[\kappa+\sigma(1+\beta)]}{\sigma} > 0$ and $B - Q = \frac{(1-\delta)(\kappa+\sigma)}{\sigma} > 0$, which, together with $P > 0$ and $Q > 0$, implies (C.21).

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C.6. Simplifying Condition (C.10)

Finally, we simplify condition (C.10) to identify the set of parameter values for which no profitable deviation for the government exists. The condition can be written as

$$\begin{aligned}0 &\leq \left(\pi_L^{NN}\right)^2 + \lambda\left(x_L^{NN}\right)^2 - \left(\pi_L^{C}\right)^2 - \lambda\left(x_L^{C}\right)^2 - \beta(1-\delta)\left(\pi_H^{C}\right)^2 - \beta(1-\delta)\lambda\left(x_H^{C}\right)^2 \\ &= -\left[A^2 - P^2 + \lambda B^2 - \lambda Q^2 + \beta(1-\delta)(\kappa^2+\lambda)\right][f(b)]^2 \\ &\quad - 2\left[A - P + \lambda(B-Q)\frac{1-\beta\delta}{\kappa}\right] f(b)\pi_L^D.\end{aligned}$$

As $\pi_L^D < 0$, $A^2 - P^2 + \lambda B^2 - \lambda Q^2 + \beta(1-\delta)(\kappa^2+\lambda) > 0$, and $\left[A - P + \lambda(B-Q)\frac{1-\beta\delta}{\kappa}\right] > 0$ (due to $A > P$ and $B > Q$), we can conclude that this expression is weakly positive for all values of $f(b)$ with $f(b) \geq 0$ that are smaller than or equal to $2\tilde{f}$, where

$$\tilde{f} := \frac{A - P + \lambda(B-Q)\frac{1-\beta\delta}{\kappa}}{A^2 - P^2 + \lambda B^2 - \lambda Q^2 + \beta(1-\delta)(\kappa^2+\lambda)}|\pi_L^D|. \tag{C.22}$$

Hence, no profitable deviation exists for the government if $f^* \leq 2\tilde{f}$, where

$$f^* = \frac{A + \lambda B\frac{1-\beta\delta}{\kappa}}{A^2 + \lambda B^2 + \beta(1-\delta)(\lambda+\kappa^2)}|\pi_L^D|. \tag{C.23}$$

■

APPENDIX D: PARAMETERS FOR SECTION 7

$$A_0 := \frac{\kappa\sigma(\kappa+\sigma+\sigma\beta)(1-\delta)}{[\sigma(1-\delta)(1-\beta\delta) - \delta\kappa][\sigma(1+\delta)(1+\beta\delta) + \delta\kappa]}. \tag{D.1}$$

$$B_0 := \frac{\sigma(\kappa+\sigma-\sigma\beta^2\delta^2)(1-\delta)}{[\sigma(1-\delta)(1-\beta\delta) - \delta\kappa][\sigma(1+\delta)(1+\beta\delta) + \delta\kappa]}. \tag{D.2}$$

$$A_1 := \frac{\kappa\delta(1-\delta)[(\kappa+\sigma)^2 + \sigma^2\beta + 2\kappa\sigma\beta + \sigma^2\beta^2(1-\delta^2)]}{[\sigma(1-\delta)(1-\beta\delta) - \delta\kappa][\sigma(1+\delta)(1+\beta\delta) + \delta\kappa]}. \tag{D.3}$$

$$B_1 := \frac{\delta(1-\delta)[(\kappa+\sigma)^2 + \kappa\sigma\beta - \sigma^2\beta^2\delta^2]}{[\sigma(1-\delta)(1-\beta\delta) - \delta\kappa][\sigma(1+\delta)(1+\beta\delta) + \delta\kappa]}. \tag{D.4}$$

$$\hat{r}_{LA} := -\frac{(1-\delta)[\kappa^2(\sigma\beta+\kappa+\sigma) + \lambda(\kappa+\sigma-\sigma\beta^2\delta^2)]}{[\kappa^2 + \lambda(1-\beta\delta)][\sigma(1+\delta)(1+\beta\delta) + \delta\kappa]}r_H. \tag{D.5}$$

$$r_{LA}^c := -\frac{\Delta_2}{\sigma\Delta_1[\sigma(1+\delta)(1+\beta\delta) + \delta\kappa]}r_H. \tag{D.6}$$

$$\begin{aligned}\Delta_1 &:= \sigma\kappa^2(\sigma\beta+\kappa+\sigma) + \sigma\lambda(\kappa+\sigma-\sigma\beta^2\delta^2)(1-\beta\delta) \\ &\quad + \beta\delta^2\kappa^2[(\sigma\beta+\kappa+\sigma)^2 - \beta\sigma^2(1+\beta\delta^2)] \\ &\quad + \beta\lambda\delta^2(1-\beta\delta)[\kappa\sigma\beta + (\kappa+\sigma)^2 - \sigma^2\beta^2\delta^2].\end{aligned} \tag{D.7}$$

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$$\begin{aligned}\Delta_2 := {} & \sigma^2(1-\delta)\kappa^2(\sigma\beta+\kappa+\sigma)^2+\sigma^2\lambda(1-\delta)(\kappa+\sigma-\sigma\beta^2\delta^2)^2\\ & +\beta\delta^3(1-\delta)\kappa^2[(\sigma\beta+\kappa+\sigma)^2-\beta\sigma^2(1+\beta\delta^2)]^2\\ & +\beta\lambda\delta^3(1-\delta)[\kappa\sigma\beta+(\kappa+\sigma)^2-\sigma^2\beta^2\delta^2]^2\\ & +\beta(\kappa^2+\lambda)[\sigma(1-\delta)(1-\beta\delta)-\delta\kappa]^2[\sigma(1+\delta)(1+\beta\delta)+\delta\kappa]^2. \qquad \textbf{(D.8)}\end{aligned}$$

APPENDIX E: MICROFOUNDATION OF THE CENTRAL BANKER'S OBJECTIVE FUNCTION

In this section, we consider only the objective function of the central bank, as the foundations of the other elements of our model are well-known from the literature [see Eggertsson (2005)]. We assume, as is standard, that the economy is populated by a continuum of identical infinitely lived households. In addition, there is an individual central banker. Hence, the central banker's individual consumption choices have no consequences for aggregate output and consumption.

We derive the central banker's intertemporal social losses with FGCs. The central banker aims at achieving a high individual utility. Moreover, he is altruistic and is also interested in furthering the well-being of the other agents. More precisely, the central banker faces one of two wage schemes. If no FGC is in effect, he receives a fixed wage $\overline{w} \geq 0$. By contrast, if an FGC was signed, the central banker is paid according to the actual choice of nominal interest rate $w_t^{CB} = \zeta(i_t) \geq 0$, where w_t^{CB} is the real wage paid to the central banker. We focus on strictly concave, twice differentiable functions $\zeta(\cdot)$ with a global maximum, $\bar{w}^{CB}$, at $i_t = 0$. Hence, the central banker faces wage reductions increasing with the level of interest rates. The central banker's wage is financed through a lump-sum tax. We note that payment to the central banker is negligible at the aggregate level, so the lump-sum tax necessary to finance his wage does not affect the households' budget constraints.

We make the assumption that the central banker is excluded from trading in financial markets. The main motivation for this assumption is that the central banker should be prevented from hedging against the variations of his income.[34]

For simplicity, we assume that the central banker is infinitely lived and has the same individual utility from consumption and discount factor β as the households. The central banker's utility from consumption is

$$u(C_t^{CB}) = \frac{(C_t^{CB})^{1-\sigma}-1}{1-\sigma} = \frac{\zeta(i_t)^{1-\sigma}-1}{1-\sigma}.$$

We evaluate this expression in the steady state with i_t equal to 0. Thus the second-order Taylor approximation delivers

$$u(C_t^{CB}) \approx \frac{\zeta(0)^{1-\sigma}-1}{1-\sigma} + \zeta(0)^{-\sigma}\zeta'(0)i_t + \frac{1}{2}\left[\frac{\partial}{\partial i_t}\left(\zeta(i_t)^{-\sigma}\zeta'(i_t)\right)\right]\Big|_{i_t=0} i_t^2.$$

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Since $\zeta'(0) = 0$, we can rewrite the approximation as

$$u(C_t^{CB}) \approx \frac{\zeta(0)^{1-\sigma} - 1}{1-\sigma} + \frac{1}{2}\zeta(0)^{-\sigma}\zeta''(0)i_t^2.$$

The first term is constant. The constant utility term can be neglected when we compute the behavior of central bankers. However, the constant utility term and thus the fixed wage $\bar{w}^{CB}$ are important to satisfy participation constraints of central bankers. Wage $\bar{w}^{CB}$ has to be set at levels at which central bankers are at least as well off as in other occupations, e.g., being a household. We assume that $\bar{w}^{CB}$ is set at levels at which the participation constraint is fulfilled.

As mentioned earlier, the central banker is also altruistic toward households. Specifically, the overall loss of the central banker in period t is

$$\alpha l_t - u(C_t^{CB}) = \frac{\alpha}{2}\left[\pi_t^2 + \lambda x_t^2 - 2\frac{\zeta(0)^{1-\sigma} - 1}{\alpha(1-\sigma)} - \frac{\zeta(0)^{-\sigma}\zeta''(0)}{\alpha} i_t^2\right], \tag{E.1}$$

with α being the weight of altruism. We scale the overall loss of the central banker by $1/\alpha$ and deduct the constant term. The resulting loss function is denoted by l_t^{CB} and given by

$$l_t^{CB} = \frac{1}{2}\left[\pi_t^2 + \lambda x_t^2 - \frac{\zeta(0)^{-\sigma}\zeta''(0)}{\alpha} i_t^2\right]. \tag{E.2}$$

We set[35] $b = -\frac{\zeta(0)^{-\sigma}\zeta''(0)}{\alpha}$ and obtain

$$l_t^{CB} = \frac{1}{2}\left(\pi_t^2 + \lambda x_t^2 + b i_t^2\right). \tag{E.3}$$

We note that the sensitivity of the wage scheme with regard to deviations of nominal interest rates from zero, $\zeta''(0)$, enters the weight b of the deviation of the interest rate from zero in the loss function of the central banker.

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APPENDIX F: LIST OF VARIABLES AND NOTATIONS

TABLE F.1. List of variables and notations (1)

Variables	Description
π_t, x_t, i_t	Inflation, output gap, nominal interest rate in period t
β	Households' discount factor
κ	Coefficient in Phillips Curve
σ	Relative risk-aversion coefficient of consumption
δ	The probability of the economy being trapped in the downturn in each period
λ	Relative weight of output-gap objective with respect to inflation objective
b	Intensity of incentives provided by FGCs
$s \in L, H$	Low and high states
r_t, r_H, r_L	Natural real interest rate in period t, in states H and L
C, N	An active contract exists, no active contract exists
π_s^D, x_s^D, i_s^D	Inflation, output gap, and interest rate in state s in discretion
π_s^C, x_s^C, i_s^C	Inflation, output gap, and interest rate in state s with an active contract
π_s^N, x_s^N, i_s^N	Inflation, output gap, and interest rate in state s without active contract π_s^N, x_s^N, i_s^N are equivalent to π_s^D, x_s^D, i_s^D
l_t, l_t^{CB}	Social loss function and the central banker's loss function
l_s^C, l_s^N	Social loss functions with and without an active contract in state s
$V_s(C), V_s(N)$	Expected discounted intertemporal social losses in state s with and without contract
$f(b)$	A function of b
$\hat{f}, \hat{b}$	Threshold values below which the zero lower bound is binding
f^*, b^*, b^{**}	Optimal designs of the contract
$\hat{r}_L$	Threshold value below which the zero lower bound is binding regardless of the value of b
r_L^c	Threshold value below which the optimal value of b is infinitely large
$\tilde{f}$	Threshold value regarding the government's behavior
A, B, P, Q	Constants
$\bar{r}_L$	The maximum possible realization of r_L in uncertainty scenario
$\tilde{b}, \tilde{r}_L, \tilde{r}_L^c, a$	Values in uncertainty scenario

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TABLE F.2. List of variables and notations (2)

Variables	Description
Alternative setup	
$\pi_{L0}^{C}, x_{L0}^{C}, i_{L0}^{C}$	Inflation, output gap, and interest rate in state L with an active contract in period 0
$\pi_{L1}^{C}, x_{L1}^{C}, i_{L1}^{C}$	Inflation, output gap, and interest rate in state L with an active contract in period 1
A_0, B_0, A_1, B_1	Constants
$V_{LA}(C)$	Expected discounted intertemporal social loss with an FGC
l_{L0}^{C}, l_{L1}^{C}	Social loss function with an active contract in state L in periods 0 and 1, respectively
$\hat{r}_{LA}$	Threshold value below which the zero lower bound is binding regardless of the value of b
r_{LA}^{c}	Threshold value below which the optimal value of b is infinite
Proof	
$\hat{\pi}_{L}^{C}, \hat{x}_{L}^{C}, \hat{l}_{L}^{C}$	Inflation, output gap, and loss function in state L with an active contract, assuming the zero lower bound is not binding
$X, Y \in C, N$	X represents whether a contract was signed in previous period; Y represents whether a contract is signed in the current period
$\pi_{s}^{XY}, x_{s}^{XY}, l_{s}^{XY}$	Inflation, output gap, and social loss function in state s
$a_1, a_2, \Delta_1, \Delta_2$	constants
$z(b)$	Function of b
Microfoundation	
$\bar{w}$	Fixed wage the central banker receives when no FGC is in effect
$w_{t}^{CB}, \bar{w}^{CB}$	Wage the central banker receives when FGC is in effect and its global maximum
$\zeta(i_t)$	A function of i_t
C_{t}^{CB}	Central banker's consumption
$u(C_{t}^{CB})$	Central banker's utility from consumption
α	The weight of central banker's altruism toward households

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