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Brillouin-shifted third-harmonic backscattering of laser in a magnetized plasma

ALIREZA PAKNEZHAD

Department of Physics, Shabestar Branch, Islamic Azad University, Shabestar, Iran

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Abstract

Third-harmonic Brillouin backscattering (3HBBS) instability is investigated in the interaction of a picosecond extraordinary laser pulse with a homogeneous transversely magnetized underdense plasma. Nonlinear coupled equations that describe the instability are derived and solved for a weakly coupled regime to find the maximum growth rate. The nonlinearity arises through the combined effect of relativistic mass increase, static magnetic field, and ponderomotive acceleration of plasma electrons. The growth rate is found to decrease as the static magnetic field increases. It also increases by increasing both plasma density and laser intensity. It is also established that the growth rate of 3HBBS instability in a magnetized plasma is lower than that of fundamental Brillouin backscattering instability.

Keywords: Growth rate; Magnetized underdense plasma; Third-harmonic Brillouin backscattering

1. INTRODUCTION

The interaction of an intense, picosecond laser pulse propagating in an underdense plasma is of fundamental interest in plasma physics, with applications to fast-ignition fusion, harmonic generation, and laser–plasma accelerator concepts (Nakajima, 2000; Hora *et al.*, 2011; 2013; Foldes *et al.*, 2013). Stimulated Raman scattering and stimulated Brillouin scattering are two important instabilities that occur in an underdense plasma, which are still a major issues in the realization and understanding of inertial confinement fusion (ICF) because the driving energy for the implosion is provided by the incident laser and the occurrence of backscattered light could greatly reduce the laser energy absorption efficiency (Bawaaneh, 2006; Paknezhad & Dorranian, 2011, Paknezhad, 2013*a*, *b*). Hence, the control of this instability is therefore of a fundamental concern for ICF. Brillouin scattering occurs in the interaction of a picosecond laser pulse with a plasma, which involves coupling of a large-amplitude light wave to a scattered light wave and an ion acoustic wave (Baton *et al.*, 1994; Froula *et al.*, 2003). Stimulated Brillouin scattering in a transversely magnetized plasma involves decaying of a pump laser wave (ω_0, k_0) into an ion acoustic upper-hybrid wave (ω, k) and two scattered upshifted (anti-Stokes) and downshifted (Stokes) light waves ($\omega_s = \omega_0 \pm \omega$, $k_s = k \pm k_0$) (Paknezhad, 2012; 2013*a*; Sharma *et al.*, 2012). Brillouin instability has been extensively studied in magnetized and unmagnetized plasmas for many years both theoretically and experimentally (Wang *et al.*, 2009; Bernhardt *et al.*, 2010; Paknezhad, 2012; 2013*a*). In an important earlier research, Salimullah and Hassan (1990) investigated the relativistic effects on the stimulated scattering of large-amplitude laser radiation in the presence of the self-generated magnetic field in a laser-produced plasma. They show that the extreme relativistic effects increase the growth rate of the stimulated scattering. Recently, Sharma and Singh (2013) have studied stimulated Brillouin backscattering of filamented hollow Gaussian beams in a collisionless plasma. They have shown that self-focusing and Brillouin backscattering decrease for higher order of a hollow Gaussian beam.

In addition to this topic, generation of harmonic radiation is also an important issue in the study of laser–plasma instabilities (Gibbon, 1997; Gupta *et al.*, 2007; Jha *et al.*, 2007). In a study in connection with this subject related to the Brillouin instability, Singh and Tripathi (2010) have carried out a numerical analysis of resonant Brillouin-shifted third-harmonic generation of a Gaussian laser pulse in a plasma. They demonstrated that the ion acoustic wave, generated in the stimulated Brillouin backscattering process, provides an uncompensated momentum between the harmonic photon and the combining fundamental photons, leading to the resonant enhancement of the harmonic power through the coupling of the laser with the ion acoustic wave.

Address correspondence and reprint requests to: Alireza Paknezhad, Department of Physics, Shabestar Branch, Islamic Azad University, Shabestar, postal code: 5381651111, Iran. E-mail: a.paknezhad@iaushab.ac.ir

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Brillouin instability has the largest growth rate in backward scattering in which only the scattered Stokes wave is important. Figure 1 shows the schematic and k-vector matching condition of the Brillouin backscattering process in a transversely magnetized plasma. During the Brillouin backscattering process, the density perturbation at ($\omega_2 = 2\omega_0 - \omega$, $k_2 = k - 2k_0$), which arises from the coupling of density perturbation associated with the ion acoustic upper-hybrid wave at (ω, k) and the second-harmonic oscillatory velocity of plasma electrons at ($2\omega_0$, $2k_0$), beats with the fundamental electron velocity at (ω_0, k_0) to generate a third-harmonic nonlinear current density at ($\omega_{3s} = 3\omega_0 - \omega$, $k_{3s} = k - 3k_0$).

Our aim in this work is to study the effect of third-harmonic generation on the growth rate of nonlinear Brillouin backscattering instability in a mildly relativistic transversely magnetized plasma. To achieve this goal, we follow the same method that has been used for the third-harmonic Raman backscattering process in our previous work (Paknezhad, 2013*c*). The present paper is organized as follows: In Section 2, we formulate a nonlinear wave equation for Brillouin scattering. In Section 3, we calculate the growth rate of third-harmonic Brillouin backscattering (3HBBS) instability in a magnetized plasma. Section 4 is devoted to results and discussion, and conclusion is presented in Section 4.

2. GENERAL FORMALISM

We now analyze the behavior of an electromagnetic wave that propagates in a homogeneous and magnetized plasma with initial density n_0. A constant magnetic field $\mathbf{B}_0$ in the y-direction is considered in the plasma channel. Here, the pump laser pulse is assumed to be a short pulse and have a high frequency, and since the duration of the short pulse is less than the plasma period, we ignore filamentation and self-focusing effects, hence is valid when the laser spot size is much bigger than c/ω_p. We consider an electromagnetic wave propagates perpendicularly to $\mathbf{B}_0$. For an extraordinary wave (defined also as X-mode) the electric field is perpendicular to $\mathbf{B}_0$, and therefore the electric field induces an electron velocity in the x–z-plane. As a result, the force $v \times \mathbf{B}_0$ introduces another velocity term in this plane. As a wave propagates through a region in which ω_p and ω_c are changing, it may encounter cutoffs and resonances. A wave generally reflected at a cutoff and absorbed at a resonance. There are two cutoffs that occur when the index of refraction goes to a zero value. The cutoffs of the extraordinary wave are found by setting k_0 equal to zero in the dispersion relation of the incident electromagnetic wave. The cutoff frequencies are $\omega_1 = \frac{1}{2}[\omega_c + (4\omega_p^2 + \omega_c^2)^{1/2}]$ and $\omega_2 = \frac{1}{2}[-\omega_c + (4\omega_p^2 + \omega_c^2)^{1/2}]$, which correspond to plasmas with $\omega_p < \omega_0$ and $\omega_p > \omega_0$, respectively. Since we consider the interaction of a high-frequency laser pulse with an underdense plasma, in which $\omega_p \ll \omega_0$ and $\omega_c \ll \omega_0$, the cutoff points therefore do not appear in our investigation.

Consider an extraordinary laser pump wave propagating in a homogeneous underdense plasma embedded in a constant external magnetic field $\mathbf{B}_0\hat{y}$ (see Fig. 2). The electric vector of the radiation field propagating along the z-direction is represented by

$$\mathbf{E}(z, t) = E_0(\hat{x} + i\alpha_0\hat{z})\, e^{i(k_0 z - \omega_0 t)}$$

where E_0, k_0, and ω_0 are amplitude, wave number, and frequency of the radiation field, respectively. Similarly, the vector potential of the radiation field is defined as $\mathbf{A}(z, t) = A_0(-i\hat{x} + \alpha_0\hat{z})\, e^{i(k_0 z - \omega_0 t)}$. Parameter α_0, which is attributable to the extraordinary property of the incident light

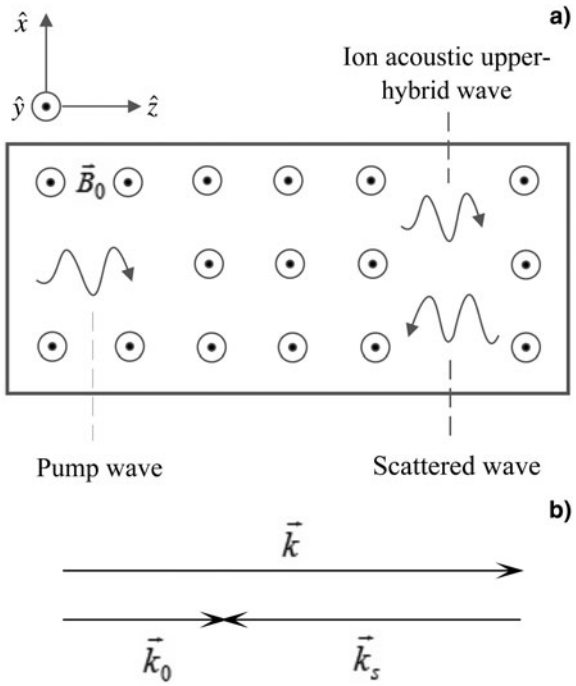


Fig. 1. (a) Schematic and (b) the k-vector diagram of the Brillouin backscattering process in a transversely magnetized plasma, where $\mathbf{k}$, $\mathbf{k}_0$, and $\mathbf{k}_s$ are the wave vectors of the excited ion acoustic upper-hybrid wave, pump wave, and scattered Stokes wave, respectively.

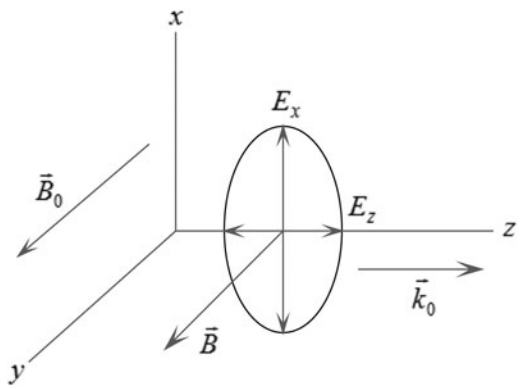


Fig. 2. The electric field components (E_x, E_z), the magnetic field $\mathbf{B}$, and the wave vector k_0 of the extraordinary pump laser wave in a transversely magnetized plasma. The electric field is elliptically polarized in a plane perpendicular to the external magnetic field $\mathbf{B}_0$.

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wave in the presence of ion motion, is given by

$$\alpha_0 = \frac{(\omega_c/\omega_0)(\omega_p^2/(\omega_0^2-\omega_c^2)) - (\Omega_c/\omega_0)(\Omega_p^2/(\omega_0^2-\Omega_c^2))}{1-(\omega_p^2/(\omega_0^2-\omega_c^2)) - (\Omega_p^2/(\omega_0^2-\Omega_c^2))},$$

where Ω_c is the ion cyclotron frequency and Ω_p is the ion oscillation frequency in the plasma (Krall & Trivelpicec, 1973). The laser beam polarization is assumed to remain unaltered while propagating through a transversely magnetized plasma. Following the same procedure from our previous work (Paknezhad, 2013*c*) and replacing α_0 with $\beta_0(=(\omega_c/\omega_0)(\omega_p^2/(\omega_0^2-\omega_{UH}^2)))$ in all computations, the *x*-component of the nonlinear wave equation of the extraordinary wave in magnetized plasma turns out to be

$$\left(\frac{\partial^2}{\partial t^2} - c^2\frac{\partial^2}{\partial z^2}\right)A_x = \frac{2\pi i n e^2}{m}\mu_0 A_0\, e^{i(k_0 z-\omega_0 t)}, \tag{1}$$

where

$$\mu_0(\omega_0) = \left(\frac{\omega_0^2+\alpha_0\omega_0\omega_c}{\omega_0^2-\omega_c^2}\right) + a_0^2\left(\frac{c^2k_0^2\omega_0 M_1}{4(\omega_0^2-\omega_c^2)^4(4\omega_0^2-\omega_c^2)} - \frac{3}{8}\frac{\omega_0 M_2}{(\omega_0^2-\omega_c^2)^4}\right)$$

with

$$M_1 = 2\alpha_0^2\omega_0^7 + \alpha_0(6\alpha_0^2-25)\omega_0^4\omega_c^3 + (26\alpha_0^2-5)\omega_0^3\omega_c^4 - (30\alpha_0^2+5)\omega_0^5\omega_c^2 + 27\alpha_0\omega_0^2\omega_c^5 - \alpha_0(6\alpha_0^2-5)\omega_0^6\omega_c + 10\omega_0\omega_c^6 + \alpha_0\omega_c^7$$

and

$$M_2 = (3\alpha_0^2+1)(\omega_0^7 + 6\omega_0^5\omega_c^2 + \omega_0^3\omega_c^4) + 4\alpha_0(\alpha_0^2+3)(\omega_0^6\omega_c + \omega_0^4\omega_c^3).$$

Equation (1) gives the nonlinear dispersion relation of the extraordinary pump laser wave in the magnetized plasma:

$$\frac{c^2k_0^2}{\omega_0^2} = 1 - \mu_0\frac{\omega_p^2}{\omega_0^2}. \tag{2}$$

By neglecting the second and fourth terms in coefficient μ_0 and then replacing it in the dispersion relation (2) and after some algebra, we recover the linear dispersion relation of the extraordinary electromagnetic wave in a non-relativistic magnetized plasma, which is known as (Krall & Trivelpicec, 1973)

$$\frac{c^2k_0^2}{\omega_0^2} = 1 - \frac{\omega_p^2}{\omega_0^2-\omega_c^2} - \frac{\Omega_p^2}{\omega_0^2-\Omega_c^2} - \frac{((\omega_c/\omega_0)(\omega_p^2/(\omega_0^2-\omega_c^2)) - (\Omega_c/\omega_0)(\Omega_p^2/(\omega_0^2-\Omega_c^2)))^2}{1-(\omega_p^2/(\omega_0^2-\omega_c^2)) - (\Omega_p^2/(\omega_0^2-\Omega_c^2))}.$$

3. THIRD-HARMONIC BRILLOUIN INSTABILITY ANALISIS

As the Brillouin instability switches into the third-harmonic scattering process, the density perturbation at $(2\omega_0 \pm \omega, k \pm 2k_0)$ couples with the electron velocity at (ω_0, k_0) to generate a third-harmonic nonlinear current density at $(3\omega_0 \pm \omega, k \pm 3k_0)$. Therefore, the wave equation for the third-harmonic scattered waves becomes

$$\left(\frac{\partial^2}{\partial t^2} - c^2\frac{\partial^2}{\partial z^2}\right)A_{x(3\omega_0\pm\omega)} = 4\pi J_{x(3\omega_0\pm\omega)}, \tag{3}$$

where $J_{x(3\omega_0\pm\omega)}$ is the third-harmonic nonlinear current density associated with the third-harmonic Brillouin scattering and is determined using the equation of continuity for density perturbation at $(2\omega_0 \pm \omega)$, which has the form

$$J_{x(3\omega_0\pm\omega)} = \frac{i}{2}ec\left(n_0 a_{3s}\mu_{3s} + \frac{1}{2}n_\omega\sigma_0\mu_0 a_0\right)e^{i[(k\pm 3k_0)z-(3\omega_0\pm\omega)t]} \tag{4}$$

with

$$\sigma_0 = -\frac{c^2a_0^2k_0^2[(\alpha_0^2+4)\omega_0^2\omega_c^2 + 2(\alpha_0^2-1)\omega_0^4 + 2\alpha_0\omega_0^3\omega_c + 4\alpha_0\omega_0\omega_c^3 + \omega_c^4]}{4(\omega_0^2-\omega_c^2)^2(4\omega_0^2-\omega_c^2)}.$$

Here, n_ω is the low-frequency density fluctuation associated with the electrostatic ion acoustic upper-hybrid wave arising in the stimulated Brillouin scattering, and a_{3s} and μ_{3s}, which are defined by replacing ω_0 with $(3\omega_0 \pm \omega)$ in the definition of parameter μ_0, are the normalized amplitude and nonlinear coefficient of the scattered wave, respectively. Hence, the linearized nonlinear wave equation governing the propagation of scattered Stokes and anti-Stokes waves with amplitude $A_{3\pm}$ associated with the Brillouin scattering in magnetized plasma is obtained as

$$(\omega_{3\pm}^2 - c^2k_{3\pm}^2 - \mu_{3s}\omega_p^2)A_{3\pm}(\omega_{3\pm}, k_{3\pm}) = \frac{2\pi e^2}{m}\mu_0\sigma_0 A_0 n_\omega(k,\omega), \tag{5}$$

where $k_{3\pm} = k \pm 3k_0$ and $\omega_{3\pm} = 3\omega_0 \pm \omega$ are the wave number and frequency of the scattered Stokes/anti-Stokes waves, respectively. Combining the equation of electron's

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motion with the continuity equation in the direction of laser wave gives the second-coupled equation associated with the Brillouin scattering of *X*-mode laser wave in a magnetized plasma

$$\frac{e}{m}\left(1+\frac{\omega_c^2}{\omega_p^2}\right)\frac{\partial\phi_s}{\partial z}=\frac{e^2}{m^2c^2}\frac{\partial}{\partial z}[\mu_{3s}\mathbf{A}_{3s}\cdot\mathbf{A}_{3x}+\chi_{3s}\mathbf{A}_{3s}\cdot\mathbf{A}_{3z}] + \frac{v_T^2}{n_0}\frac{\partial n_\omega}{\partial z}. \tag{6}$$

The second term on the right-hand side of Eq. (6) is due to the thermal pressure of plasma electrons, where $v_T\ (=\sqrt{T_e/m})$ is the electron thermal velocity. In this equation, ϕ_s is the electrostatic potential of plasma fluctuations, and $\mathbf{A}_3$, $\mathbf{A}_{3s}$ are the vector potentials of the third-harmonic incident and scattered waves, respectively. Also, μ_{3s} is defined by replacing $(3\omega_0 \pm \omega)$ with ω_0 in the definition of parameter μ_0, and parameter χ_{3s} [which is determined by replacing $(3\omega_0 \pm \omega)$ with ω_0 in the definition of parameter χ_0] is given by

$$\chi_0=\frac{\alpha_0\omega_0^2+\omega_0\omega_c}{\omega_0^2-\omega_c^2}+\frac{a_0^2}{(\omega_0^2-\omega_c^2)^4} \times\left[\frac{c^2k_0^2}{4(4\omega_0^2-\omega_c^2)}(M_1\omega_c+M_3(\omega_0^2-\omega_c^2)) -\frac{3}{8}(M_2\omega_c+M_4(\omega_0^2-\omega_c^2))\right]$$

with

$$M_3=\alpha_0(3\alpha_0^2-25)\omega_0^4\omega_c^2+(14\alpha_0^2-9)\omega_0^3\omega_c^3-(26\alpha_0^2+1)\omega_0^5\omega_c - \alpha_0(6\alpha_0^2+3)\omega_0^6+15\alpha_0^2\omega_c^4+7\omega_0\omega_c^5$$

and

$$M_4=\alpha_0(\alpha_0^2+3)\omega_0^6+(3\alpha_0^2+1)(\omega_0^3\omega_c^3+3\omega_0^5\omega_c) + \alpha_0(3\alpha_0^2+6\alpha_0+3)\omega_0^4\omega_c^2.$$

Now, combining the equations of ion motion and continuity gives (Paknezhad, 2013*c*)

$$\frac{\partial^2 n_\omega}{\partial t^2}=\frac{n_0Ze}{M}\frac{\partial^2\phi_s}{\partial z^2}-\Omega_c^2 n_\omega \tag{7}$$

where M and Z are the ion rest mass and the ion charge state respectively and Ω_c $(=e\mathbf{B}_0/Mc)$ is the ion cyclotron frequency, and we assumed that both electron and ion fluids have the same density in the plasma. By substituting ϕ_s from Eq. (6) into Eq. (7), we finally obtain a nonlinear equation for the low-frequency density fluctuation in plasma:

$$\left[\frac{\partial^2}{\partial t^2}+\Omega_c^2-c_p^2\frac{\omega_p^2}{\omega_{UH}^2}\frac{\partial^2}{\partial z^2}\right]n_\omega =\frac{\omega_p^2}{\omega_{UH}^2}\frac{n_0Ze^2}{mMc^2}\frac{\partial^2}{\partial z^2}[\mu_{3s}\mathbf{A}_{3s}\cdot\mathbf{A}_{3x}+\chi_{3s}\mathbf{A}_{3s}\cdot\mathbf{A}_{3z}], \tag{8}$$

where $c_s=(ZT_e/M)^{1/2}$ is the ion acoustic velocity. Here, $\mathbf{A}_{3x}=A_0\,e^{i[3(k_0z-\omega_0t)]}$ and $\mathbf{A}_{3z}=i\alpha_{3\omega_0}A_0\,e^{i[3(k_0z-\omega_0t)}$ are the third-harmonic vector potential components, and $A_{3sx}=A_{3\pm}e^{i[(k\pm3k_0)z-(3\omega_0\pm\omega)t)]}$ and $\mathbf{A}_{3sz}=i\alpha_{3\pm}A_{3\pm}\,e^{i[(k\pm3k_0)z-(3\omega_0\pm\omega)t)]}$ are the vector potential components of the scattered Stokes and anti-Stokes waves associated with the Brillouin scattering, where $\alpha_{3\omega_0}$ and $\alpha_{3\pm}$ are respectively determined by replacing $3\omega_0$ and $(3\omega_0\pm\omega)$ with ω_0 in the definition of α_0. Fourier transform of Eq. (8), and then combining with Eq. (5) gives the dispersion relation of the exited ion acoustic upper-hybrid wave

$$\omega^2-\Omega_c^2-k^2c_s^2\frac{\omega_p^2}{\omega_{UH}^2} =\frac{c^2k^2\Omega_p^2a_0^2\mu_0\sigma_0}{4}\frac{\omega_p^2}{\omega_{UH}^2}\left[\frac{(\mu_{3+}+\chi_{3+}\alpha_{3\omega_0}\alpha_{3+})}{D_{3+}} +\frac{(\mu_{3-}+\chi_{3-}\alpha_{3\omega_0}\alpha_{3-})}{D_{3-}}\right]. \tag{9}$$

Here, $\Omega_p\ (=\omega_p\sqrt{Zm/M})$ is the plasma ion frequency, and $D_{3\pm}$ $[=\omega_{3\pm}^2-c^2k_{3\pm}^2-\mu_{3\pm}\omega_p^2]$ is the dispersion coefficient of the third-harmonic scattered waves (Stokes/anti-Stokes) in the Brillouin scattering process.

For Brillouin instability, maximum growth rate occurs in backward scattering in which both the scattered Stocks light wave and the exited ion acoustic upper-hybrid wave are resonant modes ($\omega=\Omega$, $D_{3-}=0$), that is, when

$$(3\omega_0-\Omega)^2-c^2(k-3k_0)^2-\mu_{3-}\omega_p^2=0, \tag{10}$$

where $\Omega=(\Omega_c^2+(\omega_p^2/\omega_{UH}^2)k^2c_s^2)^{1/2}$. The frequency-matching condition for Stokes wave is $\omega_{3-}=3\omega_0-\Omega$ and the wave number k is determined from Eq. (10). Assuming $\Omega_c\ll\omega_0$, we obtain the normalized wave number of the ion acoustic upper-hybrid wave in a transversely magnetized plasma

$$\frac{k}{k_o}=3\left[\frac{1-(\omega_p/\omega_{UH})(c_s/c)(1-\mu_0(\omega_p^2/\omega_0^2))^{-1/2}}{1-(\omega_p^2/\omega_{UH}^2)(c_s^2/c^2)}\right. \cdot \left.\frac{\left[(1-(\omega_p/\omega_{UH})(c_s/c)(1-\mu_0(\omega_p^2/\omega_0^2))^{-1/2}\right)^2 + (1-(\omega_p^2/\omega_{UH}^2)(c_s^2/c^2))(((1+(\Omega_c^2/9\omega_0^2)) + \frac{-(\mu_{3-}+\chi_{3-}\alpha_{3-})(\omega_p^2/9\omega_0^2))/((1-\mu_0(\omega_p^2/\omega_0^2))-1)]^{1/2}}{1-(\omega_p^2/\omega_{UH}^2)(c_s^2/c^2)}\right]. \tag{11}$$

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By ignoring the nonlinear terms, Eq. (11) reduces to

$$\frac{k_p}{k_0} \simeq 6\left(1-\frac{\omega_p}{\omega_{UH}}\frac{c_s}{c}\right)\left(1-\frac{\omega_p^2}{\omega_{UH}^2}\frac{c_s^2}{c^2}\right)^{-1}, \tag{12}$$

which is three times the normalized linear wave number of ion acoustic upper-hybrid wave in fundamental Brillouin backscattering in a magnetized plasma (Paknezhad, 2013*a*). Note that for Brillouin backscattering, only the downshifted light wave needs to be retained (Kruer, 1998; Paknezhad, 2012; 2013*a*), and assuming the weakly coupled regime ($\Gamma \ll kc_s$), we substitute $\omega = \Omega + i\Gamma_{NBBS}$ in Eq. (9) to find

$$2i\Gamma_{NBBS}\Omega\,[-2i\Gamma_{NBBS}(\omega_0-\Omega)] = \frac{\omega_p^2}{\omega_{UH}^2}\frac{c^2k^2a_0^2\Omega_p^2\mu_0\sigma_0(\mu_{3-}+\chi_{3-}\alpha_{3\omega_0}\alpha_{3-})}{4}. \tag{13}$$

The real part of Eq. (13) gives the nonlinear growth rate of 3HBBS instability in the backscattering of an extraordinary laser wave in a weakly coupled magnetized plasma, where $\Gamma_{NBBS} \ll \omega_{UH}$ and $a_0^2 \ll 1$,

$$\Gamma_{3HBBS} = \frac{cka_0\Omega_p}{4}\frac{\omega_p}{\omega_{UH}}\sqrt{\frac{\mu_0\sigma_0(\mu_{3-}-\chi_{3-}\alpha_{3-}\alpha_{3\omega_0})}{\Omega\,(3\omega_0-\Omega)}}. \tag{14}$$

In Eq. (6), parameter σ_0 arises as a result of the effect of 3HBBS of laser in a mildly relativistic magnetized plasma. Now, we investigate the variation of the growth rate of 3HBBS instability on the basis of Eq. (14).

In Figure 3, the variation of normalized growth rate of 3HBBS instability Γ_{3HBBS}/ω_0 has been plotted against the normalized wave number k/k_0 for $a_0 = 0.02$, and two values of $\omega_c/\omega_0 = 0$ and $\omega_c/\omega_0 = 0.1$. It is demonstrated that for the given values, the starting point of k/k_0 for 3HBBS instability in the unmagnetized plasma ($\omega_c/\omega_0 = 0$) is around 5.8824, whereas it is 6 for the magnetized plasma ($\omega_c/\omega_0 = 0.1$). For both cases, as the normalized wave number decreases (by increasing plasma density), the growth rate increases and then saturates for a low value of k/k_0 (high plasma densities).

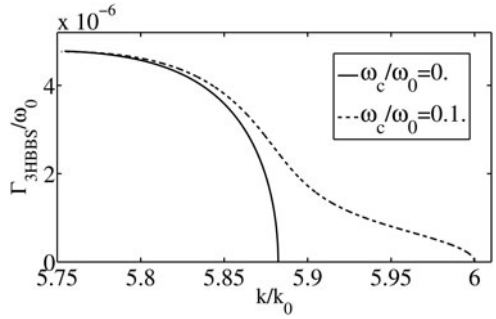


Fig. 3. Variation of the normalized growth rate of 3HBBS instability as a function of normalized wave number k/k_0 in unmagnetized ($\omega_c/\omega_0 = 0$) and magnetized ($\omega_c/\omega_0 = 0.1$) plasmas for $a_0 = 0.02$, $c_s/c = 0.02$.

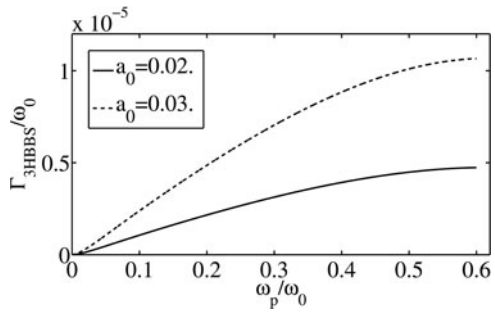


Fig. 4. Variation of normalized growth rate of 3HBBS instability as a function of normalized plasma frequency for $\omega_c/\omega_0 = 0.05$, and $c_s/c = 0.02$.

Figure 4 describes the variation of normalized growth rate of 3HBBS instability Γ_{3HBBS}/ω_0 with normalized plasma frequency ω_p/ω_0 for $\omega_c/\omega_0 = 0.05$, and two values of $a_0 = 0.02$ and $a_0 = 0.03$. It is observed that an increase in the plasma frequency (plasma density) leads to an increase in the growth rate. The growth rate also increases with laser amplitude.

Figure 5 depicts the variation of normalized growth rate of 3HBBS instability as a function of normalized cyclotron frequency ω_c/ω_0 at $a_0 = 0.02$ and $\omega_p/\omega_0 = 0.1$. Similar to the third-harmonic Raman backscattering instability (Paknezhad, 2013*c*), the growth rate decreases by increasing the external magnetic field. However, in the case of third-harmonic Raman backscattering, the growth rate has a further decrease because of the absence of ion's motion.

4. CONCLUSION

To summarize, we have developed a theoretical formalism to calculate the growth rate of nonlinear 3HBBS instability, which is observable in plasmas with a density higher than the quarter critical density. We have demonstrated that the growth rate has a reduction in magnetized plasma in comparison with the unmagnetized plasma. This result is found to be in good agreement with that obtained by Grebogy and Liu (1980) for fundamental Raman and Brillouin backscattering

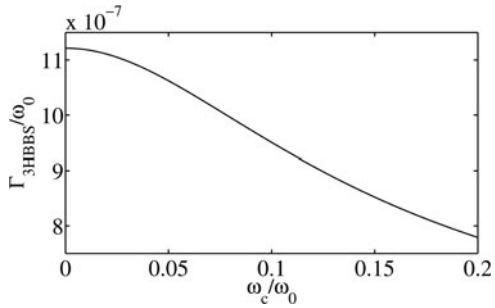


Fig. 5. Variation of normalized growth rate of 3HBBS instability as a function of normalized electron cyclotron frequency for $a_0 = 0.02$, $c_s/c = 0.02$, and $\omega_p/\omega_0 = 0.1$.

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instabilities in a magnetized plasma. However, the result differs from those of Salimullah *et al.* (1994). Additionally, the growth rate increases with plasma density and laser wave amplitude, which agrees with the results attained for fundamental Brillouin scattering in our previous work (Paknezhad, 2012); however, the growth rate of 3HBBS instability obtained in this paper is less than that of fundamental Brillouin backscattering instability. It is also less than the growth rate of third-harmonic Raman backscattering instability that has been obtained in our previous study (Paknezhad, 2013*c*).

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