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MODELING NONLINEAR AND HETEROGENEOUS DYNAMIC LINKS IN INTERNATIONAL MONETARY MARKETS

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We use daily short-term interbank interest rates of France, the United Kingdom, and the United States to examine the dynamic links of international monetary markets from 2004 to 2009. Results from vector error-correction models and smooth-transition error-correction models show strong evidence of nonlinear and heterogeneous causalities between the three interest rates. We also find that changes in the U.S. interest rate deviations from the long-run equilibrium led those in France and in the United Kingdom by one to two days. Finally, the national interest rate nexus appears to converge in nonlinear fashion toward a steady state because it is subject to structural change beyond a certain rate threshold. Our findings have important implications for the actions of leading central banks (ECB, Bank of England, and U.S. Fed) because the joint behavior of short-term interest rates can be viewed as an indicator of the degree of central banks' policy interdependence.

Keywords: International Monetary Market Relationships, Short-Term Interest Rates, Vector Error-Correction Models, Smooth-Transition Error-Correction Models

1. INTRODUCTION

With the increasing trend of global financial and economic integration, due mainly to the removal of capital controls and other investment barriers over the last three

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decades, an important body of the literature has been devoted to the investigation of the international interest rate links [Barassi et al. (2005); Wang et al. (2007)]. This issue has important policy implications for policymakers and market regulators, as it relates directly to the extent of national central banks' monetary policy independence and coordination [Kirchgassner and Wolters (1993); Benigno (2002); Cuaresma and Wójcik (2006)]. If there were strong interest rate spillover from one country to another, a particular country's interest rate would be highly sensitive to changes in the interest rates of foreign markets, and as a result an independent monetary policy might not be conducted. Thus, an enhanced understanding of these causal links makes it possible, for example, to minimize contagion in the event of an economic crisis or to make improved forecasts of interest rates. It is also commonly accepted that the links between interest rates reflect the mobility of international capital flows, which is crucial for global investors' investment decision making and the pricing of products sensitive to interest rates [Zhou (2003)].

The global financial crisis of 2007–2009, which sprang from the U.S. subprime and banking failures, has recently renewed interest in studies of both short- and long-term relationships between international interest rates, because they offer insightful guidelines for the coordination of international monetary policy. We are also motivated by the recent failures of such institutions as the International Monetary Fund and the Bank for International Settlements to foster global monetary cooperation, ensure global financial stability, and provide a prudential framework for macroeconomic policies. Central bank interventions were also poorly coordinated. Only once, as it happens, on 8 October 2008, were rate cuts announced jointly by the U.S. Federal Reserve and five other leading central banks.

In this study we attempt to investigate the interdependence of international monetary markets during the recent global financial crisis by analyzing the dynamic links of the three-month interbank-offered interest rates for France, the United Kingdom, and the United States. We rely on these rates because they capture both the time dynamics and the magnitude of central banks' monetary interventions through changes in policy rates, which have a considerable impact on financial market conditions and investment decisions [Rigobon and Sack (2003); Bernanke and Kuttner (2005); Chen (2007); Ioannidis and Kontonikas (2007)]. As a key input for economic activity and asset valuation, they reflect changes in business cycle and macroeconomic fundamentals such as inflation and exchange-rate equilibrium. We contribute to the related literature not only by looking into how different interest rates are linked in the short and long run but also by attempting to draw tentative implications for monetary policies conducted by three leading central banks—the Bank of England (BoE), the European Central Bank (ECB), and the Fed. The French short-term interest rate is used as a representative rate to mimic ECB policy action because there is little empirical evidence to support the hypothesis that one particular rate in the eurozone exerts influence over the others [Uctum (1999); Wang et al. (2007)]. In addition, the French interest rate is found to play a dominant role in international monetary markets [Awad and Goodwin (1998)].

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To assess the dynamic links among the three interest rates considered, we develop a nonlinear cointegration framework based on the smooth-transition error-correction model (STECM). This approach is more appropriate than the conventional VAR and cointegration methods used in the majority of previous studies when the dynamic adjustment of interest rates toward their common equilibrium is subject to asymmetric reactions and nonlinearities, especially during episodes of financial panic. The STECM also allows the interest rates to have regime-switching behavior whenever interest rate deviations from their long-term relationship exceed a certain threshold. Previous studies such as those of Mancuso et al. (2003), Holmes and Maghrebi (2004), Poghosyan and De Haan (2007), and Barassi et al. (2005) also take a nonlinear modeling approach to examining the dynamic links of interest rates.

Over the period from December 31, 2004, to March 19, 2009, a period chosen to cover the major events related to the global financial crisis, we find strong evidence of nonlinear causal interactions of short-term interest rates. The U.S. interest rate deviations from the long-term equilibrium established with the French and U.K. rates are found to nonlinearly lead those in France and the United Kingdom within a horizon of one to two business days. In addition, short-term interest rates in the three countries have experienced similar behavior in recent months, which may be, in our opinion, the result of greater convergence to the long-term equilibrium of policy rates set by central banks. This result is supported by the findings of Scotti (2006), who examines interest rate feedback rules between the U.S. Fed and the ECB.

The remainder of the paper is organized as follows. Section 2 provides a brief review of work on the ways national interest rates are linked. Section 3 presents our empirical method of examining nonlinear interest rate interdependence. Section 4 describes the data and discusses the results. Concluding remarks and policy implications are provided in Section 5.

2. INTERNATIONAL INTEREST RATE LINKS: A BRIEF REVIEW OF THE LITERATURE

Economic theory suggests several possible explanations for the interaction of short-term interest rates in international monetary markets [Barassi et al. (2005)]. If interest rates are treated as analogous to any other financial securities, they can then be interpreted as being driven not only by supply and demand but also by profit-seeking financial flows. In an international setting, then, any deviations from interest rate parity are likely to be exploited by arbitrage, leading to comovement of interest rates. Alternatively, short-term interest rates in a particular country may be related to those in foreign countries, as they are often used as monetary policy instruments to set exchange-rate and inflation-rate targets. Any modification or deviation from the target would cause the interest rates to deviate from parity. Therefore, national interest rates are likely to be considerably interdependent, and this interdependence is likely to grow as global financial markets integrate. It is

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also possible to make inferences on the degree of market integration by analyzing the cross-country interest rate nexus.

An extensive body of literature has investigated the international links between short-term interest rates over both the short and long run. Earlier studies including Mishkin (1984), Mark (1985), and Cumby and Mishkin (1986) focus largely on testing the hypothesis of real–interest rate equality across countries to shed light on cross-market financial integration. They generally marshal evidence against the equality of real interest rates and attribute the deviations from parity to the existence of significant currency risk. Subsequent works often make use of VAR and cointegration frameworks to investigate the short- and long-run relations among nominal and real interest rates rather than their equalization [Awad and Goodwin (1998); Bremnes et al. (2001); Anoruo et al. (2002); Yamada (2002); Baum and Barkoulas (2006); Wang et al. (2007)]. The reason is that the equality of real interest rates requires purchasing power parity (PPP) to hold, whereas there is substantial evidence against the validity of PPP.[1] The main findings of these studies indicate that short-term interest rates are tied up by a long-run phenomenon, with a dominant effect of the U.S. interest rate. Moreover, the transmission of interest rate events increases with the ongoing process of financial liberalization.

In sum, although the studies cited are useful for measuring the linear relationship between interest rates, they ignore the potential for asymmetry and nonlinearity in their dynamic adjustment toward long-run equilibrium. Consistent with, among others, Mancuso et al. (2003), Barassi et al. (2005), and Poghosyan and De Haan (2007), who find evidence of nonlinearity and structural breaks in interest rate links, we develop a threshold multivariate cointegration framework to take these stylized facts into account.

3. THRESHOLD COINTEGRATION FOR INTERNATIONAL INTEREST RATE LINKS

In a linear cointegration framework, two first-order integrated interest rate series, X_t and Y_t, are said to be cointegrated if a linear combination of them is stationary [Granger (1981); Engle and Granger (1987); Johansen (1988)]. This system converges to a long-run equilibrium over time and its symmetric, continuous, and constant adjustment process can be reproduced by a linear error-correction model (LECM). This modeling approach is inefficient, however, when adjustment incorporates asymmetric and nonlinear patterns at a time-varying speed.

Making nonlinearity and switching-regime hypotheses a part of the traditional LECM framework creates a promising class of nonlinear cointegration models, the STECMs [Granger and Teräsvirta (1993); Van Dijk et al. (2002)]. These models enable the interest rate adjustment to be nonlinear and asymmetrical with a time-varying adjustment speed. The models particularly allow the smoothness of the adjustment process to be conditional on both the magnitude and the sign of disequilibrium associated with sudden exogenous shocks. STECMs have recently been used in such studies as those by Anderson (1997), Balke and Fomby (1997),

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and Liu (2001) for interest rates; Franses and Van Dijk (2000) and Jawadi and Prat (2012) for stock prices; Jawadi et al. (2009) for insurance premia; Escribano (1997) for money demands in the United Kingdom; and Rothman et al. (2001) for relationships between production and money demands. These studies consistently provide evidence that STECMs are suitable for capturing nonlinearity and switching regimes, smoothness, persistence, structural breaks, inertia effects, and asymmetry in the adjustment dynamics induced by market frictions.

Econometrically, a two-regime STECM is a combination of two LECMs insofar as it integrates two adjustment terms reproducing the speed of adjustment in the first regime and the intensity of error correction in the second regime. The regime-switching mechanism is governed by the introduction of a nonlinear component, defined as the product of a transition function and the adjustment term of the second regime. Specifically, we can set up a two-regime STECM for interest rate deviations,

$$\Delta z_t = \phi_0 + \lambda_1 z_{t-1} + \sum_{i=1}^{p} \phi_{1,i} \Delta z_{t-i} + \lambda_2 z_{t-1} \times F(z_{t-d}, \gamma, c) + \varepsilon_t, \qquad \textbf{(1)}$$

where z_t refers to the deviation defined by $z_t = Y_t - (\alpha_0 + \alpha_1 X_t)$, which thus measures the deviations of the system of two interest rates (X_t and Y_t) from its long-run equilibrium at any point in time. λ_1 and λ_2 are the adjustment terms in the first and second regimes, respectively; z_{t-1} is the error-correction term; *F(.)* is the transition function; γ and c are the transition speed ($\gamma > 0$) and the threshold parameters, respectively; d is the delay parameter; and z_{t-d} is the transition variable.[2] Thus, the STECM explicitly describes two regimes corresponding to the extreme values of *F(.)* and an intermediate continuum state. The transition from one regime to another takes place smoothly, and the first regime is obtained when $F(.) = 0$ (i.e., the interest rate adjustment dynamics is near equilibrium), that is,

$$\Delta z_t = \phi_0 + \lambda_1 z_{t-1} + \sum_{i=1}^{p} \phi_{1,i} \Delta z_{t-i} + \varepsilon_t. \qquad \textbf{(2)}$$

The dynamics of the second extreme regime, when $F(.) = 1$, is given by

$$\Delta z_t = \phi_0 + (\lambda_1 + \lambda_2)\, z_{t-1} + \sum_{i=1}^{p} \phi_{1,i} \Delta z_{t-i} + \varepsilon_t. \qquad \textbf{(3)}$$

In all cases, λ_1 and λ_2 are the most important parameters for this specification, as their values and signs determine the adjustment dynamics of interest rates and the speed at which they converge to equilibrium [Michael et al. (1997)]. Indeed, even though λ_1 is positive, interest rates are nonlinearly mean-reverting and the STECM is stable if λ_2 and $(\lambda_1 + \lambda_2)$ are negative and statistically significant. The transition function *F(.)* can be modeled as either an exponential or a logistic function [Teräsvirta and Anderson (1992); Granger and Teräsvirta (1993); Teräsvirta

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(1994)]. A first-order logistic transition function and a first-order exponential transition function are given by the equations

$$F(z_{t-d}, \gamma, c) = [1 + \exp(-\gamma(z_{t-d} - c))]^{-1}, \quad (4)$$

$$F(z_{t-d}, \gamma, c) = 1 - \exp[-\gamma(z_{t-d} - c)^2]. \quad (5)$$

The system composed of equations (1) and (4) defines a LSTECM (logistic STECM), whereas the system combining equations (1) and (5) results in an ESTECM (exponential STECM). Even though both models allow a smooth transition between two distinct regimes and thus an asymmetric adjustment around the threshold parameter c, the ESTECM better captures the asymmetry inherent in the size of interest rate deviations, whereas the LSTECM is best suited to reproducing the asymmetry in the sign of interest rate deviations.[3] Furthermore, the two interest-rate regimes are discriminated and associated with small and large values of the transition variable z_{t-d}. In the first regime, interest rate deviations are small and may be away from the equilibrium, uncorrected, near unit root and random, whereas in the second regime large interest rate deviations will be nonlinearly mean-reverting to equilibrium when they exceed a certain threshold and then approach white noise.

The empirical modeling approach of STECMs is done in several steps [Van Dijk et al. (2002)]. It first requires the definition of the explanatory variables, the determination of the lag number, the implementation of nonlinear adjustment tests, and the choice of transition function. The nonlinear least-squares method is then used to estimate the STECM.[4]

4. EMPIRICAL RESULTS

4.1. Data and Preliminary Results

We examine the international links of short-term interest rates for France, the United Kingdom, and the United States. The data consist of the daily three-month interbank-offered interest rates obtained from Datastream International and cover the period from December 31, 2004 to March 19, 2009. Working with daily data is consistent with the fact that any monetary policy adjustments tend to be immediately incorporated into the short-term interest rates over very short time spans. This thus offers us the possibility of making inferences about the responses of monetary policymakers to uncertainties and shocks related to the recent global financial crisis. France and the United Kingdom are included in the sample because the financial crisis, originating in the United States, was quickly transmitted to these countries and made it necessary for their central banks to adjust interest rates. We depict the daily time variations of the three interest rates in Figure 1. They exhibit a higher degree of comovement after the advent of the U.S. subprime crisis.

The results of commonly used tests for the presence of unit roots, including the Dickey and Fuller (1981) augmented test, the Phillips and Perron (1988) test, the

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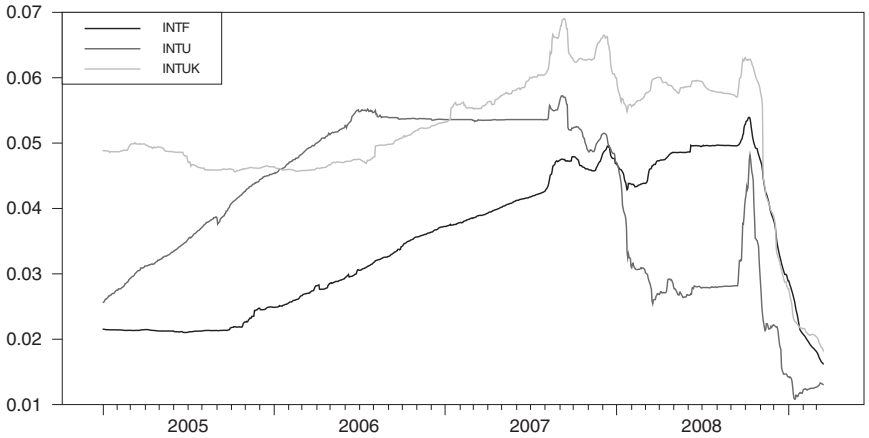


FIGURE 1. Time dynamics of the daily short-term interest rates for France (INTF1), the United States (INTU1), and the United Kingdom (INTUK1).

Zivot and Andrew (1992) test with one structural break, and the Lumsdaine and Papell (1997) test with two structural breaks confirm the common findings of the literature that interest rates are integrated of order 1.[5] For an overview of the links among the three interest rates under the effects of the recent financial crisis, we calculate the bilateral correlations between these interest rates over two subperiods (December 31, 2004–July 31, 2007; August 1, 2007–March 19, 2009) and report the results in Panel A of Table 1. We observe a substantial increase in the bilateral correlation of interest rates over the postcrisis subperiod. The greater monetary policy coordination of the central banks may well be an explanation. Although these findings are informative, more parsimonious modeling techniques must be used to check the robustness of these findings because the correlation coefficients simply reflect a linear statistical association of the variables of interest.

The descriptive statistics of interest rates, shown in Panel B of Table 1, highlight their decreasing tendency in recent months in view of the negative sign of the changes in interest rates. Skewness coefficients are negative and kurtosis coefficients are significantly greater than 3. These findings, empirically confirmed by the Jarque–Bera test for normality, suggest that changes in interest rates are not normally distributed.

4.2. Johansen Cointegration Procedure

Before we move to the investigation of interest rate dynamics via nonlinear modeling, it is opportune for us to examine the links of short-term interest rates using the well-known multivariate cointegration framework of Johansen (1988).[6] Our findings can be then compared with those from the STECM we developed in Section 3 and with those from the majority of previous works.

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TABLE 1. Correlation matrix and descriptive statistics

	DIF	DIU	DIUK
Panel A: Correlation matrices			
First subperiod: December 31, 2004–July 31, 2007			
DIF	1.00	0.02	0.07
DIU		1.00	−0.02
DIUK			1.00
Second subperiod: August 1, 2007–March 19, 2009			
DIF	1.00	0.38	0.58
DIU		1.00	0.40
DIUK			1.00
Panel B: Descriptive statistics: December 31, 2004–March 19, 2009			
Mean ($\times 10^5$)	−0.49	−1.15	−2.80
Std. dev. ($\times 10^2$)	0.02	0.04	0.04
Skewness	−1.11	−2.66	−16.13
Kurtosis	12.72	38.21	410.42
Jarque–Bera	4,542.35$^+$	57,982.80$^+$	7,634,819.00$^+$

Notes: This table reports the bilateral correlations of interest rates, as well as descriptive statistics of interest rates. DIF, DIU, and DIUK designate interest rate changes for France, the United States, and the United Kingdom.
$^+$ Rejection of the null hypothesis of normality at the 1% level.

We first apply the trace test to examine the null hypothesis of "no cointegration relationship" against its alternatives of at most one or at most two cointegration relationships, and we report the results in Table 2. The findings indicate rejection of the null hypothesis at the 5% level ($55.86 > 35.19$) and suggest the presence of at most one cointegration relationship ($12.06 < 20.26$). Table 3 reports the estimation of the cointegration relationship with the French interest rate defined as the endogenous variable in the cointegration relationship.

TABLE 2. Johansen cointegration test

Hypothesized number of CE(s)	Eigenvalue	Trace statistics	Critical value at the 5% level	Probability
None$^+$	0.039	55.86	35.19	0.00
At most 1	0.007	12.06	20.26	0.44
At most 2	0.003	3.67	9.16	0.46

Notes: The null hypothesis of no cointegration is tested against its alternatives of at least one or at least two cointegration relationships. The reported trace statistics are compared to critical values at the 0.05 level of significance to make decisions. The rejection of the null hypothesis favors the hypothesis of cointegration between three interest rate series.
$^+$ Rejection of the null hypothesis of none, at most one, and at most two cointegration relationships at the 5% level.

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TABLE 3. Cointegration relationship estimation: Normalized cointegrating coefficients of one cointegrating equation(s)

INTF	INTUK	INTUS	C
1.00	−2.41***	−0.73**	0.12***
	(−6.70)	(−2.81)	(6.67)
Log-likelihood value	22,798.18		

Notes: This table reports the estimation results of the cointegration relationship when the French interest rate is defined as the endogenous variable. INTF, INTUK, and INTUS refer to the French, British, and American interest rates. Empirical *t*-statistics are given in parentheses.
** and *** Significance at the 5% and 1% levels, respectively.

According to the cointegration results, we then estimate a VECM with three equations to examine the convergence of interest rates toward equilibrium and to apprehend their mean-reversion process. Table 4 reports the main results. The linear adjustment term appears to be significant at the 5% level only for the United States, whereas the French and British interest rates are not subject to linear mean reversion. Thus, France and the United Kingdom may subsequently

TABLE 4. Results of VECM estimation based on cointegration relationships

Error-correction estimation	*D*(ZF)	*D*(ZUS)	*D*(ZUK)
CointEq1 ($\times 10^2$)	0.03	−0.30***	0.10
	(0.29)	(−2.42)	(0.69)
D(ZF(-1))	−0.23	−0.43	0.75
	(−1.03)	(−1.40)	(3.21)***
D(ZF(-2))	0.76***	1.26***	−0.68***
	(3.35)	(4.05)	(−2.90)
D(ZUS(-1))	−0.01	0.47***	0.02
	(−0.29)	(7.04)	(0.40)
D(ZUS(-2))	−0.06	−0.04	0.08
	(−1.30)	(−0.63)	(1.58)
D(ZUK(-1))	−0.37	−0.10	0.89***
	(−1.59)	(−0.30)	(3.69)
D(ZUK(-2))	0.65***	1.16***	−0.55**
	(2.76)	(3.60)	(−2.24)
C	1.6^{-05}	1.6^{-05}	-1.7^{-05}
	(1.60)	(1.12)	(−1.59)
Adj. R^2	0.03	0.11	0.04
Log likelihood	7,190.81	6,839.36	7,138.58

Notes: We report the parameter estimates of the three-equation vector error-correction model we use to examine the dynamics of the interest rate convergence toward long-run equilibrium. *D*(ZF), *D*(ZUK), and *D*(ZUS) are the French, U.K., and U.S. interest rate deviations from their long-run equilibrium with the other rates. Empirical *t*-statistics of the estimates are shown in parentheses.
** and *** Significance at the 5% and 1% levels, respectively.

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continue to decrease their interest rates, despite the short-term spillover effects between interest rates under consideration.

Indeed, our results also highlight strong evidence of interest rate links, as for each country in the sample the short-term interest rate deviations depend not only on their previous deviations but also on those of the other countries. This significant interdependence may last for two days, reflecting the temporal shift between countries and the time required by other countries to decode, include, and react to the information contained in the interest rate changes. The slight interdependence of France and the United States may be the result of differences in the timing and magnitude of changes in monetary policy. What can be observed is that the Fed immediately decreased its policy rate in the wake of the crisis and continued to do so by stages, whereas the ECB, the supreme monetary authority for the Bank of France, kept its key interest rate constant, and even raised it, and started lowering it only on November 12, 2008.

In general, Johansen (1988)'s multivariate cointegration technique and VECM estimation results provide some evidence of significant short- and long-term relationships between interest rates. To the extent that these rates are subject to changes in the policies of central banks, this finding may support the hypothesis that the monetary policies of the Bank of England, the ECB, and the Fed are interdependent. The linear modeling discussed previously may, however, lead to biased results because of the assumption of both linearity and symmetry in the dynamic links of interest rates. A major consequence is that the adjustment process during periods of normal economic growth is similar to those during financial turmoil and crises, whereas interest rate behavior is likely to be quite different. Moreover, the dynamic patterns followed by the three interest rates seem to be neither linear nor symmetric (Figure 1). The intensity of interest rate increases preceding the subprime crisis was particularly less marked than that associated with interest rate decreases over the crisis period. The nonsymmetric and nonnormal characteristics of the distribution of interest rate deviations from their long-run equilibrium, shown in Table 5, lead to similar assessments. All in all, when the LECM is used,

TABLE 5. Descriptive statistics for interest rate deviations

	ZUK	ZUS	ZF
Mean ($\times 10^{18}$)	−5.25	−1.70	−0.93
Std. dev. ($\times 10$)	0.05	0.10	0.07
Skewness	−0.43	−0.24	−0.76
Kurtosis	3.45	1.70	2.86
Jarque–Bera	43.49^{+}	87.60^{+}	107.80^{+}

Notes: This table presents summarized statistics for the interest rate deviations from the long-run equilibrium for the French rate (ZF), U.K. rate (ZUK), and U.S. rate (ZUS). They are, on the average, negative for all three countries, indicating that the intensity of the interest rate decreases during the financial crisis.

$^{+}$ Rejection of the null hypothesis of normality at the 1% level.

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the potential for asymmetry and nonlinearity in interest rate adjustment dynamics may be neglected.

4.3. Estimation Results of Nonlinear STECM

Empirical Specifications. We now estimate the STECM, which allows both asymmetry and nonlinearity in the dynamic adjustment of interest rate deviations. In addition to the long-term cointegration relationships of interest rates, we introduce their short-term deviations into the nonlinear ECM to test how far one interest rate deviates following a change in the others. Accordingly, the formal representation of the STECM in equation (1) can be rewritten as

$$\Delta z_t^F = \phi_0 + \lambda_1 z_{t-1} + \sum_{i=1}^{p} \phi_{1,i} \Delta z_{t-i}^F + \sum_{j=0}^{p} \phi_{2,j} \Delta z_{t-j}^{\mathrm{US}} + \sum_{k=0}^{p} \phi_{3,l} \Delta z_{t-k}^{\mathrm{UK}} + \lambda_2 z_{t-1} \times F(z_{t-d}, \gamma, c) + \varepsilon_t. \quad \textbf{(6)}$$

This specification models the dynamics of the French rate, for which current and lagged U.S. and U.K. interest rate deviations are introduced as explanatory variables in the nonlinear ECM. Similar STECM representations are also given for the U.K. case (respectively the U.S. case), whereas current and lagged French and U.S. (respectively French and U.K.) interest rate deviations serve as explanatory variables.

We use the Ljung–Box test for serial correlation, partial autocorrelation functions, and the usual information criteria to determine the number of lags in the LECM for interest rate deviations. In accordance with our results, we take $p = 3$ for the United States and $p = 4$ for both France and the United Kingdom, which indicates the importance of persistence and inertia effects in interest rate adjustment dynamics. We also find that, for each interest rate in our sample data, its adjustment dynamics is significantly affected by the current and previous deviations of the other interest rates with a slight dominance of the U.S. rate. We then use the Lagrange multiplier (LM) test to check for nonlinear adjustment in interest rate deviations with respect to different values of the delay parameter d [Van Dijk et al. (2002)] and the Teräsvirta (1994) test to select the type of transition function. Table 6 summarizes the results. Our findings highlight the rejection of the linear adjustment hypothesis for interest rates in all three countries, suggesting that they are nonlinearly mean-reverting and that a nonlinear model is the most appropriate for reproducing their dynamic links. On the other hand, the Teräsvirta (1994) tests retain the exponential function, rather than the logistic function, to govern the transition between the interest rate regimes. These results are most interesting because the rejection of linearity, as well as the choice of an exponential transition function for all countries, reveals some common dynamic features of the three interest rates. Regarding the acceptance of the regime-switching hypothesis,

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TABLE 6. LM linearity test

Delay	United States	France	United Kingdom
p	3	4	4
$\hat{d}$	4	1	2
p-value	(0.00)	(0.00)	(0.00)
Teräsvirta's test conclusion	ESTECM	ESTECM	ESTECM

Notes: This table reports the results of the Lagrange multiplier test for normality. p is the number of lags in the change of deviation. $\hat{d}$ refers to the delay number defining the transition variable z_{t-d}.

it implies the existence of at least two regimes characterizing the interest rate dynamics over the study period. The activation of these regimes and the transition from one regime to another will depend closely on the intensity of interest rate changes in international monetary markets or on policy rate changes to a narrower extent, as well as on their interaction.

Results and validation. The results of the specification tests lead us to estimate an ESTECM (3,4), ESTECM (4,1), and ESTECM (4,2) for the United States, France, and the United Kingdom, respectively. The findings shown in Table 7 indicate several important facts. First, all of the autoregressive coefficients are statistically significant at conventional levels, thus confirming the persistence effects suggested by linear modeling, and suggesting continuity in interest rate cuts by central banks since July 2007. Second, for all interest rate series, both current and previous interest rate deviations have substantial information on the time dynamics of the interest rate. We also observe nonlinear continuous-time interdependence of interest rates through the significant coefficients in the second regime. In addition, the dynamic adjustment of interest rates is significantly and negatively affected by their current deviations, but positively and nonlinearly correlated with their lagged deviations. This means that in times of turbulence marked by an international financial crisis, changes in one interest rate imply an immediate adverse reaction of the others, but after a certain time lag, they get positive feedback from the others once more information about interest rate changes has been extracted.

Third, the parameters of the exponential function are statistically significant, confirming the findings of Teräsvirta's (1994) test and suggesting the presence of two regimes characterizing the dynamics of interest rate deviations: a central regime in which the interest rate may deviate from the long-run equilibrium and be uncorrected until its deviations exceed a certain threshold, and an outsider regime describing the dynamics of the interest rate when it moves back to equilibrium as a result of the activation of the nonlinear adjustment terms $\hat{\lambda}_1$ and $\hat{\lambda}_2$. It is essential to note that these coefficients, the most important parameters of the nonlinear adjustment model, are statistically significant at the 5% level. The second adjustment term $\hat{\lambda}_2$ is negative in all three cases and the sum $(\hat{\lambda}_1+\hat{\lambda}_2)$ is also

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TABLE 7. ESTECM estimation results

Variables	United States	France	United Kingdom
p	3	4	4
$\hat{d}$	4	1	2
$\hat{\gamma}$	11.42**	13.28**	100.47*
	(3.01)	(2.22)	(1.79)
c ($\times 10^2$)	−0.04*	0.11**	0.08**
	(−1.94)	(1.73)	(10.55)
$\hat{\phi}_0(\times 10^6)$	−7.60	−5.20**	2.00
	(−1.06)	(−2.50)	(−1.16)
$\hat{\lambda}_1$	−0.04	0.007**	0.03**
	(−3.36)	(2.44)	(3.65)
$\hat{\lambda}_2$	−0.05	−0.01**	−0.04**
	(−3.55)	(−2.69)	(−3.80)
$\hat{\phi}_{1,0}^{UK}$	−2.06**	−0.99**	—
	(−16.20)	(−11.70)	
$\hat{\phi}_{1,1}^{UK}$	1.34**	0.52**	0.52**
	(8.60)	(16.80)	(17.20)
$\hat{\phi}_{1,2}^{UK}$	0.61**	0.07**	0.09**
	(4.20)	(2.20)	(2.53)
$\hat{\phi}_{1,3}^{UK}$	0.12**	0.06**	0.11**
	(3.30)	(2.10)	(3.44)
$\hat{\phi}_{1,4}^{UK}$	—	0.09**	0.09**
		(3.30)	(3.30)
$\hat{\phi}_{2,0}^{US}$	—	−0.05**	−0.09**
		(−7.03)	(−16.30)
$\hat{\phi}_{2,1}^{US}$	0.52**	0.03**	0.05**
	(17.10)	(5.10)	(6.84)
$\hat{\phi}_{2,2}^{US}$	0.04	—	0.006
	(1.13)		(0.87)
$\hat{\phi}_{2,3}^{US}$	0.12**	—	0.02**
	(4.60)		(3.30)
$\hat{\phi}_{3,0}^{F}$	−0.94**	—	−0.93**
	(−7.03)		(−117.20)
$\hat{\phi}_{3,1}^{F}$	0.83**	0.49	0.51**
	(5.60)	(16.3)	(16.4)
$\hat{\phi}_{3,2}^{F}$	0.61**	0.07**	0.08**
	(4.32)	(2.20)	(2.51)
$\hat{\phi}_{3,3}^{F}$	—	0.07**	0.09**
		(2.04)	(2.81)
$\hat{\phi}_{3,4}^{F}$	—	0.01	0.09**
		(3.31)	(3.22)
$\bar{R}^2$	0.82	0.96	0.92
DW	2.04	2.02	1.99

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TABLE 7. ESTECM estimation results

Variables	United States	France	United Kingdom
ADF	−17.66	−15.25	−15.20
ARCH (q)	62.60**	49.30**	79.20**
	($q = 2$)	($q = 1$)	($q = 1$)
Number of iterations	9	32	20

Notes: This table reports the parameter estimates of the ESTECM that allows for nonlinearity and regime shifts. The values in parentheses are the t-statistics. $Q(12)$ is the Ljung-Box statistics. DW, ADF, and ARCH are the empirical statistics of the Durbin–Watson, ADF, and ARCH tests. $\bar{R}^2$ denotes the determination coefficient of the regression model.
* and ** Significance at the 10% and 5% levels, respectively.

negative. So even though the short-term interest rates may deviate in the first regime from the equilibrium (i.e., $\hat{\lambda}_1 \geq 0$), they are nonlinearly mean-reverting over the long run and the estimated ESTECMs are stable over the study period.

Fourth, we check the robustness of the results discussed previously by analyzing the statistical properties of the estimated residuals and show that they are symmetrical, stationary, and not autocorrelated. They are, however, characterized by an ARCH effect. Future research on the time-series dynamics of interest rate deviations must thus consider their time-varying volatility feature.

Finally, for more insight into the regimes characterizing the interest rate adjustment dynamics, we depict in Figure 2 the estimated transition functions (transition function with respect to transition variable) and the intertemporal transition functions (transition function with respect to time factor) for the United Kingdom, the United States, and France. The most important observations are asymmetrically distributed, confirming the choice of the exponential representation. The adjustment speed of the estimated transition functions varies and increases with the size of the interest rate deviations. The more the latter increase, the more rapid the interest rate mean-reversion is. Moreover, for all three countries, the values of the estimated transition function do not exceed 12%. The latter value is obtained in the U.K. case and reflects the intense intervention of the U.K. monetary authority in adjusting the policy interest rate and thus in moving the short-term interest rate back to its long-run equilibrium with U.S. and French rates. This finding also indicates that interest rate deviations are relatively near unit root and their dynamics may approach a random walk.

It is equally important to note that whereas the U.S. monetary market is characterized by a more volatile transition function, perhaps because of the Fed's successive interventions, the U.K. market has a higher transition function, reflecting the highest interest rate fall in 2009. A close examination of the graphs shows similar dynamics for the three interest rates, which is typically evidence of monetary policy interdependence. Furthermore, these functions consistently reflect the policy interest rate adjustments by the French, U.K., and U.S. central banks in the recent period.

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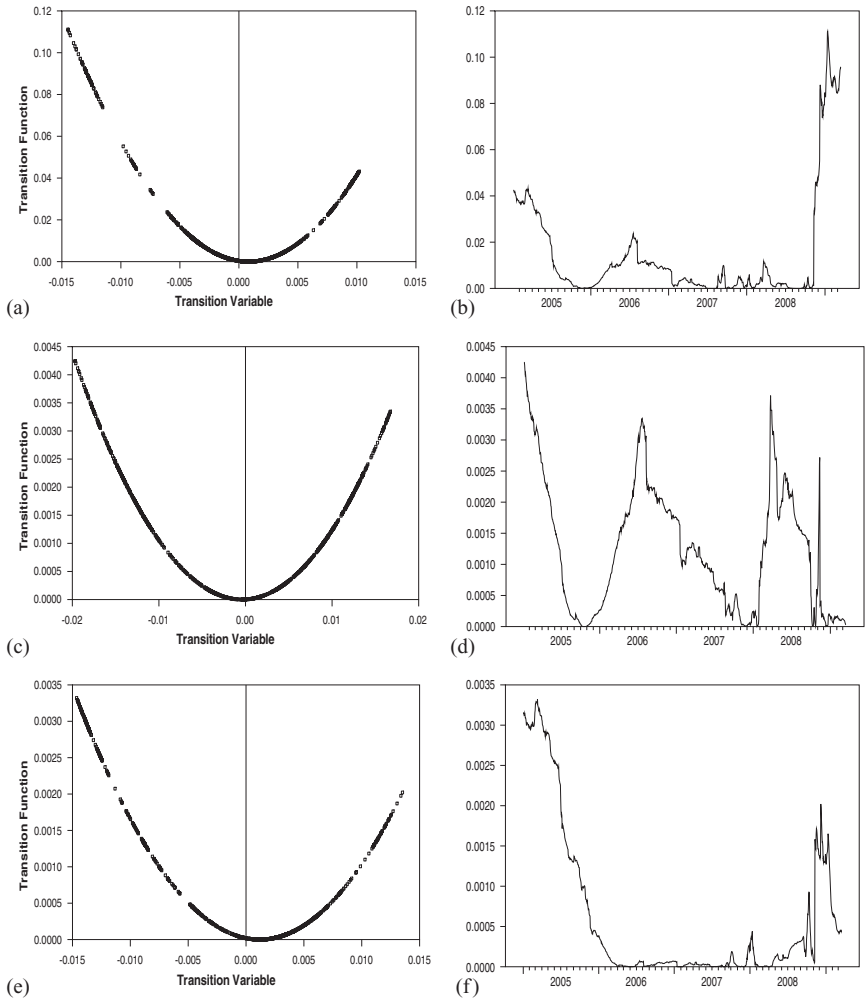


FIGURE 2. Estimated transition functions of the ESTECMs for the U.K., U.S., and French interest rates: (a) transition function for the United Kingdom; (b) intertemporal transition function for the United Kingdom; (c) transition function for the United States; (d) intertemporal transition function for the United States; (e) transition function for France; (f) intertemporal transition function for France.

Interest rate undervaluation and overvaluation periods. The empirical model used in this paper also permits us to gauge the interest rate undervaluation and overvaluation phases, as well as the speed of interest rate mean reversion and the rhythm of correction of interest rate misalignments. To do so, we use the two

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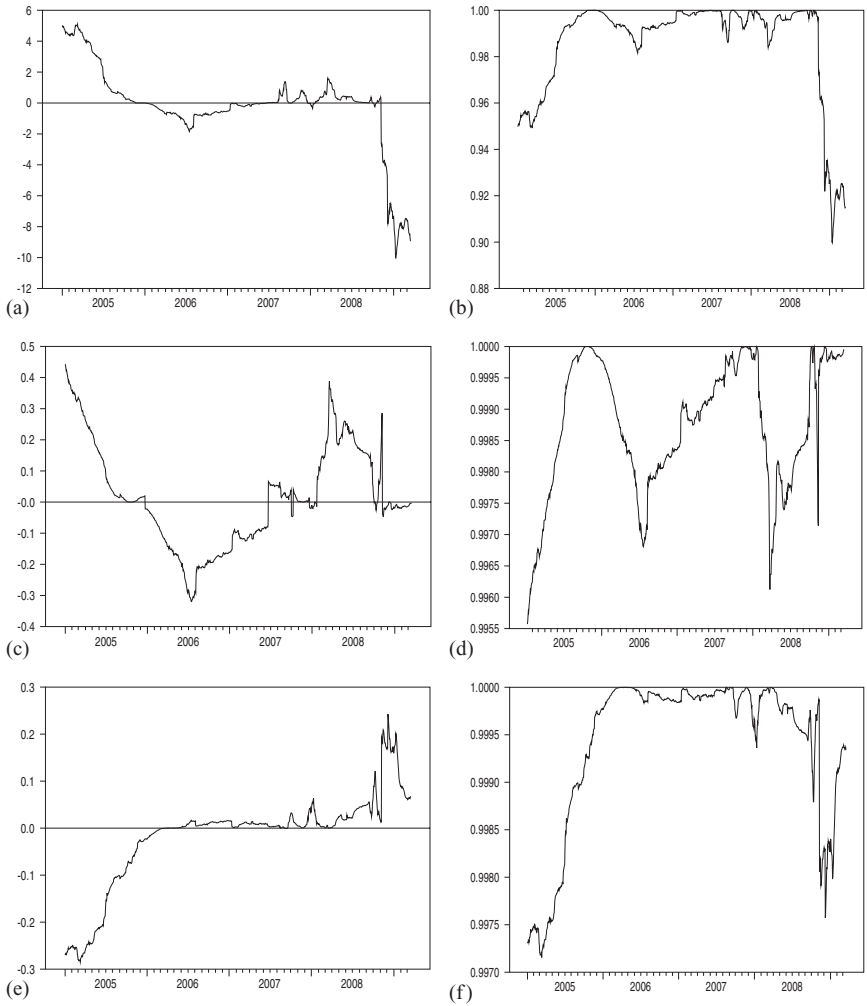


FIGURE 3. Undervaluation and overvaluation phases for the U.K. (a), U.S. (c), and French (e) interest rates and mean reversion in the interest rate adjustment dynamics for the U.K. (b), U.S. (d), and French (f) interest rates.

indicators developed by Peel and Taylor (2000).[7] The first indicator corresponds to

$$\Omega(z_t) = 100 \times F(z_t) \times \text{sign}(z_t),\ \text{sign}(z_t) \equiv \frac{z_t}{|z_t|},\ -100 \leq \Omega(z_t) \leq 100. \quad (7)$$

This indicator reproduces the magnitude of the interest rate deviations from equilibrium. Following Peel and Taylor (2000), a positive value of this indicator implies

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that interest rates are overevaluated and a negative value that they are undereval-uated. Interest rates are in equilibrium when this indicator converges toward zero.

The second indicator is given by

$$\Gamma(z_t) = 1 - F(z_{t-d}),\ 0 \leq \Gamma(z_t) \leq 1. \quad (8)$$

This indicator evaluates the interest rate adjustment speed. The more $\Gamma(z_t)$ tends toward 1, the more the interest rate deviations z_t approach a random walk, the more $\Gamma(z_t)$ convergence toward the speed of adjustment increases, and the more z_t converges toward white noise [Jawadi and Prat (2012)].

The two indicators discussed are computed for each of the three countries in our sample and presented in Figure 3. As can be observed, the evolution of the second indicator highlights a high adjustment speed, thus indicating that interest rate adjustment is highly activated for all three monetary markets because it is close to one almost all the time. This result also confirms the low estimated values for the transition functions. Moreover, we observe clear phases of interest rate overvaluation for the United States and the United Kingdom, and, in 2005, an undervaluation phase for France. These phases are then followed by a correction phase, which, however, is more turbulent at the end of the estimation period, probably because of the effects of the global financial crisis. It is also worth noting that the most recent readings of the undervaluation and overvaluation indicator have shown several similarities in interest rate adjustment dynamics for France, the United Kingdom, and the United States, similarities that once again suggest that monetary policies of these countries have been more closely linked in recent years.

5. CONCLUDING REMARKS

In this paper, multivariate cointegration techniques, VECMs, and STECMs were used to study the dynamic links of short-term interest rates for France, the United Kingdom, and the United States. Using daily data from December 31, 2004 to March 19, 2009, we find strong evidence of nonlinear and heterogeneous links between the three interest rates. Exogenous shifts in the U.S. short-term interest rate are found to lead those in France and in the United Kingdom over a horizon from one to two business days. The results also show that, over the study period, the time deviations of interest rates are subject to regime-switching behavior and that their multivariate nexus converges toward a common long-run equilibrium in a nonlinear way. If we can assume that deviations of short-term interest rates reasonably reflect changes in target interest rates, the convergence of short-term interest rates toward a common equilibrium in recent years may be explained by greater coordination by the ECB, the U.K. central bank, and the Fed. An examination of the interdependence of monetary policy surprises could be done to confirm this hypothesis.

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NOTES

1. See Rogoff (1996) for a survey of theoretical and empirical evidence on PPP.

2. The difference between the STAR (smooth-transition autoregression) and the STECM is the transition variable, z_{t-d}. In a STAR model, z_{t-d} can be a lagged dependent variable, Y_{t-d}, and in a STECM, z_{t-d} is represented by the error-correction term.

3. See Teräsvirta (1994) for more details regarding the statistical properties of these transition functions.

4. See Jawadi and Prat (2012) for more details concerning the STECM modeling, nonlinear adjustment tests, and estimation issues.

5. Results of unit root tests are available from the corresponding author.

6. Engle and Granger's (1987) analysis of cointegration, although widely used in previous studies, makes it possible to examine one cointegrating vector at a time. Therefore, for our study, the cointegration testing procedure of Johansen (1988) seems to be more appropriate, because we can simultaneously test for several cointegration relationships, and thus prevent the loss of information about the real interest rate links due to the restriction of a bilateral relationship.

7. Peel and Taylor (2000) develop these indicators to investigate the exchange-rate adjustment to PPP, whereas they are applied by Jawadi and Prat (2012), for example, to examine the undervaluation and overvaluation of stock prices with respect to their fundamentals.

REFERENCES

Anderson, H.M. (1997) Transaction costs and nonlinear adjustment towards equilibrium in the US Treasury bill market. *Oxford Bulletin of Economics and Statistics* 59, 465–484.

Anoruo, E., R. Sanjay, and H.F. Thiewes (2002) International linkage of interest rates: Evidence from the emerging economies of Asia. *Global Finance Journal* 13, 217–235.

Awad, M.A. and B.K. Goodwin (1998) Dynamic linkages among real interest rates in international capital markets. *Journal of International Money and Finance* 17, 881–907.

Balke, N.S. and T. Fomby (1997) Threshold cointegration. *International Economic Review* 38, 627–643.

Barassi, M.R., G.M. Caporale, and S.G. Hall (2005) Interest rate linkages: A Kalman filter approach to detecting structural change. *Economic Modelling* 22, 253–284.

Baum, C. and J. Barkoulas (2006) Dynamics of intra-EMS interest rate linkages. *Journal of Money, Credit and Banking* 38, 469–482.

Benigno, P. (2002) A simple approach to international monetary policy coordination. *Journal of International Economics 57*, 177–196.

Bernanke, B. and K.N. Kuttner (2005) What explains the stock market's reaction to Federal Reserve policy. *Journal of Finance* 60, 1221–1257.

Bremnes, H., O. Gjerde, and F. Soettem (2001) Linkage among interest rates in the United States, Germany and Norway. *Scandinavian Journal of Economics* 103, 127–145.

Chen, S.S. (2007) Does monetary policy have asymmetric effects on stock returns? *Journal of Money* 39, 667–668.

Cuaresma, J.C. and C. Wójcik (2006) Measuring monetary independence: Evidence from a group of new EU member countries. *Journal of Comparative Economics* 34, 24–43.

Cumby, R.E. and M.S. Mishkin (1986) The international linkage of real interest rates: The European–US connection. *Journal of International Money and Finance* 5, 5–23.

Dickey, D.A. and W.A. Fuller (1981) Likelihood ratio statistics for autoregressive time series with a unit root. *Econometrica* 49, 1057–1072.

Engle, R.F. and C.W.J. Granger (1987) Cointegration and error correction: Representation, estimation and testing. *Econometrica* 55, 251–276.

Escribano, A. (1997) Nonlinear Error-Correction: The Case of Money Demand in the UK (1878–1970). Working paper 96–55, Universidad Carlos III De Madrid.

https://doi.org/10.1017/S136510051100037X Published online by Cambridge University Press

Franses, P.H. and D. Van Dijk (2000) *Non-linear Time Series Models in Empirical Finance*, 1st ed. Cambridge, UK: Cambridge University Press.
Granger, C.W.J. (1981) Cointegrating Variables and Error Correcting Models. Working paper, University of California, San Diego.
Granger, C.W.J. and T. Teräsvirta (1993) *Modeling Nonlinear Economic Relationships*, 1st ed. New York: Oxford University Press.
Holmes, M.J. and N. Maghrebi (2004) Asian real interest rates, nonlinear dynamics, and international parity. *International Review of Economics and Finance* 13, 387–405.
Ioannidis, C. and A. Kontonikas (2007) The impact of monetary policy on stock prices. *Journal of Policy Modeling* 30, 33–53.
Jawadi, F., C. Bruneau, and N. Sghaier (2009) Nonlinear cointegration relationships between non-life insurance premium and financial markets. *Journal of Risk and Insurance* 76, 753–783.
Jawadi, F. and G. Prat (2012) Arbitrage costs and nonlinear adjustment in the G7 stock markets. *Applied Economics* 44, 1561–1582.
Johansen, S. (1988) Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control* 12, 231–254.
Kirchgassner, G. and J. Wolters (1993) Does the DM dominate the Euro market? An empirical investigation. *Review of Economics and Statistics* 75, 773–778.
Liu, Y. (2001) Modelling Mortgage Rate Changes with a Smooth Transition Error Correction Model. Working paper 23, Bank of Canada.
Lumsdaine, R.L. and D.H. Papell (1997) Multiple trend breaks and the unit-root hypothesis. *Review of Economics and Statistics* 79, 212–218.
Mancuso, A.J., B.K. Goodwin, and T.J. Grennes (2003) Nonlinear aspects of capital market integration and real interest rate equalization. *International Review of Economics and Finance* 12, 283–303.
Mark, N.C. (1985) Some evidence on the international inequality of real interest rates. *Journal of International Money and Finance* 4, 189–208.
Michael, P., A.R. Nobay, and D.A. Peel (1997) Transactions costs and nonlinear adjustment in the real exchange rates: An empirical investigation. *Journal of Political Economy* 105, 862–879.
Mishkin, F.S. (1984) Are real interest rates equal across countries? An empirical investigation of international parity conditions. *Journal of Finance* 39, 1345–1357.
Peel, D.A. and M.P. Taylor (2000) Nonlinear adjustment, long-run equilibrium and exchange rates fundamentals. *Journal of International Money and Finance* 19, 33–53.
Phillips, P.C.B. and P. Perron (1988) Testing for unit root in a time series regression. *Biometrika* 75, 335–346.
Poghosyan, T. and J. de Haan (2007) Interest Rate Linkages in EMU Countries: A Rolling Threshold Vector Error-Correction Approach. CESifo working paper series.
Rigobon, R. and B. Sack (2003) Measuring the reaction of monetary policy to the stock market. *Quarterly Journal of Economics* 118, 639–669.
Rogoff, K. (1996) The purchasing power parity puzzle. *Journal of Economic Literature* 34, 647–668.
Rothman, P., D. Van Dijk, and P.H. Franses (2001) A multivariate STAR analysis of the relationships between money and output. *Macroeconomic Dynamics* 5, 506–532.
Scotti, C. (2006) A Bivariate Model of Fed and ECB Main Policy Rates. International finance discussions paper, Board of Governors of the Federal Reserve System.
Teräsvirta, T. (1994) Specification, estimation and evaluation of smooth transition autoregressive models. *Journal of the American Statistical Association* 89, 208–218.
Teräsvirta, T. and H.M. Anderson (1992) Characterizing nonlinearities in business cycles using smooth transition autoregressive models. *Journal of Applied Econometrics* 7, 119–136.
Uctum, M. (1999) European integration and asymmetry in the EMS. *Journal of International Money and Finance* 18, 769–798.
Van Dijk, D., T. Teräsvirta, and P.H. Franses (2002) Smooth transition autoregressive models—A survey of recent developments. *Econometric Reviews* 21, 1–47.
Wang, Z., J. Yang, and Q. Li (2007) Interest rate linkages in the Eurocurrency market: Contemporaneous and out-of-sample Granger causality tests. *Journal of International Money and Finance* 26, 86–103.

https://doi.org/10.1017/S136510051100037X Published online by Cambridge University Press

Yamada, H. (2002) On the linkage of real interest rates between the US and Canada: Some additional empirical evidence. *Journal of International Financial Markets, Institutions and Money* 12, 279–289.
Zhou, S. (2003) Interest rate linkages within the European monetary system: New evidence incorporating long-run trends. *Journal of International Money and Finance* 22, 571–590.
Zivot, E. and D.W.K. Andrews (1992) Further evidence on the great crash, the oil price shock and the unit root hypothesis. *Journal of Business and Economic Statistics* 10, 251–270.

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