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Interaction between parallel Gaussian electromagnetic beams in ionic collision dominated plasmas with thermal conduction

MAHENDRA SINGH SODHA[1],
SUJEET KUMAR AGARWAL[2]
and ASHUTOSH SHARMA[2]

[1] Disha Academy of Research and Education, Disha Crown, Katchna Road, Shankar Nagar, Raipur-492007, India
(msodha@rediffmail.com)

[2] DST Project, Department of Education Building, Lucknow University, Lucknow-226007, India

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Abstract. In this communication the interaction between two Gaussian electromagnetic beams in an ionic collision dominated plasma has been investigated, when the axes of the two beams are initially ($z=0$) parallel along the z-axis in the xz plane; the beams are initially propagating in the z-direction. Taking into account the loss of electron energy by collisions and by thermal conduction, the energy balance equation for electrons has been solved to obtain the space dependence of the electron temperature and the dielectric function has been expressed as a function of the electron temperature; this expression for the dielectric function has been substituted in the wave equation and a solution of the resulting nonlinear equation obtained in the paraxial approximation. Second-order coupled ordinary differential equations have been obtained for the distance between the centers of the beams and the beam widths in the x- and y-directions as a function of the distance of propagation along the z-axis. The equations have been solved numerically for a range of parameters and a discussion of the results is presented for the case when the two beams have the same axial irradiance, frequency and width. From simple considerations it is seen that the beams attract each other when $2x_o < wr_o$ and in this situation beams are close enough for the paraxial approximation to be valid.

1. Introduction

The interaction between electromagnetic beams (which are not co-axial) has been investigated recently (Ren et al. 2000, 2002; Dong et al. 2002); part of the motivation of these studies is to use the results as a first step towards understanding the interaction between individual filaments of a beam or individual speckles in a random phase plate beam. Ren et al. (2000) used the variational method as well as a three-dimensional particle-in-cell model of laser light in a plasma to demonstrate the mutual attraction between two skew electromagnetic beams, which may cause the beams to spiral around each other. The physical mechanism considered in this investigation is the relativistic dependence of the electronic mass on the quiver

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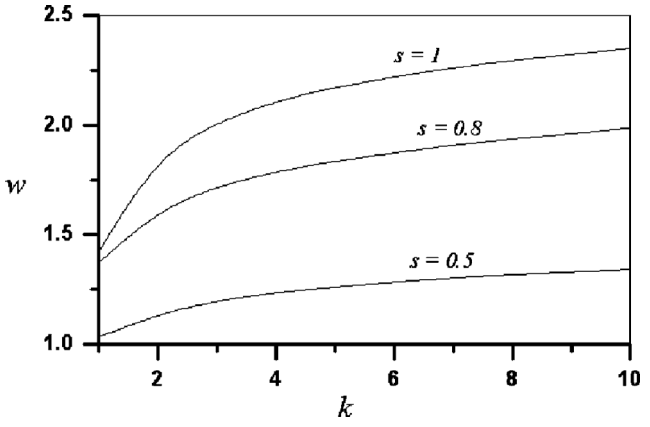


Figure 1. Variation of the parameter w with the ratio k of the axial irradiances of the two beams for different ratios s of the widths of the two beams.

speed of the electrons; however, this nonlinearity was approximated by a quadratic expression. Ren et al. (2002) have made an exhaustive study of the interaction between two laser beams, taking into account the relativistic nonlinearity and the nonlinearity caused by density modulations from a plasma wake. For incoherent beams and coherent beams, linearly polarized in perpendicular directions or circularly polarized, the beams are attracted to each other. For coherent beams, linearly polarized in directions at an angle other than $\pi/2$, the beams attract or repel depending on whether the beams are in phase or antiphase. In this analysis the relativistic nonlinearity was also taken as being quadratic in nature. Dong et al. (2002) studied the time evolution of two interacting beams in a plasma, taking into account the relativistic as well as the ponderomotive nonlinearity. Shukla et al. (2006) have recently investigated the nonlinear interaction between two weakly relativistic crossing laser beams in plasmas. A set of nonlinear mode coupled equations and nonlinear dispersion relations have been derived and analyzed for Raman (backward and forward) scattering instabilities as well as Brillouin and modulation self-focusing instabilities.

These analyses, based on numerical simulation, do not explicitly bring out the underlying physics and are not very appropriate for parametric studies on account of the complexity, and the corresponding necessary efforts, in the numerical computations. Sodha et al. (2006a) have highlighted the basic physics of interaction for the case of identical incoherent (or circularly polarized or cross polarized) beams; the argument can be extended to the general case.

It can be readily seen that the irradiance distribution resulting from two parallel incoherent Gaussian beams at a distance $2x_o$, having a ratio of axial irradiances k and widths of beams r_o and sr_o, has (in between the beams) a maximum when $2x_o < wr_o$, and a minimum when $2x_o > wr_o$; the parameter w as a function of k and s can be determined from the conditions $(dI/dx = 0)$ and $(d^2I/dx^2) = 0$ (where $I(x, wr_o)$ is the sum of the irradiance of the two beams) and it is illustrated in Fig. 1. Hence, in a nonlinear plasma (in which the effective dielectric function is an increasing function of the irradiance) the dielectric function has a maximum or minimum in the space between the two beams, depending on whether $2x_o < wr_o$ or $2x_o > wr_o$. Since light bends towards regions of higher refractive index (i.e.

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dielectric function) the beams move towards the maximum irradiance region, i.e. towards each other when $2x_o < wr_o$; this is substantiated by the analysis in this paper. Similarly, when $2x_o > wr_o$ the beams should repel each other; however, the paraxial theory developed herein is not applicable in this regime of $2x_o$ and hence is not relevant to validation of this result.

Sodha et al. (2006a) have recently published a paraxial theory of interaction between two initially parallel beams, considering three distinct mechanisms of nonlinearity, *viz*. collisional, ponderomotive and relativistic. In accounting for the collisional nonlinearity, the loss of electron energy by thermal conduction was ignored, which is justified only when $\delta r_o^2/l^2 \gg 1$, where δ is the fraction of excess energy lost by an electron in collision with heavy particles and l is the mean free path of electrons; this inequality is not valid over a large range of parameters for plasmas.

In this paper a paraxial theory of interaction between two initially parallel electromagnetic beams in an ionic collision dominated plasma is presented. A collisional plasma is defined by the condition that the mean free path of the electrons is much less than the dimensions of the plasma, perturbed by the wave. For such plasmas the ponderomotive nonlinearity is negligible as compared to the collisional nonlinearity; the ponderomotive nonlinearity is 10^{-3} to 10^{-4} times the collisional nonlinearity in the ionosphere (Gurevich 1978). For such plasmas the relativistic nonlinearity is still weaker. Hence, ponderomotive and relativistic nonlinearities (e.g., those investigated by Shukla and Stenflo (1984)) have not been considered. The physical mechanism of nonlinearity is as follows. The ohmic heating of the electrons by the non-uniform field of the beams and the loss of electron energy by the collisions and thermal conduction results in a non-uniform distribution of the electron temperature, which creates pressure gradients of the electron and ion gases. The balance of these pressure gradients by the space charge field in the steady state results in a non-uniform field-dependent distribution of the electron density and hence the dielectric function. The space-dependent distribution of the electron temperature and thereby the dielectric function has been obtained in a way similar to that in a recent study (Sodha et al. 2006b) of self-focusing of a single beam in a collisional plasma with significant thermal conduction.

The paraxial limitation of the theory is not a serious one because in the interesting situation when the beams attract each other, they are close enough for the paraxial approximation to be valid.

2. Analysis

2.1. Radial distribution of electron density

On account of the non-uniform radial distribution of irradiance in the beam, there is a corresponding distribution of the electron temperature, which leads to the distribution of the electron and ion gas pressures; in the steady state this is balanced by a space charge field. This phenomenon, along with the assumptions of charge neutrality and efficient energy exchange between the ions and neutral species, leads to an expression for the electron density in terms of the electron temperature; thus, following earlier works (e.g., Sodha and Sharma 2006) one obtains

$$\frac{N_{\mathrm{e}}}{N_{eo}} = \frac{2T}{T_{\mathrm{e}} + T}, \tag{1}$$

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where N_{eo} is the electron density and T the temperature of electrons, ions and neutral species in regions of the plasma, that are not affected by the beam. The premise that the ionic temperature is equal to that of the neutral species temperature and that it remains constant is based on the assumptions that $\delta_{ei}\nu_{ei} \ll \nu_{im}$ (where ν_{ei} and δ_{ei} are the frequency and fraction of excess energy exchanged in electron–ion collisions and ν_{im} is the frequency of ion–neutral species collisions) and that the heat capacity (number density) of the neutral gas is much larger than the heat capacity (number density) of electrons/ions. Furthermore, it is implied that the electron density does not change significantly with increased electron temperature, at least in the duration of interaction of the two beams.

2.2. *Propagation of two initially parallel Gaussian electromagnetic beams in plasmas*

The irradiance of two Gaussian beams propagating along the z-axis can at $z=0$ be represented by

$$E_1 E_1^* = E_{10}^2 \exp\left[-\frac{(x+x_o)^2}{r_1^2} - \frac{y^2}{r_1^2}\right] \tag{2a}$$

and

$$E_2 E_2^* = E_{20}^2 \exp\left[-\frac{(x-x_o)^2}{r_2^2} - \frac{y^2}{r_2^2}\right], \tag{2b}$$

where E_1 and E_2 are the complex amplitudes of the electric vectors of the two beams, $2x_o$ is the distance between the axes of the beams at $z=0$ and r_1 and r_2 are the widths of the two beams at $z=0$. In what follows the two beams are considered to be either (i) incoherent or (ii) coherent, with linearly polarized electric vectors at right angles to each other or both circularly polarized.

The nonlinear dielectric function ϵ_j in an isotropic inhomogeneous medium can in general be expressed as

$$\epsilon_j = \epsilon_j(z, E_1 E_1^*, E_2 E_2^*),$$

where E_j is the amplitude of the electric vector of the jth beam ($j=1,2$). The imaginary part of the dielectric function is negligible, which is justified when the electron collision frequency is much less than the wave frequencies.

Since $E_j E_j^*$ is, in general, a function of x and y, one can expand ϵ_j in powers of x and y and in the paraxial approximation retain terms only up to those having x^2 and y^2. Thus, in the paraxial approximation one obtains

$$\begin{aligned}\epsilon_j = {} & \epsilon_{oj}(z, E_{10}E_{10}^*, E_{20}E_{20}^*) - x\epsilon_{1j}(z, E_{10}E_{10}^*, E_{20}E_{20}^*) \\ & - x^2\epsilon_{2xj}(z, E_{10}E_{10}^*, E_{20}E_{20}^*) - y^2\epsilon_{2yj}(z, E_{10}E_{10}^*, E_{20}E_{20}^*),\end{aligned} \tag{3}$$

where suffix $j=1,2$ refers to the two beams.

In the steady state the amplitude of the electric vector E_j satisfies the scalar wave equation

$$\nabla^2 E_j + \frac{\omega_j^2}{c^2}\epsilon_j E_j = 0. \tag{4}$$

Equation (4) has a solution of the form

$$E_j(x,y,z) = A_j(x,y,z)\exp\left[-i\int_o^z k_j(z)\,dz\right] \tag{5a}$$

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where $A_j(x,y,z)$ is the complex amplitude of the electric field and

$$k_j^2 = \frac{\omega_j^2}{c^2}\epsilon_{oj}. \tag{5b}$$

Substituting for $E_j(x,y,z)$ from (5a) in (4) and neglecting $\partial^2 A_j/\partial z^2$ one obtains

$$\frac{\partial^2}{\partial x^2}A_j + \frac{\partial^2}{\partial y^2}A_j - 2ik_j\frac{\partial A_j}{\partial z} - iA_j\frac{\partial k_j}{\partial z} - k_j^2 A_j + \frac{\omega_j^2}{c^2}\epsilon_j(x,y,z)A_j = 0. \tag{6}$$

For a nearly spherical wave front

$$A_j(x,y,z) = A_{jo}(x,y,z)\exp[-ik_j(z)S_j(x,y,z)] \tag{7a}$$

and the eikonal

$$S_j = \frac{x_j^2}{2}\beta_{1j}(z) + x_j\beta_{2j}(z) + \frac{y^2}{2}\beta_{3j}(z) + \Phi_j(z), \tag{7b}$$

where

$$x_1 = x + x_oF_1 \quad \text{and} \quad x_2 = x - x_oF_2. \tag{7c}$$

For convenience the functions $\beta_{11}(z)$, $\beta_{21}(z)$, $\beta_{31}(z)$, $\beta_{12}(z)$, $\beta_{22}(z)$ and $\beta_{32}(z)$ may be expressed as

$$\begin{aligned}\beta_{11} &= \frac{1}{f_{1x}}\frac{df_{1x}}{dz}, \quad \beta_{21} = -x_o\frac{dF_1}{dz}, \quad \beta_{31} = \frac{1}{f_{1y}}\frac{df_{1y}}{dz},\\ \beta_{12} &= \frac{1}{f_{2x}}\frac{df_{2x}}{dz}, \quad \beta_{22} = x_o\frac{dF_2}{dz}, \quad \beta_{32} = \frac{1}{f_{2y}}\frac{df_{2y}}{dz}.\end{aligned} \tag{7d}$$

Substituting for $A_j(x,y,z)$ and S_j from (7) and ϵ_j from (3) into (6) and equating the real and imaginary parts on both sides of the resulting equation, one obtains

$$\begin{aligned}&2\frac{\partial S_j}{\partial z} + \left[\left(\frac{\partial S_j}{\partial x}\right)^2 + \left(\frac{\partial S_j}{\partial y}\right)^2\right] + \frac{2S_j}{k_j(z)}\frac{dk_j(z)}{dz}\\ &\quad = \frac{1}{k_j^2 A_{jo}}\left(\frac{\partial^2}{\partial x^2}A_{jo} + \frac{\partial^2}{\partial y^2}A_{jo}\right) - x\frac{\epsilon_{1j}(z)}{\epsilon_{oj}(z)} - x^2\frac{\epsilon_{2xj}(z)}{\epsilon_{oj}(z)} - y^2\frac{\epsilon_{2yj}(z)}{\epsilon_{oj}(z)}\end{aligned} \tag{8}$$

and

$$\frac{\partial}{\partial z}A_{jo}^2 + A_{jo}^2\left(\frac{\partial^2}{\partial x^2}S_j + \frac{\partial^2}{\partial y^2}S_j\right) + \left(\frac{\partial A_{jo}^2}{\partial x}\frac{\partial S_j}{\partial x} + \frac{\partial A_{jo}^2}{\partial y}\frac{\partial S_j}{\partial y}\right) = -\frac{A_{jo}^2}{k_j(z)}\frac{dk_j(z)}{dz}. \tag{9}$$

The solution of (9) is

$$\begin{aligned}A_{jo}^2(x,y,z) &= \text{constant} \times \sqrt{\frac{\epsilon_{oj}(0)}{\epsilon_{oj}(z)}}\frac{1}{f_{jx}(z)f_{jy}(z)}\mathfrak{I}\left(\frac{x_j}{r_jf_{jx}(z)}, \frac{y}{r_jf_{jx}(z)}\right),\\ &= \left[\frac{E_{jo}^2(z)}{f_{jx}(z)f_{jy}(z)}\right]\mathfrak{I}\left[\frac{x_j}{r_jf_{jx}(z)}, \frac{y}{r_jf_{jy}(z)}\right],\end{aligned} \tag{10}$$

where $\mathfrak{I}$ is an arbitrary function and $E_{jo}^2(z)$ is the axial ($x = (-1)^j x_oF_j$) irradiance.

At $z = 0$, the beam intensity is given by

$$E_jE_j^* = A_{jo}^2 = E_{joo}^2\exp\left(-\frac{x_{jo}^2}{r_j^2} - \frac{y^2}{r_j^2}\right) \tag{11}$$

where $x_{10} = x + x_o$ and $x_{20} = x - x_o$.

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Comparing (10) and (11) at $z=0$, one obtains

$$E_1E_1^* = A_{10}^2(x,y,\xi) = \frac{E_{100}^2}{f_{1x}(\xi)f_{1y}(\xi)}\sqrt{\frac{\epsilon_{01}(0)}{\epsilon_{01}(\xi)}}\exp\left(-\frac{\{x+x_oF_1(\xi)\}^2}{r_1^2f_{1x}^2(\xi)}-\frac{y^2}{r_1^2f_{1y}^2(\xi)}\right) \tag{12a}$$

and

$$E_2E_2^* = A_{20}^2(x,y,\xi) = \frac{E_{200}^2}{f_{2x}(\xi)f_{2y}(\xi)}\sqrt{\frac{\epsilon_{02}(0)}{\epsilon_{02}(\xi)}}\exp\left(-\frac{\{x-x_oF_2(\xi)\}^2}{r_2^2f_{2x}^2(\xi)}-\frac{y^2}{r_2^2f_{2y}^2(\xi)}\right), \tag{12b}$$

where $\xi = cz/(\omega_1 r_1^2)$ is the dimensionless distance of propagation and $F_1 = F_2 = 1$ at $z=0$.

From (12a) and (12b) it is evident that the curves $x=-x_oF_1(\xi)$ and $x=x_oF_2(\xi)$ in the zx-plane ($y=0$) represent the axes of the two beams.

Substituting for A_{jo}^2 from (12a) and (12b) and S_j from (7b) into (8), and equating the coefficients of x, x^2 and y^2 on both sides for both the beams ($j=1,2$), one obtains

$$\frac{d^2F_1}{d\xi^2} = -\frac{\rho_1^2}{\epsilon_{01}}[\epsilon''_{2x1}-\epsilon'_{11}] - \frac{1}{2\epsilon_{01}}\frac{d\epsilon_{01}}{d\xi}\frac{dF_1}{d\xi}, \tag{13a}$$

$$\frac{d^2f_{1x}}{d\xi^2} = -\frac{\rho_1^2}{f_{1x}\epsilon_{01}}\left[\epsilon'_{2x1}-\frac{1}{\rho_1^2f_{1x}^2}\right] - \frac{1}{2\epsilon_{01}}\frac{d\epsilon_{01}}{d\xi}\frac{df_{1x}}{d\xi}, \tag{13b}$$

$$\frac{d^2f_{1y}}{d\xi^2} = -\frac{\rho_1^2}{f_{1y}\epsilon_{01}}\left[\epsilon'_{2y1}-\frac{1}{\rho_1^2f_{1y}^2}\right] - \frac{1}{2\epsilon_{01}}\frac{d\epsilon_{01}}{d\xi}\frac{df_{1y}}{d\xi}, \tag{13c}$$

$$\frac{d^2F_2}{d\xi^2} = -\frac{\rho_2^2}{\epsilon_{02}}ps^4[\epsilon''_{2x2}+\epsilon'_{12}] - \frac{1}{2\epsilon_{02}}\frac{d\epsilon_{02}}{d\xi}\frac{dF_2}{d\xi}, \tag{13d}$$

$$\frac{d^2f_{2x}}{d\xi^2} = -ps^4\frac{\rho_2^2}{f_{2x}\epsilon_{02}}\left[\epsilon'_{2x2}-\frac{1}{\rho_2^2f_{2x}^2}\right] - \frac{1}{2\epsilon_{02}}\frac{d\epsilon_{02}}{d\xi}\frac{df_{2x}}{d\xi}, \tag{13e}$$

$$\frac{d^2f_{2y}}{d\xi^2} = -ps^4\frac{\rho_2^2}{f_{2y}\epsilon_{02}}\left[\epsilon'_{2y2}-\frac{1}{\rho_2^2f_{2y}^2}\right] - \frac{1}{2\epsilon_{02}}\frac{d\epsilon_{02}}{d\xi}\frac{df_{2y}}{d\xi}, \tag{13f}$$

where $\rho_1 = r_1\omega_1/c$ and $\rho_2 = r_2\omega_2/c$ are the dimensionless widths of the two beams at $z=0$,

$$p = \frac{\omega_1^2}{\omega_2^2}, \quad s = \frac{r_1}{r_2},$$

$$\epsilon'_{2x1} = r_1^2f_{1x}^2\epsilon_{2x1}, \tag{14a}$$

$$\epsilon'_{2yl} = r_l^2f_{1y}^2\epsilon_{2yl}, \tag{14b}$$

$$\epsilon''_{2x1} = r_1^2F_1^2\epsilon_{2x1} = \frac{F_1}{f_{lx}^2}\epsilon'_{2xl}, \tag{14c}$$

$$\epsilon'_{11} = \frac{r_1^2}{2x_o}\epsilon_{11}, \tag{14d}$$

$$\epsilon'_{2x2} = r_2^2f_{2x}^2\epsilon_{2x2}, \tag{14e}$$

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$$\epsilon'_{2y2} = r_2^2 f_{2y}^2 \epsilon_{2y2}, \tag{14f}$$

$$\epsilon''_{2x2} = r_2^2 F_2 \epsilon_{2x2} = \frac{F_2}{f_{2x}^2} \epsilon'_{2x2}, \tag{14g}$$

and

$$\epsilon'_{12} = \frac{r_2^2}{2x_o} \epsilon_{12}. \tag{14h}$$

Adding (13a) and (13d) one gets

$$\frac{d^2}{d\xi^2}(F_1 + F_2) = -\frac{\rho_1^2}{\epsilon_{01}}[\epsilon''_{2x1} - \epsilon'_{11}] - \frac{\rho_2^2}{\epsilon_{02}} ps^4 [\epsilon''_{2x2} + \epsilon'_{12}] - \frac{1}{2\epsilon_{o1}} \frac{d\epsilon_{o1}}{d\xi} \frac{dF_1}{d\xi} - \frac{1}{2\epsilon_{02}} \frac{d\epsilon_{o2}}{d\xi} \frac{dF_2}{d\xi}. \tag{15}$$

2.3. Evaluation of $\epsilon_{oj}, \epsilon_{1j}, \epsilon_{2xj}$ and ϵ_{2yj}

The dielectric function ϵ_j (a meaningful parameter in the case when the Debye length of the plasma is much less than the wavelength of the radiation) in a plasma, corresponding to electromagnetic radiation of frequency ω_j, is given by

$$\epsilon_j = 1 - (\omega_{\mathrm{p}}^2/\omega_j^2), \tag{16}$$

where $\omega_{\mathrm{p}} = (4\pi e^2 N_{\mathrm{e}}/m)^{1/2}$ is the plasma frequency, e is the electronic charge, N_{e} is the electron density and m is the electronic mass.

The energy balance of the electrons in a plasma in the presence of a number of parallel Gaussian beams may be expressed as

$$-\frac{3}{2} N_{\mathrm{e}} k_{\mathrm{B}} (T_{\mathrm{e}} - T) \nu_{\mathrm{e}} \delta + \left[\frac{\partial}{\partial x} \left(\chi \frac{\partial T_{\mathrm{e}}}{\partial x} \right) + \frac{\partial}{\partial y} \left(\chi \frac{\partial T_{\mathrm{e}}}{\partial y} \right) \right] + \frac{e^2 N_{\mathrm{e}}}{2m} \nu_{\mathrm{e}} \sum (E_j E_j^* / \omega_j^2) = 0, \tag{17}$$

where δ is the effective fraction (averaged over heavy species of molecules, atom and ions) of excess energy lost by an electron in a collision, ν_{e} is the electron collision frequency ($\nu_{\mathrm{e}} \ll \omega_j$) and χ is the electronic thermal conductivity.

The first term in the above equation represents the power lost by the electrons in collisions per unit volume, the second term corresponds to the power lost by the electrons by thermal conduction and the third term is the ohmic power per unit volume gained by the electrons from the fields of the beams.

For further analysis an ionic collision dominated plasma is considered. Thus the electronic collision frequency with ions ν_{ei} and the electronic thermal conductivity are given by (Shkarofsky et al. 1966; Spitzer 1968)

$$\nu_{\mathrm{e}} = \nu_{\mathrm{ei}} = \nu_{\mathrm{eio}} (N_{\mathrm{e}}/N_{eoo}) (T_{\mathrm{e}}/T)^{3/2} \tag{18a}$$

and

$$\chi = \chi_o (T_{\mathrm{e}}/T)^{5/2}, \tag{18b}$$

where

$$\nu_{\mathrm{eio}} = \frac{5.5 N_{eo}}{T^{3/2}} \ln \Lambda \ \mathrm{s}^{-1},$$

$$\chi_o = 13 \times 10^{-5} / \ln \Lambda \ \mathrm{ergs\ cm^{-1}\ s^{-1}\ K^{-1}},$$

and

$$\Lambda = 220 T / N_{eoo}^{1/3};$$

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all the parameters are expressed in the CGS system of units. Thus, (17) can be simplified to

$$\left(\frac{T_e}{T}-1\right)-\frac{2}{3k_BT\delta}\frac{\chi}{N_e\nu_e}\times\left[\frac{5}{2T_e}\left\{\left(\frac{\partial T_e}{\partial x}\right)^2+\left(\frac{\partial T_e}{\partial y}\right)^2\right\}+\left(\frac{\partial^2}{\partial x^2}T_e+\frac{\partial^2}{\partial y^2}T_e\right)\right]=\sum\alpha_jE_jE_j^* \tag{19}$$

where $\alpha_j=e^2/3m\delta\omega_j^2k_BT$.

As the electron temperature T_e is a function of $\alpha_jE_jE_j^*$, in the paraxial approximation T_e can be expanded in powers of x and y and the series terminated at the terms having x^2 and y^2. Thus,

$$T_e=T_{eo}-\frac{x}{r_o}T_{e1}-\frac{x^2}{r_o^2}T_{e2x}-\frac{y^2}{r_o^2}T_{e2y} \tag{20}$$

where r_o is a normalizing parameter of the order of the width of the beams.

From the radial profile of the beams and the energy balance equation for the electrons, the radial distribution of the electron temperature is evaluated in the paraxial approximation by expanding the irradiance and the electron temperature in powers of x and y and retaining terms up to x^2 and y^2; the electron thermal conductivity is a function of the electron temperature and may be expanded likewise. Thus, T_{eo}, T_{e1}, T_{e2x} and T_{e2y} can be expressed as functions of irradiance on the axis. Knowing $T_e(x,y,z)$, the electron density $N_e(x,y,z)$ may be evaluated from (1). Hence, $N_e(x,y,z)$ can be expressed as

$$N_e=N_{eo}-xN_{e1}-x^2N_{e2x}-y^2N_{e2y}. \tag{21}$$

Substituting for ν_{ei} and χ from (18a) and (18b), for N_e/N_{eo} and T_e from (1) and (20) and for $E_1E_1^*$ and $E_2E_2^*$ from (12a) and (12b) into (19), expanding the terms in powers of x and y and equating the coefficients of terms with x, x^2 and y^2, and those independent of x and y on both sides one obtains

$$(T'_{eo}-1)-DT'^3_{eo}(1+T'_{eo})^2[2.5T'^2_{e1}-2T'_{eo}(T'_{e2x}+T'_{e2y})]=\sum\frac{\alpha_jE_{jo}^2}{f_{jx}f_{jy}}\exp\left(-\frac{x_o^2F_j^2}{r_j^2f_{jx}^2}\right), \tag{22a}$$

$$T'_{e1}+DT'^3_{eo}(1+T'_{eo})T'_{e1}\times\left[10T'_{e2x}(1+T'_{eo})-\frac{2.5T'^2_{e1}(5T'_{eo}+3)}{T'_{eo}}+4(T'_{e2x}+T'_{e2y})(3T'_{eo}+2)\right]=\frac{2x_or_o}{r_1^2}\frac{F_1}{f_{1x}^2}\frac{\alpha_1E_{10}^2}{f_{1x}f_{1y}}\exp\left(-\frac{x_o^2F_1^2}{r_1^2f_{1x}^2}\right)-\frac{2x_or_o}{r_2^2}\frac{F_2}{f_{2x}^2}\frac{\alpha_2E_{20}^2}{f_{2x}f_{2y}}\exp\left(-\frac{x_o^2F_2^2}{r_2^2f_{2x}^2}\right), \tag{22b}$$

$$T'_{e2x}+2.5DT'^3_{eo}\left[\left(4T'^2_{e2x}-\frac{15T'^2_{e1}T'_{e2x}}{T'_{eo}}+\frac{3T'^4_{e1}}{T'^2_{eo}}\right)(1+T'_{eo})^2+(1+T'_{eo})T'^2_{e1}\left(\frac{6T'^2_{e1}}{T'_{eo}}-10T'_{e2x}\right)+T'^4_{e1}\right]+2D(T'_{e2x}+T'_{e2y})T'^4_{eo}\times\left[\frac{2(1+T'_{eo})^2}{T'_{eo}}\left(2T'_{e2x}-\frac{3T'^2_{e1}}{T'_{eo}}\right)+(1+T'_{eo})\left(2T'_{e2x}-\frac{8T'^2_{e1}}{T'_{eo}}\right)-T'^2_{e1}\right]=\sum\frac{r_o^2}{r_j^2f_{jx}^2}\left(1-\frac{2x_o^2F_j^2}{r_j^2f_{jx}^2}\right)\frac{\alpha_jE_{jo}^2}{f_{jx}f_{jy}}\exp\left(-\frac{x_o^2F_j^2}{r_j^2f_{jx}^2}\right), \tag{22c}$$

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$$T'_{e2y} + DT'^{3}_{eo}T'_{e2y}(1+T'_{eo}) \times \left[(1+T'_{eo})\left(8T'_{e2x} + 18T'_{e2y} - \frac{7.5T'^{2}_{e1}}{T'_{eo}}\right) + 4T'_{eo}(T'_{e2x}+T'_{e2y}) - 5T'^{2}_{e1}\right] = \sum \frac{r_o^2}{r_j^2 f_{jy}^2}\frac{\alpha_j E_{jo}^2}{f_{jx}f_{jy}} \exp\left(-\frac{x_o^2 F_j^2}{r_j^2 f_{jx}^2}\right), \tag{22d}$$

where

$$D = \left(\frac{\chi_o}{3k_B \delta\nu_{eio} N_{eoo} r_o^2}\right),$$

$$T'_{eo} = \frac{T_{eo}}{T}, \quad T'_{e1} = \frac{T_{e1}}{T}, \quad T'_{e2x} = \frac{T_{e2x}}{T}, \quad T'_{e2y} = \frac{T_{e2y}}{T}.$$

The parameters T'_{eo}, T'_{e1}, T'_{e2x} and T'_{e2y} may be evaluated from the above four coupled equations.

Substituting for N_e from (1) into (16), and using (20) for T_e, one obtains

$$\epsilon_{oj} = 1 - \frac{\omega_{po}^2}{\omega_j^2}\frac{2}{(1+T'_{eo})}, \tag{23a}$$

$$\epsilon_{1j} = \frac{\omega_{po}^2}{\omega_j^2}\frac{2T'_{e1}}{r_o(1+T'_{eo})^2}, \tag{23b}$$

$$\epsilon_{2xj} = \frac{\omega_{po}^2}{\omega_j^2}\frac{2}{r_o^2(1+T'_{eo})^2}\left[T'_{e2x} + \frac{T'^{2}_{e1}}{(1+T'_{eo})}\right] \tag{23c}$$

and

$$\epsilon_{2yj} = \frac{\omega_{po}^2}{\omega_j^2}\frac{2T'_{e2y}}{r_o^2(1+T'_{eo})^2}, \tag{23d}$$

where $\epsilon_{oj}, \epsilon_{1j}, \epsilon_{2xj}$ and ϵ_{2yj} are defined by (3) and the dependence of T_{eo}, T_{e1}, T_{e2x} and T_{e2y} on the field of the beams is given by (22).

2.4. Evaluation of f_{jx}, f_{jy} and F_j ($j=1,2$)

Equations (13a)–(13f) can be numerically solved for f_{jx}, f_{jy} and F_j ($j=1,2$) by substituting for the components of the dielectric function ϵ from (23a)–(23d) and for components of T_e as given by (22a)–(22d). The boundary conditions are $F = f_x = f_y = 1$ at $\xi = 0$ (as per the definition) and $(dF/d\xi) = (df_x/d\xi) = (df_y/d\xi) = 0$ at $\xi = 0$ (for parallel identical beams with parallel wavefront).

3. Numerical results and discussion

As discussed in the introduction, two parallel beams in a nonlinear plasma will move towards each other when the resulting irradiance distribution of the two beams has a maximum in the space between the two beams; this condition corresponds to the distance between the two beams $2x_o < wr_o$, where r_o and sr_o are the widths of the two beams and w is a function of r_o and the ratio of the irradiances of the two beams k. The dependence of w on s and k is illustrated in Fig. 1. It is seen that w is an increasing function of k and s; however, the paraxial theory presented in this paper may not be valid for all the values of w thus obtained.

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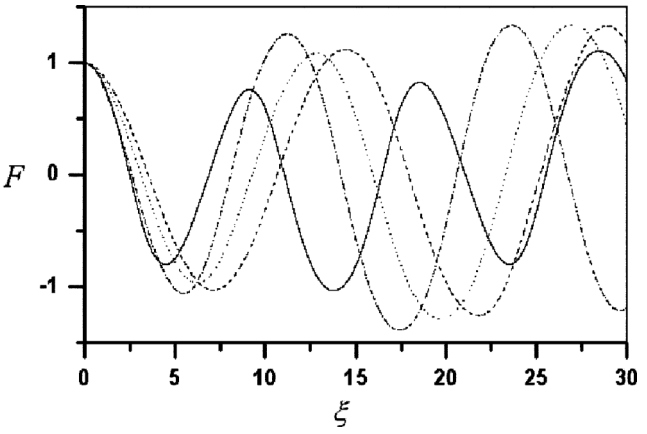


Figure 2. Dependence of the parameter F, representing the axial separation of each beam from the z-axis, with distance of propagation $\xi = (c/\omega_1 r_1^2)z$ for different values of the parameter $D = \chi_o/(3k_B\delta v_{eio}N_{eoo}r_o^2)$, corresponding to $\alpha_1 E_{100}^2 = \alpha_2 E_{200}^2 = 5, \Omega = 0.8, p = 1, s = 1, r_1^2 = r_2^2 = r_o^2, \rho_1^2 = 10$ and $\mu = 0.3$. The variations of F for $D = 0, D = 0.01, D = 0.1$ and $D = 1$ are respectively shown by solid, dashed, dotted and dash-dotted curves.

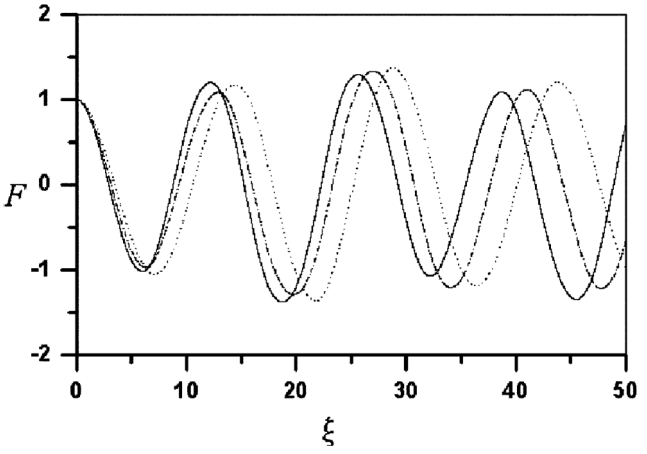


Figure 3. Variation of the axial separation F of each beam from the z-axis with distance of propagation $\xi = (c/\omega_1 r_1^2)z$ for various axial irradiances αE_{oo}^2 of the beams, corresponding to $D = 0.1, \Omega = 0.8, p = 1, s = 1, r_1^2 = r_2^2 = r_o^2, \rho_1^2 = 10$ and $\mu = 0.3$. The variations of F for $\alpha_1 E_{100}^2 = \alpha_2 E_{200}^2 = 3, \alpha_1 E_{100}^2 = \alpha_2 E_{200}^2 = 5$ and $\alpha_1 E_{100}^2 = \alpha_2 E_{200}^2 = 10$ are respectively shown by solid, dashed and dotted curves.

Figures 2–6 correspond to incoherent beams of the same frequency, width and axial irradiance. Figure 2 shows the dependence of F ($x_o F$ being the distance of the axis of the beam from the z-axis) on D, the parameter which characterizes the ratio of the electron energy loss by thermal conduction to that by collisions. It is seen that the distance between the two consecutive points where the axes of the beams intersect each other increases with increasing D. This can be readily explained by the fact that increasing thermal conductivity leads to enhanced energy

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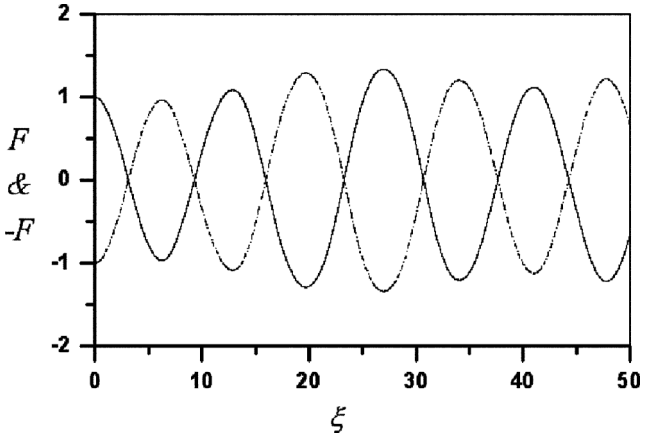


Figure 4. Variation of the parameters F and $-F$ representing the axial separation from the z-axis (or axes) of the two beams with distance of propagation $\xi = (c/\omega_1 r_1^2)z$ corresponding to $\alpha_1 E_{100}^2 = \alpha_2 E_{200}^2 = 5, \Omega = 0.8, p = 1, s = 1, r_1^2 = r_2^2 = r_0^2, \rho_1^2 = 10, D = 0.1$ and $\mu = 0.3$. The variations of F and $-F$ are respectively represented by solid and dashed curves.

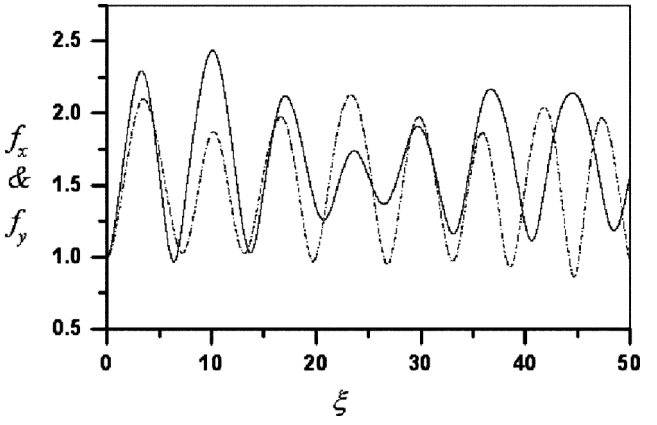


Figure 5. Variation of f_x and f_y representing the beam width parameters corresponding to the x-axis and y-axis respectively with distance of propagation $\xi = (c/\omega_1 r_1^2)z$ corresponding to $\alpha_1 E_{100}^2 = \alpha_2 E_{200}^2 = 5, \Omega = 0.8, p = 1, s = 1, r_1^2 = r_2^2 = r_o^2, \rho_1^2 = 10, D = 0.1$ and $\mu = 0.3$. The variations of f_x and f_y are respectively shown by solid and dashed curves.

loss by ohmically heated electrons, and consequently lower electron temperature and associated nonlinearity.

Furthermore, it is also seen that $D \propto 1/r_o^2$, which implies that thermal conduction becomes more important when r_o is small, corresponding to larger thermal gradient.

Figure 3 illustrates the dependence of the parameter F on the axial irradiance of the two beams (which is taken as equal for both beams). It is seen that the distance between two consecutive points of intersection of the two beams increases with increasing irradiance of the beams. This can be understood by the fact that the focusing term (decreasing F, corresponding to bringing the beams closer) is

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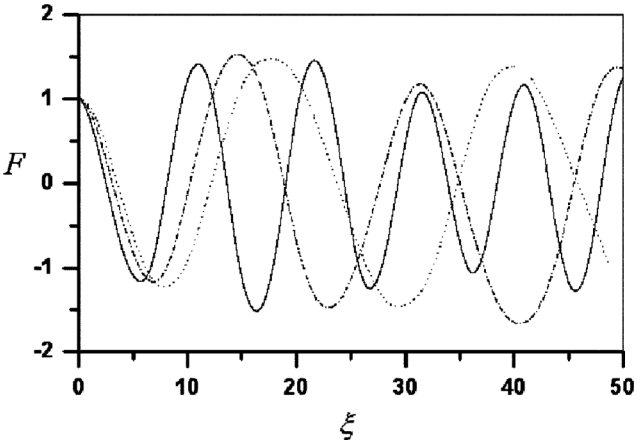


Figure 6. Dependence of the axial separation F of each beam from the z-axis on distance of propagation $\xi = (c/\omega_1 r_1^2)z$ for various $\mu = (x_o^2/r_o^2)$ of the beams, corresponding to $\alpha_1 E_{100}^2 = \alpha_2 E_{200}^2 = 5, \Omega = 0.8, p = 1, s = 1, r_1^2 = r_2^2 = r_o^2, \rho_1^2 = 10$ and $D = 0.1$. The variations of F for $\mu = 0.1, \mu = 0.3$ and $\mu = 0.4$ are respectively shown by solid, dashed and dotted curves.

proportional to the first derivative of the nonlinear dielectric function (with respect to the irradiance) rather than the nonlinearity.

Figure 4 is a representation of the axes of the two beams; for the specific case of equal irradiance, width and frequency of the two incoherent beams, the axes are mirror reflections of each other in the yz-plane.

Figure 5 indicates that f_x and f_y are significantly different as ξ increases, which corresponds to the symmetric Gaussian beam turning into an elliptical Gaussian one. Furthermore, the difference in the position of the maximum and minimum of f_x and f_y increases with increasing ξ (at least up to $\xi \approx 40$), beyond which it should decrease and show a periodic behavior.

Figure 6 shows the dependence of F on the initial separation of the beams; it can be seen that the interaction of the beams becomes stronger as they get closer, which is an obvious conclusion (qualitatively).

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