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Logicism and Formal Necessity: Reflections on *Kant's Modal Metaphysics*

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Abstract

Logicism is Stang's name for the Leibnizian doctrine that all necessary truths are derivable from identities and definitions. Stang shows that the early Kant opposed this doctrine because he thought the proposition *God exists* was a counterexample to it; I raise some non-theological counterexamples as well. *Formal necessity* is the necessity that attaches to a proposition when its truth is grounded in our categories and forms of intuition. Stang treats it as one of several sui generis kinds of necessity in Kant, all of them falling short of logical or metaphysical necessity. I raise several questions for Stang's account, including the following: Can our having the forms we do really explain the necessity of geometry? Is our possession of those forms self-grounding in an objectionable way? How can our forms ground general truths without grounding particular instances of them?

Keywords: necessity, analyticity, possibility, necessity grounded in our forms of intuition, geometrical truth, Kant, Leibniz, Nicholas Stang

Nick Stang's *Kant's Modal Metaphysics* (Stang 2016) is a tour de force. It weaves together a deep knowledge of Kantian texts from the beginning to the end of Kant's career with an equally deep command of modal logic and the metaphysical issues associated with it. There are brilliant chapters on the following topics among others: the Leibnizian programme of reducing all necessities to logical principles; Kant's reasons, some not previously well known, for rejecting this programme; versions in Kant's major and minor predecessors of the ontological argument for the existence of God; the ontology of merely possible objects and its role in undergirding the ontological argument; Kant's reasons for rejecting merely possible objects and the ontological argument along with them;

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Kant's own early attempt to prove the existence of God from purely modal considerations (God as the ground of all possibility); a catalogue of Kant's various notions of possibility and necessity, exhibiting for each how it is grounded in the actual; the factors unifying the items in this catalogue; Kant's reasons late in his career for going over to the view that modal notions are artefacts of how we represent things. All of these topics are treated with unflagging philosophical ingenuity and technical virtuosity.

In what follows I have room to discuss only two of the topics above, Stang's treatment of logicism and his reconstruction of Kant's notion of formal necessity (that is, necessity grounded in our forms of intuition).

1. Logicism

Three 'isms' are of central importance in Stang's first chapter. *Logicism*, espoused by Leibniz, Wolff and Baumgarten, is the thesis that a proposition p is necessary iff there is a finite demonstration of p from identical propositions and definitions of the constituents of p.[1] *Ontotheism* is the thesis that God exists in virtue of his essence; it implies that there is a sound ontological proof of the existence of God. *Possibilism* is the thesis that there are or could be merely possible objects, objects that do not exist; in symbols, $\Diamond \exists x \sim Ex$.

We know from the *Critique of Pure Reason* that Kant was a staunch opponent of logicism, since his synthetic *a priori* propositions are necessary propositions that are *not* demonstrable from identical propositions and definitions. But Stang finds already in Kant's pre-Critical writings the makings for an argument against logicism different from anything given in the *Critique*. It involves the following chain of implications: existence is not a real predicate => possibilism is false => ontotheism is false => logicism is false.[2] Kant accepts the third implication because in his pre-Critical days, he believed that *God exists* is a necessary truth, which on logicist principles it could not be unless God's existence were derivable from the concept or definition of God, as it is according to the various ontological arguments. I found it intriguing that two Kantian doctrines that survive into the Critical period – existence is not a real predicate, the ontological argument is not valid – team up with another doctrine that did not survive – God necessarily exists – to yield an early argument for the synthetic *a priori*.

Except for expounding the argument above, Stang pretty much gives logicism a free pass. In the remainder of this section, I want to mention some further objections to it.

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Stang offers two formulations of logicism. Here is the first, with my label:

> (L1) For any proposition P, it is necessary that P if and only if ~P entails a contradiction. (p. 16)

Here is the second, from Leibniz:

> (L2) For any proposition P, it is necessary that P iff there is a finite demonstration of P from identical propositions and definitions of the constituents of P. (p. 19)

Stang says the definitions are equivalent (p. 19). I think it is worth pointing out that they are *not* equivalent unless one introduces a special sense of entailment much narrower than the usual one.

By an identical proposition, Leibniz means a proposition either of the form 'All A is A' (a complete identity) or of the form 'All AB is A' (a partial identity, where B itself may be the conjunction of two or more predicates). A good example of a proposition demonstrable from identities and definitions is Kant's paradigm of an analytic proposition, 'All bodies are extended' (A7/B11).[3] By the definition of a body as an extended impenetrable substance, that proposition reduces to 'all extended impenetrable substances are extended', which is a partial identity. Starting with the partial identity and moving in the opposite direction, we would have a demonstration of the original proposition. In effect, Leibnizian demonstrations allow us to use only one rule – *definienda* may be substituted for their *definientia* and vice versa – and one type of axiom, the class of identities.

What now of logicism under Stang's first formulation of it, L1? Suppose we understand 'P entails Q' in the way proposed by C. I. Lewis and now standard in modal logic: P entails Q iff it is necessary that if P, then Q.[4] We immediately secure that P is necessary iff ~P entails a contradiction. If P is necessary, its negation is impossible, and it is a theorem even in very weak systems of modal logic that an impossible proposition entails any proposition whatever – one of the once so-called 'paradoxes' of strict implication. So of course ~P will entail a contradiction whenever P is necessary.

Now let us focus on L2, which I maintain makes more stringent demands on necessity and is not equivalent to L1. At any rate, the two are not equivalent unless we introduce some notion of entailment specially

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tailored to make them equivalent, such as 'P entails Q iff Q is deducible from P using only definitional substitutions in the deduction'. And if we do that, we will have made L1 as demanding as L2; we will have made it no longer obvious that the negation of every necessary truth entails a contradiction.

Is a contradiction deducible from 'not all bodies are extended' using just definitions? Suppose we define '(x)' as '$\sim\exists x\sim$'; then we can pass from '$\sim(x)(Bx \rightarrow Ext(x))$' to '$\sim\sim\exists x\sim(Bx \rightarrow Ext(x))$'. I believe the reader who continues the proof to an explicit contradiction – such as '*a* is extended & *a* is not extended' – will find that he needs to use (besides the definition of bodies) the following logical rules: double negation elimination, the law that $\sim(P \rightarrow Q)$ is intersubstitutable with $P \,\&\, \sim Q$, existential instantiation, conjunction elimination, and conjunction introduction. None of these is mentioned as one of Leibniz's resources for deducing contradictions.

I go into this because the equation of L1 and L2 lends specious plausibility to logicism. We can be lulled into accepting it under the L1 formulation and a broad notion of entailment, but under the L2 formulation it is really quite audacious. I question whether even some of the most obvious necessary truths live up to its demands.

How about such an elementary and obvious necessity as 'Every three-sided plane figure has three angles'? May we deduce it from identities and definitions? We may start with the partial identity 'every plane figure that has three sides has three sides'. We must now deduce the target proposition from our starting point with appropriate definitions of sides or angles. One definition I found of 'angle' is 'the space between two sides'. Well, where does the space end – an inch out from one of the vertices, two inches out, or where? There are indefinitely many spaces enclosed by two sides, unless we count only the maximal space, which reaches all the way to the third side. But then any three-sided figure has just *one* angle (its entire interior), not three. The most promising definition of an angle that occurred to me is this: an angle is a pair of sides meeting in a vertex. We could then prove that a figure with three sides has three angles, provided we help ourselves to the auxiliary premise that from any three things, *a*, *b* and *c*, we can form exactly three distinct pairs, {*a*,*b*}, {*b*,*c*} and {*c*,*a*}. That is an obvious and necessary truth. But it is not an identity; nor is it derivable in any way I can see just from definitions and identities. The same is true of many other necessary truths, such as Euclid's theorem that there is no greatest prime number.

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My quarrel in this section is not so much with Stang as with Leibniz and the logicists. In Kant's language from the first *Critique*, they hold that all necessary truths are analytic (A6–7/B10–11).[5] Kant was right: many necessary truths are synthetic.

2. Formal Necessity

A central theme of Stang's book is that Kant recognizes several varieties of 'real' possibility and necessity, each distinct from logical possibility and necessity and each having its own ground in the actual. The varieties of real possibility and necessity include the formal, the empirical-causal, the noumenal-causal and the nomic. What they have in common is the following two features, which we may express by letting 'Pp' say it is really possible that p in one of the senses and '◇p' say that it is logically possible that p: (i) ◇p is not sufficient for Pp (equivalently, its being formally necessary that p is not sufficient for its being logically necessary that p), and (ii) if Pp, the fact that Pp has a real ground in some actual object or principle. The logical necessity here spoken of is the necessity championed by the logicists of the preceding section – a matter of the opposite's entailing a contradiction in some suitably narrow sense of 'entails' – and logical possibility is its dual. I believe that in addition to saying that logical possibility is insufficient for real possibility, Stang would also say that metaphysical possibility is not sufficient for real possibility in most of the Kantian senses.

Formal necessity is the kind of necessity anchored or grounded in our having the forms of intuition we do and the intellectual apparatus we do, including the twelve categories. Stang defines it as follows:

> It is formally necessarily that p iff our forms of intuition and our intellectual forms wholly ground the fact that p. (p. 203)

He defines formal possibility as the dual of formal necessity:

> It is formally possible that p iff it is not the case that our forms of intuition and our intellectual forms wholly ground the fact that ~p. (p. 205)

The grounding here spoken of is the relation much discussed under that name in contemporary metaphysics. It is the converse of 'holding in virtue of', and it is a relation that can hold asymmetrically even between logically or metaphysically equivalent propositions. Stang makes a good case that Kant recognized a relation with these properties (pp. 125–6).

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It is not logically necessary that triangles have angle sums of 180 degrees or that every event have a cause – these are Kantian paradigms of the synthetic *a priori*. Yet they are necessary in *some* sense for Kant. As Stang interprets Kant, they are formally necessary, their truth being grounded in or made true by our having the forms of intuition and intellection that we do.[6]

I now offer a series of queries, objections or observations about Stang's account of formal possibility and necessity.

First, is there a way to characterize formal necessity otherwise than in terms of groundedness in our forms of intuition and thought? It would be nice if there were. The necessity of geometrical theorems could then be said to be *explained* by the fact that their truth is grounded in our forms, rather than consisting in this fact.

Second, why does Stang make the connection he does in the following paragraph?

> If we analyzed formal possibility and necessity in terms of logical grounding, it would follow that geometrical axioms logically entail geometrical theorems. … But Kant holds that geometric reasoning is irreducibly synthetic because it relies on construction in pure intuition. Consequently, the relation between forms of experience and what is formally necessary cannot be assimilated to the relation of logical entailment; the formal necessities do not follow *logically* from the forms of experience. (p. 207)

When I reconstruct that argument, I come up with something like the following. Premise: geometrical axioms do not logically entail geometrical theorems, in Kant's view. The derivations rely on peculiarities of our form of intuition. Conclusion from the premise: 'If A(xioms) then T(heorem)' conditionals are not logically necessary; they are only formally necessary, like the axioms themselves. Supposition for *reductio*: To be formally necessary is to be logically entailed by our having the forms of intuition we do. Conclusion from the foregoing: In that case, 'If A then T' conditionals would be logically necessary after all, contradicting the initial premise.

I do not see how the second conclusion follows from the stated premises. It would follow with the help of one additional premise: that it is logically necessary that we have the forms we do. One could then invoke the

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principle that what follows with logical necessity from facts that are logically necessary is itself logically necessary. But the additional premise is false and Stang agrees; it is *not* logically necessary that we have the forms we do. So what am I missing in the argument I have quoted?

Third, not only does Stang not say that it is logically necessary that we have the forms we do; he also says positively that it is *formally* necessary that we have them. The formal necessities include not only truths of geometry, but also the fact that we have such and such forms (pp. 207 and 209). I find the positive claim problematic. If Stang had defined formal necessity simply as a matter of being entailed, either logically or metaphysically, by our having the forms we do, this claim would have been trivial and harmless. But he defines formal necessity as being *grounded in* our having the forms we do. To say it is formally necessary that we have these forms is therefore to say that our having them is grounded in our having them – a violation of the irreflexivity that is normally taken to be an essential feature of grounding.[7] Kant himself says that no being, not even God, can be its own ground (cf. 'New Elucidation' II, Prop. VI; 1: 394).

Fourth, are the two main features of real possibility identified above, Non-logicality and Groundedness, independent of each other, or does the first imply the second? Stang always lists them separately, but I think it is a plausible conjecture that, for Stang's Kant, the first does imply the second.

Suppose the conjecture is correct. Then, since the grounding relation between our forms and the geometrical truths they ground is non-logical, it would have to be grounded in some actuality. In what actuality? The most plausible candidate, I believe, is those very forms. We would then arrive at a view isomorphic to one defended by Karen Bennett for grounding at large, in which the ultimate grounds ground not only the entities or truths above them, but also their own grounding of those truths (Bennett 2011).[8]

Fifth, Stang says the grounds of formal possibility and necessity are 'immanent' rather than 'transcendent', meaning they are either phenomena or conditions of phenomena rather than noumena (p. 200). Our forms of experience are not phenomena, but conditions of phenomena. But is not the distinction between phenomena and noumena exhaustive (p. 182)? And must not conditions of phenomena that are not themselves phenomena therefore be noumenal? And would our forms of experience

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not therefore have to be noumenal, making the grounds of formal possibility transcendent after all? Here I am pressing on a chronic sore spot in Kantian philosophy, which according to some critics violates its own strictures against the knowability of the noumenal realm when it gives its transcendental explanation of *a priori* knowledge.

To evade what I just said, Stang might say that the conditions of phenomena are themselves phenomenal. But by both his reckoning and mine, phenomena are mind-dependent, which I take to imply that they are entities grounded in the acts or features of noumenal minds. If this is granted, all relations among phenomena, including relations of grounding, would have to be grounded in noumenal minds as well. So whether at one remove or two, formal necessity is anchored in noumena.

Sixth, the formally necessary truths include not only the truths of geometry, which are grounded in our forms of intuition, but also truths such as *every event has a cause*, which is grounded in our operating with the categories we do. Now Kant makes a point of saying that the necessity enjoyed by the general causal principle is not shared by particular causal laws. It is necessary (and knowable *a priori*) that if wax melts, there must be *some* cause of it, that is, some prior event upon which the melting follows according to a law. But it is not necessary or knowable *a priori* that the prior event was the heating of the wax by the sun, or that the heating of wax is always followed by its melting (A766/B794). Our forms do not ground these particular truths, or else the particular truths would be formally necessary just as the general causal principle is. My question is how this combination is possible: how can our forms ground general principles without grounding any particular instances of them? This question arises for Stang as well as for Kant, since Stang says that the category of cause and effect ensures that we experience events as governed by causal laws, but does not dictate which causal laws in particular govern these events (p. 202).

Let me explain my puzzlement in further detail. Kant states the general causal principle thus: 'Everything that happens (that is, begins to be) presupposes something upon which it follows according to a rule' (A189). That is, for every event e that occurs, there is an event c such that (i) c preceded e, and (ii) c and e are of types such that whenever an event of c's type occurs, an event of e's type follows. Somehow or other, we or our intellectual forms fully ground the truth of that proposition. But how do we do it without grounding any particular instances that make the general proposition true? If a dog sneezes, how do we make it the case that

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the sneeze was preceded by some event that it follows according to a law without making it the case that (a) it was preceded by pepper inhalation (let us say) and (b) pepper inhalation is always followed by sneezing? Yet one would have thought that our forms do not busy themselves in such matters.

I have a guess as to how Stang would answer one half of my puzzlement. He tells us that one variety of necessity in Kant is nomic necessity, the type of necessity possessed by laws, and that nomic necessity is grounded in the natures of the kinds that are linked by the laws (chapter 8). To make it true, then, that the sneeze was preceded by something upon which it follows according to law, we do not need to make any law true; we just need to make happen an antecedent event of some appropriate kind, a kind linked by law to the consequent event's kind, and the kinds themselves will 'take over' in grounding the truth of the law. But that still leaves the other half of my puzzlement – how we ensure the occurrence of an appropriate antecedent without ensuring the occurrence of a token of the right type and, indeed, without ensuring the occurrence of that very token.

If e is some event that happens and c its cause, perhaps I do not ground the occurrence of c directly, but instead make happen the entire causal series of which c is a part. This extraordinary suggestion fits in with a line of thought that sometimes seems to be hinted at in Kant's solution to the Third Antinomy. How can I be responsible for telling a malicious lie if my doing so is the empirically causally necessary upshot of my history up until then? Answer: it was 'noumenally causally' possible for me not to have told the lie, because I (as a noumenal being) had the power to make the entire empirical order not include that event (pp. 224–5). I cannot have changed any laws, since those are fixed by the natures of kinds, but I could have made antecedent conditions different all the way back by laying down an entirely different history of the universe. This suggestion resonates with some of the things Kant and Stang say, but it still leaves me wondering why they are not stuck with saying that particular occurrences are formally necessary. How do I make an entire series happen without making at least some of its members happen?

Seventh, does the notion of formal possibility let Kant escape a famous objection levelled against him by Russell, namely, that in accounting for the necessity of geometry by saying that geometrical truth is grounded in us, Kant is abolishing necessity rather than explaining it? Here is Russell:

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> The thing to be accounted for is the necessity that the facts must always conform to geometry and arithmetic. To say that geometry and arithmetic are contributed by us does not account for this. Our nature is as much a fact of the existing world as anything, and there can be no necessity that it will remain constant. It might happen, if Kant is right, that tomorrow our nature would so change as to make two and two become five. This possibility seems never to have occurred to him. (Russell 1999: 61–2)[9]

May we invoke the distinction among varieties of necessity in reply to Russell? Here is what occurs to me: we concede to Russell that it is logically and metaphysically possible that our natures change overnight, and we concede thereby that it is logically and metaphysically possible that two and two be five or that triangles have non-Euclidean angle sums. But we continue to insist that non-standard arithmetics and geometries are formally impossible nonetheless. Formal necessities and impossibilities are grounded in the forms we actually possess, and it is one of the marks of formal necessity that it is not sufficient for logical necessity. I am fairly sure that Stang would also say that it is not sufficient for metaphysical necessity. So even if we concede that Russell's nature-altering and math-altering scenario is metaphysically possible, we can still say that Kant has staked out a sense in which arithmetic and geometry are necessary.

I cannot help but think myself that Kant was after bigger game. When he offers up necessity as a mark of the *a priori*; when he says it is necessary that two lines cannot enclose a space; when he says that seven and five cannot be anything but twelve – in all these instances, I take him to be declaring these things to be necessary in some absolute or metaphysical sense. That the necessary facts are grounded in our forms of intuition is supposed to explain this necessity, not constitute a sui generis kind of necessity. Of course, if Russell is right, the explanation fails. But I prefer to see Kant as involved in a spectacular failure rather than (as with Stang) a contrived success.

Notes

1 This is not the same as the doctrine known as logicism in the philosophy of mathematics, but it would arguably imply it.

2 The case for the first implication in brief is this. As Stang explicates Kant, a *real* or *determining* predicate is a predicate P that determines at least one concept, where a

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predicate P *determines* a concept C iff $\Diamond \exists x(Cx \,\&\, Px)$ and $\Diamond \exists x(Cx \,\&\, \sim Px)$. It follows from these definitions that existence (E for short) is a real predicate iff $\Diamond \exists x \sim Ex$, hence that if existence is not a real predicate, possibilism is false. The case for the second implication is roughly this. One of the premises of the ontological argument is 'God has every perfection'. That premise, critics allege, either presupposes the existence of God (by predicating something of him) or must be understood conditionally (if anything is God, it has all perfections). In the former case, the argument begs the question, and in the latter case, it only reaches a conditional conclusion – that if anything is God, it exists. The only escape from the dilemma is possibilism, which goes between the horns by saying properties may be predicated categorically of *possibilia* without presupposing their existence. Ontotheists are also committed to possibilism if they wish to say God is the *only* being whose existence may be proved from its essence.

3 Citations from Kant will be in standard A/B format, with translations those of Kant 1965.

4 For expository convenience I use 'P' and 'Q' sometimes as terms and sometimes as formulas.

5 Analytic propositions are propositions in which the predicate is contained in the subject either overtly (as in complete and partial identities) or covertly (as in propositions reducible to the former class by definitional substitutions).

6 The full story is a bit more complicated. In the first instance, what our forms ground is our being able to entertain certain contents but not others – e.g. our being able to intuit triangles only as having any two of their sides longer than the third. But this fact about what *contents* are possible in turn grounds a fact about what *objects* are possible – the only triangles there can *be* are triangles with two sides longer than the third. As appearances, triangles have only the properties they can be intuited as having. See p. 206.

7 Irreflexivity could be honoured if the necessity of our having the first-level forms we do were grounded in our having certain forms at the second level – but that way lies a Third Man regress.

8 An infinity breaks out here: A grounds not only G, but also A's grounding of G, A's grounding of A's grounding G and so on. But I believe the regress is not vicious. It violates neither the irreflexivity of grounding (as did the supposition that it is formally necessary that we have the forms we do) nor the requirement of well-foundedness for the grounding relation.

9 I have substituted 'geometry' twice for 'logic' and 'necessity' twice for 'certainty', which I think is entirely in the spirit of Russell's objection.

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