Macroeconomic Dynamics, **16** (Supplement 2), 2012, 267–283. Printed in the United States of America.
doi:10.1017/S1365100511000435

PERSISTENT DEFICIT, GROWTH, AND INDETERMINACY

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In this paper, we look for long-run and short-run effects of fiscal deficits on economic growth in an endogenous growth model with productive public spending that may be financed by public deficit and debt. The model shows a multiplicity of long-run balanced growth paths (a high-growth and a low-growth steady state) and a possible indeterminacy of the transition path, which may be consistent with the empirical literature, which exhibits strong nonlinear responses of economic growth to fiscal deficits. Starting from the high-growth steady state, a positive impulse in the deficit ratio exerts an adverse effect on economic growth in the long run, after an initial rise. Starting from the low-growth steady state, the situation may be radically undetermined, and the effect of fiscal deficit impulses is subjected to "optimistic" or "pessimistic" views on public-debt sustainability.

Keywords: Deficit, Productive Government Spending, Endogenous Growth, Nonlinear Effects of Fiscal Policy, Public Debt

1. INTRODUCTION

Since the mid-seventies, OECD countries have been characterized by persistent deficits, with an average deficit ratio of about 2.5% from 1970 to 2009. Such persistent deficits have probably affected the balanced growth path (hereafter BGP) of these countries, but there is no unanimous theoretical or empirical answer to this question. Effectively, a higher deficit today means a higher debt interest burden tomorrow, with a possible crowding-out effect on the BGP. But a higher deficit today may also provide resources for productive public expenditures, which may improve the BGP. In discussing the fiscal rules set in the Stability and

We are indebted to the Editors (W. Barnett and F. Jawadi) and to two anonymous referees for very valuable comments on a previous version of the paper. We are grateful to A. d'Autume, J. Bullard, J.-B. Chatelain, T. Faehn, J. Feigenbaum, S. Ghatak, A. Greiner, A. Kirman, M. Kumhof, K. Lansing, X. Guo, J. Weimark, and participants to conferences in ISCEF (Sousse), SNDE (Atlanta), LAGV (Marseille), IIPF (Warwick), FEAC (Paris), SCE (Montreal), CEA (Halifax), PET (Nashville) and SES (Perth). We thank FERDI (the Fondation pour les Etudes et Recherches sur le Développement International) for financial support. Usual disclaimers apply. Address correspondence to: Alexandru Minea, CERDI (University of Auvergne), 65 Boulevard François Mitterrand, B.P. 320, 63009 Clermont-Ferrand Cedex 1, France; e-mail: alexandru.minea@u-clermont1.fr.

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https://doi.org/10.1017/S1365100511000435 Published online by Cambridge University Press

Growth Pact, for example, some authors[1] have pointed out the risk that these rules may permanently reduce public investment and the rate of economic growth, and have suggested that the public debt may be used as a way of financing public infrastructure. Furthermore, an important econometric literature highlights the existence of strong nonlinearities between fiscal impulses and economic growth [see, for example, Giavazzi et al. (1999)], particularly in a context of high public-debt ratios [Perotti (1999)].

To study the long-run implications of persistent deficits in a model in which public infrastructure may be debt-financed, we extend the Barro (1990) endogenous growth model by allowing public deficits as a method of government finance for productive public spending. To our knowledge, only a few papers deal with persistent deficits in growth models. Futagami et al. (2008) construct an endogenous growth model with public debt, but are interested in debt rules, and not in persistent deficits. Greiner and Semmler (2000), Minea and Villieu (2009, 2010) and Groneck (2010) deal with deficit rules close to those in to the present paper, but with no reference to any form of nonlinearity or indeterminacy.

In our paper, persistent deficits are introduced *via* a "deficit rule," namely a deficit-to-output ratio m. Our results are as follows. First, we show that a "Barro-type" balanced-budget rule ($m = 0$) generates two BGPs: the "Barro solution," with zero public debt, and a "Solow solution" [Solow (1956)], with zero growth. Allowing deficits ($m > 0$) also yields two BGPs, a high-growth BGP and a low-growth BGP. Second, as regards transitional dynamics, the multiplicity[2] of BGPs may generate nonlinear effects of fiscal impulses. We show that the low-growth BGP is unstable and that the high-growth BGP is a stable saddlepoint.

If we are only concerned with the stable BGP, raising deficits is always growth-reducing in the long run, as in Minea and Villieu (2009, 2010). The reason is straightforward: If Ponzi finance is forbidden, governments are not allowed to finance the interest payment on debt by issuing new debt (deficit). Therefore, deficit rules such as $m > 0$ always generate a higher interest burden than the resources they provide, with an associated crowding-out effect on productive expenditures and growth in the steady state.

However, if we are interested in the multiplicity of BGPs and transition paths, this result must be amended. Our model shows that, starting from the (unstable) low-growth BGP, long-run economic growth may respond to fiscal deficit impulses positively or negatively, depending on the initial level of the public debt. Consequently, the effect of a fiscal deficit impulse on long-run economic growth is nonlinear. This finding somewhat contradicts the results in Minea and Villieu (2009), based on the properties of stable BGPs. Furthermore, our model produces indeterminacy in the neighborhood of the low-growth BGP: depending on the initial public-debt ratio, the transition path may be subjected to "optimistic" or "pessimistic" views about debt sustainability.

Section 2 presents the model. Section 3 deals with steady-state issues and discusses the effect of fiscal deficits in the long run. Section 4 describes the

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transitional dynamics of the model, Section 5 depicts the nonlinear character of fiscal deficit impulses, and Section 6 concludes the paper.

2. THE MODEL

We consider a closed economy with a private sector and a government. The private sector consists of a producer–consumer infinitely lived representative agent, who maximizes the present value of a discounted sum of instantaneous utility functions based on consumption $c_t > 0$, with $\rho > 0$ the discount rate and $S \equiv -u_{cc}c_t/u_c > 0$ (with $u_c \equiv du(c_t)/dc_t$) the consumption elasticity of substitution:

$$U = \int_0^\infty u\,(c_t) \exp\,(-\rho t) dt, \quad \text{with } u\,(c_t) = \begin{cases} \frac{S}{S-1}\left((c_t)^{\frac{S-1}{S}} - 1\right), & for \;\; S \neq 1 \\ Log\,(c_t)\,, & for \;\; S = 1 \end{cases}. \tag{1}$$

For the intertemporal utility U to be bounded, we also have to ensure that $(S-1)\gamma_c < S\rho$, with γ_x the long-run growth rate of variable x.[3]

As a producer, the representative agent generates per capita output y_t using per capita private capital k_t and per capita productive public expenditures g_t. Population is normalized to unity and $0 < \alpha < 1$ is the elasticity of output to private capital:

$$y_t = k_t^\alpha g_t^{1-\alpha}. \tag{2}$$

Public expenditures g_t enter into the production function as a flow and no congestion is present, so that (2) is comparable to the production function of Barro (1990).

The representative agent budget constraint is

$$\dot{k}_t + \dot{b}_t = r_t b_t + (1-\tau)\, y_t - c_t - \delta k_t, \tag{3}$$

where we define $\dot{x}_t \equiv dx_t/dt, \forall x_t$.

Households use their income (y_t) to consume (c_t); to invest $(\dot{k}_t + \delta k_t)$, with δ the rate of private capital depreciation; and to pay taxes (we assume a flat tax rate on output τ). Households can buy government bonds (b_t), which return the real interest rate r_t. In equilibrium, r_t equals the marginal productivity of private capital.

The government taxes output, provides productive public expenditures, and borrows from households whenever current spending exceeds current income, in which case it will have to pay interest on the issued debt:

$$\dot{b}_t = r_t b_t + g_t - \tau y_t. \tag{4}$$

Notice that (4) is an extension of the Barro (1990) government constraint $g_t = \tau y_t$. In our model, by using deficits, the government can make productive

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expenditures eventually higher than fiscal revenues τy_t. However, public deficits generate unproductive public expenditures, in the form of interest payments on debt, which may crowd out productive expenditures in the government budget constraint.

The maximization of (1) subject to (2)–(4), given k_0 and the transversality condition $\lim_{t\to\infty}(\exp(-\int_0^t r_s ds)(k_t + b_t)) = 0$, gives rise to the familiar Keynes–Ramsey relation

$$\gamma_c \equiv \frac{\dot{c}}{c} = S\left[\alpha(1-\tau)\left(\frac{g}{k}\right)^{1-\alpha} - \delta - \rho\right], \tag{5}$$

which describes the rate of growth of consumption (time indices will be henceforth omitted).

Next, we write (3) and (4) in a more convenient form by displaying the rate of growth of private capital (the *IS* equilibrium) and of public debt:

$$\gamma_k \equiv \dot{k}/k = (g/k)^{1-\alpha} - c/k - g/k - \delta, \tag{6}$$

$$\gamma_b \equiv \dot{b}/b = \alpha\,(1-\tau)\,(g/k)^{1-\alpha} - \delta - \tau\frac{(g/k)^{1-\alpha}}{b/k} + \frac{g/k}{b/k}. \tag{7}$$

Relations (5)–(7) form a system with three equations in four variables; thus a free variable remains in the government budget constraint (7). To reproduce persistent deficits, we specify a deficit rule, namely that the ratio of deficit to output is equal to some value m:

$$\frac{\dot{b}}{y} = m \geqslant 0. \tag{8}$$

To obtain a BGP, the ratio g/k has to be constant in the long run. This is the case in Barro (1990), because the balanced-budget hypothesis ($g = \tau y$) ensures a constant g/k ratio in the steady state. Here, we proceed in a similar way, except that the g/k ratio will depend on τ *and* m in the long run.[4] Notice that a constant deficit rule such as $m \geqslant 0$ (for example, $m = 2.5\%$) is consistent with OECD data in the long run, whereas the balanced-budget rule ($m = 0$) generally postulated in the literature would not be.

Using the debt evolution rule (4), we can write (8) in a more convenient way:

$$g/k = (m+\tau)\,(g/k)^{1-\alpha} - \left[\alpha\,(1-\tau)\,(g/k)^{1-\alpha} - \delta\right](b/k). \tag{9}$$

To search for endogenous growth solutions we define the three modified variables $c_k \equiv c/k$, $g_k \equiv g/k$, and $b_k \equiv b/k$. Equations (5)–(7) and (9)

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become

$$\begin{cases} \text{(a)} \quad \dfrac{\dot{c}_k}{c_k} = \dfrac{\dot{c}}{c} - \dfrac{\dot{k}}{k} = S\left[\alpha(1-\tau)\left(g_k\left(b_k\right)\right)^{1-\alpha} - \delta - \rho\right] \\ \qquad\qquad +g_k\left(b_k\right) + c_k + \delta - \left(g_k\left(b_k\right)\right)^{1-\alpha}, \\ \text{(b)} \quad \dfrac{\dot{b}_k}{b_k} = \dfrac{\dot{b}}{b} - \dfrac{\dot{k}}{k} = \dfrac{m\left(g_k\left(b_k\right)\right)^{1-\alpha}}{b_k} + g_k\left(b_k\right) + c_k + \delta - \left(g_k\left(b_k\right)\right)^{1-\alpha}, \\ \text{(c)} \quad g_k = (m+\tau)g_k^{1-\alpha} - rb_k, \quad with\ r = \alpha(1-\tau)g_k^{1-\alpha} - \delta. \end{cases} \tag{10}$$

3. FISCAL DEFICITS IN THE LONG RUN

We find the BGPs by imposing $\dot{c}_k = \dot{b}_k = 0$ in system (10). We obtain constant values of c_k, b_k, and g_k, which means that the four initial variables (c, k, g, and b) grow at the same constant rate γ. Putting $\dot{c}_k = \dot{b}_k = 0$, we eliminate c_k by extracting (10b) from (10a). Reintroducing the obtained expression of b_k into (10c) provides an implicit expression in g_k (note that γ is an increasing function of g_k in (5)), with parameters S,α, δ, ρ, τ, and m:

$$F\left(g_k\right) = \gamma\left(g_k\right)\left((m+\tau) - g_k^{\alpha}\right) - m\left(\frac{\gamma\left(g_k\right)}{S} + \rho\right) = 0. \tag{11}$$

Because $\gamma = \gamma_c$ in steady state, we can write (11) as a system:

$$\begin{cases} \text{(a)} \quad \gamma^1\left(g_k\right) = S\left[\alpha(1-\tau)g_k^{1-\alpha} - \delta - \rho\right], \\ \text{(b)} \quad \gamma^2\left(g_k\right) = m\rho\left[m\left(\frac{S-1}{S}\right) + \tau - g_k^{\alpha}\right]^{-1}, \quad if\ g_k^{\alpha} \neq m\left(\frac{S-1}{S}\right) + \tau. \end{cases} \tag{12}$$

PROPOSITION 1 [Multiplicity of BGPs].

(A) *If $m = 0$, the system (12) has two solutions. The first is the "Solow solution" corresponding to a zero long-run growth rate, and the second is the "Barro solution."*

(B) *If $m > 0$, one can find a set of admissible parameters $S, \alpha, \delta, \rho, \tau, m$ such that the system (12) has two solutions leading to positive long-run growth rates.*

Proof. See Figure 1. ■

Figure 1 describes the steady-state rates of economic growth for $S = 1$ (results are trivially unchanged for $S \neq 1$). The $\gamma^2(g_k)$ curve is a hyperbola, with asymptote $g_k = \tau^{1/\alpha}$. The $\gamma^2(g_k)$ curve is a concave increasing curve. The steady-state solutions are defined as the points of intersection between the two curves $\gamma^1(g_k)$ and $\gamma^2(g_k)$.

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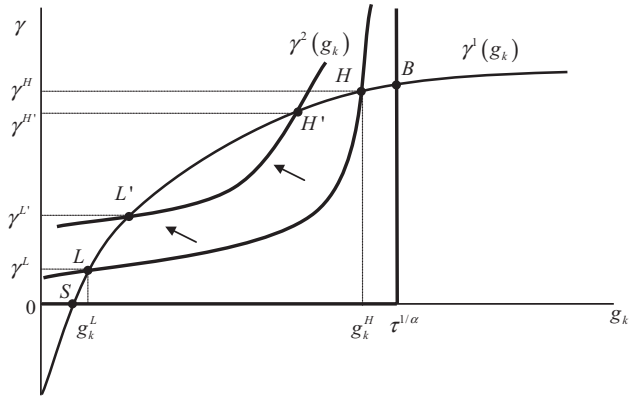


FIGURE 1. Multiplicity of BGPs.

3.1. The (A) Case ($m = 0$)

A balanced-budget rule means that public debt is constant ($\dot{b} = 0$). Notice that this case does not perfectly match the Barro (1990) model. The difference is that whereas in Barro (1990) public debt is *always* at zero, in our model, public debt may be constant and *positive* in steady state, even with a balanced-budget rule ($rb + g = \tau y$). This slight change has important consequences, because two BGPs emerge.

Putting $m = 0$ eventually makes the γ^2 curve (12b) undetermined,[5] but we easily find the two solutions from (11). The first solution is the Barro (1990) one, for $g_k^B = \tau^{1/\alpha}$: $\gamma^B = S[\alpha(1-\tau)(g_k^B)^{1-\alpha} - \delta - \rho] > 0$. This solution is the B point in Fig. 1 and corresponds to a zero stock of public debt in the steady state ($b_k^B = 0$). The second solution corresponds to a zero-growth BGP $\gamma^S = 0$, with

$$g_k^S = \left(\frac{\delta + \rho}{\alpha\,(1-\tau)}\right)^{\frac{1}{1-\alpha}}.$$

We call this steady state the "Solow solution" (S point of Fig. 1), which is reached for a positive stock of public debt (from (10c)):

$$b_k^s = \frac{\left(g_k^s\right)^{1-\alpha}}{\rho}\left[\tau - \left(g_k^s\right)^{\alpha}\right] > 0.$$

This second solution comes from the fact that the constant level of public debt forces private capital to be constant in the long run to achieve a constant steady-state ratio b_k^S. Thus, economic growth disappears in the long run ($\gamma^S = 0$).

The intuition for this no-growth solution is that, contrary to Barro (1990), *not* all public expenditures are productive in the steady state, if public debt is *positive*. Interest payments on debt generate unproductive public expenditures and crowd out productive expenditures. Because, in the steady state, tax revenues

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must finance productive expenditures plus the interest payments on public debt, b_k^S corresponds to a level of debt such that interest payments absorb a great part of government resources: remaining resources for productive expenditures cannot generate a positive rate of economic growth, which explains why a "Solow solution" emerges in our model.

3.2. The (B) Case ($m > 0$)

With persistent deficits, three cases may occur. The most interesting of them, illustrated in Fig. 1, exhibits two points of intersection between γ^1 and γ^2, namely L and H, both leading to positive long-run growth rates (further called the *low* and the *high* growth rates, γ^L and γ^H). By changing parameters, one can obtain a unique solution for the system (12), or even no solution, when γ^1 passes below γ^2.

Let us try an intuitive interpretation of the (B) case (when multiplicity occurs), compared to the (A) case. With a deficit rule ($m > 0$), public debt expands at a positive stationary growth rate ($\dot{b}/b = my/b$), forcing the long-run growth rate of the economy to move away from the Solow solution (the low steady-state solution moves from γ^S to γ^L). Similarly, introducing deficit generates a positive level of public debt in the long run, which requires interest payments. Therefore the Barro solution can no longer be reached. In this case, the high steady-state solution goes from γ^B to γ^H.

Let us now study comparative statics in response to a change in the deficit ratio.

PROPOSITION 2 [Effect of an Increase in the Deficit Ratio (m)].

(a) *Any increase in the deficit-to-GDP ratio, ceteris paribus, lowers the high-growth (stable) BGP and raises the low-growth (unstable) BGP.*
(b) *It is impossible for the long-run growth rate to exceed the Barro solution, obtained with a balanced-budget rule (and no public debt in steady state), which is the highest long-run rate of economic growth.*

Proof. From (12b):

$$\left.\frac{d\gamma^2}{dm}\right|_{g_k \text{ given}} = \frac{\rho\left(\tau - g_k^\alpha\right)}{\left[m\left(\frac{S-1}{S}\right) + \tau - g_k^\alpha\right]^2} \geqslant 0,$$

if $m \geqslant 0$. Thus, an increase in the deficit ratio moves the γ^2 curve toward the left. Because γ and g_k are positively linked in the γ^1 curve, we have $\gamma^{H'} < \gamma^H$ and $\gamma^{L'} > \gamma^L$. Notice in particular that $g_k^H < g_k^B = \tau^{1/\alpha}$. ■

Proposition 2a shows that, a priori, no statement can be made about the effect of a deficit impulse, because of multiplicity. However, as we shall see in Section 4, the low-growth BGP is unstable; therefore the rise of this BGP in response to deficit impulses is not relevant per se. Consequently, public deficits reduce growth, if we are exclusively concerned with *long-run stable solutions*. Proposition 2b extends this finding, and shows that the maximal feasible BGP is

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associated with the balanced-budget rule. This result is due to the transversality condition, which constraints the debt growth rate to be lower than the real interest rate in the long run. Because, in the long run, all variables grow at the same rate, the transversality condition $\gamma < r$ means that $\dot{b}/b < r$, and further that $my < rb$. Thus, in the government budget constraint, the debt service necessarily exceeds the new resources provided by the deficit rule in the steady state, generating a crowding-out effect on productive public spending: $g/y < \tau$ in equation (4) because $my < rb$. This is a very general result, which simply comes from the fact that fiscal deficits are not a way of government finance in the long run: the permanent flow of new resources from deficits ($\dot{b}$) is always lower than the permanent flow of new expenditures resulting from the debt burden (rb) in steady state.

4. TRANSITIONAL DYNAMICS

In addition to the previous steady-state results, our model exhibits nontrivial transitory dynamics. In this section we discuss the impact of deficit rules along the transition path. As we shall see, in contrast to their negative effect on long-run growth (if we are exclusively interested in stable BGPs), deficit-financed increases in productive expenditures may improve economic growth in the short run.

To check for the stability of the two equilibria, we examine the transitional dynamics of the variables (c_k, b_k) in system (10). First we derive some analytical results from the case $m = 0$; then we offer a graphical representation for the case $m > 0$. Our analysis is supplemented with numerical simulations.

4.1. A Balanced-Budget Rule ($m = 0$)

For $m = 0$ (no deficit) we find the two BGPs expounded in the preceding section. First, the Barro BGP is reached for $b_k = b_k^B = 0$, so that $\dot{b}_k = 0$ in (10b). When public debt is zero, productive public expenditures are $g_k = g_k^B = \tau^{1/\alpha}$, and the associated steady-state consumption-to-capital ratio is $c_k = c_k^B = (g_k^B)^{1-\alpha} - \gamma^B - g_k^B - \delta$, which makes $\dot{c}_k = 0$ in (10a). Second, we find the Solow BGP for $g_k = g_k^S$, thus $b_k = b_k^S$ (both values are defined earlier), and $c_k = c_k^S = (g_k^S)^{1-\alpha} - g_k^S - \delta$, so that $\dot{c}_k = \dot{b}_k = 0$ with zero long-run growth.

Graphically, the stability locus of c_k ($\dot{c}_k = 0$) describes an inverted-U curve with a maximum at $\hat{b}_k$, as represented in Fig. 2. The stability locus of b_k ($\dot{b}_k = 0$) consists of two curves: the vertical curve $b_k = b_k^B = 0$ and a decreasing curve in b_k, namely $c_k = (g_k(b_k))^{1-\alpha} - g_k(b_k) - \delta$. The B point is the Barro solution and the S point is the Solow solution (no growth); on the right-hand side of b_k^S economic growth is negative, whereas it is positive on the left-hand side. System (10) shows that the $\dot{b}_k = 0$ curve is always above the $\dot{c}_k = 0$ curve, except when the long-run rate of growth is zero (point S) or negative (on the right of b_k^S). Because b_k is predetermined and c_k jumps, the dynamics clearly show that B is a saddle point, whereas S is unstable, as confirmed by analytical results in the Appendix.

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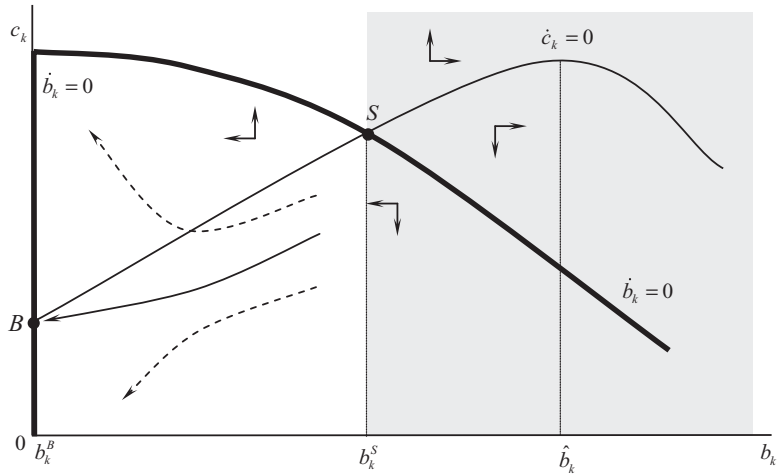


FIGURE 2. Phase diagram in the case of a balanced-budget rule ($m = 0$).

4.2. A Deficit Rule ($m > 0$)

When the government runs into debt ($m > 0$), analytical results are delicate to obtain. We can find a set of admissible parameters such that the phase diagram changes slightly with respect to the case $m = 0$.[6] The major changes concern the $\dot{b}_k = 0$ locus, which now has an inverted-U shape (with a maximum at $\tilde{b}_k$)[7] and presents an asymptote at $b_k = b_k^B = 0$, so that the B steady state can no longer be reached.

In order to locate the different BGPs associated with different deficit rules, we construct the steady-state curve for different values of the deficit ratio by extracting m from the $\dot{c}_k = 0$ relation and substituting it into the $\dot{b}_k = 0$ relation. This steady-state locus takes the form of an increasing curve, represented by the SS curve in Fig. 3. The two extreme points of this curve are the Barro BGP (B point, for $b_k = 0$), and the Solow BGP (S point, associated with b_k^S and no growth). These two extrema correspond to a balanced-budget rule ($m = 0$), whereas BGPs located inside the segment BS (like points L and H) correspond to a deficit rule ($m > 0$); the multiplicity of steady states implies that each value of m is related to two points on the SS curve.

The phase diagram (Fig. 3) points out that, similarly to the $m = 0$ case, the high BGP (H) is a saddle point and the low BGP (L) is unstable. Because b_k is a predetermined variable, whereas c_k is a jumpable variable, the saddle path that converges to H (denoted AA) can be reached by an initial jump of the consumption ratio, provided that the (predetermined) debt ratio is "not too high" ($b_{k0} < \bar{\bar{b}}_k$). Nevertheless, as we can remark in Fig. 3, the paths of variables turn around L, following an unstable spiral. Thus, an economy starting with a very high debt stock ($b_{k0} > \bar{\bar{b}}_k$) cannot reach any steady state, because there is no initial jump of c_k

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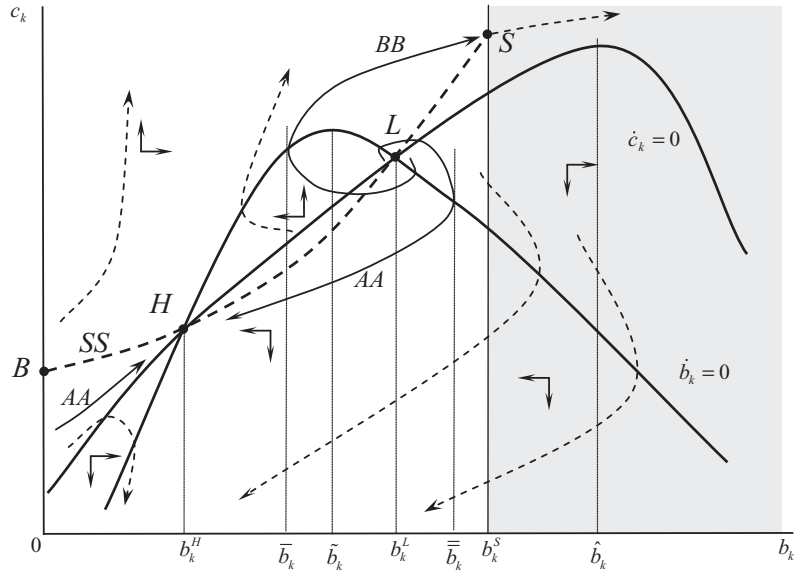


FIGURE 3. Phase diagram in the case of a deficit rule ($m > 0$).

that puts the economy on a saddle path. To avoid such unstable paths, in which the public-debt ratio explodes, we impose a superior bound for b_k ($b_k \leqslant b_k^{\max}$), which in turn defines a certain maximum "sustainable" level of public debt, namely the highest debt ratio that the society is prepared to finance.[8]

Consequently, at some date T (to be endogenously determined), the debt-to-capital ratio will reach $b_k^{\max}$, and at this point in time the government must adopt the deficit rule m that makes T a steady state. In what follows, we define $b_k^{\max}$ as the highest debt ratio that is consistent with the existence of a steady state; in Fig. 3 this ratio corresponds to the Solow solution ($b_k^{\max} = b_k^S$). Thus, the deficit rule that must be adopted at T is $m = 0$, and public debt is seen as "sustainable" as long as $\gamma \geqslant 0$.

This setup produces indeterminacy. In our model, indeterminacy has two meanings. First, it takes the form of multiple (two) BGPs: an endogenous growth path (H solution) and a "poverty trap" without growth (S solution) obtained when the public-debt sustainability condition is reached. Second, indeterminacy means that one cannot choose between the multiple (two) transition paths to steady-state equilibrium. Fig. 3 exhibits three kinds of solutions, depending on the initial public-debt ratio. An economy initially endowed with a low public-debt ratio ($b_{k0} < \bar{b}_k$) converges to the high BGP (H) via the saddle path AA. In contrast, an economy initially endowed with a very high public-debt ratio ($b_{k0} > \bar{\bar{b}}_k$) converges to the "poverty trap" solution (S) via the explosive path BB. Because this path is explosive, government must change the deficit rule at time T, to make sure that S is a steady state. Thus, the deficit rule $m > 0$ cannot be permanent, and

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government must adopt the balanced-budget rule $m = 0$ on the interval $[T, +\infty[$. The BB path in Fig. 3 is precisely the only path that reaches the point S at time T; so if the government adopts the new rule $m = 0$ at this date, the economy remains at this steady state forever.

In these two cases, knowing the initial level of the public-debt ratio is enough to define the BGP that will be reached in the future, and the transition path is reached by an initial jump of c_{k0}, which rules out any other paths.

However, for economies initially endowed with an intermediate public-debt ratio ($\bar{b}_k < b_{k0} < \bar{\bar{b}}_k$), the transition path is radically indeterminate. These economies may reach either the convergent path to the high equilibrium H or the path leading to the "no-growth trap" S. Nothing in the model ensures selection of a particular transition path AA or BB, and the two perfect foresight paths that converge to the high-growth BGP or to the "poverty trap" are both feasible equilibrium paths. This phenomenon illustrates the possibility of self-fulfilling prophecies: if everyone thinks that the economy will end up in high BGP, then it will, whereas if the low BGP is expected, it will also be sustained by self-fulfilling expectations.

This situation recalls the "history versus expectations" scenario of Krugman (1991): for countries with an initial high or low public debt, "history," in the form of the initial public-debt ratio, determines the equilibrium path, whereas for countries with an intermediate initial public-debt ratio, "expectations," in the sense of self-fulfilling prophecies, play the decisive role. In this case, the transition path is subject to "optimistic" or "pessimistic" views on debt sustainability.

5. NONLINEAR EFFECTS OF FISCAL DEFICIT ON ECONOMIC GROWTH

Assume now an upward jump of the deficit-to-output ratio, from m to $m' > m$. The two steady states (L and H) shift respectively to L' and H' on the SS curve, as in Fig. 4.

If the economy starts from the H steady state, for a predetermined public-debt ratio ($b_{k0} = b_k^H$), g_{k0} jumps up in (10c) and the rate of economic growth increases initially. Thus, debt-financed productive expenditures lead to a higher economic growth rate in the short run, whereas they lower the BGP in the long run. Following a positive impulse on the deficit ratio, the consumption ratio initially jumps down at c_{k0}^3 in Fig. 4, and the economy reaches the new BGP (H') with higher public debt and lower economic growth in the long run.

If the economy initially departs from the low BGP (L), indeterminacy occurs. First notice that, because L' is unstable, there is no transition path that converges to it. For a predetermined public-debt ratio ($b_{k0} = b_k^L$), the initial consumption ratio can either jump upward (to c_{k0}^1), if agents anticipate that public debt will become unsustainable at some future date T, or jump downward (to c_{k0}^2), if agents are "optimistic" about public-debt sustainability. In the first case, for a steady state to be reached at date T (to be endogenously determined), the initial deficit impulse must be temporary, and the government must adopt the following rule: $\dot{b}/y = m'$

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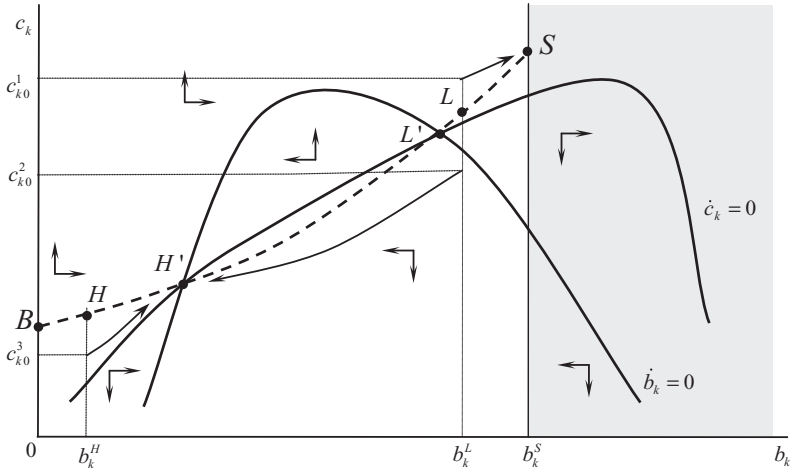


FIGURE 4. Transitional dynamics following a positive impulse on deficit (m increases).

from $t = 0$ to $t = T$, then $\dot{b}/y = 0$ from $t = T$ to infinity. Thus, the economy converges to the no-growth trap S. In the second case, the deficit impulse can be permanent, because the economy does not bump into the public-debt sustainability constraint, and the economy converges to the high BGP (H').

In both cases, economic growth increases initially, but, because of the collapse of the consumption-to-capital ratio, it increases much more in the second case. In Table 1, the rate of economic growth jumps from $\gamma^L = 0.8\%$ to $\gamma^2_{y0} = 1.81\%$ in the second case, but only to $\gamma^1_{y0} = 0.84\%$ in the first case, with capital accumulation going through similar changes.

These initial jumps in consumption and in the growth rate of output or capital accumulation determine the transition path that will be reached by the economy. With a downward jump of consumption, the economy is able to produce a sufficiently high rate of capital accumulation to reduce the debt-to-capital ratio. Consequently, all variables converge to the high steady state H', with declining public-debt ratio

TABLE 1. Simulation results for an increase in the deficit-to-GDP ratio from $m = 1\%$ to $m = 2\%$—Starting from L steady state

(%)	b_k	c_k	g_k	γ_y	γ_k	γ_c	γ_{b_k}	γ_{g_k}
Initial steady state	53.57	25.01	11.97	$\gamma^L = 0.80$			0	
Initial jump of variables (case 1, case 2)	53.57	25.12	12.60	0.84	0.95	1.44	0.68	−0.27
		24.28	12.60	1.81	1.78	1.44	−0.15	0.00
Final steady state	13.35	19.44	21.52	$\gamma^{H'} = 8.13$			0	
(H', S)	54.60	25.46	11.21	$\gamma^S = 0$			0	

Note: For simulation values $\alpha = 0.6$, $\rho = 0.1$, $S = 2$, $\tau = 0.4$, $\delta = 0.05$. For these values, $T = 2.35$.

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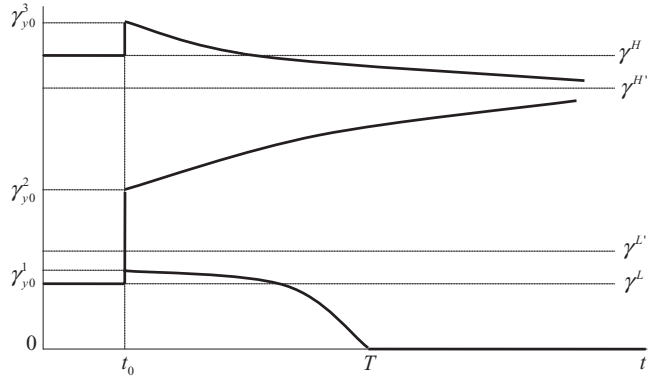


FIGURE 5. Nonlinear effects of fiscal deficit on economic growth.

and self-enforcing economic growth. With an upward jump in consumption, in contrast, the initial jump of economic growth does not allow public debt to decline; the economy moves to the poverty trap (the S point), which it reached at time T.

These initial jumps in the consumption ratio explain why self-fulfilling prophecies appear in the model. If agents are pessimistic about debt sustainability, a no-growth trap is expected in the future, and there is no incentive to save, because economic growth roughly determines the return of saving. Thus, consumption increases initially, and the growth rate does not rise enough to put the economy on the "good" transition path. Consequently, the no-growth trap becomes the equilibrium BGP and fiscal policy will really become unsustainable in the future, because of these self-fulfilling prophecies. In contrast, if agents are optimistic, they expect high rates of economic growth in the future and are encouraged to save. As a result, consumption jumps down initially, and economic growth is boosted from now to infinity. The increase in fiscal deficit is henceforth sustainable, and leads the economy to the high BGP.

Our model reveals the nonlinear characteristic of a fiscal deficit impulse in the medium and long run. In all three cases of Fig. 5, public deficit is positively linked to economic growth in the short run. In contrast, in the medium or long run, fiscal deficit may be positively or negatively related to economic growth, *with an unchanged set of parameters*, according to the initial public debt–to–capital ratio.

Growth and deficits are negatively related in the neighborhood of the high steady state, where the public debt–to–capital ratio is small (Case 3). In the neighborhood of the low steady state, with a high public debt–to–capital ratio, the relation between economic growth and fiscal deficit depends on expectations. If the fiscal policy stance is seen to become unsustainable at some future date, the economy converges to the no-growth solution: once more, economic growth and fiscal deficits are negatively related (Case 1). If agents are "optimistic," however, the initial cut in consumption is enough to generate a permanent positive growth

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effect and the economy converges to the high BGP. In this case, the link between public deficit and economic growth is positive (Case 2).

Thus, our model can reproduce some empirical results describing strong nonlinear effects of deficit impulses on economic growth, which are generally associated with expectations switches and the initial debt level.[9] When the public debt ratio is high, optimistic or pessimistic views of the future may reverse the effect of deficits on economic growth and fiscal impulses may have positive or negative effects, depending on the stance of public finance. What produces this nonlinearity is an expectation effect on the credibility of fiscal policy, when the initial debt ratio is very high, in the spirit of the empirical work of Perotti (1999), for example. If households judge public debt to be sustainable, the rise of public deficit is credible and the economy grows to the high solution. In contrast, if the deficit rise is seen to turn unsustainable, the economy is condemned to a poverty trap.

6. CONCLUSION

Does higher public deficit, increasing the burden of public debt, generate a crowding-out effect on economic growth or does it ensure the development of productive infrastructures that promote long-run growth? In a very stylized model of endogenous growth, we show that both effects may occur, depending fundamentally on (i) the initial debt ratio and (ii) self-fulfilling prophecies about government debt sustainability. Although analytically simple, our model exhibits some complex dynamics, allowing for indeterminacy and nonlinear effects of fiscal policy.

As in Minea and Villieu (2009), if the initial fiscal stance is close to equilibrium, allowing for debt-financed productive public expenditures does not ensure that a shortage of public investment will be filled in the long run. On the contrary, the shortage of public investment may be the consequence of excessive government borrowing, either for productive or unproductive expenditures. However, if the initial fiscal stance is much damaged, our model produces another effect: a deficit-financed impulse in productive expenditures may (or may not) generate self-fulfilling optimistic expectations, which allow the economy to leave the low-growth BGP and to catch up with the high-growth BGP. Unfortunately, the alternative between optimistic or pessimistic views of the future is not a matter of choice for the government, and the risk of such a debt policy is condemning the economy to a no-growth trap.

NOTES

1. See, e.g., Modigliani et al. (1998) and the critical view of Minea and Villieu (2009).

2. In this model, *multiplicity* refers to the existence of a *finite* number of solutions, and *indeterminacy* stands for the case in which we cannot select a determinate solution or trajectory.

3. This condition corresponds to a no–Ponzi game constraint $\gamma_c < r$, with r the real interest rate to be defined.

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4. Thus, as in Barro (1990), the g/k ratio is endogenous in our model. The fact that productive expenditures are the adjustment variable in the government budget constraint is well documented in, e.g., Alesina and Perotti (1997), or in Oxley and Martin (1991, p. 161): "Most countries have offset such increases by winding back public investment, reflecting the political reality that it is easier to cut-back or postpone investment spending than it is to cut current expenditures."

5. If $m = 0$ and $g_k^{\alpha} = \tau$, (12b) produces a 0/0 case of indeterminacy. Thus, for $m = 0$, the γ^2 curve is either $\gamma = 0$ or $g_k = \tau^{1/\alpha}$, as in Fig. 1.

6. Recall that m is small, ordinarily less than 5%. The Appendix presents simulations of local stability.

7. $\tilde{b}_k$ is implicitly defined by $g\left(\tilde{b}_k\right) = (1-\alpha)^{1/\alpha} > g(\hat{b}_k)$ if $S < 2/\alpha\,(1-\tau)$, which we assume, and $\hat{b}_k$, the maximum of the curve $\dot{c}_k = 0$, is defined by $g(\hat{b}_k) = ((1-\alpha)\,(\alpha\,(1-\tau)\,S-1))^{1/\alpha}$. In Fig. 2, we assume (without loss of generality) that $\tilde{b}_k = 0 = b_k^B$ (this is obtained for $\tau = 1-\alpha$), so that $\tilde{b}_k < b_k^S < \hat{b}_k$.

8. Such an upper bound of public debt is often assumed when the public debt path is unstable [see, e.g., Sargent and Wallace (1981)]. In this model, the *sustainability* constraint is justified by the fact that no BGP would exist without this condition, if the initial level of debt were very high. This finding is close to that of Yakita (2008), who shows, in a different context with overlapping generations, that if the initial public debt is greater than some threshold value, fiscal policy is not sustainable, and the government must change the fiscal policy to reduce the budget deficit. In Yakita (2008), the adjustment variable in this problem of dynamic sustainability is the tax rate, whereas in our model it is the deficit rule. We thank an anonymous referee for pointing out this reference.

9. See, e.g., Perotti (1999) and Giavazzi et al. (2000).

REFERENCES

Alesina, Alberto and Roberto Perotti (1997) Fiscal Adjustments in OECD Countries: Composition and Macroeconomic Effects. IMF Staff Papers 44, 210–248.

Barro, Robert (1990) Government spending in a simple model of economic growth. *Journal of Political Economy* 98, S103–S125.

Futagami, Koichi, Tatsuro Iwaisako, and Ryoji Ohdoi (2008) Debt policy rules, productive government spending, and multiple growth paths. *Macroeconomic Dynamics* 12, 445–462.

Giavazzi, Francesco, Tullio Japelli, and Marco Pagano (2000) Searching for non-linear effects of fiscal policy: Evidence from industrial and developing countries. *European Economic Review* 44, 1259–1289.

Greiner, Alfred and Willi Semmler (2000) Endogenous growth, government debt and budgetary regimes. *Journal of Macroeconomics* 22, 363–384.

Groneck, Max (2010) A golden rule of public finance or a deficit regime? Growth and welfare effects of budget rules. *Economic Modelling* 27, 523–534.

Krugman, Paul (1991) History versus expectations. *Quarterly Journal of Economics* 106, 651–667.

Minea, Alexandru and Patrick Villieu (2009) Borrowing to finance public investment? The GRPF reconsidered in an endogenous growth setting. *Fiscal Studies* 30, 103–133.

Minea, Alexandru and Patrick Villieu (2010) Endogenous growth, government debt and budgetary regimes, by A. Greiner and W. Semmler: A corrigendum. *Journal of Macroeconomics* 32, 709–711.

Modigliani, Franco, Jean-Paul Fitoussi, Beniamino Moro, Dennis Snower, Robert Solow, Alfred Steinherr, and Paolo Sylos Labini (1998) An economists' manifesto on unemployment in the European Union. *BNL Quarterly Review* 206, 327–361.

Oxley, Howard and John Martin (1991) Controlling government spending and deficits: Trends in the 1980s and prospects for the 1990s. *OECD Economic Studies* 17, 145–189

https://doi.org/10.1017/S1365100511000435 Published online by Cambridge University Press

Perotti, Roberto (1999) Fiscal policy when things are going badly. *Quarterly Journal of Economics* 64, 1399–1436.

Sargent, Thomas and Neil Wallace (1981) Some unpleasant monetarist arithmetics. *Federal Reserve Bank of Minneapolis Quarterly Review* 5, 1–17.

Solow, Robert (1956) A contribution to the theory of economic growth. *Quarterly Journal of Economics* 70, 65–94.

Yakita, Akira (2008) Sustainability of public debt, public capital formation, and endogenous growth in an overlapping generations setting. *Journal of Public Economics* 92, 897–914.

APPENDIX: THE LOCAL STABILITY OF STEADY STATES

In the neighborhood of the Barro steady state, we have, with $\psi(b_k) = g_k(b_k) + \delta - (g_k(b_k))^{1-\alpha}$,

$$J^B = \begin{bmatrix} c_k^B & c_k^B \left[\alpha S (1-\tau)(1-\alpha) \left(\left. \frac{dg_k}{db_k} \right|_B \right) \left(g_k^B\right)^{-\alpha} + \left. \frac{d\psi}{db_k} \right|_B \right] \\ 0 & -\gamma^B \end{bmatrix}.$$

Clearly $\text{Det}(J^B) = -c_k^B \gamma^B < 0$, so that the Barro steady state is a saddle path.

In the neighborhood of the Solow steady state, we have

$$J^S = \begin{bmatrix} c_k^S & c_k^S \left[\alpha S (1-\tau)(1-\alpha) \left(\left. \frac{dg_k}{db_k} \right|_S \right) \left(g_k^S\right)^{-\alpha} + \left. \frac{d\psi}{db_k} \right|_S \right] \\ b_k^S & b_k^S \left. \frac{d\psi}{db_k} \right|_S \end{bmatrix}.$$

Clearly

$$\text{Det}\left(J^S\right) = -\alpha S (1-\tau)(1-\alpha) \left(\left. \frac{dg_k}{db_k} \right|_S \right) c_k^S b_k^S \left(g_k^S\right)^{-\alpha} > 0.$$

To show that

$$\text{Tr}\left(J^S\right) = c_k^S + b_k^S \left. \frac{d\psi}{db_k} \right|_S > 0,$$

note that

$$b_k \frac{d\psi}{db_k} = \frac{r b_k (g_k)^{1-\alpha} \left[(1-\alpha) - (g_k)^{\alpha} \right]}{\alpha g_k + (1-\alpha)\delta b_k}$$

and $c_k = (g_k)^{1-\alpha} - g_k - \delta = (1-\tau)(g_k)^{1-\alpha} - \delta + r b_k = r + (1-\tau)(g_k)^{1-\alpha} + r b_k$, so that

$$c_k + b_k \frac{d\psi}{db_k} = r + (1-\tau)(g_k)^{1-\alpha} + \frac{(1-\alpha) r b_k \left((g_k)^{1-\alpha} + \delta b_k - g_k \right)}{\alpha g_k + (1-\alpha)\delta b_k} > 0.$$

Table A.1 provides simulations of the local stability of the steady states for different deficit values.

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TABLE A.1. Local stability of steady states if $m > 0$ (λ_i^j are the eigenvalues of J^j)

(%)	$\mathrm{Det}(J^H)$	$\mathrm{Det}(J^L)$	$\mathrm{Tr}(J^H)$	$\mathrm{Tr}(J^L)$	$(\lambda_1^H, \lambda_2^H)$	$(\lambda_1^L, \lambda_2^L)$
$m = 1$	−1.48	1.83	10.03	27.47	18.17, −8.14	16.18, 11.29
$m = 2$	−1.24	1.47	10.92	25.39	17.85, −6.93	16.41, 8.98
$m = 3$	−0.91	1.04	12.29	22.83	17.51, −5.22	16.56, 6.27

Notes: For simulation values $\alpha = 0.6$, $\rho = 0.1$, $S = 2$, $\tau = 0.4$, $\delta = 0.05$. These values yield the dynamics displayed in Fig. 4, but changing parameters may lead to complex eigenvalues, with dynamics as in Fig. 3.

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