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Symposium on Models of Path Dependence

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The symposium develops statistical models and methods for the study of path dependence. In this introductory essay, the connections between key areas in the path dependence and statistical literatures are illuminated. And some ways in which familiar time series and regression models embody these ideas are explained. The arguments in the articles in the symposium then are summarized and compared. Finally, directions for additional, statistically grounded research on path dependence are discussed.

1 Introduction

The term "path dependence" has become ubiquitous in the social sciences. Page (2006, 88) provides an excellent summary of the range of disciplines and settings in which path dependence is invoked to describe some phenomenon, going back to David's (1985) explanation for the persistence of the QWERTY keyboard. Path dependence often is used to mean history matters. But, history always matters. The issue is which aspect of history and how. Effective use of the term to distinguish some historical processes from others and to make predictions about the likely course of future events for path-dependent and non–path-dependent processes requires rigorous conceptual frameworks and statistical methodologies.

Unfortunately, in these regards, progress in developing a rigorous definition of path dependence in all its variants has been uneven. Page (2006, 88) comments that, ". . . the concept of path dependence seems almost metaphorical." Historical sociologists like Mahoney (2000) clarified the importance of initial conditions—"contingent events"—in path-dependent processes. But Mahoney proposed narrative methods for discerning path dependence in observed histories. Bennett and Elman (2006), in the pages of *Political Analysis*, developed a more rigorous qualitative methodology for the study of path dependence. They explained how techniques like process tracing and case selection can help illuminate path dependency. But, like Mahoney, they restricted their attention to narratives not to time series data.

In an effort to render the literature more systematic, Page (2006) clarified the nature of path dependence and related it to key ideas in the study of system dynamics and human decision making. Page's insightful analysis and examples, however, are based on discrete and unique stochastic processes with simple rules for changing states, more specifically, selecting and replacing balls from and into urns. Page made little progress connecting his concepts and ball-urn processes to statistical ideas or in developing a methodology for identifying different kinds of processes in social time series. More recently, Vergne and Durand (2010, 749–51) offered some suggestions about how to make the study of path dependence more "empirically tractable." But they make only passing reference to the data analysis of path-dependent processes.[1]

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[1] In addition, Vergne and Durand (2010, 746–7, 752) are confusing in the way they use the terms "structural causes," "exogenous shocks," and "shocks." It is unclear, for example, what functional forms Vergne and Durand have in mind when they refer to regression models or how (when) those functional form embody path dependence.

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This essay and the articles in this symposium aim to develop models and statistical methods for the study of path dependence. They grew out of discussions of a recent contribution by Jackson and Kollman (2010) and of a conference on the topic. The conference's first goal was to extend the formal models of path dependence beyond the discussions of drawing balls from urns such as the Polya process and of case studies, which all three articles do.[2] The second objective was to develop a statistically grounded, conceptual framework for the empirical study and testing of these models. This framework incorporates key ideas in regression, time series, and other branches of statistics and, at the same time, captures the important distinctions made by scholars like Mahoney and Page. We sought to show how well-known univariate and multivariate statistical models are (not) useful for analyzing path dependence. In what follows, we sketch each goal and then explain how the three articles published here contribute to the achievement of one or both of our objectives.[3]

2 Conceptualization

Two concepts are at the heart of the study of this topic: equilibrium dependence and outcome dependence. The concept of an equilibrium has different meanings across the various disciplines in the social sciences. A Nash equilibrium in game theory is not the same thing as the equilibrium (cointegrating) relationship between two nonstationary time series, which is not the same thing as an equilibrium in a dynamic system. In our context, an equilibrium means the limiting distribution of the outcomes of some temporal process. For discrete outcomes such as those described by the familiar Markov process or the ball-urn process discussed by Page, an equilibrium is the long-run distribution of probabilities of the different states of the world, such as the proportion of blue or yellow balls in an urn.

For a continuous outcome as observed with many time series and denoted by y_t, an equilibrium can be defined by the possible mean, variance, and lagged autocovariances of the y_t's as t becomes large. These are denoted by, respectively, $E(y_t) = \mu_t$, $E[(y_t - \mu_t)^2] = \sigma_t^2$, and $E[(y_t - \mu_t)(y_{t-j} - \mu_{t-j})] = \gamma_j$. A process has an equilibrium distribution if both μ_t and σ_t^2 are finite, if μ_t approaches a constant value μ as $t \to \infty$, and if the process is stationary, meaning that γ_j is independent of t and declines as j increases. The article by Jackson and Kollman provides more formal descriptions of this definition of an equilibrium. In a companion piece, Freeman (2011) analyzes how the concept of ergodicity as applied by students of path dependence (long-run behavior of a discrete Markov chain) and some time series statisticians (e.g., the relation between ensemble and long-run time averages) embodies the idea of equilibrium independence. He also compares the ways scholars in the two literatures use the concepts of stochastic stability and stationary (density) distribution.

Outcome dependence, as used by Page (2006), means that current realizations of y_t depend on previous realizations, $y_{t-s}, s = 1, \ldots, t$. For example, the standard first-order autoregressive (AR(1)) process exhibits outcome dependence. Outcome dependence has two forms, equilibrium independence and equilibrium dependence. In an equilibrium-independent process, the mean of the limiting distribution μ is not a function of the initial or early realizations of the process. If there are forcing or exogenous variables denoted by X_t, the limiting distribution defining an equilibrium is conditioned on a value for X, say X^*, that is reached and maintained as t increases. In an equilibrium-independent process, μ is only a function of X^* such as $\mu = X^*B$ in a linear system.

With equilibrium dependence, by contrast, the mean of the limiting distribution is also a function of some set of the previous outcomes. Most usage of the term path dependence coincides with equilibrium dependence.[4] There are several distinct kinds of equilibrium dependence. An interesting case for our purposes is when the current value of a process depends upon its initial conditions. Historical sociologists such as Mahoney (2000) and also Goldstone (1998) define path dependence in terms of the impact of

[2] Page (2006, Example 2, 98) defines the Polya process for the case in which there is one maroon (M) and one brown (B) ball: Initially M = B = 1. In any period, if a brown (respectively maroon) ball is selected, then it is put back in the urn together with an additional ball of the same color. Not only can this process converge to more than one ratio of brown and maroon balls, it can also converge to any ratio of brown and maroon balls.

[3] The complete list of participants and paper titles can be found on the polmeth server (http://polmeth.wustl.edu/conferences.php?i=4). Some of the original papers can be found on this site. Some revised versions of the papers can be found there as well.

[4] Page (2006) defines path dependence in a way that may include equilibrium dependence as well as equilibrium independence. We advocate restricting the use of the term path dependence to processes that are equilibrium dependent.

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initial conditions on current outcomes; "[path dependent] outcomes are related stochastically to initial conditions" (Goldstone 1998, 834). Frequently, this definition is extended to include the early outcomes of the process in addition to initial conditions. In pure path dependence, the sequence of these early outcomes affects the mean of the limiting distribution. In what Page (2006) humorously refers to as phat dependence, only the set but not the sequence of early outcomes matters.[5] As we show below, familiar time series and regression models may embody all these types of equilibrium dependence—dependence on initial conditions and path and phat dependence.[6]

To illustrate the difference between equilibrium independence and path or equilibrium dependence, recall the previous definition of a temporal process $y_t, t = 1, \ldots, T$, and its limiting equilibrium distribution. Now consider a second series of observations for y_t, a rerun of history if you will. And another series, and so on. Denote each of these histories as y_t^i, where $i = 1, \ldots, I$, giving I different histories, or what Hamilton (1994) calls an ensemble. Restrict, again, these histories to stationary processes with a limiting distribution, meaning each series has a mean, μ^i. Now denote by y_0^i the initial value and by $y_1^i, y_2^i, \ldots, y_s^i$ the outcomes for the first s periods in series i. In an equilibrium-independent process, as described above, each series' equilibrium limiting distribution has the same mean and variance, denoted by $\mu^i = \mu$ and $\sigma^{2,i} = \sigma^2$ for all i. In an equilibrium- or path-dependent process, the mean and (or) variance of the limiting distributions will be different, $\mu^i \neq \mu^j$, and (or) $\sigma^{2,i} \neq \sigma^{2,j}, i \neq j$. Further, each mean μ^i will be a function of $y_0^i, y_1^i, y_2^i, \ldots, y_s^i$. In a sequence-dependent process, the precise sequence $y_1^i, y_2^i, \ldots, y_s^i$ matters, whereas in a phat-dependent process, the set of previous y values matter but not their order.

In outcome-dependent processes without a single limiting distribution, current values of y_t embody some or all of its history; the set or sequence of past outcomes will have an impact on y_t. But when such processes do not have a limiting distribution, they are not stationary processes and therefore do not have an equilibrium as we define the term here. A random walk is a classic example of such an outcome-dependent but nonequilibrating process.

These same concepts and definitions apply in a multivariate context that includes exogenous (forcing) variables, X. That is, in both equilibrium independence and dependence, the mean of the limiting distribution will be a function of the steady-state values of X, denoted by X^*. The difference is that in the equilibrium-dependent process, the means will be functions of the initial and/or early outcomes as well as of X^*, meaning $\mu^i = F(X^*, y_0^i, y_1^i, \ldots, y_s^i)$. This contrasts with an equilibrium-independent process where $\mu^i = \mu = F(X^*)$. Again, Jackson and Kollman offer more formal descriptions of the difference between equilibrium independence and dependence.

These two concepts, those of an equilibrium as a limiting distribution of a stationary process with a fixed mean and finite variance and that of outcome dependence in its two different forms, are at the core of the formal discussions and models of path dependence presented here. Bednar, Page, and Toole (2012) model the impact on the limiting distribution if it is possible to revise or alter some previous outcome. Franzese, Hays, and Kachi (2012) analyze path dependence in the context of a Markov type network model. Jackson and Kollman examine the equilibrium properties of different nonlinear dynamic models to distinguish more clearly and precisely the differences between various equilibrium-independent and -dependent processes.

3 Statistical Evidence

Data analysis that distinguishes among the different outcome- and equilibrium-dependent processes is as important as conceptualizing them. Without sound data analysis, one stands a high probability of choosing the wrong statistical model and of drawing incorrect inferences about the nature of dynamic processes. The challenge, as always, is to identify the true data-generating process and to interpret it correctly.

[5] The common urn process discussed by Arthur, Ermoliev, and Kaniovski (1987) and Page (2006) where a ball is drawn and then is returned to the urn along with another ball of the same color is an example of a phat- but not path-dependent process.

[6] Page's Founder Process—a ball-urn process in which one of two balls is chosen, replaced, and then the other ball is removed—illustrates early outcome dependence. Page's Polya process and Burden of History processes illustrate phat and path outcome dependence, respectively.

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Time series analysis expressly focuses on historical dependence—on "unpacking historical causality" (Page 2006). Freeman's (2011) companion essay on time series analysis and the detection of path dependence, summarized here and available on the conference Web site, addresses this problem. He analyzes, for instance, how different time series models capture dependence on initial conditions. His investigation is based on a four-fold classification of time series models, linear and nonlinear, and univariate and multivariate. The Appendix to this Introduction contains Table A1 that summarizes his arguments.[7]

One important distinction among these time series models in the table is whether the model is weakly or strongly restricted. The weaker the restrictions in the model, the more the researcher is letting the data select a functional form as well as the parameters. Conversely, the stronger the restrictions the more structure that is being imposed on the process and the data by the researcher. A vector autoregression is an example of a weakly restricted model. The models of Jackson and Kollman (2010, 2012) of path dependence are examples of strongly restricted models. Heckman (1991) discusses this distinction in the context of distinguishing between the effects of initial conditions and heterogeneity in duration models. He concludes that externally imposed restrictions, such as a functional form, are required. But that may not always be the case. The decision will be influenced by the data available as the fewer the restrictions the richer and more informative the data must be to develop results with useful confidence levels.

Freeman's essay shows that some weakly restricted, linear time series models embody dependence on the sequence of most recent outcomes and also a form of equilibrium independence. Another model, the random walk, captures the effects of initial or early conditions and phat dependence. Because its distribution varies *in time*, the random walk embodies not just initial condition and phat outcome dependence but also some of the characteristics of equilibrium dependence. Unit root tests therefore can yield insights about these elements of path dependence. On the other hand, there are concepts in the study of linear time series models that appear to have no clear parallel in the path dependency literature, in particular, the idea of a (correction to) moving equilibrium in phat-dependent processes (cointegration).

Nonlinear time series models exhibit forms of outcome and equilibrium dependence. For example, threshold autoregressive (TAR) models that entail switching between rates of persistence or between two different univariate AR(1) processes can connote sequential outcome independence but still embody a form of equilibrium independence: geometric ergodicity. A band-TAR, on the other hand, for a period of time, can switch into a random walk process and hence into phat outcome-dependent and equilibrium-dependent modes. The notion of switches between different paths of adjustment to a moving equilibrium in phat-dependent processes is captured by nonlinear error correction models. Nonlinear regression models where the autoregressive parameters vary temporally can exhibit equilibrium dependence under certain conditions. The implication is that existing time series methods give us the tools to study a wide range of concepts associated with the idea of path dependence. Analysts simply must be clear about which concepts are embodied in each model.

A second topic in time series analysis is the analysis of pooled time series, cross-sectional data (TSCS) where there are longitudinal data, such as monthly, yearly, or biannually, on a number of different units, such as election districts, states, or countries. Denote by y_{ti} the outcome in period t, $t = 1, \ldots, T$, for unit i, $i = 1, \ldots, I$. These outcomes are then functions of previous values for y and any exogenous variables X_{ti}. There is an important parallel between TSCS models and the concept of an ensemble used earlier to define an equilibrium distribution and to discuss the distinctions between equilibrium-independent and equilibrium-dependent processes. The correspondence is not exact as the realizations for y_{ti} for the different units may not be generated by the same process as was assumed in the case of ensembles. These units differ in their values for X_{ti} and not just in their initial values and early outcomes. Consequently, the mean of the limiting distributions will be different for each unit for reasons other than equilibrium dependence. But proper specification of the variables that comprise X_{ti} and of the form of the model will make these equilibrium mean values conditional on the X's and enable one to infer whether differences in these equilibrium means are related to initial and early conditions as well as to recent and current values for X_{ti}. If they are, the results are consistent with an outcome-dependent process. Pooled TSCS data are central to the analyses in two of the symposium's papers.

[7] Freeman (2011) contends that although there are clear parallels between the ways students of path dependence and time series statisticians conceive and analyze various forms of outcome dependence, there are some important differences in how they analyze equilibrium dependence. Hence, the last column in his table uses the term "limiting behavior."

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4 Symposium Content

In the previous article, Jackson and Kollman (2010) articulate some necessary conditions for path dependence in single equation, multivariate dynamic regression models. Here they extend their analysis to distinguish between equilibrium, or path, dependence and equilibrium independence. They restrict path dependence to be processes that are equilibrium dependent.[8] Jackson and Kollman illustrate how equilibrium-independent processes can appear to be equilibrium dependent and vice versa. They then use time series, cross-sectional data on health expenditures in Organization for Economic Cooperation and Development countries to demonstrate a possible estimation strategy for slow-moving equilibrium-independent processes. Jackson and Kollman show that a very slow-moving, or sticky, equilibrium-independent process might be mistaken for one that is equilibrium dependent. Making the distinction between the two types of dependence requires a well-specified nonlinear dynamic model, rich time series data, and carefully selected estimation methods. In the end, Jackson and Kollman conclude that path dependence is an important feature of comparative political economy and public policy, but it is also likely rarer than most scholars realize.

Franzese, Hays, and Kachi study path dependence as a by-product of spatial interdependence. They associate path dependence with multiple steady states of a process that is composed of external and internal stimuli, contagion, and selection. They analyze multiple steady states with a discrete time Markov model of network behavior and coevolution. The important feature of their model is that individuals' states are connected to those of other individuals due to both spatial and network factors. The state transition probabilities evolve over time, satisfying one of Page's criteria for equilibrium dependence. Franzese et al. then develop an estimator for this system and show its properties with simulated data. The performance of their estimator compares very favorably with that used by proponents of the simulation investigation for empirical network analysis (SIENA) model. Franzese et al. then apply their model to great power conflict and alliances behavior, showing that the respective behaviors can be explained as a coevolutionary network. An important implication of the article by Franzese et al. is that the evolution of behavior among cross-sectional units may be path dependent under specific conditions.

Despite employing a similar logic of explaining how it emerges within a particular multivariate causal mechanism, Franzese et al. and Jackson and Kollman reach different conclusions about the prevalence of path dependence. Both sets of scholars allow for first-order temporal persistence (in the form of a first-order Markov process and the first-order autoregressive functional form). Both allow external exogenous shocks to change the nature of their causal mechanisms (to change the (c) parameters for coevolution and the level of the forcing (X) variable, respectively).[9] And both make effective use of simulation to chart social dynamics, for example, to illuminate how phat dependence occurs and how it manifests itself over time. But they employ somewhat different conceptions of path dependence, Franzese et al. as equilibrium in distributions of types and Kollman and Jackson as differences in the expected value of distributions. Also, Franzese et al. stress cross-unit processes in their modeling, whereas Jackson and Kollman emphasize temporal persistence. And, most important, their articles reach different conclusions about when path dependence emerges: Franzese et al. say conditions of pure persistence of individuals types and of past ties ($c_0 = c_1 = 1, c_2 = 1$) *eliminates* path dependence; the parallel conditions in Jackson and Kollman's mechanism, the condition of complete persistence, $\rho_t = 1$, is what produces path dependence. Finally, the former end up arguing that path dependence is relatively common, whereas the latter argue it is rare.[10]

Bednar, Page, and Toole introduce what they call revision to the analysis of equilibrium dependence. Revision means that a previous outcome can be altered or revised. This revision can change the future evolution of a process. An example is a ball already in a Polya type urn process could be "flipped" to the opposite color as a consequence of a current draw. The analogy is that a past court decision can be reinterpreted in the present, and this reinterpretation then alters future judicial decisions. Bednar et al.

[8] Recall that Page (2006) includes path dependence within his broad group of outcome-dependent processes.

[9] In fact, Franzese et al. say that shocks to their c parameters are similar to shocks in an exogenous variable in a regression equation.

[10] As readers will see, there is an important difference between the models in the articles by Franzese et al. and Jackson and Kollman. In the article by Franzese et al., outcomes are discrete binary choices, whereas in the article by Jackson and Kollman, outcomes are measures of a continuous interval variable. The difference in the type of behavior modeled by the two sets of scholars leads to substantially different specifications and applications.

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show that for processes with binary outcomes, such as the classic Polya process, even small probabilities of revision can dramatically alter the set of equilibrium outcomes and hence the possibility of path dependence. Under one condition, in the urn process, the equilibrium could be balls all of one color. In another condition, the equilibrium outcome is a set of balls the colors of which are evenly distributed. Only under a knife-edge condition does the process generate the "anything is possible" equilibrium often claimed for path-dependent processes. This is a profoundly different description of binary processes than one observes with the traditional Bernoulli or Polya models.

The analysis of Bednar et al. resembles, in some ways, those in the other two articles. Bednar et al. also start from a general causal mechanism, in their case a combination of three processes: a familiar urn process of the Polya type, a process that sometimes produces random inputs (shock), and a process that sometimes produces revision. Like Jackson and Kollman and Franzese et al., they then derive the conditions under which this causal mechanism produces path dependence. Bednar et al.'s mechanism also has temporal persistence but of a different kind. Their two main parameters, p and q, are similar to those that cause the shocks in the c parameters in the model of Franzese et al. and in the X variable in the model of Jackson and Kollman. Both Bednar et al. and Franzese et al. emphasize the idea of multiple limiting distributions connoting path dependence. For the former, this includes the possibilities of obtaining distributions composed of either all or both types of colored balls over the long term; for the latter, it is steady states composed of different distributions of actor types. Like Jackson and Kollman, Bednar et al. conclude that path dependence is relatively rare, a knife-edge condition produces it.

There are clear differences in the three articles as well. Bednar et al. motivate their model from historical agent-centered processes like judicial decision making, whereas Franzese et al. and Jackson and Kollman motivate their investigations from more macro-historical processes like alliance formation and conflict and long-term economic growth. Like Franzese et al., Bednar et al. suggest the idea of changepoints. Just like sudden changes in the c parameters in the former's model, the latter allow for sudden episodes of revision. Kollman and Jackson allow for sudden shocks in their exogenous variable but focus mostly on the idea of smooth changes in their autoregressive parameter, ρ_t.[11] Bednar et al. do not simulate their causal process; unlike the other sets of scholars, they do not produce any analysis of the rates of convergence to the three kinds of equilibria suggested in their main result (Claim 3). And Bednar et al. do not explain how one could estimate their p and q parameters for judicial or any other social process, although they do provide some insights about how such estimates—if they somehow could be produced—should be interpreted. Finally, the results of Bednar et al. are developed for processes with discrete outcomes, such as an urn with two colors of balls. In this example, it is easy to say that revision means a blue ball was drawn instead of a yellow one and hence a blue ball replaces a yellow ball in the urn. It is not clear what revision means, or implies, for processes with continuous outcomes, such as economic performance or health expenditures, as described by Jackson and Kollman. How large a difference in gross domestic product growth or in the public share of health expenditures is needed for there to be "revision?"

5 Going Forward

Together the articles illuminate several avenues for future research. First, it is important to understand more fully how the causal mechanisms in the three works are related to one another. Is the path revision process in the model of Bednar et al. related somehow to the *combination* of behavioral contagion and homophilic selection in the investigation by Franzese et al.? In what sense? The different accounts of path dependence in the analyses of Jackson and Kollman and Franzese et al. need to be reconciled. Are the conditions for temporal persistence, $\rho_t = 1$, that produce path dependence in the former and the parallel condition that eliminates path dependence in the latter, $c_0 = c_1, c_2 = 1$, inconsistent or complementary? The larger question is whether Bednar et al. and Jackson and Kollman are correct that path dependence is rare. Can we classify social processes into those for which path dependence is and is not likely?

Empirically, the articles illuminate many challenges. Changepoint detection is one of them. The articles clearly show how changes in parameters can alter the character of a social process, eliminating or producing path dependence. Franzese et al. suggest that such break points are common. The three articles

[11] At the same time, Jackson and Kollman note that sufficiently large shocks to exogenous variables can create large shifts in ρ_t yielding a large change in the relationship between y_t and y_{t-1}.

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are less clear about how we detect such changes or whether extant methods of changepoint detection are useful (Park 2011a,b; Spirling 2007).

A related problem is observational equivalence. The articles are relatively clear about what we should expect it the way of long-run behavior of each causal mechanism and when (if) this behavior connotes path dependence. And, the simulations in the articles by Jackson and Kollman and Franzese et al. give us some useful insights about how their mechanisms behave in time. The former spell out some seemingly stringent conditions for discerning changes from equilibrium-independent to (from) equilibrium-dependent processes. The latter show the advantages of their model over a well-established alternative model, SIENA. But the deeper question remains of whether processes out of equilibrium can be distinguished meaningfully from processes that are equilibrium dependent with data that are often highly spatially and temporally aggregated and hence do not exhibit a lot of variance. The Jackson and Kollman (footnote 11) difficulty in estimating models with complete persistence, $\rho_t = 1$, is illustrative.

Third is the problem of model scale. Franzese et al. acknowledge the problems of estimating models for higher order social processes and high event rates. Would latent variable approaches be more useful in these cases, for instance?

The usefulness of human subjects research in motivating and understanding mechanism that produce path dependence is a fourth issue. Bednar et al. demonstrate how human subjects experiments can be used effectively to motivate revision. Whether such experiments also could be used to refine and extend the analyses in the investigations by Jackson and Kollman and Franzese et al. is unclear.

A profound implication of the articles by Jackman and Kollman and Franzese et al. is that one should carefully consider the possibility of path dependence in traditional cross-sectional studies. The social outcomes on which such studies focus represent random distributions about some equilibrium conditioned on the values of a set of exogenous variables. If the respective process is path dependent, say as in the case of the network of Franzese et al. with coevolution, these cross-sectional results, even if each observation is at or near its equilibrium, will be partially dependent upon the history and initial conditions. The model of Franzese et al. combines time series and cross-sectional data, which is how they can identify path dependence. Many models for studying networks and the related models for studying spatial interdependence, however, are estimated only with cross-sectional data and omit the possible presence of path dependence. Omission of variables measuring these properties may lead to the classic omitted variable bias. At a minimum, it means that cross-sectional analysis will not be able to recover the structure of the true data-generating process. For an illustration of this problem, see Jackson (1996). What is needed are explicit arguments that the process is equilibrium independent and that all deviations from the equilibrium values of the limiting distribution are white noise. Or, one must explicitly exclude variables representing initial conditions and(or) early outcomes in the process.

Page (2006) identifies one very important condition for equilibrium dependence that is then elaborated by these papers. He makes the point that equilibrium dependence requires time-varying transition probabilities or coefficients. In discrete Markov models, this means variations in the transition matrix. In other models, it means that coefficients relating outcomes at time t to previous outcome must exhibit some variation over time. Freeman's (2011) companion piece identifies some time series models that connote a kind of equilibrium dependence. An example is the univariate random walk, the distribution of which changes in time (due to its time-dependent variance). Other examples are the band TAR model, the nonlinear error correction model, and nonlinear regression model. This property is evident in the models of Franzese et al. and Jackson and Kollman with equilibrium dependence. Tests for equilibrium dependence in any process will require the application of more complex estimators as well. Interest in path dependence is likely to lead to more intensive exploration of these time-varying and likely nonlinear models and their statistical underpinnings. It may also prove fruitful to reexamine some of the well-known time series studies with these estimators to ascertain if reestimation with these more encompassing methods will lead to altered inferences about some of our important dynamic processes. For example, almost all studies of macropartisanship have been done with models with fixed coefficients.[12] It is possible that some of the conclusions about macropartisanship may be modified by a more thorough exploration along the lines proposed in these papers.

[12]An exception is Jackson and Kollman (2011, 2012) that includes a nonlinear time-varying autoregressive term.

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Appendix

Table A1 Time series models, outcome dependency, and related concepts

Model	*Impact of initial condition(s) on outcomes*	*Phat outcome dependence*	*Sequence outcome dependence*	*Limiting behavior*
Univariate, linear				
Stationary autoregression	Yes	No	Yes	For Gaussian errors, ergodic in moments; stochastically stable
Nonstationary (random walk)	Yes	Yes	No	Fixed expected value, changing distribution in time
Multivariate linear models				
Single equation time series regression (stationary errors)	No	No	Yes	Depends on forcing function
Singe equation ECM	Yes	Yes	Yes	Fixed expected value repeated moves into/out of "equilibrium"
Stationary vector autoregression	No	No	Yes	Fixed vector of expected values
VECM (less than full rank)	Yes	Yes	Yes	Fixed vector of expected values; repeated moves into/out of "equilibrium"
Nonlinear time series				
Simple TAR	No	No	Yes	Under certain conditions, geometrically ergodic
Basic TAR	No	No	Yes	Under certain conditions, geometrically ergodic
Band TAR	Yes	Yes	Yes	In part, changing distribution in time
Nonlinear ECM	Yes	Yes	Yes	Fixed vector of expected values; switches between rates of error correction movements into/out of "equilibrium"
Nonlinear least squares regression	No	No	Yes	Depends on forcing functions

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