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Summary

We have discussed the interpretation of hadronic scattering, at large momentum transfers, in terms of a stationary-phase approximation to the diffraction integral over impact parameters $\boldsymbol{b} = (b_x, b_y)$. For a spherically symmetric potential, the two-dimensional diffraction integral reduces to a trivial integral along the impact parameter b_y perpendicular to the scattering plane, and a nontrivial one in b_x, the component in the scattering plane. The integration over b_y essentially amounts to evaluating the integrand for $b_y = 0$, i.e., in the scattering plane.

In this limit of large momentum transfers, which admits a semiclassical approximation, the whole diffraction integral over b_x can be replaced by the sum of a few terms (typically two), corresponding to the partial amplitudes for scattering at these discrete impact parameters, determined by the stationary-phase condition

$$\boldsymbol{q} = \nabla_b \chi(\boldsymbol{b}), \tag{16.1}$$

or, with $b_y = 0$:

$$q = X'(b_x), \tag{16.2}$$

where $X(b_x) \equiv \chi(b_x, 0)$.

In the simplest possible case, that of Coulomb scattering from a point charge, the approach furnishes an exact result, up to an overall, screening-dependent and unmeasurable phase factor. For other real phase shift functions, at low momentum transfers the method typically yields two points of stationary phase. At higher momentum transfers these approach each other, and at some critical value they merge. This is rainbow scattering. Mathematically, it corresponds to $X'(b_x)$ having a maximum. In the asymptotic approach the scattering amplitude is divergent.

Beyond the rainbow (at larger momentum transfers) one stationary point has moved up into the complex plane and become irrelevant, whereas the other has moved down into the lower half-plane and yields a scattering amplitude that is

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exponentially damped. The reason the point of stationary phase that is in the upper complex b_x-plane is irrelevant is that the path of integration does not pass through it.

Furthermore, it is actually not difficult to find examples exhibiting more than one rainbow. Then $X'(b_x)$ must have both maxima and minima, at least one of each.

If the interaction is absorptive, then $\chi(\boldsymbol{b})$ is complex for real values of $\boldsymbol{b}$. In this case, it is clear that $\boldsymbol{b}$ (in particular, b_x) must be *complex* in order to satisfy Eqs. (16.1) and (16.2) for real $\boldsymbol{q}$.

We have illustrated characteristic features of a variety of phase shift functions having Gaussian and exponential edges, Coulomb scattering from extended charge distributions, and combinations of absorption and Coulomb scattering. Various two-scale phase shift functions have been explored, as well as phase shift functions having singularities of different degrees.

This approximation in terms of a discrete sum of terms is found to be in excellent quantitative agreement with the diffraction integral for a vast variety of phase shift functions, provided the stationary points are well separated. It offers a very physical understanding of the diffraction pattern in terms of interference between these amplitudes, often just two. For the case of pure absorption, the interpretation is particularly simple: The two points of stationary phase are symmetrically located with respect to the imaginary axis: $b_{\alpha x} = -b^*_{\beta x}$. The separation of these two stationary points, $b_{\alpha x} - b_{\beta x} = 2\,\mathrm{Re}\, b_{\alpha x}$ is then a measure of the period of oscillation, whereas $\mathrm{Im}\, b_{\alpha x}$ determines the overall slope or envelope of the oscillation pattern.

The usefulness of this observation lies in the fact that at large momentum transfers the stationary points will be located near the *singularities* of the phase shift function, as dictated by Eq. (16.1). Thus, if the diffraction pattern has a particular change or evolution with increasing momentum transfer, then this must be reflected in the movement of the points of stationary phase, attracted to one or more sets of singularities.

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