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The order-bound topology

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1. *Introduction.* Let (E, C) be a partially ordered vector space with positive cone C. The *order-bound topology* $\mathscr{P}_b$(6) (*order topology* in the terminology of Schaefer(9)) on E is the finest locally convex topology for which every order-bounded subset of E is topologically bounded.

Gordon(3) and Peressini(8) have established some relations between the relative uniform convergence and the order-bound topology. In this note, we first give, by means of the concept of the relative uniform convergence, some characterizations of linear mappings from (E, C) into a locally convex space $(F, \mathscr{T})$ which map order-bounded subsets of E into $\mathscr{T}$-bounded (Proposition 2·4), and then we obtain a result due to Gordon((3), Theorem 3·6). On the other hand, using Köthe's idea, we are able to obtain a dual characterization of the order-bound topology (Theorem 2·8). Recall that a subset V of E is said to be *solid* ((7) and (11)) if $V = \cup \{[-u, u]: 0 \leqslant u \in V\}$, and that a locally convex topology $\mathscr{P}$ on (E, C) is said to be *locally solid* if it admits a neighbourhood-base of 0 consisting of convex and solid sets in E. One of Namioka's results ((6), Theorem 5·5) can be generalized as follows: If (E, C) is a weakly Riesz space (i.e. E has the Riesz decomposition property and C is generating), then the order-bound topology $\mathscr{P}_b$ on E is the finest locally solid topology (Theorem 2·9). The final section is devoted to giving some characterizations for when the topology on a bornological ordered space is the order-bound topology.

2. *Relative uniform convergence.* We shall assume throughout this note that the scalar field for vector spaces is the field $\mathbf{R}$ of real numbers, that (E, C) is a partially ordered vector space with positive cone C, that C^* is the set of all positive linear functionals on E, and that E^b is the *order-bound dual* of E, namely E^b is the set of all order-bounded linear functionals on E.

A sequence $\{x_n\}$ in (E, C) is called a *relative uniform null-sequence* if there exist positive numbers α_n with $\alpha_n \to \infty$, such that $\{\alpha_n x_n\}$ is an order-bounded subset of E. It is clear that $\{x_n\}$ is a relative uniform null-sequence if and only if there exist positive numbers ϵ_n, with $\epsilon_n \to 0$, and an order-bounded subset B of E such that $x_n \in \epsilon_n B$ for all n. A relative uniform null-sequence $\{x\}_n$ is also said to be *relatively uniformly convergent* to 0 by other authors. For further information about the relative uniform convergence we refer the reader to (1), (3), and (8).

It should be noted that in a partially ordered vector space (E, C) we shall not assume that the cone C is generating, therefore the following result is an improvement of Gordon's result ((3), Theorem 3·4).

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LEMMA. 2·1. *Let V be a convex and symmetric subset of (E, C). Then V absorbs all order-bounded subsets of E if and only if it absorbs all relative uniform null-sequences in E.*

Proof. Let $\{x_n\}$ be a relative uniform null-sequence in E, and let $\{\alpha_n\}$ be a sequence of positive numbers with $\alpha_n \to \infty$ such that $\{\alpha_n x_n\}$ is order-bounded. Then V absorbs $\{\alpha_n x_n\}$, and so, by the boundedness of $\{1/\alpha_n\}$, V must absorb $\{x_n\}$. Conversely, if B is an order-bounded subset of E such that the assertion $B \subset n^2 V$ is false for all n, then we have $x_n \in B$ for which $x_n \notin n^2 V$, therefore V does not absorb the relative uniform null-sequence $\{x_n/n\}$.

By an *ordered convex space* we shall mean a partially ordered vector space equipped with a locally convex topology. Let $(E, C, \mathscr{P})$ be an ordered convex space. Throughout this note, E' will denote the topological dual of $(E, \mathscr{P})$, $\sigma(E', E)$ (resp. $\beta(E', E)$ and $\tau(E', E)$) will denote the weak (resp. the strong and the Mackey) topology on E', E^* will denote the algebraic dual of E, C' will denote the set of all $\mathscr{P}$-continuous positive linear functionals on E, and E^{tb} the set of all linear functionals on E which are bounded on each $\mathscr{P}$-bounded subset of E. For convenience, E^{tb} is referred to as the *topologically bounded dual* of $(E, \mathscr{P})$.

Let $(F, \mathscr{T})$ be a locally convex space, and let T be a linear mapping of (E, C) into F. Let us say temporarily that T is *locally order-bounded* if it maps each order-bounded subset of E into a $\mathscr{T}$-bounded subset of F.

We recall that the *order-bound* topology $\mathscr{P}_b$ on (E, C) is the finest locally convex topology $\mathscr{P}$ for which every order-bounded set in E is $\mathscr{P}$-bounded. Therefore $\mathscr{P}_b$ has a neighbourhood-base of 0 consisting of all convex, symmetric subsets of E which absorbs all order-bounded subsets of E. It should be noted that the order-bound topology need not be Hausdorff. For further information on the order-bound topology we refer the reader to (4), (6), (8), (9), and (10).

PROPOSITION 2·2. *Let $(E, C, \mathscr{P})$ be an ordered convex space, and let $\mathscr{P}_b$ be the order-bound topology on E. Then the following statements are equivalent.*

(1) *$\mathscr{P}$ is finer than $\mathscr{P}_b$.*

(2) *Each convex, symmetric subset of E which absorbs all relative uniform null-sequences in E is a $\mathscr{P}$-neighbourhood of 0.*

(3) *Each locally order-bounded linear mapping of $(E, C, \mathscr{P})$ into any locally convex space $(F, \mathscr{T})$ is continuous.*

The proof is straightforward and will be omitted.

There is an immediate consequence of the preceding result:

COROLLARY 2·3. *Let $(E, C, \mathscr{P})$ be an ordered convex space. Then the following statements are equivalent.*

(1) *$\mathscr{P}$ is the order-bound topology $\mathscr{P}_b$.*

(2) *Each convex and symmetric subset of E which absorbs all relative uniform null-sequences in E is a $\mathscr{P}$-neighbourhood of o, and each order-bounded subset of E is $\mathscr{P}$-bounded.*

(3) *Every locally order-bounded linear mapping of $(E, C, \mathscr{P})$ into any locally convex space $(E, \mathscr{T})$ is continuous, and each order-bounded subset of E is $\mathscr{P}$-bounded.*

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We recall that a sequence $\{y_n\}$ in a locally convex space $(F, \mathscr{T})$ is called a local *null-sequence* if there exists positive numbers α_n, with $\alpha_n \to \infty$, such that $\{\alpha_n y_n\}$ is a $\mathscr{T}$-bounded subset of F.

PROPOSITION 2·4. *Let (E, C) be a partially ordered vector space, let $(F, \mathscr{T})$ be a locally convex space, and let T be a linear mapping of E into F. Then the following statements are equivalent.*

(1) *T is locally order-bounded.*

(2) *T maps every relative uniform null-sequence in E into a local null-sequence in F.*

(3) *T maps every relative uniform null-sequence in E into a sequence which is convergent to 0 with respect to $\mathscr{T}$.*

(4) *T maps every relative uniform null-sequence in E into a $\mathscr{T}$-bounded subset of F.*

Proof. Let $\{x_n\}$ be a relative uniform null-sequence in E. There exist positive numbers α_n, with $\alpha_n \to \infty$, and an order-bounded subset B of E such that $\alpha_n x_n \in B$ for all n. Suppose that T is locally order-bounded. Then $T(B)$ is a $\mathscr{T}$-bounded subset of F, and so $\{T(x_n)\}$ is a local null-sequence in F, this proves the implication (1) $\Rightarrow$ (2). The implications (2) $\Rightarrow$ (3) $\Rightarrow$ (4) are obvious. It remains to show that (4) implies (1). Suppose that T is not locally order-bounded. Then there exists an order-bounded subset B of E such that $T(B)$ is not $\mathscr{T}$-bounded. Let V be a convex and symmetric $\mathscr{T}$-neighbourhood of 0 in F such that the assertion $T(B) \subset n^2 V$ is false for all n. For each n, let x_n, in B, be such that $T(x_n) \notin n^2 V$. Then we have found a relative uniform null-sequence $\{x_n/n\}$ in E such that $\{T(x_n/n)\}$ is not $\mathscr{T}$-bounded, this gives a contradiction. The proof is complete.

COROLLARY 2·5 (Gordon ((3), Theorem 3·6)). *Let (E, C) be a partially ordered vector space, and let f be in E^*. Then $f \in E^b$ if and only if $f(x_n) \to 0$ whenever $\{x_n\}$ is a relative uniform null-sequence in E.*

It is easily seen that each relative uniform null-sequence in E is convergent to 0 with respect to the order-bound topology $\mathscr{P}_b$. Then the following result is an immediate consequence of Propositions 2·2 and 2·4.

COROLLARY 2·6. *Let $\mathscr{P}_b$ be the order-bound topology on (E, C), and let $(F, \mathscr{T})$ be a locally convex space. A linear mapping T of $(E, C, \mathscr{P}_b)$ into $(F, \mathscr{T})$ is continuous if and only if it is sequentially continuous, and if and only if it satisfies one of the conditions* (1)–(4) *of Proposition* 2·4.

The preceding Corollary was proved by Peressini ((8), IV, 2·3) in the special case when E is a Riesz space and F is the real line $\mathbf{R}$.

Let (F, P) be a partially ordered vector space, and let T be a linear mapping of (E, C) into (F, P). T is said to be *positive* if $T(C) \subset P$. If $\mathscr{T}$ is a locally convex topology on F such that each order-bounded subset of F is $\mathscr{T}$-bounded, then each positive linear mapping of E into $(F, P, \mathscr{T})$ is locally order-bounded. Therefore by making use of Corollary 2·6 and a result due to Namioka ((6), Theorem 5·4), we obtain:

COROLLARY 2·7 (Namioka and Nachbin ((6), Corollary 5·5)). *Let $\mathscr{P}$ be a metrizable locally convex topology on (E, C) such that E is of the second category and C is $\mathscr{P}$-complete*

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and generating. Suppose that $\mathscr{T}$ is a locally convex topology on a partially ordered vector space (F, P) such that each order-bounded subset of F is $\mathscr{T}$-bounded; in particular, $\mathscr{T}$ is the order-bound topology. Then each positive linear mapping of $(E, C, \mathscr{P})$ into $(F, P, \mathscr{T})$ is continuous.

Let $(E, C, \mathscr{P})$ be an ordered convex space such that each order-bounded subset of E is $\mathscr{P}$-bounded. Then each relative uniform null-sequence $\{x_n\}$ in E is $\mathscr{P}$-bounded, and so the polar, taken in E', of the set $\{x_n\}$ is absorbing; we denote the topology of uniform convergence on all relative uniform null-sequences in E by $\mathfrak{T}'_{r_0}$.

We shall employ Köthe's argument ((5), section 28, 5(4)) to prove a dual characterization of the order-bound topology.

THEOREM 2·8. *Let $(E, C, \mathscr{P})$ be an ordered convex space for which C is $\mathscr{P}$-closed and generating. Then $\mathscr{P}$ is the order-bound topology $\mathscr{P}_b$ if and only if $\mathscr{P}$ satisfies the following conditions*:

(i) *$\mathscr{P}$ is the Mackey topology $\tau(E, E')$,*

(ii) *each order-bounded subset of E is $\mathscr{P}$-bounded, and*

(iii) *E' is $\mathfrak{T}'_{r_0}$-complete.*

Proof. Necessity. It is clear that (i) and (ii) are satisfied by $\mathscr{P}_b$. It remains to verify that E' is $\mathfrak{T}'_{r_0}$-complete. Let f_τ be a $\mathfrak{T}'_{r_0}$-Cauchy net in E'. Then f_τ is a $\sigma(E', E)$-Cauchy net since $\{x/n\}$ is a relative uniform null-sequence in E for any $x \in E$, and so there exists $f \in E^*$ such that f_τ converges to f pointwise on E. Let $\{x_n\}$ be any relative uniform null-sequence, and let V be the set consisting of $\{x_n\}$. Then f_τ converges to f uniformly on V, and each f_τ is bounded on V by Proposition 2·4, so that f is bounded on V; it then follows from $E' = E^b$, that $f \in E'$, and hence that f_τ converges to f with respect to $\mathfrak{T}'_{r_0}$. Therefore E' is $\mathfrak{T}'_{r_0}$-complete.

Sufficiency. The condition (ii) implies that $E' \subseteq E^b$. We now show that $E^b \subseteq E'$. Suppose that $\{x_n\}$ is a relative uniform null-sequence in E, and that V is the convex symmetric hull of $\{x_n\}$. Since E' is $\mathfrak{T}'_{r_0}$-complete, by Grothendieck's completeness theorem, it is enough to show that the restriction of each $f \in E^b$ to the set $\overline{V}$ is $\sigma(E, E')$-continuous. In fact, since C is generating, there exist positive numbers λ_n, with $\lambda_n \to 0$, and $e \in C$ such that $x_n \in \lambda_n[-e, e]$, and so $x_n \in \mu[-e, e]$ for some $\mu > 0$ because of $\lambda_n \to 0$, consequently $\overline{V} \subset \mu[-e, e]$ since C is $\mathscr{P}$-closed. On the other hand, suppose that $E_e = \bigcup_n n[-e, e]$, and that $\|\,.\,\|$ is the gauge of $[-e, e]$ in E_e. Then $\|\,.\,\|$ is an order-unit norm in E_e, and $[-e, e]$ is the closed unit ball of $(E_e, \|\,.\,\|)$, i.e. $[-e, e] = \{x \in E_e : \|x\| \leqslant 1\}$. Observe that V is a precompact subset of $(E_e, \|\,.\,\|)$, and that $[-e, e]$ is $\sigma(E', E')$-closed, it is known from Köthe ((5), §28, 5(2)) that the norm topology $\|\,.\,\|$ and the topology $\sigma(E, E')$ coincide on V, and hence that the $\|\,.\,\|$-closure of V in E_e and the $\sigma(E, E')$-closure of V in E_e are the same by (5), §28, 5(3), consequently $\overline{V}$ is the $\|\,.\,\|$-closure of V in E_e. Therefore $\overline{V}$ is a precompact subset of $(E, \|\,.\,\|)$, and the norm topology $\|\,.\,\|$ and the topology $\sigma(E, E')$ coincide on $\overline{V}$. Finally, f(in E^b) is bounded on $[-e, e]$, the restriction of f to E_e is a continuous linear functional on $(E_e, \|\,.\,\|)$, and so the restriction of f to $\overline{V}$ is $\|\,.\,\|$-continuous, and *a fortiori* $\sigma(E, E')$-continuous. This completes the proof.

Let V be a subset of (E, C), and let

$$S(V) = \bigcup\{[-u, u] : 0 \leqslant u \in V\}.$$

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We say that V is *solid* if $S(V) = V$. A locally convex topology $\mathscr{P}$ on E is said to be *locally solid* if $\mathscr{P}$ admits a neighbourhood-base of 0 consisting of convex and solid sets in E, and $(E, C, \mathscr{P})$, equipped with a locally solid topology $\mathscr{P}$, is called a *locally solid space*. It is clear that if E is a Riesz space then V is solid if and only if the following property is satisfied:

$$|y| \leqslant |x| \quad \text{with} \quad x \in V \quad \text{then} \quad y \in V,$$

where $|x|$ denotes the absolute value of x. For further information about solid sets we refer the reader to (11) and (7).

Choquet(2) calls a partially ordered vector space (E, C) a *weakly Riesz space* if C is generating and E has the Riesz decomposition property, i.e.

$$[0, u+w] = [0, u] + [0, w]$$

whenever u and w are in C.

THEOREM 2·9. *Let (E, C) be a weakly Riesz space. Then the order-bound topology $\mathscr{P}_b$ on E is the finest locally solid topology.*

Proof. The statement that $\mathscr{P}_b$ is locally solid can be deduced from Schaefer's result (cf. (9)), Jameson's result ((4), (3.10.2)), and the author's result ((11), Theorem 3·1); but we present here a more direct and elementary proof. Let V be a convex and symmetric $\mathscr{P}_b$-neighbourhood of 0. We wish to find a convex, solid $\mathscr{P}_b$-neighbourhood U of 0 such that $U \subset V$. Let $S = \cup \{[-u, u] : u \in C \text{ and } [0, u] \subseteq V\}$. If $x, y \in S$ and if $0 \leqslant \lambda \leqslant 1$, there exist $u, w \in C$ with $[0, u] \subseteq V$ and $[0, w] \subseteq V$, such that $-u \leqslant x \leqslant u$ and $-w \leqslant y \leqslant w$, and so $-[\lambda u + (1-\lambda) w] \leqslant \lambda x + (1-\lambda) y \leqslant \lambda u + (-\lambda) w$; by the Riesz decomposition property, we have

$$\begin{aligned}[0, \lambda u + (1-\lambda) w] &= [0, \lambda u] + [0, (1-\lambda) w] \\ &= \lambda [0, u] + (1-\lambda) [0, w] \subseteq \lambda V + (1-\lambda) V = V\end{aligned}$$

and so $\lambda x + (1-\lambda) y \in S$. Thus S is convex. Clearly S is solid. On the other hand, if $x \in S$, there exists $u \in C$, with $[0, u] \subseteq V$, such that $-u \leqslant x \leqslant u$; it then follows that $\frac{1}{2}(x+u)$ and $\frac{1}{2}(u-x)$ are in $[0, u]$, and hence from $x = \frac{1}{2}(x+u) - \frac{1}{2}(u-x)$ that $x \in 2V$, consequently $S \subseteq 2V$. Clearly $\{u \in C : [0, u] \subseteq V\} \subseteq S \cap C$. Suppose that $x \in S \cap C$. Then there exists $u \in C$, with $[0, u] \subseteq V$, such that $-u \leqslant x \leqslant u$, and so

$$[0, x] \subseteq [0, u] \subseteq V, \quad \text{that is} \quad x \in \{u \in C : [0, u] \subseteq V\}.$$

Therefore $S \cap C = \{u \in C : [0, u] \subseteq V\}$. Finally, we show that S absorbs all elements in C; from this S absorbs all order-bounded subsets of E since C is generating and S is solid. Suppose not, there exists $x \in C$ such that $x \notin nS$ for all n, it then follows from

$$S \cap C = \{u \in C : [0, u] \subseteq V\} \quad \text{that} \quad [0, x] \subset nV$$

is false for all n, this gives a contradiction because V absorbs all order-bounded subsets of E. Therefore S is a convex, solid $\mathscr{P}_b$-neighbourhood of 0 for which $S \subseteq 2V$, and so $U = \frac{1}{2}S$ has the desired property.

Since each order-bounded subset of E is bounded with respect to any locally solid topology on E, it follows that $\mathscr{P}_b$ is the finest locally solid topology. This completes the proof.

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The preceding result was proved by Naimoka ((6), Theorem 8·5) in the special case when E is a Riesz space (i.e. vector lattice).

3. *Relationships between the order-bound topology and the topology on a bornological ordered space.* Let $(E, C, \mathscr{P})$ be an ordered topological space. A semi-norm p on E is said to be *locally bounded* (resp. *locally order-bounded*) if p is bounded on each $\mathscr{P}$-bounded (resp. on each order-bounded) subset of E. Each subset of C is said to be *positive.*

The following result is a generalization of parts of (10), Theorem 7.

PROPOSITION 3·1. *Let $(E, C, \mathscr{P})$ be an ordered convex space for which every order-bounded subset of E is $\mathscr{P}$-bounded. If $(E, \mathscr{P})$ is bornological, then the following statements are equivalent.*

(1) *$\mathscr{P}$ is the order-bound topology $\mathscr{P}_b$.*

(2) *Each locally order-bounded semi-norm on E is locally bounded.*

(3) *Each positive $\sigma(E^b, E)$-bounded subset of E' is $\mathscr{P}$-equicontinuous and $E^b \subseteq (E, \mathscr{P})'$.*

Proof. It is clear that (1) is equivalent to $E^b \subseteq E'$; and so (1) and (2) are trivially equivalent. On the other hand, it is easy to see that (3) implies (1). It remains to verify that (2) implies (3). We first show that $E^b \subseteq (E, \mathscr{P})'$. For any $f \in E^b$, we write

$$p_f(x) = |f(x)| \quad (x \in E).$$

Then p_f is a locally order-bounded semi-norm on E, and so p_f is locally bounded. From this it follows that $V = \{x \in E : p_f(x) < 1\}$ absorbs all $\mathscr{P}$-bounded subsets of E. Since $(E, \mathscr{P})$ is bornological, then V is a $\mathscr{P}$-neighbourhood of 0, and thus $f \in (E, \mathscr{P})'$.

On the other hand, let B be any positive $\sigma(E^b, E)$-bounded subset of E^b. We claim that $U = (-B \cup B)^0$ absorbs all order-bounded subsets of E. Suppose that M is any order-bounded subset of E, and that $M \subset [a, b]$ for some a, b in E with $a \leqslant b$. Since B is $\sigma(E^b, E)$-bounded, there exists $\lambda > 0$ such that $B \subset \lambda(\{-a, b\})^0$; it follows from the positivity of B that $-B \cup B \subseteq \lambda[a, b]^0 \subseteq \lambda M^0$, and hence that $\lambda U \supseteq M$. Let p be the gauge of U. Then p is a locally order-bounded semi-norm on E, and so p is locally bounded, consequently $\{x \in E : p(x) < 1\}$ is a $\mathscr{P}$-neighbourhood of 0 because $(E, \mathscr{P})$ is bornological. Observe that

$$\{x \in E : p(x) < 1\} \subset U = (-B \cup B)^0.$$

Then $-B \cup B$ is $\mathscr{P}$-equicontinuous; in particular, B is $\mathscr{P}$-equicontinuous.

The following result gives a connexion between the associated bornological space and the order-bound topology.

COROLLARY 3·2. *Let $(E, C, \mathscr{P})$ be an ordered convex space for which each order-bounded subset of E is $\mathscr{P}$-bounded. Then the following statements are equivalent.*

(1) *$E^{tb} = E^b$.*

(2) *The Mackey topology $\tau(E, E^{tb})$ is the order-bound topology.*

(3) *Each locally order-bounded semi-norm on E is locally bounded.*

(4) *Each positive $\sigma(E^b, E)$-bounded subset of E^b is $\tau(E, E^{tb})$-equicontinuous and*

$$E^b \subseteq E^{tb}.$$

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Suppose that $(E, \mathscr{P})$ is a locally convex space with the topological dual E'. Then the topology, on E', of uniform convergence on all local null-sequences in E is denoted by $\mathfrak{T}'_{c_0}$.

PROPOSITION 3·3. *Let $(E, C, \mathscr{P})$ be an ordered convex space for which C is $\mathscr{P}$-closed and generating. Suppose that $(E, \mathscr{P})$ is bornological and that each order-bounded subset of E is $\mathscr{P}$-bounded. If each $\mathfrak{T}'_{r_0}$-Cauchy net in E' is $\mathfrak{T}'_{c_0}$-Cauchy net, then $\mathscr{P}$ is the order-bound topology $\mathscr{P}_b$.*

Proof. By making use of Theorem 2·8, we only have to show that E' is $\mathfrak{T}_{r_0}$-complete. Suppose that $\{f_\tau\}$ is a $\mathfrak{T}'_{r_0}$-Cauchy net. Then it is a $\mathfrak{T}'_{r_0}$-Cauchy net. Since $(E, \mathscr{P})$ is bornological, it follows from Köthe ((5), 28, 5(1)) that E' is $\mathfrak{T}'_{c_0}$-complete. There exists f in E' such that f_τ converges to f with respect to $\mathfrak{T}'_{c_0}$. On the other hand, since each order-bounded subset of E is $\mathscr{P}$-bounded, then $\mathfrak{T}'_{r_0}$ is coarser then $\mathfrak{T}'_{c_0}$, and so f_τ converges to f with respect to $\mathfrak{T}'_{r_0}$. This completes the proof.

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