Processing Models for Very High Accuracy GPS Positioning

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This paper addresses two aspects of high accuracy GPS positioning: single point positioning and real-time kinematic positioning. In both cases, it is shown that use of the correct models significantly improves the quality of the resulting positions. In the first case, emphasis is placed on physical models, especially the satellite orbit, and in the second case, emphasis is placed on the stochastic model, i.e. understanding the statistics of the GPS observables.

1. INTRODUCTION. That GPS has such an enormously wide range of applications is due partly to the fact that it is able to deliver positions with such a variety of qualities and costs. In fact, the quality of GPS positions can vary by up to five orders of magnitude (from 100 m to almost 1 mm) depending on a wide range of factors including (*inter alia*):

(i) whether positioning is absolute or relative
(ii) the length of the baseline in the case of relative positioning
(iii) the prevailing atmospheric (especially ionospheric) conditions
(iv) whether phase or code observables are being used
(v) whether single or dual frequency receivers are being used
(vi) the sophistication of processing models
(vii) the local environmental conditions (especially multipath)
(viii) the type of antenna being used
(ix) whether static or kinematic data collection techniques are adopted
(x) whether or not results are required in real time.

In general, the highest accuracies (single millimetre) are achieved, and greatest costs incurred, when several dual frequency phase measuring receivers with choke-ring antennas are deployed over small areas of the Earth's surface (just a few kilometres), data is collected over several days and post-processed using sophisticated mathematical models. In contrast to this, the lowest accuracies (around 100 m) can be delivered inexpensively in real time using the simplest hand-held receivers operating completely independently. In between there is an enormous variety of possibilities covering the whole spectrum of possible accuracies. In fact, the simple question 'how accurate is GPS?' is extremely difficult to answer. It is worth noting that modern geodetic GPS receivers can resolve phase very accurately, with a resolution of at least one thousandth of a cycle. Since the wavelengths of the L1 and L2 carriers are 190 mm and 244 mm respectively, this leads to a resolution of the order 0·2 mm. The challenge for GPS

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processing research is to refine the functional and stochastic models to exploit fully this very high potential accuracy.

This paper concentrates on very high accuracy GPS and looks specifically at the computational models that are needed for two specific scenarios where single-centimetre accuracy is required:

(i) high accuracy static point positioning
(ii) short-range real-time kinematic (RTK) positioning.

The first situation arises when it is required to position a single point within a global reference frame (such as WGS84 or an ITRF). This might be required, for instance:

(i) to provide a reference point for a local navigation system (e.g. at an airport or port)
(ii) to provide control for a local survey (e.g. geophysical or topographical) that needs to be linked to other such surveys or to local mapping
(iii) to position offshore (or otherwise remote) structures (such as oil or gas drilling platforms).

The classical way to solve this problem is to deploy several receivers simultaneously, one or more of which is located at an already known point, and to carry out relative positioning. The disadvantages of this approach (from a cost perspective) are the need to purchase or rent more than one high-specification receiver and the need (in some countries) to purchase the coordinates of the known points. There may also, in some countries, be access problems to the known points. The alternative method discussed here is to deploy just one receiver (at the unknown point) and to use publicly available GPS precise positioning products from the International GPS Service for Geodynamics (IGS) with sophisticated processing models such as those used in software packages like GIPSY, BERNESE and GAMIT.

The second problem addressed in this paper is that of RTK GPS. RTK GPS has made enormous strides over the past five years or so and is now a routine tool for many engineering and mapping applications. It essentially relies on prior knowledge of the so-called integer ambiguities for the observed phase differences (so that they can be converted to range differences). This issue is not considered here but is discussed in detail in many places, e.g. Leick (1995) and Kleusberg & Teunissen (1996). Once the ambiguities are known, the quality of positioning depends only on the quality of the GPS phase measurements and the satellite geometry. The former unfortunately varies extremely rapidly in most practical environments due to the interference of reflected GPS signals (multipath). In this paper, the importance of this variation is discussed in detail, and it is shown that its proper understanding can lead to the development of a correct stochastic model which, in turn, not only improves the quality of the positioning but also leads to more correct quality measures.

2. HIGH ACCURACY POINT POSITIONING. As explained in section 1, high accuracy point positioning is achieved by deploying a dual frequency phase measuring receiver at an unknown location and recording all four observables

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(two phase and two code) for periods of several hours to several days. Processing is then carried out using IGS products, but since the IGS is not well known to those outside of the academic and research community, some details are given in the following.

2.1. INTERNATIONAL GPS SERVICE FOR GEODYNAMICS. The International GPS service for Geodynamics (IGS), through a global system of satellite tracking stations, data centres and analysis centres, provides high quality geodetic data products to meet the objectives of a wide range of scientific and engineering applications and studies [IGS, 1997]. Planning of the IGS began in 1990, and it was established by the International Association of Geodesy (IAG) in 1993. Although formal operation did not begin until January 1994, the IGS has been delivering products since June 1992.

2.1.1. *Data products*. The GPS data from approximately 150 permanently operated IGS stations is made available in RINEX format (Receiver INdependent EXchange format) [Gurtner, 1994]. The data sets are used by the IGS to generate the following data products:

(i) high accuracy GPS satellite ephemerides
(ii) coordinates and velocities of the IGS tracking stations
(iii) GPS satellite and tracking station clock information
(iv) earth rotation parameters
(v) ionospheric information
(vi) tropospheric information

The IGS accomplishes its objectives through several key components:

(i) network of tracking stations
(ii) data centres (DCS)
(iii) analysis centres (ACS)
(iv) analysis coordinator
(v) associate analysis centres (AACS), global and regional network
(vi) central bureau (CB)
(vii) governing board (GB)

Data from the IGS network of continuously operating GPS stations (Fig. 1) is collected at IGS data centres (DCS). Currently seven analysis centres (ACS) (see Table 1) independently with different methods, analyse the data and produce IGS

TABLE 1. IGS ANALYSIS CENTRES (ACS)

Analysis centre	ID
Centre for Orbit Determination in Europe, Berne University, Switzerland	COD
National Resource Canada Geodetic Survey, Ottawa, Canada	EMR
European Space Agency	ESA
GeoForchungsZentrum, Potsdam, Germany	GFZ
Jet Propulsion Laboratory, Pasadena, USA	JPL
National Geological Survey, Silver Springs, USA	NGS
Scripps Institute of Oceanography	SIO

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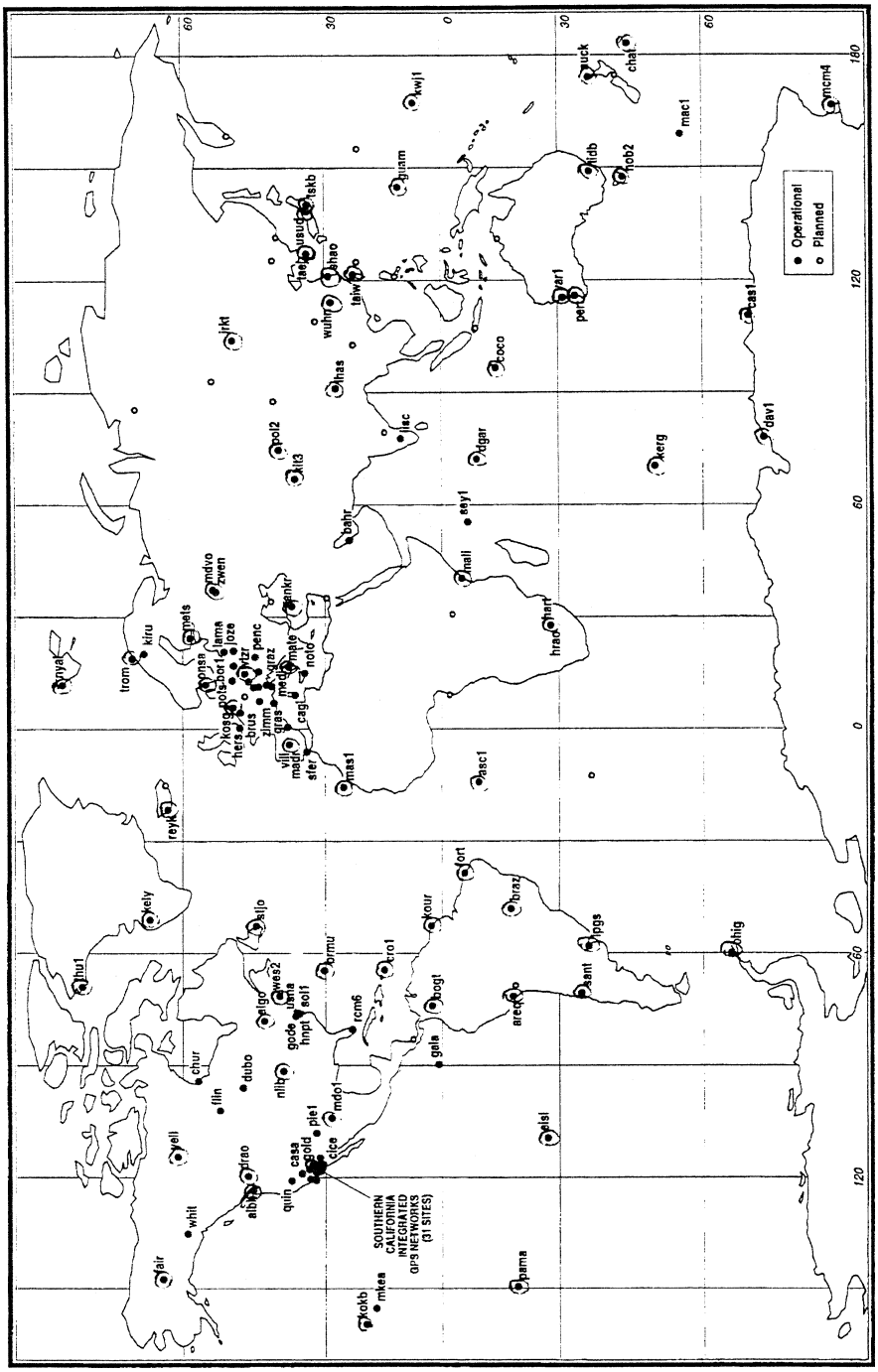


Fig. 1. IGS tracking stations

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Table 2. Accuracy of ephemerides

Ephemeris	Availability	Orbit error	Position error
Broadcast	Real time	5 m	10^{-7}–10^{-6} L
IGS Predicted (IGP)	0·5 h before start of day	1 m	10^{-8}–10^{-7} L
IGS Rapid (IGR)	24 h after obs.	0·3 m	$< 10^{-8}$ L
IGS Precise (IGS)	11 days after obs.	0·10 m	$< 10^{-8}$ L

products. The independent ACs products are compared and combined by the analysis coordinator, producing the improved and unified IGS official orbit. The combined IGS orbit is now available 11 days after the observations are made. There is also a rapid IGS orbit (IGR) available within 24 hours, which agrees with the final orbit to about 20 cm rms, and since March 1997 3-day predicted orbits (IGP), with rms of 50–80 cm 24 hour and 1–2 m for 48 hour predicted, became available. A unique global network coordinate solution is produced in a similar way to the IGS orbit. The global network associate analysis centres (AACs) combine network solutions from the ACs and incorporate regional solutions from the regional network AACs. The Department of Geomatics at Newcastle University is an AAC. To complete the component description the central bureau, located at JPL, is responsible for the day-to-day management and the governing board, consisting of 15 members makes policy decisions.

2.1.2. *Utilizing IGS products.* Using the first three products (orbits, coordinates and clocks), surveyors can position GPS receivers precisely in the reference frame, ITRF (Blewitt, 1996). They can be retrieved from Global IGS data centres from the internet simply using a Web browser *via* File Transfer Protocol (FTP). The most commonly used products are precise ephemerides, IGS station data and ITRF station coordinates.

(i) *Precise ephemerides*

The error in relative coordinates ϵ, between two stations separated by distance L can be expressed as:

$$\epsilon_r / L = k\epsilon_s / A \qquad (1)$$

where ϵ_s is the orbit error, A is the orbit altitude, k is a number between 0·2 and 1 (depending on DOP, estimation strategy, orbit improvement).

The table 2 gives the approximate ppm error in the distance L for different types of ephemerides.

If broadcast ephemerides from the navigation message are used, sub-centimetre geodetic accuracy is limited to distances of 10–20 km. The use of the IGS precise orbits allows relative positioning accuracy to a few millimetres over distances spanning hundreds of kilometres.

(ii) *IGS tracking station data and ITRF coordinates*

The GPS data at IGS stations consists of dual frequency carrier phase and pseudo range observations recorded at a 30 second interval. Herstmonceux (HERS), in East Sussex, is currently the only IGS station in Britain; but a permanent GPS site at Morpeth (MORP), operated by the Department of Geomatics at Newcastle University, has been proposed as a new IGS station. The ITRF coordinates of IGS

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stations have errors at the level of 1 cm or better at any time, with velocities to account for plate tectonic motion. Using high precision orbits, IGS station data and precise ITRF coordinates allows relative positioning accuracy as high as 1 part in 10^8 (Blewitt, 1996).

2.1.3. *Research activities of the IGS.* The accuracies of the products are sufficient to support current scientific objectives [IGS, 1997]. They include:

(i) improvement of the International Terrestrial Reference Frame (ITRF)
(ii) monitoring deformations of the solid earth
(iii) monitoring variations in the liquid earth (sea level, ice-sheets, etc.)
(iv) monitoring earth rotation
(v) scientific satellite orbit determinations
(vi) ionospheric monitoring
(vii) climatological research (weather prediction)

Further information can be found on the IGS Web site at http://igscb.jpl.nasa.gov/

2.2. POINT POSITIONING USING IGS PRODUCTS. The technique of precise point positioning has been developed at JPL and requires only a single receiver and no ground control (Zumberge *et al* 1997). The methodology behind the relatively new strategy is to estimate the orbits and satellite clocks once, in a high-quality global fiducial network, and to use the information to estimate station parameters (position, clock and wet troposphere) anywhere in the world (Gregorius, 1996). Through analysis at JPL, fixed orbits, satellite clocks and time, polar motion and Earth orientation estimates as well as eclipse information are used in the approach. The accuracy of station coordinate estimates are comparable to a global network solution (few mm to cm) using 24 hours of data. With a few hours of data, precise point position produces absolute ITRF coordinates with an accuracy of 1–2 cm [Blewitt, 1996]. One processing run on the technique only takes a couple of minutes using the GIPSY high precision processing software. The only limitations of the technique are that the position estimates of the station may not indicate the actual quality of the results [Zumberge et. al 1997]. This is due to the fact that station correlations cannot be accounted for and orbits are assumed to be perfect.

Investigations to see the accuracy of the precise point positioning using just 2 hours of data have been conducted by Gregorius (1997). First, a precise point position solution using Hartebeesthoek IGS station in South Africa was computed using 24 hours of data (accurate to 1–2 cm). Six precise point position solutions were then computed using 2-hour slots spread across the 24-hour period and compared with the 24-hour solution. Table 3 gives the average repeatability obtained in latitude, longitude and height.

Table 3. Repeatability of precise point positioning using 2 hours of data

Latitude	4·1 cm
Longitude	4·2 cm
Height	16·9 cm

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Clearly only decimetre-level accuracy can be achieved with 2 hours of data using the precise point positioning technique, although much greater accuracy is possible with a longer period.

3. STOCHASTIC MODELLING FOR DOUBLE DIFFERENCE CARRIER PHASE GPS. In standard least squares theory, we require a functional model and a stochastic model for the observations. The functional model tells us how we compute the observations while the stochastic model describes the statistics of the expected errors. There are essentially two equivalent approaches in the modelling of observation errors, the inclusion of additional functional model parameters or modification of the stochastic model (Blewitt, 1997). Therefore the stochastic model is dependent on the choice of functional model; hence any errors not included in the functional model should be described stochastically. A correct stochastic model is the key to obtaining meaningful quality measures and also improving the parameter estimates, as this paper will show. GPS carrier phase double difference observations are contaminated by several kinds of errors and biases; namely, residual orbital errors, residual ionospheric and tropospheric delays, multipath error and system noise. Due to the complex nature of these errors, rigorous modelling, in general, cannot be achieved. The remaining residual error components must be modelled stochastically through the least squares stochastic model (the observation covariance matrix). The variances of the stochastic model describe the statistical quality while the covariances describe the correlation of the observations. Two types of correlation are present in GPS double difference carrier phase measurements, mathematical and physical.

3.1. MATHEMATICAL CORRELATION. Differencing the carrier phase observation causes mathematical correlation to be introduced. This type of correlation can be easily accounted for using Gauss's law of error propagation. The carrier phase double difference observation for one epoch may be written as (Hofmann-Wellenhof et al., 1993)

$$\Delta\nabla \quad = a \tag{2}$$

where $\Delta\nabla$ represents the double difference operator, a is a differencing operator matrix and is the vector of carrier phase observations. Applying Gauss's law of error propagation to (2) yields the covariance matrix of the carrier phase double difference observations M

$$\mathrm{M} = a\mathrm{C} \quad a^{T} \tag{3}$$

where C is the covariance matrix of the undifferenced observations. In the simplest case all the observations are assumed to be independent and have the same precision; C thus takes the form of an identity matrix multiplied by a scale factor.

3.2. PHYSICAL CORRELATION. The measurement environment associated with the four GPS observations in a double difference is similar to a certain degree. It is therefore valid to expect that the residual errors show a certain degree of temporal and/or spatial correlation. Physical correlation is the term used to describe both temporal and spatial correlation, as shown in Fig. 2. Physical correlation between double difference observations is difficult to

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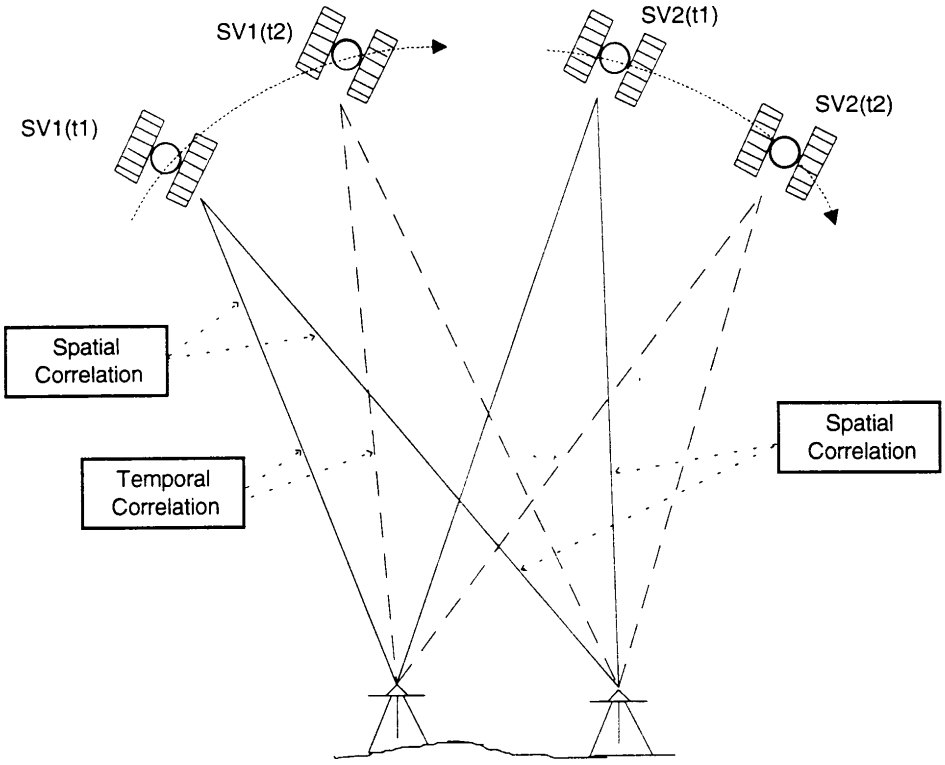


Fig. 2. Physical correlation in GPS double differencing

estimate and usually neglected (Hofmann-Wellenhof *et al.*, 1993). However, a complete knowledge of physical correlations within GPS double difference measurements is essential in order to derive a correct stochastic model.

3.3. STOCHASTIC MODELLING OF MULTIPATH. One of the main limiting factors in short baseline kinematic or short occupation GPS is multipath. An investigation conducted by the authors shows that multipath can be modelled stochastically giving improved solution and quality estimates. In order to obtain the correct stochastic properties of double difference carrier phase GPS, we need to know the 'true' position to enable the computation of 'true' errors in the observations. The 'true' position will never be known, but a position with accuracy two orders of magnitude greater will be sufficient for a truth. For instance, phase solutions can be used to benchmark DGPS [Roberts et al., 1997]. However, a truth for phase GPS is difficult to obtain specially in a kinematic environment. Therefore the aim of the investigation was to determine a correct stochastic model for a static single epoch fixed ambiguity L1 phase solution, utilizing the GASP software (Corbett & Cross, 1995).

3.3.1. *Description of the data set.* Static data was collected on the roof of the Bedson building, Newcastle University for a period of 3 hours with an elevation angle of zero degrees and a recording interval of 1 Hz. The receivers used were Trimble 4000 SSE's and the baseline length was approximately 10 m. Previous

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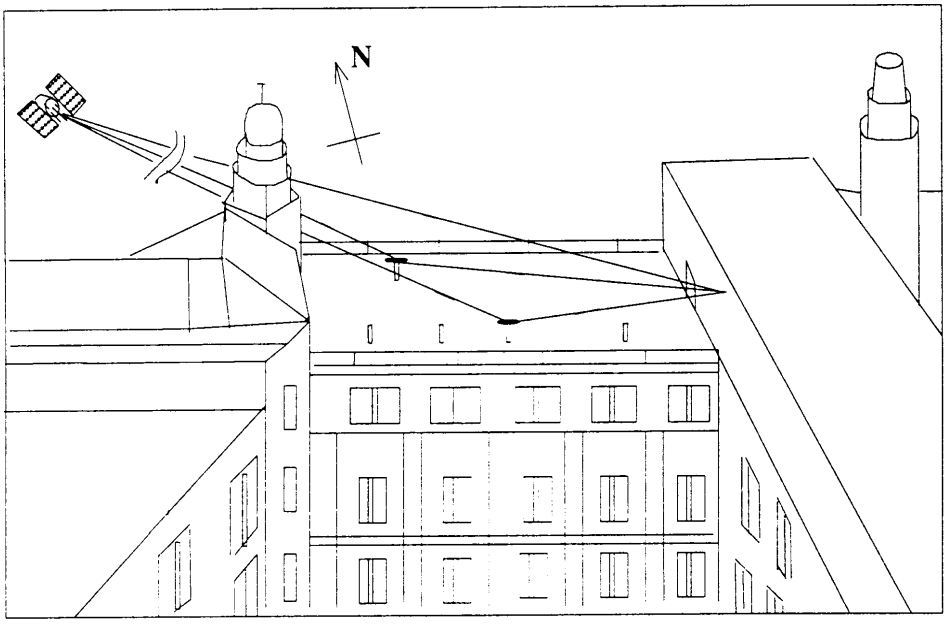


Fig. 3. Possible Multipath source on the roof of the Bedson Building, Newcastle University

data collected on the roof has shown significant multipath and Fig. 3 illustrates possible multipath sources from the many reflective surfaces.

3.2.2. *Determination of 'true' double difference errors.* A 1-hour period was selected for analysis from the 3-hour data period. The coordinates of the roof pillars, one a SCINET point, have been well determined by many previous campaigns. Using these coordinates as truth, the 'true' double difference can be computed since the satellite coordinates are known, and thus the 'true' double difference observation errors can be found. The errors in the double difference observations will be dominated by multipath and also receiver noise because over such short distances tropospheric and ionospheric errors cancel. It should be noted that the true errors in the double difference carrier phase GPS cannot usually be computed in this way. Fig. 4 shows two satellite L1 phase double difference residual (negated error) time series. The approximate elevation path of SV4 is 20-3 degrees, SV6 50–80 degrees and the differencing SV20 90–60 degrees. Two observations can be noted:

(i) Non random series – the errors are heavily contaminated by multipath with characteristic low frequency components.
(ii) Elevation dependency – lower elevation satellites tend to have greater multipath and noise.

3.3.3. *Determination of variances and covariances.* Using the true carrier phase double difference errors, variances and covariances were computed for all satellites using an eleven second data window centered on the current epoch.

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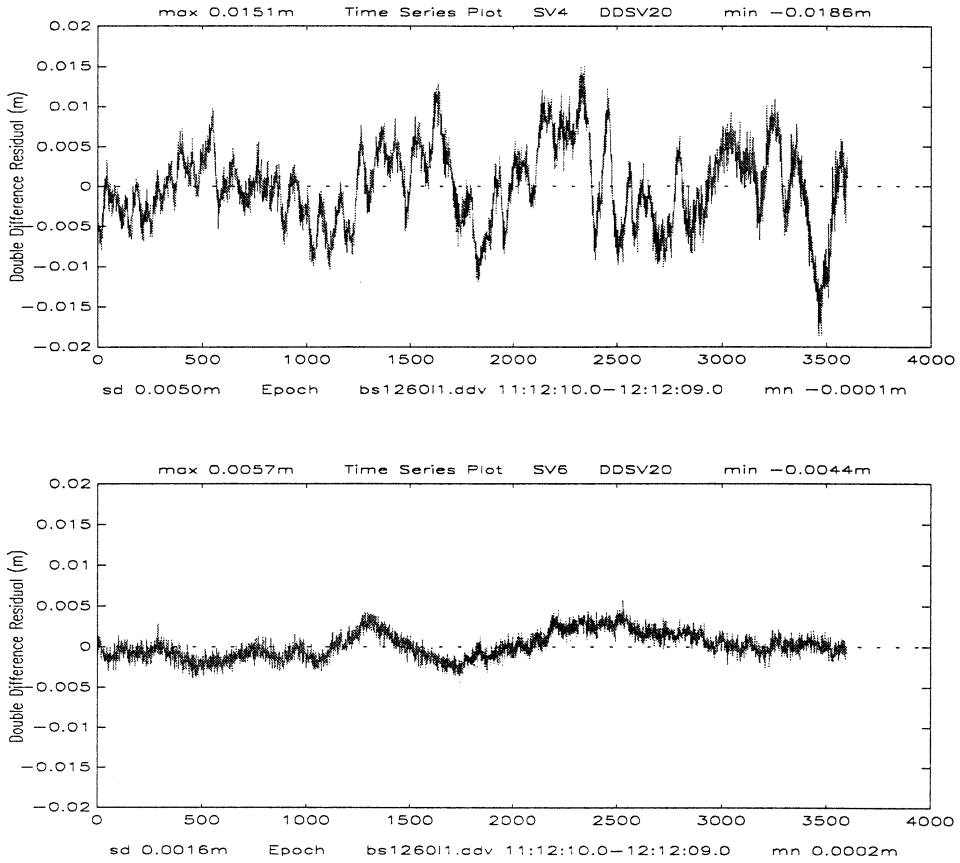


Fig. 4. 'True' L1 double difference residuals for SV4 (20–23 deg.) and SV6 (50–80 deg.)

3.3.4. *Candidate stochastic models*. Using the GASP software, different candidate stochastic models were used to see the effect on the quality of the position solution and associated quality estimates. These stochastic models were:

(i) An identity stochastic model scaled so that all phase observations had a standard deviation of 1·73 mm.
(ii) A cosine elevation dependent diagonal stochastic model with 1 mm as the standard deviation for an observation at the zenith.
(iii) A full correct stochastic model, which accounts for the spatial correlations populated from variance/covariance analysis of the 'true' errors.

For the different stochastic model scenarios above, a single epoch L1 fixed phase solution was computed for the hour-long GPS data set with the elevation mask set to zero degrees. The results now follow. Note, no unit variance scaling was carried out.

3.3.5. *GASP solution using a scaled identity stochastic model*. Fig. 5 shows time series plots of the 3D position error vector, the 95 percent 3D error eclipse (in

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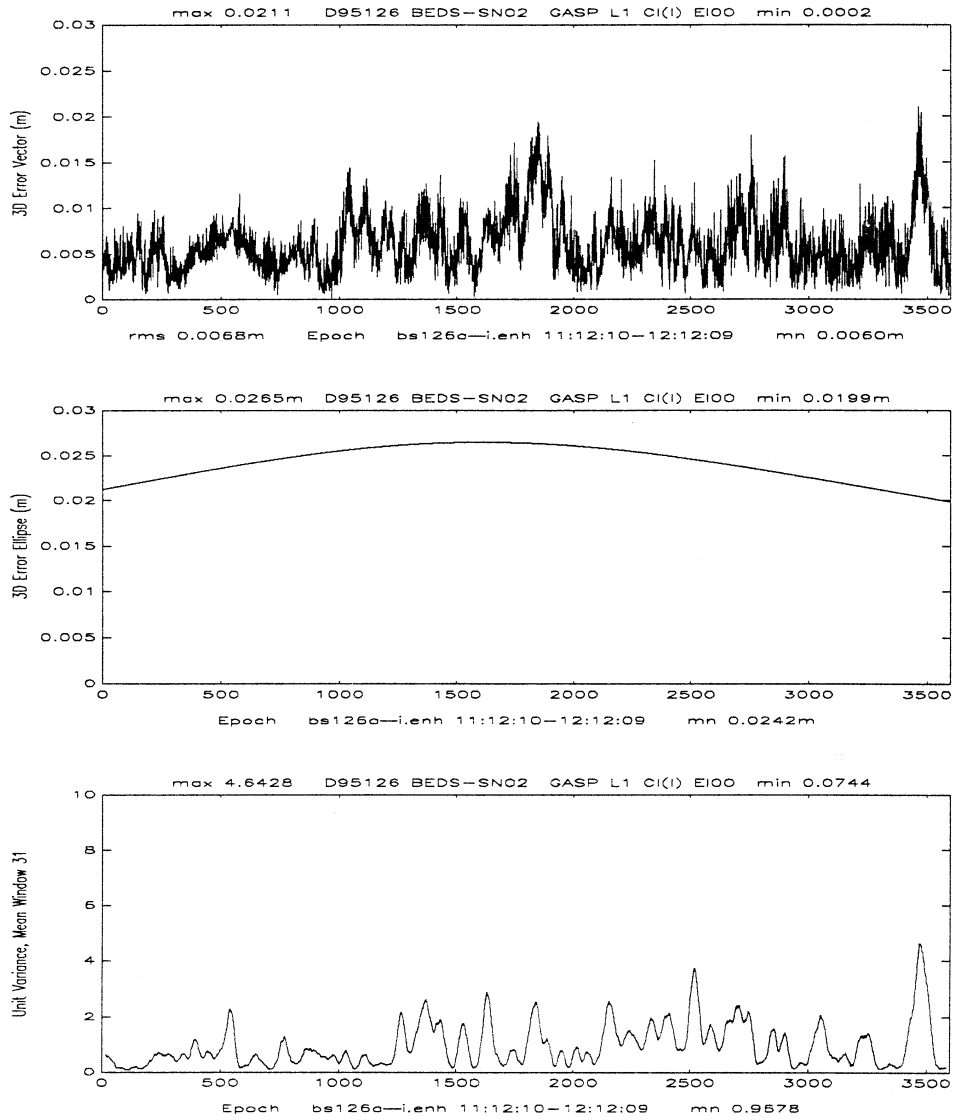


Fig. 5. GASP results for a scaled identity stochastic model

the maximum direction) and the mean unit variance (average over a 31-second window).

Commentary. The overall 3D position error rms is 6·8 mm and the time series is clearly contaminated by multipath with positional errors as large as 2·1 cm. The 95 percentage error ellipse does not follow trends in the 3D positional error time series and is largely dependent on satellite geometry. The average unit variance (using a moving window of 31) oscillate wildly, but the mean of the entire series is close to unity.

3.3.6. *GASP solution using an elevation dependent stochastic model*. Fig. 6 shows time series plots of the 3D position error vector, the 95 percent 3D error ellipse

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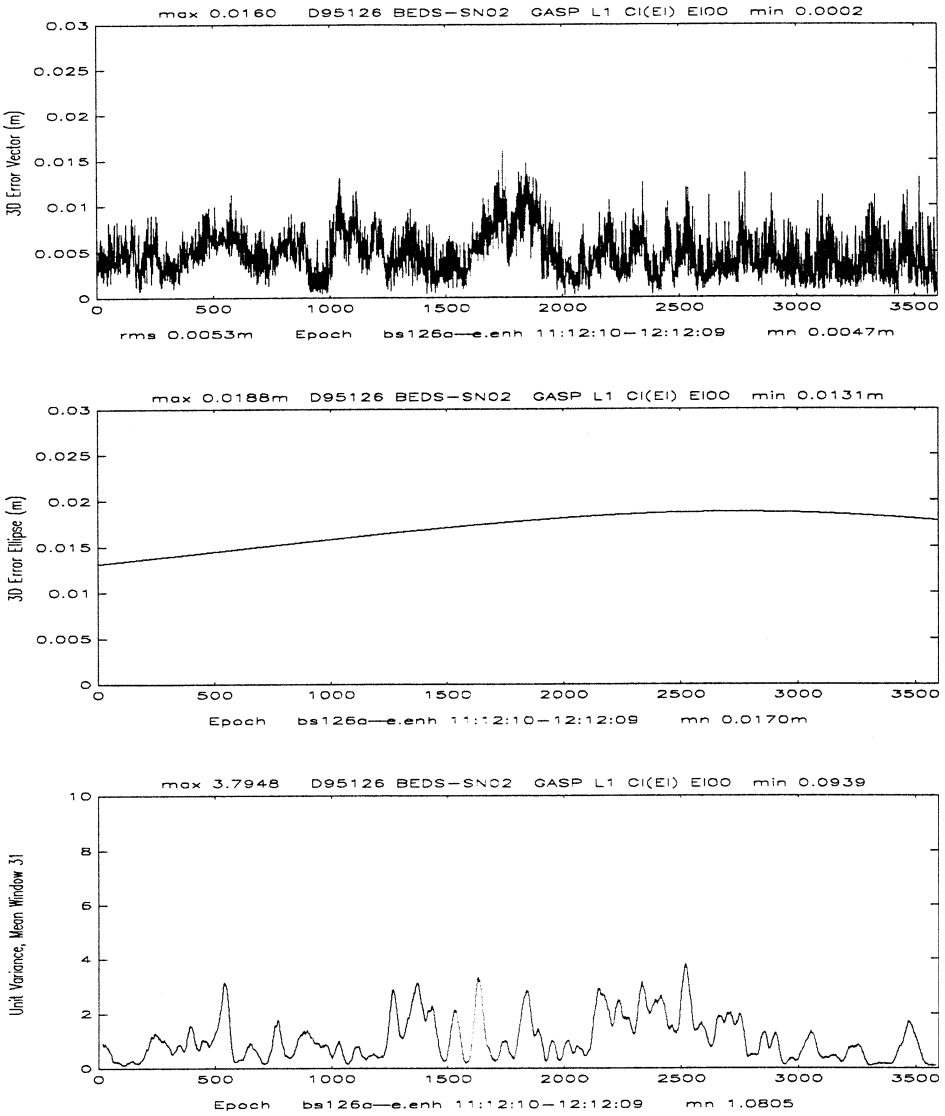


Fig. 6. GASP results for an elevation dependent stochastic model

(in the maximum direction) and the mean unit variance (average over a 31-second window).

Commentary. The overall 3D position error rms is 5·3 mm; approximately 20 percent improvement over the solution in section 3.3.5, and maximum positional error is now 1·6 cm. However, the solution still shows non-random patterns, characteristic of multipath error. The 95 percent error ellipse like section 3.3.5 is largely geometry-dependent and does not follow small trends in the series. The average unit variance (using a moving window of 31) oscillates wildly, but the mean of the entire series is close to unity.

3.3.7. *GASP solution using a correct full stochastic model*. Fig. 7 shows time series

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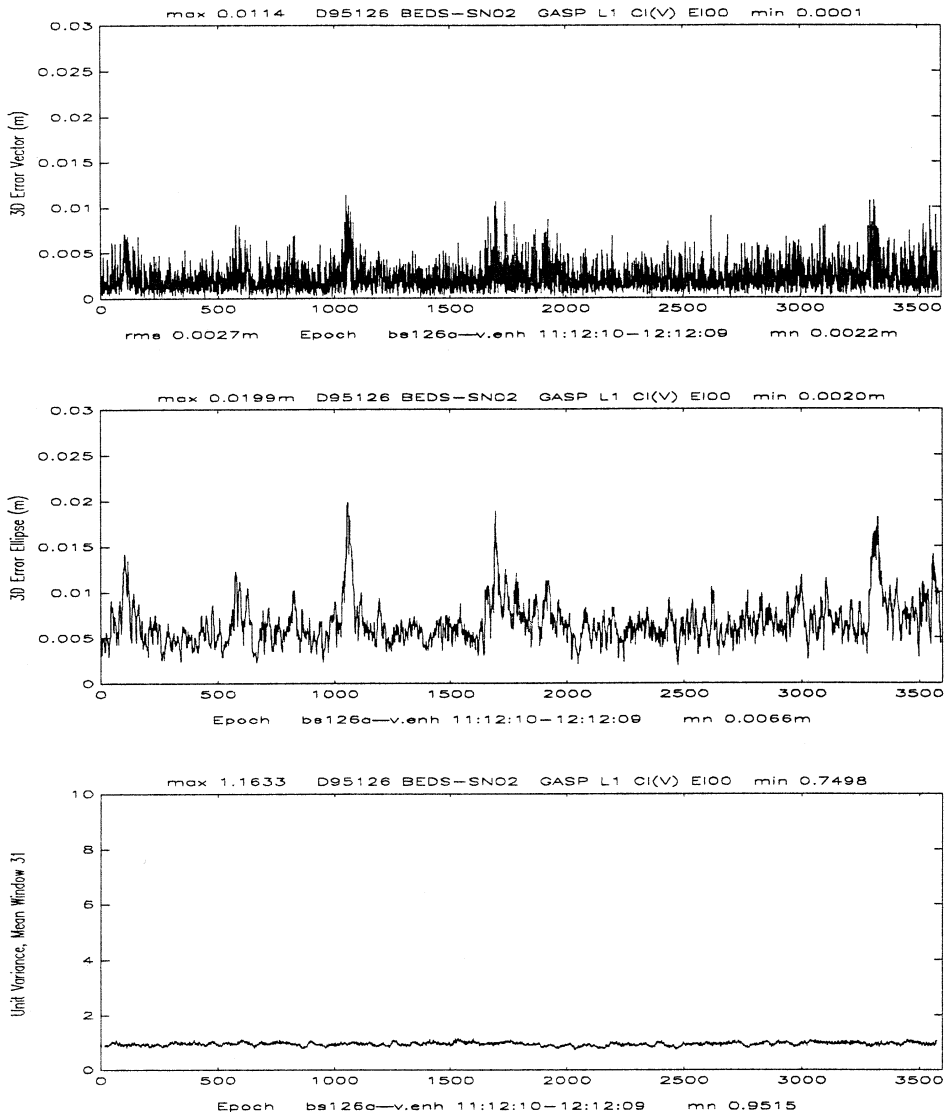


Fig. 7. GASP results for a full correct stochastic model

plots of the 3D position error vector, the 95 percent 3D error ellipse (in the maximum direction) and the mean unit variance (average over a 31-second window).

Commentary. The overall 3D position error rms is 2·7 mm; almost a 50 percent improvement in the 3D position rms of section 3.3.6, and the maximum positional error is reduced to 1·1 cm. The series exhibits more random behaviour with occasional increases in noise. The 95 percent error ellipse follows trends in the 3D error vector and gives a true indication of position quality. The values do appear to be slightly larger than expected and an indication that the stochastic model is slightly pessimistic. The average unit variance (using a moving window

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of 31) no longer oscillates wildly as in sections 3.3.5 and 3.3.6 but is steady around unity. The overall average unit variance is less than unity and again indicates a slightly pessimistic stochastic model.

3.4. CONCLUSIONS. The investigation has demonstrated the power of the stochastic model for the improvement of solution estimates and quality measures. The research challenge is now to find indicators of phase quality and correlations for carrier phase data in real time, enabling improved RTK solutions and quality measures.

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