5

Complex Trajectories

There are excellent reasons, as we have seen, to consider the possibility of complex stationary points, even when the particle interactions are purely real. When we account for the influence of inelastic transitions by regarding them as absorption and thereby making the phase shift function $\chi(\boldsymbol{b})$ complex, the more general consideration of complex trajectories becomes inevitable. It is therefore worth reconsidering, for a moment, some of the calculations we have carried out in Chapter 3 in terms of real variables.

In these situations when classical trajectories are absent, there remains a quantum mechanical scattering amplitude which can be represented by venturing into the complex plane. Let us again assume that the phase shift function $\chi(\boldsymbol{b})$ has rotational symmetry and so depends only on $|\boldsymbol{b}| = b$. Then the stationary phase point will still have $b_y = 0$, according to Eq. (3.11b). We must deal with the fact that the roots of Eq. (3.11a) for b_x, however, may lie off the real axis in the complex b_x-plane. But how do we even define $\chi(b)$ in general for complex values of b_x? At any stationary point we have $b_y = 0$, and therefore we need only discuss the phase function $\chi(b_x, 0)$, which is always an even function of b_x since $b = \sqrt{b_x^2 + b_y^2}$,

$$\chi(-b_x, 0) = \chi(b_x, 0), \tag{5.1}$$

(while $\chi(b)$ need not be an even function of b, regarded as a complex variable). The condition to determine the b_x components of the stationary phase points, Eq. (3.8), is then still

$$\frac{\partial}{\partial b_x}\chi(b_x, 0) = q. \tag{5.2}$$

We shall assume that both the real and imaginary parts of $\chi(b_x, 0)$ are analytic for values of b_x along the real axis, i.e., we can write

$$\chi(b_x, 0) = \mathcal{R}(b_x) + i\mathcal{I}(b_x), \tag{5.3}$$

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where $\mathcal{R}(b_x)$ and $\mathcal{I}(b_x)$ are real-valued analytic functions for real b_x. Then both of these functions can be continued analytically into the complex b_x-plane, where they will then obey the reflection principle

$$\mathcal{R}(b_x^*) = \mathcal{R}^*(b_x), \qquad \mathcal{I}(b_x^*) = \mathcal{I}^*(b_x). \tag{5.4}$$

The phase shift $\chi(b_x, 0)$ as a whole, however, since it is complex, will *not* obey the reflection principle, i.e.,

$$\chi^*(b_x, 0) = \mathcal{R}^*(b_x) - i\mathcal{I}^*(b_x) \neq \chi(b_x^*, 0). \tag{5.5}$$

The symmetries in Eq. (5.4) are nonetheless useful in locating the roots of Eq. (5.2).

For purely real phase shifts χ, for example, we can take the complex conjugate of the equation to determine the stationary point,

$$\frac{\partial}{\partial b_x}\mathcal{R}(b_x) = q \tag{5.6}$$

by writing

$$\frac{\partial}{\partial b_x^*}\mathcal{R}^*(b_x) = \frac{\partial}{\partial b_x^*}\mathcal{R}(b_x^*) = q. \tag{5.7}$$

If b_x is a root of Eq. (5.6) then so is b_x^*. For real interactions then the roots can come in complex conjugate pairs, much as we found in the examples of Section 4.2.

For purely imaginary phase shifts, on the other hand, a root must satisfy the equation

$$i\frac{\partial}{\partial b_x}\mathcal{I}(b_x) = q. \tag{5.8}$$

If we take the complex conjugate of this equation and avail ourselves of the symmetry conditions in Eqs. (5.1) and (5.4), we see that

$$-i\frac{\partial}{\partial b_x^*}\mathcal{I}^*(b_x) = -i\frac{\partial}{\partial b_x^*}\mathcal{I}(b_x^*) = -i\frac{\partial}{\partial b_x^*}\mathcal{I}(-b_x^*) = q. \tag{5.9}$$

Hence for each b_x that satisfies Eq. (5.8) there will be another root $-b_x^*$, which is its reflection in the imaginary axis.

To proceed with the evaluation of the scattering amplitudes we shall assume again as we did in Chapter 3 that the function $\chi(\boldsymbol{b}) - \boldsymbol{q}\cdot\boldsymbol{b}$ can be expanded quadratically about its stationary points $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$. We can then use the same summation convention we used in Chapter 3, i.e., in connection with Eqs. (3.20) and (3.21) to define the required integrals. The integration over b_y still has a stationary point at $b_y = 0$ and essentially takes the form of Eq. (3.21a). It can still be carried out over the b_y axis from $-\infty$ to ∞, but its convergence requires that the imaginary part of α_y, defined by Eqs. (3.17) and (3.18b) not be negative. If $\boldsymbol{b}_j$ is the j-th stationary

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phase point then, to carry out the b_y integration for this point we require, according to Eq. (3.18b)

$$\operatorname{Im}\alpha_y = \frac{q}{2}\operatorname{Im}\left(\frac{1}{b_{jx}}\right) \geq 0. \tag{5.10}$$

This important constraint shows that the b_y integrals are convergent only for stationary points b_{jx} lying in the lower half of the b_x-plane.

The integration over b_x, however, has two stationary points at which the modulus of the integrand has maxima. If the stationary points are sufficiently far apart, or the maxima at these points are narrow enough, we can approximate that integral as the sum of two Gaussian integrals similar to Eq. (3.21b).

The case of two or more stationary points is best discussed for the absorptive case. Then the argument of the exponential can be expanded to second order in $b_x - b_{jx}$ around each stationary point, with a coefficient (earlier denoted as α_x) that has a positive imaginary part. It is that positive imaginary part which provides convergence. When this coefficient is sufficiently large, the contributions to the integral over b_x will be very localized, and there exists a point b_{0x} somewhere between the two stationary points b_{1x} and b_{2x}, such that it is meaningful to split the range of integration in Eqs. (3.20) and (3.21b) into two:

$$\begin{aligned}
&\lim_{\epsilon\to 0}\int\int e^{-\epsilon(b_x^2+b_y^2)-iqb_x+i\alpha_y b_y^2+i\chi(|b_x|)}\,\mathrm{d}b_x\,\mathrm{d}b_y \\
&\simeq \lim_{\epsilon\to 0}\int_{-\infty}^{\infty}\sqrt{\frac{i\pi}{\alpha_y}}\,e^{-\epsilon b_x^2-iqb_x+i\chi(|b_x|)}\,\mathrm{d}b_x \\
&= \lim_{\epsilon\to 0}\left[\int_{-\infty}^{b_{0x}}+\int_{b_{0x}}^{\infty}\right]\sqrt{\frac{i\pi}{\alpha_y}}\,e^{-\epsilon b_x^2-iqb_x+i\chi(|b_x|)}\,\mathrm{d}b_x \\
&\simeq e^{-iqb_{1x}+i\chi(|b_{1x}|)}\sqrt{\frac{i\pi}{\alpha_{1y}}}\int_{-\infty}^{b_{x0}}e^{i\alpha_{1x}(b_x-b_{1x})^2}\,\mathrm{d}b_x \\
&\quad + e^{-iqb_{2x}+i\chi(|b_{2x}|)}\sqrt{\frac{i\pi}{\alpha_{2y}}}\int_{b_{x0}}^{\infty}e^{i\alpha_{2x}(b_x-b_{2x})^2}\,\mathrm{d}b_x.
\end{aligned} \tag{5.11}$$

Here, α_{1x} and α_{2x} refer to the second derivatives of the phase shift function at the points of stationary phase,

$$\alpha_{1x} \equiv \tfrac{1}{2}\chi''(|b_{1x}|), \quad \alpha_{2x} \equiv \tfrac{1}{2}\chi''(|b_{2x}|). \tag{5.12}$$

Likewise, α_{1y} and α_{2y} refer to the second derivatives of the phase shift function with respect to b_y, as discussed in Chapter 3, and evaluated at the stationary points b_{1x} and b_{2x}. Furthermore, the assumed convergence will allow us to extend the

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first integral up to $b_x = \infty$ and the second one down to $b_x = -\infty$, leading to two distinct contributions, each of the form given in Eq. (3.21b).

5.1 A Purely Absorptive Interaction

When short-range inelastic interactions such as particle production processes completely dominate the elastic ones, as they do for collisions at sufficiently high energies, it has often been found useful to regard the overall interaction as purely imaginary or absorptive.

As an analytically simple example of an absorptive interaction, let us assume that the phase shift function takes the form

$$\chi(b) = \frac{iA}{\cosh(b/\beta)}, \tag{5.13}$$

where β is the range of the interaction and A is a positive-valued opacity parameter. We shall see this example as a useful one to consider in detail as a model for the calculations to be undertaken later. Our choice of an even function of b, for example, means that $\chi(b_x, 0)$ is a single-valued function of b_x,

$$\chi(b_x, 0) = \frac{iA}{\cosh(b_x/\beta)}. \tag{5.14}$$

It is furthermore periodic under imaginary displacements of b_x with period $2\pi\beta$.

The stationary points are the roots of Eq. (5.2), that is to say the roots of

$$-\frac{iA}{\beta}\frac{\sinh(b_x/\beta)}{\cosh^2(b_x/\beta)} = q. \tag{5.15}$$

If we introduce the parameter

$$\lambda = \frac{A}{2q\beta}, \quad (\lambda > 0), \tag{5.16}$$

then Eq. (5.15) can be written as the quadratic equation

$$\sinh^2(b_x/\beta) + 2i\lambda \sinh(b_x/\beta) + 1 = 0, \tag{5.17}$$

which has the solutions

$$\sinh(b_x/\beta) = i\left\{-\lambda \pm \sqrt{1+\lambda^2}\right\}. \tag{5.18}$$

To find the stationary phase points b_x we can let

$$\frac{b_x}{\beta} = x + iy.$$

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Then we find

$$\sinh(b_x/\beta) = \sinh x \cos y + i \cosh x \sin y, \tag{5.19}$$

and we see from Eq. (5.18) that the real term in this expression must vanish,

$$\sinh x \cos y = 0. \tag{5.20}$$

Hence either $\cos y$ or $\sinh x$ must vanish. If $\cos y = 0$ we must have

$$y = (n - \tfrac{1}{2})\pi \tag{5.21}$$

for any integer value of n, and thus

$$\cosh x = (-1)^{n+1}\left(-\lambda \pm \sqrt{1+\lambda^2}\right). \tag{5.22}$$

Since λ is intrinsically positive, and we require x to be real, this equation will only have solutions for even values of n. In those cases the roots are given by

$$x = \pm \cosh^{-1}\left(\lambda + \sqrt{1+\lambda^2}\right), \tag{5.23a}$$

$$y = (2m - \tfrac{1}{2})\pi \quad m = -2, -1, 0, 1, 2, \ldots \tag{5.23b}$$

If, on the other hand, it is $\sinh x$ that vanishes, it follows that $x = 0$ and in that case y must satisfy

$$\sin y = -\lambda \pm \sqrt{1+\lambda^2}. \tag{5.24}$$

The only solutions for real y are those for which $\sin y$ takes the positive value, or those for which we have the pair of values

$$x = 0, \tag{5.25a}$$

$$y = \left(2m - \tfrac{3}{2}\right)\pi \pm \cos^{-1}\left(\sqrt{1+\lambda^2} - \lambda\right), \quad m = -2, -1, 0, 1, 2, \ldots \tag{5.25b}$$

Both the sets of roots in Eqs. (5.23) and (5.25) possess the required reflection symmetry in the imaginary axis.

For small values of λ, or large values of the momentum transfer, the roots of Eq. (5.15) are quite naturally found to be clustered in pairs, one pair near each of the poles of $\chi(b)$. (See Fig. 5.1.) For large λ on the other hand, or small momentum transfers, the pairs of roots that are off the imaginary axis tend to be spread far apart in their x coordinates. Those that are on the imaginary axis tend to approach the values $y = 2\pi n i$ where n is an integer. (See Fig. 5.2.)

We have found the stationary points, in fact an infinite number of them, and so we turn next to the problem of how they are to be used in approximating the diffraction integral, Eq. (3.4). Let us recall, first of all, that convergence of the integral over b_y has put a strong restriction on the values of b_x. We see in Eq. (5.10) that at the

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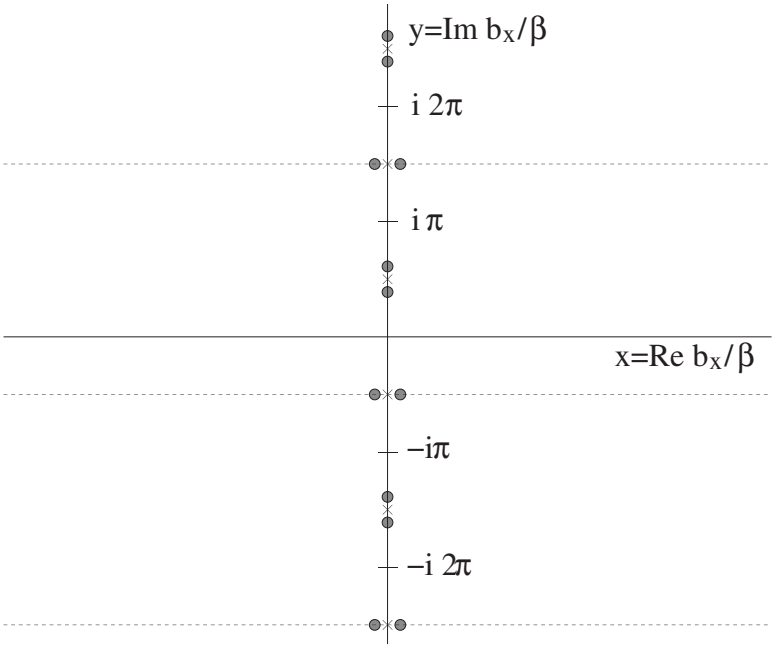


Figure 5.1 Solid circles are the roots of Eq. (5.15) for large momentum transfers, or $\lambda = A/2q\beta \ll 1$. Note that they are clustered in pairs about the poles (crosses) of the function $\chi(b)$ given by Eq. (5.13).

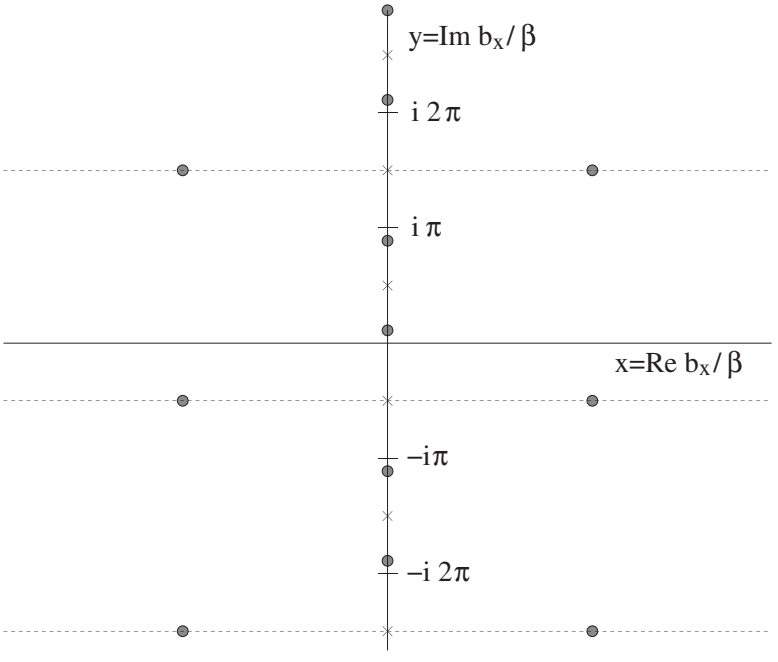


Figure 5.2 Solid circles are the roots of Eq. (5.15) for small momentum transfers or $\lambda = A/2q\beta \gg 1$.

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stationary points which lie on the path of integration, b_x must not lie in the upper half-plane. The requirement

$$\operatorname{Im} b_{jx} \leq 0 \tag{5.26}$$

eliminates half the roots we have found, but still leaves an infinite number of candidates for possible use.

5.2 Choice of Integration Path

The integral that we seek, according to Eq. (3.4), is to be carried out over the real b_x-axis. Given the analytic properties we have assumed for $\chi(b_x, 0)$, it will be possible in general to deform the path of integration into the lower half-plane without changing the value of the integral. That will be true as long as $\chi(b_x, 0)$ is analytic in the region between the new path and the real axis. If we are to use the approximation methods illustrated in Chapter 4, we must steer the integration path through stationary phase points in such a way that the dominant contribution to the integral come just from those points. That may seem, at first, a strange assignment, but we have no need, of course, to use all of the stationary points that are available, and in the present case two will do.

Once the b_y integration has been carried out in Eq. (3.4) the function we must integrate over b_x is

$$h(b_x) = \exp\{-\epsilon b_x^2 - iqb_x + i\chi(b_x, 0)\}. \tag{5.27}$$

To determine an appropriate contour of integration we note that $\chi(b_x, 0)$ goes exponentially to zero as $\operatorname{Re} b_x \to \pm\infty$. Then $h(b_x)$ goes strongly to zero as well for $|\operatorname{Re} b_x| \to \infty$, thanks to the ϵb_x^2 term in the exponent. The integral of $h(b_x)$ then, taken over any finite segment of path, will vanish in the limit in which the segment lies at distances for which $|\operatorname{Re} b_x| \to \infty$.

If we deform the path, for example, to the one shown in Fig. 5.3, the two vertical segments will give vanishing contributions in the limit as they recede to infinity,

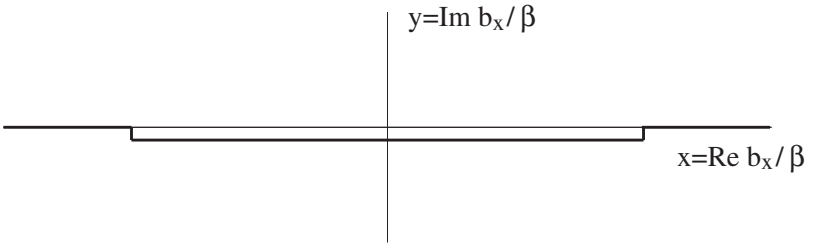


Figure 5.3 Displacement of the path of integration away from the real b_x-axis.

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and the integral over the horizontal segment will be the same as the integral over the real axis—provided $h(b_x)$ is analytic in the strip between them. So we are free to displace the straight-line path downwards, for example, but we must pay attention to the poles of $\chi(b_x, 0)$ along the imaginary axis, which are essential singularities of $h(b_x)$.

Let us translate the straight-line path into the lower half-plane until it passes through the two stationary points labelled by $m = 0$ in Eq. (5.23). That is as far as we can translate it, in fact, without passing the singularity at $b_x = -i\frac{\pi}{2}\beta$. Along this line, that is to say for $y = -\frac{\pi}{2}$, we have

$$\chi(b_x, 0) = -\frac{A}{\sinh x}, \tag{5.28}$$

and if we let $\epsilon \to 0$, we can write

$$h(b_x) = \exp\left\{-\frac{\pi q\beta}{2} - i\left[q\beta x + \frac{A}{\sinh x}\right]\right\}. \tag{5.29}$$

The modulus of the function h is constant along this line. Only its phase varies. The variation of the phase along the line is shown in Fig. 5.4. It exhibits explicitly the two stationary points given by Eq. (5.23a). The straight path $b_x = \beta(x - \frac{i\pi}{2})$, in other words, represents just the sort of contour we want for the stationary phase approximation.

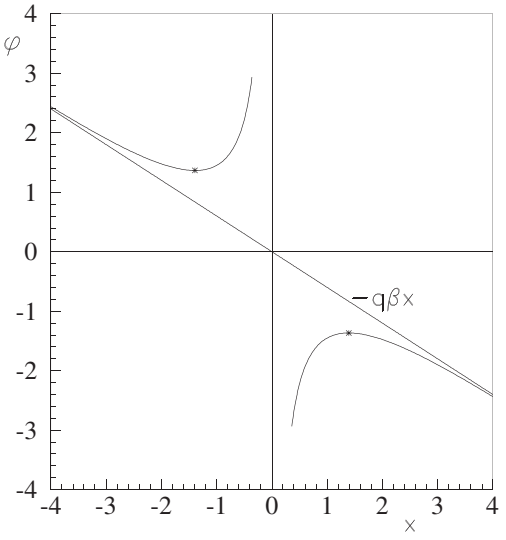


Figure 5.4 Variation of the phase $\varphi = \arg h(b_x)$ of Eq. (5.29) along the line $b_x = \beta(x - i\pi/2)$. The stationary phase points indicated by $*$ are given by Eq. (5.23a).

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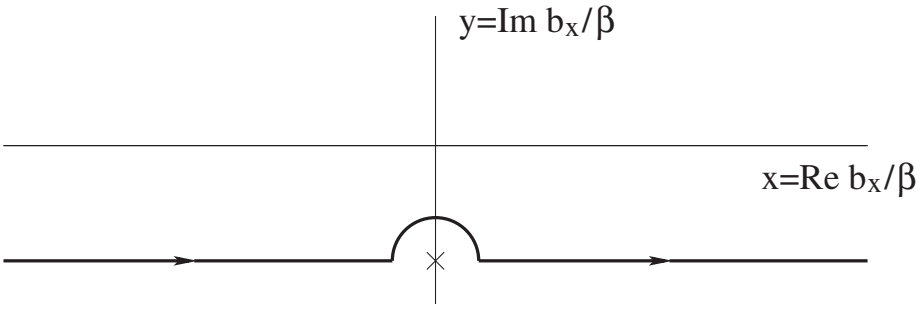


Figure 5.5 Deformation of the straight-line contour to avoid the singularity at $b_x = -i\pi/2$.

It should be noted that in picking this contour we are choosing to integrate through an essential singularity of $h(b_x)$ at $x = 0$, $y = -\frac{\pi}{2}$. Mathematically sensitive souls may regard that as risky if not absolutely wicked, so a few more words may be in order. We can easily deform the contour to follow an infinitesimal semicircle above the singularity and thereby avoid it altogether. (See Fig. 5.5.) On this semicircle we can write

$$b_x = \beta\left(-\frac{i\pi}{2} + \delta e^{i\theta}\right) \tag{5.30}$$

for an infinitesimal value of the radius δ, and any phase angle θ. The phase function $\chi(b_x, 0)$ then has an imaginary part

$$\text{Im}\,\chi(b_x, 0) \sim \frac{A\sin\theta}{\delta} \tag{5.31}$$

which becomes positively infinite for angles θ such that $0 < \theta < \pi$. The modulus of $h(b_x)$ thus vanishes on the semicircle, and that justifies the more brutal procedure of integrating straight through the singularity.

There is, of course, an infinite variety of contours that all yield the same integral for an analytic function. The question is simply how suitable they are for various methods of approximation. Could we have chosen the integration path differently for our example? The answer is obviously "yes". We have made no mention thus far of any more general properties of the stationary phase points defined by Eq. (5.2). They are points at which the derivative of $h(b_x)$ vanishes (for $\epsilon = 0$) in the complex analytic sense, and are therefore saddle points in the modulus of $h(b_x)$. A contour plot of the function $\text{Im}\{qb_x - \chi(b_x, 0)\}$ which determines the contours of $|h(b_x)|$ is shown in Fig. 5.6 for $q\beta = 1.2$. The stationary phase points are seen to be saddle points as well. Instead of using the method of stationary phase to approximate the integral of $h(b_x)$, we could, with equal justice, have used the closely related method of steepest descents [17]. Had we done that, we should formally have chosen the

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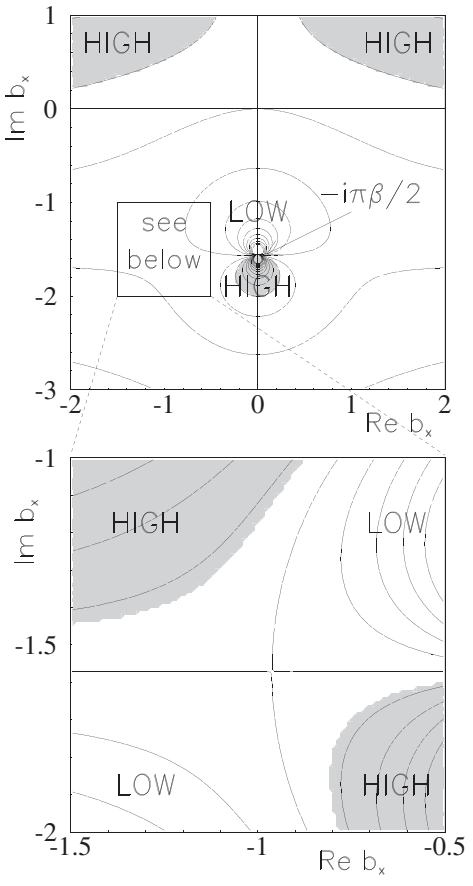


Figure 5.6 Contour plots of the function $\psi = \mathrm{Im}\{qb_x - \chi(b_x, 0)\}$. In the upper part a larger area is shown, including the singularity at $b_x = -i\pi\beta/2$. Shaded: $\psi > 0$. In the lower part we show details of the saddle point to the left of the singularity (shaded: $\psi > -1.8$). There is a similar saddle point to the right of the singularity, for $\mathrm{Re}\, b_x > 0$.

contour that comes in from $b = -\infty - i\infty$ along the low "valley" in the lower left of Fig. 5.6 (note that there is no contribution from $b_x = -\infty$ to $b_x = -\infty - i\infty$), goes over the saddle point, into the deep region between the singularity on the imaginary axis and the origin, and out again in the "valley" (not shown) towards $b_x = \infty - i\infty$. But in practice we should have approximated the integrals through the saddle points by integrations over straight lines tilted at $\pm 45^\circ$ with respect to the real axis.

The results then would be precisely the same as those we find by using the method of stationary phase and the more convenient straight-line contour.

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We have already made implicit use of this alternative method, in fact, in our discussion of rainbow behavior in Chapter 4. There the integration of a function that is oscillating rapidly for $q < q_R$ turned magically into an integration over decreasing Gaussian functions for $q > q_R$. The method of stationary phase had in effect become the method of steepest descents because of the appearance of a factor of i in the coefficient α_x of Eq. (3.18a).

As long as we transit the stationary points along straight paths, and use quadratic approximations to the behavior of $\chi(b_x, 0)$ near those points, the two methods are effectively the same. We can even make a continuum of compromises between them without altering the results. There will always be a 90° interval of angles within which the path of integration can transit the stationary point so that the point represents a maximum of the modulus rather than a minimum. We shall often find it convenient to use such intermediate paths when the behavior of the function $\chi(b_x, 0)$ is less symmetric in the complex plane.

The technique of approximating the integral along the line $\operatorname{Im} b_x = -\pi/2$ is no different from that we have used before for integrating along the real axis. There are two stationary points on the line, which we label b_{1x} and b_{2x} respectively. These are given by

$$\operatorname{Re} b_{1x} = -\operatorname{Re} b_{2x} = \beta \cosh^{-1}\left(\lambda + \sqrt{1+\lambda^2}\right) \tag{5.32a}$$

$$\operatorname{Im} b_{1x} = \operatorname{Im} b_{2x} = -\pi\beta/2 \tag{5.32b}$$

with λ defined by Eq. (5.16). The integral in Eq. (3.4) is then approximated as a sum of two terms, one contributed by each of the stationary points. These partial scattering amplitudes are given, as in Eq. (3.23), by

$$f_j(\boldsymbol{k}', \boldsymbol{k}) = \frac{k}{i}\left\{\frac{-b_{jx}}{q\chi''(b_{jx}, 0)}\right\}^{\frac{1}{2}} e^{-iqb_{jx} + i\chi(b_{jx}, 0)}, \quad (j = 1, 2). \tag{5.33}$$

The overall scattering amplitude is then

$$f(\boldsymbol{k}', \boldsymbol{k}) = f_1(\boldsymbol{k}', \boldsymbol{k}) + f_2(\boldsymbol{k}', \boldsymbol{k}). \tag{5.34}$$

The scattering amplitudes f_j possess a symmetry worth noting. Since $b_{2x} = -b_{1x}^*$, we also have, for the phase shift function in Eq. (5.13),

$$\chi(b_{2x}, 0) = -\chi^*(b_{1x}, 0) \tag{5.35}$$

$$\chi''(b_{2x}, 0) = -\chi''^*(b_{1x}, 0). \tag{5.36}$$

It follows then that

$$f_2(\boldsymbol{k}', \boldsymbol{k}) = -f_1^*(\boldsymbol{k}', \boldsymbol{k}), \tag{5.37}$$

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and that the total scattering amplitude, Eq. (5.34), is purely imaginary, as it must be for a purely absorptive interaction.

We shall not undertake the numerical evaluation of the scattering amplitude in Eq. (3.23) here, but will save it for Chapter 8, where we begin the analysis of a number of analogously shaped interactions, and include the possibility of refraction as well as absorption. We shall show there that for appropriate ranges of parameters our approximate scattering amplitude adheres quite closely in numerical terms to the behavior of the general expression for the diffraction amplitude, Eq. (2.1), from which we began.

Even before substituting any numbers however, we can see certain general features of the scattering described by Eq. (3.23). The dominant dependence on q in the amplitude f_j will lie in the factors

$$\exp(-iqb_{jx}) = \exp\left\{-iq\,\mathrm{Re}\,b_{jx} - \frac{\pi q\beta}{2}\right\}. \tag{5.38}$$

Both of the amplitudes for $j = 1$ and 2 will decrease in modulus exponentially with increasing q. Each will correspond then approximately to the amplitude transmitted by a small diffuse aperture of the kind we have discussed in Chapter 1. The diffuseness parameter we called β there is replaced by $\pi\beta/2$ here.

The phase factors contained in the exponentials in Eq. (5.38) also depend strongly on q. When the sum of the amplitudes is squared, we may expect it to show considerable oscillation. If $\mathrm{Re}\,b_{1x}$ were independent of q, in fact, it would play precisely the role of the parameter c in Eqs. (1.6) and (1.7), that is to say $2\,\mathrm{Re}\,b_{1x}$ would represent the distance apart of the two diffuse slits that effectively furnish the diffraction pattern. We find in practice that $\mathrm{Re}\,b_{1x}$ given by Eq. (5.32a) doesn't vary very strongly with q, and the distribution of scattered intensities does look a great deal like that of two fixed diffuse slits. But to the extent that $\mathrm{Re}\,b_{1x}$ is indeed found to decrease somewhat with increasing q, the oscillations of the scattered intensity will be found to increase slowly in period as q increases. That is already an indication that the oscillations of the differential cross section do not remain precisely periodic at large momentum transfers.

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