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Drift-wave trapping by drift vortices

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The possibility for a drift dipole vortex to trap free drift waves is demonstrated. Drift perturbations can be trapped near the centre of the vortex or at its sides. The localization domain and eigenfrequencies of trapped modes are obtained.

1. Introduction

Drift-wave turbulence in tokamaks is considered to be the main reason for anomalous heat conductivity, connected, in particular, with the ion–temperature–gradient driven mode (Connor 1995). It is widely thought that the strong drift-turbulence plasma state can be regarded as a set of regular vortex structures and weakly correlated drift waves (Horton 1989; Horton *et al.* 1990; Crotinger and Dupree 1992; Pavlenko and Weiland 1993). Computer simulations show that monopolar and dipolar vortices can appear even from random initial data, but the latter are transient (Horton *et al.* 1990; Crotinger and Dupree 1992; Koniges *et al.* 1992; Jovanovic and Horton 1993). The simplest model in which such vortices arise is the Hasegawa–Mima equation (Hasegawa and Mima 1978; Petviashvili and Yan'kov 1989; Infeld and Rowlands 1992), which describes low-frequency drift waves in plasmas as well as large-scale motion of the atmosphere, particularly nonlinear Rossby waves (in the latter case it is often referred to as the Charney equation). This model has been widely used in theoretical analysis of the influence of the coherent structures on transport processes in tokamaks. Nevertheless, it is not yet quite clear why dipole vortices, the simplest known solutions of the Hasegawa–Mima equation (Meiss and Horton 1983), which are outside Čerenkov resonance for drift waves, have been much more rarely observed in numerical simulations than monopole vortices, for which the Čerenkov-resonance conditions are fulfilled (Crotinger and Dupree 1992). We suppose that this can be connected with the possibility for the dipole vortex to trap free drift waves.

This paper is organized as follows. A theoretical model for the description of this process is proposed in Sec. 2. Further, we show that electrostatic potential of the drift dipole vortex looks like a single potential well at the centre of the vortex (Sec. 3) or like two wells at both of its sides, where free drift waves with a discrete number of frequencies can be trapped (Sec. 4). Some conclusions will be drawn in Sec. 5.

2. Basic equation

We consider a non-uniform plasma with equilibrium density $n_0(x)$ immersed in a homogeneous magnetic field $\mathbf{B}_0$, which is oriented along the z axis. Our analysis is based on the Hasegawa–Mima equation

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$$\frac{\partial}{\partial t}(1-\rho^2\Delta_\perp)\,\phi+v_d\frac{\partial\phi}{\partial y}=c_s\rho^3\{\phi,\Delta_\perp\phi\},\tag{1}$$

where $v_d=D/L_n$ is the electron diamagnetic drift speed, L_n is the density scale length, $D=cT_e/eB_0$, c_s is the ion–acoustic velocity, $\rho=c_s/\omega_{ci}$, ω_{ci} is the ion cyclotron frequency, $\phi=e\Phi/T_e$, Φ is the electrostatic potential,

$$\Delta_\perp=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2},$$

and the Poisson bracket is defined by

$$\{\phi,g\}=\frac{\partial\phi}{\partial x}\frac{\partial g}{\partial y}-\frac{\partial\phi}{\partial y}\frac{\partial g}{\partial x}.$$

Equation (1) has a well-known exact solution in the form of a solitary dipole vortex (modon) propagating with the speed u in the y direction (Meiss and Horton 1983):

$$\phi_V=\frac{au}{D}\cos\theta\begin{cases}\left(1+\dfrac{\beta^2}{\gamma^2}\right)\dfrac{r}{a}-\dfrac{\beta^2}{\gamma^2}\dfrac{J_1(\gamma r/a)}{J_1(\gamma)} & (r\leqslant a),\\[2ex] \dfrac{K_1(\beta r/a)}{K_1(\beta)} & (r\geqslant a),\end{cases}\tag{2}$$

where $x=r\cos\theta$, $y-ut=r\sin\theta$,

$$\beta^2=\frac{a^2}{\rho^2}\left(1-\frac{v_d}{u}\right)>0,$$

and γ is determined by

$$\frac{K_2(\beta)}{\beta K_1(\beta)}=-\frac{J_2(\gamma)}{\gamma J_1(\gamma)}\equiv\delta.$$

In (2), J_n and K_n are Bessel and McDonald functions of order n.

The main assumption at the basis of many transport models is that the nonlinear interaction between coherent vortex structures and uncorrelated drift fluctuations is negligible. We try to verify this assumption for the widely used simplest model of Hasegawa and Mima.

Equation (1) also describes small-amplitude free drift waves with dispersion relation

$$\omega_{\mathbf{k}}=\frac{k_y v_d}{1+k_\perp^2\rho^2}.\tag{3}$$

Let us describe an 'elementary interaction act' between a single vortex (2) and a small-amplitude drift wave with fixed wave vector $\mathbf{k}$ and frequency (3). We represent the potential in the form

$$\phi=\phi_V(x,y-ut)+\phi_w,\tag{4}$$

where ϕ_V is given by (2) and ϕ_w is assumed to be small, $\max|\phi_w|\ll\max|\phi_V|$. Substituting (4) into (1) and linearizing, we obtain for ϕ_w

$$\frac{\partial}{\partial t}(1-\rho^2\Delta_\perp)\,\phi_w+v_d\frac{\partial\phi_w}{\partial y}=\{\phi_V,\Delta_\perp\phi_w\}+\{\phi_w,\Delta_\perp\phi_V\}.\tag{5}$$

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In the absence of the vortex, any solution ϕ_w is a linear combination (Fourier integral) of drift waves with different ω. Now the amplitude of a drift wave with dispersion relation (3) will change in space owing to interaction with the vortex ϕ_V. We shall consider the initial stage of this interaction, neglecting the nonlinear term $\{\phi_w, \Delta_\perp \phi_w\}$. Equation (5) can describe the scattering of a drift wave by the vortex or a trapping.

It is convenient to write (5) in a system of coordinates moving with the vortex speed u. Then we have

$$\left(\frac{\partial}{\partial t} - u\frac{\partial}{\partial y}\right)(1-\rho^2\Delta_\perp)\,\phi_w + v_d\frac{\partial \phi_w}{\partial y} = \{\phi_V, \Delta_\perp \phi_w\} + \{\phi_w, \Delta_\perp \phi_V\}. \tag{6}$$

In (6), $\phi_V = \phi_V(x, y)$ does not depend on time, and we can seek a solution of (6) in the form

$$\phi_w = \phi(x, y)\exp(-\mathrm{i}\omega t),$$

where the drift wave amplitude $\phi(x, y)$ satisfies the equation

$$-i\omega(1-\rho^2\Delta_\perp)\,\phi + (v_d - u)\frac{\partial \phi}{\partial y} + u\rho^2\Delta_\perp\frac{\partial \phi}{\partial y} = \{\phi_V, \Delta_\perp \phi\} + \{\phi, \Delta_\perp \phi_V\}. \tag{7}$$

For some values of ω, called eigenvalues, (7) possibly has solutions vanishing at infinity (localized or trapped modes). Three localized solutions of (7) are the so-called translational modes $\phi_1 = \partial\phi_V/\partial x$, $\phi_2 = \partial\phi_V/\partial y$ and $\phi_3 = \partial\phi_V/\partial a$, with eigenvalue $\omega = 0$. As is well known, these simply correspond to translation of the vortex along the x and y directions and to variation of its radius a. Modes with $\omega \neq 0$ are not so trivial, and correspond to drift-wave trapping.

We shall restrict ourselves to the case of a single modon (2) with the minimum value $\gamma = \gamma_1$ for the fixed β ($J_1(\gamma_1) < 0$, $J_2(\gamma_1) > 0$ and $3.8 < \gamma_1 < 5.1$ for any β), and this solution is expected to be the most stable (Meiss and Horton 1983). Equation (7) still seems too complicated. Let us make the substitution $\phi = \Psi(x, y)\exp(iQy)$ and choose the constant Q so that the coefficient of $\partial\Psi/\partial y$ vanishes. This gives a connection between ω and Q:

$$\omega = \frac{v_0 - 3Q\rho^2 u}{2\rho^2 Q}, \tag{8}$$

where $v_0 = v_d - u$. We can simplify (7) further if

$$|\boldsymbol{\nabla}_\perp \ln \Psi| \ll |Q| \tag{9}$$

and

$$\beta^2 \gg Q^2 a^2. \tag{10}$$

In this case (8) takes the form

$$\frac{uQ}{\omega} = -\frac{2Q^2a^2}{\beta^2}. \tag{11}$$

Then the equation for $\Psi(x, y)$ can be reduced to the Schrödinger equation

$$\Delta_\perp \Psi + V(x, y)\,\Psi = 0, \tag{12}$$

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with

$$V(x,y)=\left(Q^2-\frac{1}{\rho^2}\right)+\chi(a-r)\frac{uQ}{\omega}$$
$$\times\left\{Q^2\left(1+\frac{\beta^2}{\gamma^2}\right)-\frac{\beta^2\left(Q^2-\frac{\gamma^2}{a^2}\right)}{\gamma J_1(\gamma)}\left[\frac{dJ_1(z)}{dz}\cos^2\theta+\frac{J_1(z)}{z}\sin^2\theta\right]\right\}$$
$$+\chi(r-a)\frac{uQ}{\omega}\left\{\frac{\beta\left(Q^2-\frac{\gamma^2}{a^2}\right)}{K_1(\beta)}\left[\frac{dK_1(g)}{dg}\cos^2\theta+\frac{K_1(g)}{g}\sin^2\theta\right]\right\}, \tag{13}$$

where

$$z=\frac{\gamma r}{a},\quad g=\frac{\beta r}{a},\quad \chi(\xi)=\begin{cases}1 & (\xi\geqslant 0),\\ 0 & \xi<0).\end{cases}$$

The form of the potential well in (12) in the region $r<a$ depends on the sign of $Q^2-\gamma^2/a^2$. It can be seen that inside the vortex ($r<a$) the effective potential in (12) has a single minimum at the centre ($x=0$, $y=0$) if $Q^2<\gamma^2/a^2$ or two local minima at the side points $x=\pm x_0$, $y=0$ if $Q^2>\gamma^2/a^2$.

3. Trapped drift modes in the centre of the vortex

First let us consider the case of drift-wave trapping in the centre of the vortex, supposing that $Q^2<\gamma^2/a^2$. In this case we use an expansion of the Bessel functions near $z=0$ and obtain approximately

$$\Delta_\perp\Psi+(E-px^2-qy^2)\Psi=0, \tag{14}$$

where

$$E=Q^2-\frac{1}{\rho^2}+2Q^2a^2\left[\frac{Q^2-\gamma^2/a^2}{2\gamma J_1(\gamma)}-Q^2\left(\frac{1}{\gamma^2}+\frac{1}{\beta^2}\right)\right], \tag{15}$$

$$q=\frac{\gamma Q^2(Q^2-\gamma^2/a^2)}{8J_1(\gamma)},\quad p=3q. \tag{16}$$

Note that $J_1(\gamma)<0$, and the necessary conditions for the trapping

$$E>0,\quad p,q>0 \tag{17}$$

are fulfilled. Equation (14) has the localized eigenfunctions $\Psi_{ln}=H_l(p^{1/2}x)H_n(q^{1/2}y)$, where

$$H_n(\xi)=(-1)^n e^{\xi^2/2}\left(\frac{d^n}{d\xi^n}\right)e^{-\xi^2},$$

under the condition

$$E_{ln}=p^{1/2}(2l+1)+q^{1/2}(2n+1)=q^{1/2}\lambda_{ln}, \tag{18}$$

where $\lambda_{ln}=3^{1/2}(2l+1)+2n+1$. Equation (16) determines a discrete series of values Q_{ln}. Then the eigenfrequencies ω_{ln} can be found from (9). Equation (18) can be rewritten in the form

$$\left[\frac{1}{\gamma|J_1(\gamma)|}+\frac{1}{2}\left(\frac{1}{\gamma^2}+\frac{1}{\beta^2}\right)\right]Q^4a^4-Q^2a^2\left[1+\frac{\gamma}{|J_1(\gamma)|}\right]+\frac{a^2}{\rho^2}+q^{1/2}\lambda_{ln}=0. \tag{19}$$

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Next we consider two limiting cases. Let Q^2 be close to γ^2/a^2, so that $Q^2 = \gamma^2/a^2 - s^2$, where $s^2 \ll \gamma^2/a^2$. Then we have

$$\left[\frac{\gamma}{|J_1(\gamma)|}+\frac{\gamma^2}{\beta^2}\right]a^2s^2-\left[\frac{\gamma^3}{8|J_1(\gamma)|}\right]^{1/2}\lambda_{ln}\,as+\frac{\gamma^2}{2}-\frac{a^2}{\rho^2}-\frac{\gamma^4}{2\beta^2}=0. \tag{20}$$

For $\beta^2 \gg \gamma^2$ (see the condition (10)), we obtain

$$sa = [\gamma|J_1(\gamma)|]^{1/2}\left[\frac{\lambda_{ln}}{8^{1/2}}+\left(\frac{\lambda_{ln}^2}{32}-\frac{1}{2}+\frac{a^2}{\gamma^2\rho^2}\right)^{1/2}\right]. \tag{21}$$

We see from (21) that the condition $s \ll \gamma/a$ is fulfilled for

$$a^2 \lesssim \gamma^2\rho^2 \quad \text{and} \quad \lambda_{ln} \ll \left[\frac{8\gamma}{|J_1(\gamma)|}\right]^{1/2}, \tag{22}$$

that is, for several of the lowest modes. The condition (10), or equivalently $q^{1/2} < Q^2$, is also easily fulfilled in this case. The width of the localization domain of the modes $\delta x \approx \delta y \approx q^{-1/4}$ should be less than $2a/\gamma$ (near this point, the function $J_1(z)$ has its maximum). The latter condition is valid if

$$q^{1/2} = \left[\frac{\gamma}{8|J_1(\gamma)|}\right]^{1/2}\frac{\gamma}{a}s > \frac{\gamma^2}{a^2}\frac{\lambda_{ln}}{8} > \frac{\gamma^2}{4a^2},$$

whence we get $\lambda_{ln} > 2$. This holds in our case, where the smallest values of λ_{ln} are $\lambda_{00} = 2.77$, $\lambda_{01} = 4.77$, $\lambda_{10} = 6.31$, $\lambda_{11} = 8.33$, $\lambda_{20} = 9.85$, ... Thus, under the condition (22), we have the equation for the eigenfrequencies of the trapped modes:

$$Q_{ln}^2\,a^2 = \gamma^2 - \tfrac{1}{8}\gamma|J_1(\gamma)|\,\lambda_{ln}^2. \tag{23}$$

For the opposite limit $Q^2 \ll \gamma^2/a^2$, we can neglect the first term in (19) to obtain

$$\left(1+\frac{\gamma}{|J(\gamma)|}\right)Q^2a^2-\gamma\left[\frac{\gamma}{8|J_1(\gamma)|}\right]^{1/2}\lambda_{ln}\,Qa-\frac{a^2}{\rho^2}=0.$$

So we have in this case

$$Qa = \left[\frac{\gamma|J_1(\gamma)|}{8}\right]^{1/2}\left\{\frac{\lambda_{ln}}{2}+\left[\frac{\lambda_{ln}^2}{4}+\frac{8a^2|J_1(\gamma)|}{\rho^2\gamma^3}\right]^{1/2}\right\} \approx [\gamma|J_1(\gamma)|]^{1/2}\,\lambda_{ln}. \tag{24}$$

The requirement that $Q^2a^2 \ll \gamma^2$ is fulfilled for the modes with $\lambda_{ln}^2 \ll 8\gamma/|J_1(\gamma)|$, which coincides with (22).

So, for each λ_{ln} that is less than some critical value, we have the two positive roots Q^2a^2 of (19). The smaller root (24) grows with λ_{ln}, and the larger one (23) decreases with λ_{ln}, but for the lower branch (24) the condition $q^{1/2} < Q^2$ is violated. Equation (23) approximately gives us the eigenvalues Q_{ln} of several trapped modes under the condition (22). The expression (23) is valid until $Q^2a^2 \approx \frac{1}{2}\gamma^2$, so that $\frac{1}{2}\gamma^2 < Q^2a^2 < \gamma^2$. The relation (11) gives for the eigenvalues ω_{ln} the restrictions

$$\frac{u^2\beta^4}{4a^2\gamma^2} < \omega_{ln} < \frac{u^2\beta^4}{2a^2\gamma^2}. \tag{25}$$

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4. Trapped drift modes at the vortex sides

As was noted in Sect. 2, trapping at the vortex sides is possible if $Q^2 > \gamma^2/a^2$. The effective potential in (12) now has a local maximum at the centre $x = y = 0$. The potential is symmetric under both of the transformations $x \to -x$ and $y \to -y$, and we shall consider its behaviour for $x \geqslant 0$ and $y \geqslant 0$. As can be seen from (13), on the x axis the form of the potential in the region $x < a$ is determined by the function $dJ_1(z)/dz$. This function has a minimum value (approximately 0.4) at the point $x_0 \approx 3.5a/\gamma$. On the y axis (and for $y < a$) the potential depends only on the function $J_1(z)/z$, which has a maximum at $z = 0$ and a minimum at $z_0 \approx 5.1$. This minimum, however, lies outside the circle $r \leqslant a$. When $y > a$, the potential decreases monotonically as the function $K_1(\beta y/a)/y$. Therefore we find that in this case the trapping potential has minima in the vicinity of the two points $x = \pm x_0$, $y = 0$, that is, at the 'sides' of the vortex. Near the point $x = x_0$, $y = 0$, it can be approximated by a paraboloid, so that (12) takes the form

$$\Delta_\perp \Psi + [W - p_1(x - x_0)^2 - q_1 y^2]\Psi = 0, \tag{26}$$

where

$$W = Q^2 - \frac{1}{\rho^2} - \frac{2Q^2a^2}{\beta^2}\left[Q^2\left(1 + \frac{\beta^2}{\gamma^2}\right) - \frac{0.4\beta^2(Q^2 - \gamma^2/a^2)}{\gamma|J_1(\gamma)|}\right],$$

$$p_1 = \frac{0.175\gamma}{|J_1(\gamma)|}Q^2\left(Q^2 - \frac{\gamma^2}{a^2}\right), \qquad q_1 = \frac{p_1}{z_0}.$$

The dispersion relation for the eigenvalues Q_{ln} is now

$$W = \lambda_{ln}\, p_1^{1/2}, \tag{27}$$

where

$$\lambda_{ln} = (2l + 1) + \frac{1}{z_0^{1/2}}(2n + 1).$$

We again consider two limiting cases. If Q is close to γ/a, that is, $Q^2 = \gamma^2/a^2 + s^2$ with $s^2 \ll \gamma^2/a^2$, we obtain from (27) (assuming $\beta^2 \gg \gamma^2$)

$$s = \frac{[\gamma|J_1(\gamma)|]^{1/2}}{a}\left[0.42\lambda_{ln} + \left(\frac{0.175\lambda_{ln}^2}{4} - \frac{1}{2} + \frac{a^2}{\gamma^2\rho^2}\right)^{1/2}\right]. \tag{28}$$

If $a^2 \lesssim \gamma^2\rho^2$, the condition $s \ll \gamma/a$ is fulfilled for the modes with $\lambda_{ln} \lesssim 10$. The condition $q_1^{1/2} < \mathrm{Q}^2$ and the restriction on the width of the localization domain $\delta x \approx p_1^{-1/4} < 1.5a/\gamma$ are also fulfilled for several of the lowest values of λ_{ln}. Note that $Q_{ln}^2 a^2$ increases with s^2, which is almost proportional to λ_{ln}^2, and for $\lambda_{ln} > 10$ the approximation $s \ll \gamma/a$ is violated.

In the opposite limiting case $Q^2 \gg \gamma^2/\mathrm{a}^2$ we have $q_1 = 0.175\gamma Q^2/|J_1(\gamma)|$, and (26) takes the form

$$Q^4a^4\left[\frac{1}{\gamma|J_1(\gamma)|} - \frac{1}{2}\left(\frac{1}{\beta^2} + \frac{1}{\gamma^2}\right)\right] - Q^2a^2\left\{\frac{\gamma}{|J_1(\gamma)|} + \left[\frac{0.175\gamma}{|J_1(\gamma)|}\right]^{1/2}\lambda_{ln} - 1\right\} - \frac{a^2}{\rho^2} = 0,$$

and can be easily solved. We can see that with further increase in λ_{ln}, Q_{ln} increases approximately linearly. In the limit as $\lambda_{ln} \to \infty$, the condition p_1, $q_1 \ll Q^4$ is hardly fulfilled. Large values of λ_{ln} are also restricted by the condition (10).

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Thus we see that drift-wave trapping is possible for $Q_{ln}^2 \gtrsim \gamma^2/a^2$, that is, for eigenfrequencies satisfying

$$\omega_{ln}^2 < \frac{u^2}{4a^2}\frac{\beta^4}{\gamma^2} = \frac{a^2(u-v_d)^2}{4\rho^4\gamma^2} \tag{29}$$

near the points $x = \pm x_0 \approx 3.5a/\gamma$, $y = 0$. Trapped modes with small l and n are localized in the region $\delta x \approx p_1^{-1/4}$, $\delta y \approx (z_0/p_1)^{1/4}$.

5. Conclusions

We have shown that in the framework of the Hasegawa–Mima model there is a finite set of localized drift modes trapped by the potential well of the vortex. Their electric potential has the form $\phi_w = \Psi(x,y)\exp(iQy - i\omega t)$, with the function $\Psi(x,y)$ localized inside the circle $r = a$. Under the conditions (9) and (10) discussed above, the function $|\Psi|^2$ has a maximum at the centre of the vortex ($x = 0$, $y = 0$) for a few modes with eigenfrequencies in the region (25), and two maxima at both sides of the vortex ($x = \pm x_0$, $y = 0$) in the region of frequencies (29). Near the maxima, the curves of equal $|\Psi|^2$ have the form of ellipses slightly more prolonged in the y direction. Outside the localization region $|\mathbf{r} - \mathbf{r}_0| \gg a/\gamma$, we have $\Psi \sim r^{-1/2}\exp(i\chi r)$, with $\chi = (Q^2 - 1/\rho^2)^{1/2}$ if $Q\rho > 1$, and $\Psi \sim r^{-1/2}\exp(-\sigma r)$ with $\sigma = (1/\rho^2 - Q^2)^{1/2}$ if $Q\rho < 1$. In the first case the mode in fact radiates to infinity and can be regarded as a quasi-trapped mode.

We have not considered the final stage of the vortex interaction with the trapped or quasi-trapped modes. However, it is very likely that they trend to smooth (annihilate) the original dipole vortex if they are trapped at the centre, or tend to smooth the maximum and minimum of the vortex field if the perturbations are trapped at the sides. In the latter case the dipole vortices should be transformed into a monopole vortex. In general, this depends on the sign of the initial perturbation and on its form. This picture is in accordance with numerical simulations, which indicate splitting of the dipole vortex into two monopoles, one of which rapidly vanishes (Horton *et al.* 1990; Nycander and Isichenko 1990; Spatschek *et al.* 1990; Crotinger and Dupree 1992; Koniges *et al.* 1992).

In conclusion, we note that the possibility of mode trapping indicates that the interaction between vortices and free drift waves can be rather strong, and, at least in the regions (25) and (29), the drift waves cannot be considered independently from the vortices.

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