Probability in the Engineering and Informational Sciences, **35**, 2021, 75–91.
doi:10.1017/S0269964818000566

OPTIMAL ENERGY DISTRIBUTION WITH ENERGY PACKET NETWORKS

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We use the Energy Packet Network (EPN) to investigate an optimal energy distribution problem for the computer-communication system which is powered by intermittent renewable energy sources. The objective is to find an optimal energy distribution to minimize the proposed cost function which computes penalty costs caused by the overall average response time of jobs and the energy loss. In this EPN system, we consider the energy can be lost through storage leakages, or due to empty workstations which will consume energy even no job needs to be processed. Related numerical examples with different sets of parameter values are presented in the paper to evaluate the system performance and to examine the obtained analytical solution. Then a special case is considered to study the optimal system performance when the total energy harvesting rate is sufficiently large.

Keywords: energy distribution, energy harvesting, energy packet network, G-networks, optimization, product-form solution, renewable energy

1. INTRODUCTION

Although conventional fossil fuels are still the primary energy sources in the world today, demands on the renewable and environmentally friendly energy sources have triggered many investigations into energy-saving technologies and energy harvesting (EH) systems. Some recent researches [48,54] have addressed the challenges and the opportunities of developing EH technologies in the power systems. Moreover, energy-saving technologies and EH systems also play an important role in computer-communication systems.

The critical energy consumption increase in information and communication technologies (ICT) [23] creates a huge need for the parsimonious energy usage in ICT systems. Thus early research [28,29] has addressed the need for power-aware routing in wireless networks where the number of hops that a packet traverses, including in simple opportunistic networks [37], also increase the network's energy consumption [18]. The interplay of routing and energy consumption in wired networks has been studied in [31,34]. Energy savings in Cloud Computing has been studied in [1], where adaptive task scheduling methods can also improve energy savings [30,55]. Whether the distributed or the centralized EH system can provide the lowest average overall response time to a fixed workload of jobs is investigated in [24]. Optimizing the energy consumption and the quality of service subject to the finite

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energy storage and the causality constraints is studied in [49,56] where the EH technologies are applied in the wireless communication systems. A wireless communication base station fully powered by renewable energy sources is modeled in [5]. In [38], a definition of "self-sustainability" is provided for an EH system, in terms of the mathematical characterization of the time until a system's energy may be depleted, causing an outage of the system. A two-stage optimization approach is developed in [44] for the power-adaptive computer system which can use energy harvested from solar panels. A stochastic model of the Energy Packet Network (EPN) is detailed in [25] to investigate the corporation among energy prosumers. This model can compute the system steady-state by a product-form solution (PFS). Its utility functions that consider both the needs of the consumers and the amount of energy reserved for potential future needs are presented to evaluate the model performance. The recent contributions in EH wireless communication up to 2015 are summarized in [53].

Motivated by these recent work and considerations, we consider an energy distribution problem for the computer-communication system with EH nodes and energy storage nodes. The system consists of the workstations (WSs), each of which is powered by the associated energy store (ES). Every ES is charged by the EH unit which can harvest energy from intermittent energy sources at a fixed rate. The objective is to investigate how to optimally distribute the harvested energy flow to every ES so as to minimize the cost function which computes penalty costs caused by the overall average response time of jobs and the energy loss.

We use the EPN [19–21] as an approach to evaluate both the performance and the energy consumption for the complex interconnected computer-communication system. The EPN is a discrete state-space framework belonging to a family of queueing model known as G-networks [12,13,15,27] which were first developed in the early 1990s. Up to today, a series of work regarding G-networks with batch removals [14], multi-class G-networks [27], triggers [13], resets [26], and adders [8] has been published. In the EPN model, computer jobs or data are in the form of packets, and energy is also represented in energy packets (EPs), each of which contains a certain amount of energy in the discretized form. According to the G-network theory, the EPN model has the PFS which is a steady-state joint probability distribution of the number of jobs[1] and EPs waiting in the WSs and the ESs, respectively.

Recently various applications have used the EPN model to evaluate the performance and the energy consumption. An on-line routing algorithm is developed in [33] to reduce the overall energy consumption and the overall response time of data packets in the wired networks. In [22], the EPN models the performance of backhaul networks which are powered by intermittent and renewable energy sources. Sensor networks powered by EH technologies are developed by means of the EPN in [11,32,41,42,57]. As mentioned, the EPN is also detailed in [24,25]. Moreover, another approach known as Power Packets is developed in [50,51]. The power packet is a discrete representation that combines the energy and the data in a single packet. Then a power packet routing system is designed and experimentally verified in [52].

For references, the recent applications and extensions of G-networks should be briefly discussed. A new type of customers named "resets" was first developed in [26]. When resets arrive at an empty queue, the queue length is reset to a random number whose probability distribution is identical to the stationary distribution at that queue. If the queue is not empty, the reset does not affect the queue. A similar form of resets developed in [39] can propagate through a sequence of nodes. When "adder" [8] which is another type of customer arrives at a service center, it acts as a load regulator to probabilistically change the queue length at the service center. G-networks have also been applied to restart problems [9,

[1] The generic term "jobs" refers to both job packets and data packets.

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10] where the user can abort and resubmit a running job which exceeds a deadline. The tandem G-networks were investigated to study the response time probability distribution in [40] and the computer networks with blocking in [36]. A closed network of queues with external signals was developed in [7]. In the network, the average queue size and average waiting time can be determined without computing the probabilities. A G-network with depended services was developed in [3], in which the target customer can be "killed" by the arrived negative customer according to the probability distribution and the queue type. The number of customers in the queues of G-networks which are under heavy traffic [43] can be approximated by a reflected stochastic differential equation. G-networks with signals which can change the sub-network availability were investigated in [6] to model the networks with failed components. The paper [45] reviews the PFS and summarizes some classes of G-networks. The paper [4] introduces theoretical developments of G-networks and its applications to machine learning, EPNs, and Cloud management over the Internet. Recently, G-networks have been also applied to the large-scale evacuation [2], the expected revenues [46,47], the cybersecurity [16], and the probabilistic gene regulatory networks [17].

Our contributions are as follows:

1. In the previous EPN models, a WS can either send one job to another WS for more processing or remove this job from the system when one EP arrives at the WS. Hence, only one single WS can be processed by expanding one EP. Here the EPN model we consider is more general. One single EP can be expanded to process a variable number of jobs, which follows a probability distribution.
2. The proposed cost function computes penalty costs incurred by both the overall average response time and the energy loss. Rewards (negative penalty costs) may be given to encourage the small overall average response time. Note that penalty costs can be very tangible, e.g. the US dollar, or just a score that evaluates the system performance.
3. The energy distribution problem is solved analytically, instead of being solved by the gradient-related methods.
4. We consider a special case in which the total EH rate is sufficiently large. An approximated solution can be solved by the Lagrangian multiplier method.

The paper is organized as follows. In Section 2, we present the EPN system and the PFS to evaluate the complex computer-communication system. The optimization problem and its cost function are given in Section 3. We solve the optimization problem analytically in Section 4. Comments are given for every numerical example. In Section 5, we analyze the special case. Conclusions are in Section 6.

2. ENERGY PACKET NETWORK MODEL

We consider an EPN system which is an open network with $2N$ queues presented schematically in Figure 1. Each WS is modeled as a queue that contains the jobs waiting to be processed. Jobs, which are ordinary customers to WSs, first arrive at a given WS i according to a Poisson process with parameter λ_i jobs/sec. Similarly, EPs, each of which represents a certain amount of energy harvested by the EH unit from intermittent energy sources, arrive at a given ES i at a rate of γ_i EPs/sec, which also follows a Poisson process. Note that each ES is also modeled as a queue containing EPs. In this EPN system, we have N WSs and N ESs. Hence, the ith queue represents the ith WS, and the $(i + N)$th queue represents the ith ES, where $i \in \{1, \ldots, N\}$.

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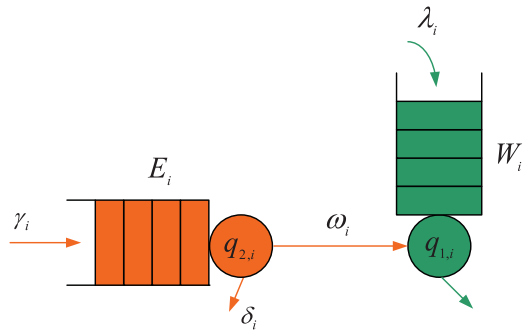


FIGURE **1.** Schematic representation of the pair of the ith WS and the ith ES.

Three types of customers: *positive customers*, *negative customers*, and *triggers* are in the EPN system. Each of them has the following manner:

1. **Positive customers**: positive customers behave like ordinary customers that wait at a queue to be processed. After being processed, these positive customers may either be removed from the system or be moved to another queue according to a given probability matrix. Note that only positive customers can wait at queues.
2. **Negative customers**: negative customers do not require any service or process. When a negative customer arrives at a non-empty queue at time t, the negative customer instantaneously removes $\max[k_i(t), X_i]$ positive customers waiting at the queue. The $k_i(t)$ denotes the number of positive customers at the queue at time t. The X_i, a random variable, is the maximum number of positive customers that can be processed by one EP. After the removal, the negative customer disappears spontaneously. If the queue is empty, the negative customer does not affect the queue.
3. **Triggers**: triggers also do not require any service or process. When a trigger arrives at a non-empty queue, the trigger moves the positive customer at the head of the queue to another queue according to another given probability matrix, instead of removing a batch of positive customers from the system. The trigger disappears spontaneously after the movement. If the queue is empty, the trigger also does not affect the queue.

We denote the number of jobs at WS i by $K_i(t)$, while the number of EPs at ES i is denoted by $B_i(t)$ at time t. The EPs at ES i are in the following manner:

1. We assume that ES i interconnects with only WS i. Hence, WS i is only powered by ES i, and the EPs at ES i cannot be moved to other ESs.
2. If ES i is non-empty at time t, i.e. $B_i(t) > 0$, energy will either leak at a rate $\delta_i > 0$ EPs/sec or be sent to WS i at a rate w_i EPs/sec. For ES i, we model the successive EP leakage time as independent and identically distributed (i.i.d.) random variables which follow a exponential distribution with parameter δ_i. Similarly, the successive time that one EP is sent to WS i is also modeled as i.i.d. random variables following a exponential distribution with parameter w_i. Hence, the number of EPs at ES i is $B_i(t) - 1$ after a time of average value $(\delta_i + w_i)^{-1}$.

When an EP arrives at WS i at time t, it has the following manner:

1. If WS i is non-empty at time t, i.e. $K(t) > 0$, the EP is instantaneously expanded to process $\max[K_i(t), X_i]$ jobs. Since each job may consume different amounts of

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energy, as mentioned, the maximum number of jobs that can be processed by one EP is modeled to a random variable X_i following a given probability distribution $\pi_{i,s}$, which is

$$\pi_{i,s} = \Pr[X_i = s], \quad s = 1, 2, \ldots, \tag{1}$$

and

$$\sum_{s=1}^{\infty} \pi_{i,s} = 1 \quad \text{for all} \quad i = 1, \ldots, N. \tag{2}$$

Hence the average number of jobs processed by one single EP is $E[X_i] = \sum_{s=1}^{\infty} s\pi_{i,s}$. Then the processed jobs are removed from WS i and the EP disappears, i.e.

$$K_i(t^+) = \begin{cases} 0, & \text{if } X_i \geq K_i(t) \\ K_i(t) - X_i, & \text{otherwise} \end{cases}. \tag{3}$$

Note that the EP behaves like a negative customer at WSs.

2. If WS i is empty at time t, i.e. $K(t) = 0$, the EP will be expanded to keep WS i working. Then the EP disappears. We consider this EP, which arrives at an empty WS, is wasted.
3. Since we assume that WSs are not connected, the EP cannot act as a trigger to move the jobs at WS i to another WS. In another word, these jobs can only be processed by WS i locally.

2.1. Energy Harvesting Process of the EPN

Here we present how the EPN system harvests and distributes energy from intermittent renewable energy sources. One of the most significant challenges in EH is the intermittent nature of renewable energy. The output power of these renewable energy sources, e.g. wind, solar, and tide varies with time, location, and even meteorological conditions, which cannot be controlled by the system. To tackle these uncontrollable factors, energy storage systems are used to store harvested energy so as to distribute stored energy to loads, which are WSs in the EPN system. Unlike those uncontrollable factors, the energy distribution process can be controlled by the system. Therefore, we use ESs in the EPN system.

Recently, much work has focused on stochastic EH, which models the variability of renewable energy output as a random process. Nowadays, considerable data of variables can be recorded for a specific energy source to observe relations between these variables and the output power. Through these data, we have prior knowledge of the output power (which is a random variable) for certain variables, e.g. time and location. Then we can model the EH process based on the knowledge of data. In this EPN system, we model the EH process to a Poisson process with parameter γ. Hence, the EH unit can generate average γ EPs every second from a given energy source. Then the energy flow is distributed to each ES according to the probability vector p. The energy distributed to the ES will be stored until it is sent to the interconnect WS to keep it working.

2.2. Product-Form Solution of the EPN Model

We recall a remarkable property of G-networks known as the PFS which can help us to analyze the EPN system steady-state. Regarding G-networks presented in [14], the EPN

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system has a set of *traffic equations*, which is

$$\begin{cases} \Lambda_{1,i}^{+} & = \lambda_i, \\ \Lambda_{1,i}^{-} & = q_{2,i+N} w_i \left[\frac{1-\sum_{s=1}^{\infty} q_{1,i}^{s} \pi_{i,s}}{1-q_{1,i}} \right], \\ r_{1,i} & = 0, \\ \Lambda_{2,i+N}^{+} & = \gamma_i, \\ \Lambda_{2,i+N}^{-} & = 0, \\ r_{2,i+N} & = w_i + \delta_i, \end{cases} \tag{4}$$

and

$$q_{1,i} = \frac{\Lambda_{1,i}^{+}}{r_{1,i} + \Lambda_{1,i}^{-}}; \quad q_{2,i+N} = \frac{\Lambda_{2,i+N}^{+}}{r_{2,i+N} + \Lambda_{2,i+N}^{-}}, \tag{5}$$

where $i \in \{1, \ldots, N\}$. The index "1" denotes Class 1 customers, which are jobs in the EPN system, while Class 2 customers, which are EPs, are denoted by the index "2" in the traffic equations. The $q_{c,i}$ denotes the steady-state probability that the ith queue has at least one job (if $c = 1$) or one EP (if $c = 2$).

For $i = 1, \ldots, N$, we have $q_{1,i} \geq 0$ and $q_{2,i} = 0$ since EPs cannot wait at WSs. For $i = N+1, \ldots, 2N$, we have $q_{2,i} \geq 0$ and $q_{1,i} = 0$ since jobs cannot arrive at ESs. The steady-state joint probability distribution of the number of positive customers at queues can be presented as a PFS given as below:

THEOREM 1: *Let* $K(t) = (K_1(t), \ldots, K_N(t))$ *and* $B(t) = (B_1(t), \ldots, B_N(t))$. *If an unique non-negative solution of the traffic equations given in (4) exists such that the conditions* $0 < q_{1,i} < 1$ *and* $0 < q_{2,i+N} < 1$ *for* $i = 1, \ldots, N$ *hold, then the PFS:*

$$\lim_{t \to \infty} \Pr[(K(t), B(t)) = (k_1, \ldots, k_N, b_1, \ldots, b_N)] = \prod_{i=1}^{N} q_{1,i}^{k_i} (1 - q_{1,i}) q_{2,i+N}^{b_i} (1 - q_{2,i+N}), \tag{6}$$

exists.

PROOF: Proof is given in [35]. ■

3. ENERGY DISTRIBUTION OPTIMIZATION PROBLEM

As mentioned, we assume that the EH harvests energy at a fixed rate γ EPs/sec. In this case, the harvested energy flows into ES i at rate $\gamma_i = p_i \gamma$ EPs/sec, where probability $0 < p_i < 1$ is the control variable for the energy distribution. Then we address a specific constraint, which is $\sum_{i=1}^{N} p_i = 1$.

The cost function, which computes penalty costs incurred by the overall average response time and the energy loss, is given as follows

$$C = a \times (W - T_W) + b \times E, \tag{7}$$

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where

$$W = \sum_{i=1}^{N} \frac{\lambda_i}{\sum_{i=1}^{N} \lambda_i} \frac{(r_{1,i} + \Lambda_{1,i}^{-})^{-1}}{1 - q_{1,i}},$$

$$= \frac{1}{\sum_{i=1}^{N} \lambda_i} \sum_{i=1}^{N} \frac{q_{1,i}}{1 - q_{1,i}}. \tag{8}$$

$$E = \sum_{i=1}^{N} q_{2,i+N} \delta_i + q_{2,i+N} w_i (1 - q_{1,i}). \tag{9}$$

By Little's Formula, we can write W which represents the overall average response time of jobs, while E denotes the overall energy loss. The first term of E represents the overall energy leakage flow, and the second term is the overall energy flow to empty WSs. To unify W and E in the cost function, we use two constant positive factors, a and b which can convert flow rates (in jobs/sec or EPs/sec) into penalty costs in cents/sec.

In the cost function, we also consider a simple service level agreement by introducing a threshold named T_W for the overall average response time:

1. If $W > T_W$, the system is punished by charging penalty costs.
2. If $W < T_W$, rewards are given to encourage the small overall average response time.
3. If $W = T_W$, no penalties or rewards will be given.

The optimization problem is to find optimal distribution probability vector $p = (p_1, \ldots, p_N)$ for the given values of the system parameters, so as to minimize the cost function.

In following Section 4, we first make a geometric assumption regarding the probability distribution $\pi_{i,s}$. Under the geometric assumption, we analyze the EPN system with the case of $N = 2$ and the case of $N \geq 3$. We also present Proposition 1 that guarantees the existence of a globally optimal solution under the geometric assumption.

4. ANALYSIS OF THE OPTIMIZATION PROBLEM

According to the traffic equations given in (4) and (5), and the specific constraint, i.e. $\sum_{i=1}^{N} p_i = 1$, for $1 \leq i \leq N$, we have:

$$q_{1,i} = \frac{\lambda_i}{q_{2,i+N} \cdot w_i [((1 - \sum_{s=1}^{\infty} q_{1,i}^{s} \cdot \pi_{i,s})/(1 - q_{1,i}))]}, \tag{10}$$

$$q_{2,i+N} = \frac{\gamma p_i}{w_i + \delta_i}. \tag{11}$$

Then we make a geometric assumption regarding the probability distribution $\pi_{i,s}$, which is

$$\pi_{i,s} = \frac{(1 - u_i) u_i^{s}}{u_i}, \tag{12}$$

and

$$\sum_{s=1}^{\infty} (1 - u_i) u_i^{s-1} = 1, \tag{13}$$

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where $0 < u_i < 1$ is a real number. With the assumption, it is convenient to analyze the optimization problem analytically. Substituting (12) into (10), we have

$$\begin{aligned} q_{1,i} &= \frac{\lambda_i}{q_{2,i+N} w_i} \left[\frac{1 - \sum_{s=1}^{N} q_{1,i}^s u_i^s ((1-u_i)/(u_i))}{1 - q_{1,i}} \right]^{-1} \\ &= \frac{\lambda_i}{q_{2,i+N} w_i} \left[\frac{1 - (((1-u_i) q_{1,i})/(1 - u_i q_{1,i}))}{1 - q_{1,i}} \right]^{-1} \\ &= \frac{\lambda_i}{u_i \lambda_i + q_{2,i+N} w_i}. \end{aligned} \tag{14}$$

Then the overall average response time and the energy loss become, respectively:

$$W = \sum_{i=1}^{N} \frac{1}{\lambda^+} \cdot \frac{\lambda_i}{\lambda_i (u_i - 1) + \sigma_i \gamma p_i}, \tag{15}$$

$$E = \gamma - \lambda^+ + \sum_{i=1}^{N} \frac{\lambda_i^2 u_i}{\lambda_i u_i + \sigma_i \gamma p_i}, \tag{16}$$

where $\sigma_i = (w_i)/(w_i + \delta_i)$ denoting the energy efficiency regarding the leakages at ES i. Moreover, $\lambda^+ = \sum_{i=1}^{N} \lambda_i$ is a constant.

Remark 1: *The optimization problem is subject to the specific constraint, i.e.* $\sum_{i=1}^{N} p_i = 1$. *Hence, for any* $j \neq i$, *we must have* $p_j = 1 - \sum_{i=1, i \neq j}^{N} p_i$, *and*

$$\frac{\partial p_j}{\partial p_i} = -1, \ \forall i \neq j. \tag{17}$$

This remark is a useful tool that helps us to analyze the following derivative equations.

4.1. The Case of $N = 2$

In the case of $N = 2$, we have two pairs of WSs and ESs. The cost function can be written as

$$\begin{aligned} C = {} & b(\gamma - \lambda^+) - aT_W + \frac{1}{\lambda^+} \frac{a\lambda_1}{\lambda_1 (u_1 - 1) + \sigma_1 \gamma p_1} \\ & + \frac{b\lambda_1^2 u_1}{\lambda_1 u_1 + \sigma_1 \gamma p_1} + \frac{1}{\lambda^+} \frac{a\lambda_2}{\lambda_2 (u_2 - 1) + \sigma_2 \gamma (1 - p_1)} + \frac{b\lambda_2^2 u_2}{\lambda_2 u_2 + \sigma_2 \gamma (1 - p_1)}, \end{aligned} \tag{18}$$

where $p_2 = 1 - p_1$. The specific constraint is removed by replacing p_2 with $(1 - p_1)$. The derivative of C with respect to p_1 is

$$\begin{aligned} \frac{dC}{dp_1} = {} & \frac{-a\lambda_1 \sigma_1 \gamma}{\lambda^+ [\lambda_1 (u_1 - 1) + \sigma_1 \gamma p_1]^2} + \frac{-b\lambda_1^2 \sigma_1 u_1 \gamma}{[\lambda_1 u_1 + \sigma_1 \gamma p_1]^2} \\ & + \frac{a\lambda_2 \sigma_2 \gamma}{\lambda^+ [\lambda_2 (u_2 - 1) + \sigma_2 \gamma (1 - p_1)]^2} + \frac{b\lambda_2^2 \sigma_2 u_2 \gamma}{[\lambda_2 u_2 + \sigma_2 \gamma (1 - p_1)]^2}. \end{aligned} \tag{19}$$

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TABLE 1. Parameters of the EPN with $N = 2$

Parameters	Values	Parameters	Values
$u_1,\ u_2$	$0.2, 0.2$	$\lambda_1,\ \lambda_2,$	$50, 60$ jobs/sec
$\delta_1,\ \delta_2$	$10, 6$ EPs/sec	$w_1,\ w_2,$	$100, 80$ EPs/sec

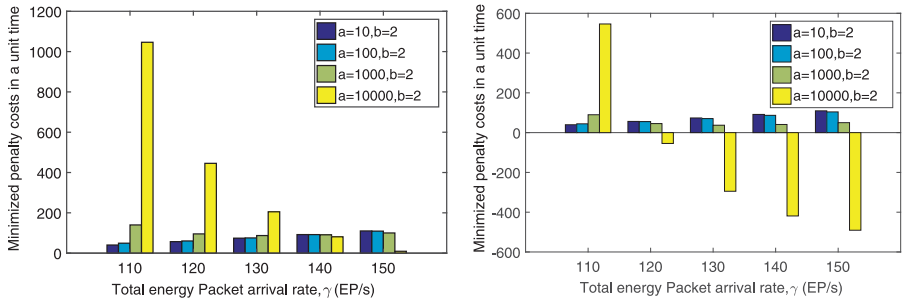


FIGURE 2. The minimized cost function with the different total energy packet arrival rates. The threshold is set to 0.05 (left) and 0.1 (right) respectively. The negative penalty costs represent rewards.

Let $dC/dp_1 = 0$. Then we have a function:

$$\frac{a\lambda_2\sigma_2}{[\lambda_2(u_2-1)+\sigma_2\gamma-\sigma_2\gamma p_1]^2}+\frac{b\lambda_2^2\sigma_2u_2\lambda^+}{[\lambda_2u_2+\sigma_2\gamma-\sigma_2\gamma p_1]^2}$$
$$=\frac{a\lambda_1\sigma_1}{[\lambda_1(u_1-1)+\sigma_1\gamma p_1]^2}+\frac{b\lambda_1^2\sigma_1u_1\lambda^+}{[\lambda_1u_1+\sigma_1\gamma p_1]^2}. \tag{20}$$

By solving (20) with respect to $p_1 \in [0,1]$, we can obtain the optimal solution $p^* = (p_1^*, 1-p_1^*)$. In order to illustrate the analytically obtained optimal solution, we consider a set of numerical examples with two pairs of WSs and ESs, and their parameters are shown in Table 1.

The first numerical example result is presented in Figure 2. We use five different total EH rates, i.e. $\gamma = 110,\ 120,\ 130,\ 140,\ 150.$ EPs/sec to investigate the trends of minimal penalty costs.

COMMENT 1: *From Figure 2, we can observe the trends of minimal penalty costs with respect to the ratio a/b:*

1. *When the ratio a/b is low, we observe that the penalty cost is relatively high at high γ. Intuitively, the overall average response time is small when the total EH rate is very high; hence, the smaller response time will cause much fewer penalty costs. Nevertheless, the penalty costs on the energy loss increase because the probability that one EP is sent to empty WSs increases.*
2. *When the ratio a/b is high, the penalty cost is high at low γ. The overall average response time becomes high if energy is not sufficient. Hence, a large amount of penalty on high W is charged. Moreover, the probability that one EP is sent to empty WSs decreases so that the energy loss will not cause significant penalty costs.*
3. *We observe rewards in Figure 2 when the threshold is set to a higher value. The less strict threshold make achieving "small" W easier.*

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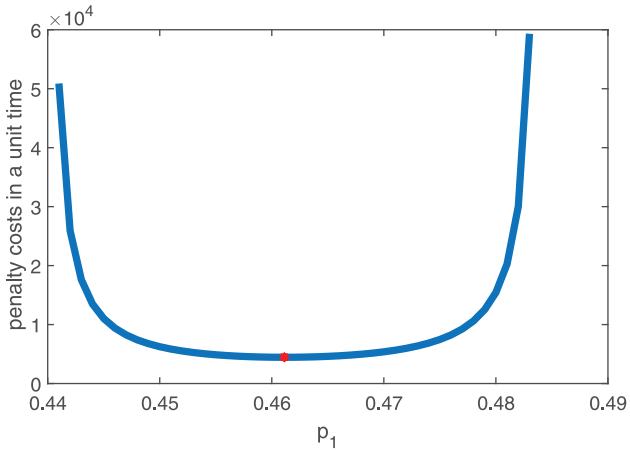


FIGURE **3.** The cost function of the EPN system with all (p_1, p_2) pairs. The red point designates the optimal solution by solving (20). Note that the range p_i for all i is not $[0, 1]$ due to constraints. The threshold is set to $T_W = 0.05$. The a and b are set to 10000 and 2, respectively. The optimal solution $p^* = (0.4611, 0.5389)$ produces the minimal cost function $C = 4452.6$ cents/sec, $E = 11.2191$ EPs/sec, and $W = 0.4930$ seconds.

The results we observed in Figure 2 reveal the penalty cost trends in the EPN system. The trends also follow the realistic EPN system dynamics as we discuss in Comment 1.

Now the total EH rate is fixed to 100 EPs/sec in the second numerical example to investigate the system with insufficient harvesting energy. We first have to avoid system instabilities by guaranteeing the condition $q_{c,i} < 1$ holds for all c and i. Hence, we must have

$$\frac{\lambda_i(1-u_i)}{\gamma\sigma_i} < p_i < \frac{w_i + \delta_i}{\gamma}, \tag{21}$$

From (21), we can find the lower bound of the rate γ, which is

$$\gamma > \sum_{i=1}^{N} \frac{\lambda_i(1-u_i)}{\gamma_i}. \tag{22}$$

The lower bound, which is 95.6 EP/sec in this example, gives the minimal EH rate for the EPN system to work normally. According to (21), the numerical conditions are

$$0.4400 < p_1 < 1 \text{ and } 0.5160 < p_2 < 0.8600,$$

and $p_1 + p_2 = 1$. Then we calculate all the values of the cost function subject to these numerical conditions and compare them to the optimal solution given by (20). The results are shown in Figure 3 in which the x-axis is p_1 while p_2 follows from $p_2 = 1 - p_1$. The results are given in Figure 3.

COMMENT 2: *From Figure 3, we observe the maximum value of W and E are* 5.9799 *seconds and* 11.2593 *EP/sec, respectively. Compared with the optimal solution, the overall average response time is reduced by* 5.4869 *seconds, while we also can save* 0.0402 *EPs every second. In summary, the penalty is reduced by 54869 cents/sec. As mentioned, the penalty on the overall average response time is high if energy is not sufficient. We significantly reduce the penalty by keeping the response time small.*

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4.2. The Case of $N \geq 3$

Here we consider a more general case of $N \geq 3$. The cost function C can be rearranged to

$$C = b(\gamma - \lambda^+) - aT_W + \frac{a}{\lambda^+}\frac{q_{1,1}}{1-q_{1,1}} + b\lambda_1 u_1 q_{1,1} + \sum_{i=2}^{N} \frac{a}{\lambda^+}\frac{q_{1,i}}{1-q_{1,i}} + b\lambda_i u_i q_{1,i}. \tag{23}$$

By the chain rule, the partial derivatives of C with respect to $q_{1,i}$ for all $i \geq 2$ are

$$\frac{\partial C}{\partial q_{1,i}} = \frac{a}{\lambda^+(1-q_{1,i})^2} + b\lambda_i u_i + \left(\frac{a}{\lambda^+(1-q_{1,1})^2} + b\lambda_1 u_1\right)\left(-\frac{\sigma_1\lambda_i}{\sigma_i\lambda_1}\left(\frac{q_{1,1}}{q_{1,i}}\right)^2\right). \tag{24}$$

Then let $\partial C/\partial q_{1,i} = 0$. Here we have $N-1$ equations for $i = 2, \ldots, N$:

$$\frac{a}{\lambda^+(1-q_{1,i})^2} + b\lambda_i u_i = f_i\left(\frac{1}{q_{1,i}}\right)^2. \tag{25}$$

where

$$f_i = \left(\frac{a}{\lambda^+(1-q_{1,1})^2} + b\lambda_1 u_1\right)\frac{\sigma_1\lambda_i}{\sigma_i\lambda_1}(q_{1,1})^2. \tag{26}$$

Rearranging (25), it becomes

$$\frac{f_i}{b\lambda_i u_i} = q_{1,i}^4 - 2q_{1,i}^3 + \frac{b\lambda_i u_i\lambda^+ + a - f_i\lambda^+}{b\lambda_i u_i\lambda^+}q_{1,i}^2 + \frac{2f_i}{b\lambda_i u_i}q_{1,i}. \tag{27}$$

The optimal solution is obtained by solving $N-1$ equations for $i \in \{2, \ldots, N\}$ given in (27) and one constraint, which is

$$\sum_{i=1}^{N} p_i = \sum_{i=1}^{N}\frac{\lambda_i}{\sigma_i\gamma}\left(\frac{1}{q_{1,i}} - u_i\right) = 1, \tag{28}$$

simultaneously. Now we present a sufficient condition which guarantees the existence of the globally optimal solution in Proposition 1.

PROPOSITION 1: *Under the geometric assumption, the global optimal solution of the composite cost function C exists if the PFS given in Theorem 1 exists.*

PROOF: In [35], Gelenbe and Schassberger prove the PFS exists if $q_{c,i} < 1$ for all $c,\ i$ holds. Thus conditions

$$q_{1,i} = \frac{\lambda_i}{\lambda_i u_i + \gamma\sigma_i p_i} < 1, \quad q_{2,i+N} = \frac{\gamma p_i}{w_i + \delta_i} < 1, \tag{29}$$

must hold. Hence, the PFS of the EPN exists if the condition

$$\frac{\lambda_i(1-u_i)}{\gamma\sigma_i} < p_i < \frac{w_i + \delta_i}{\gamma}, \tag{30}$$

holds for all i. Then the Hessian matrix $\nabla_{pp}C$ is an diagonal matrix with N diagonal entries, which are

$$\frac{\partial C^2}{\partial^2 p_i} = \frac{2a\lambda_i\gamma^2\sigma_i^2}{\lambda^+[\gamma\sigma_i p_i - \lambda_i(1-u_i)]^3} + \frac{2b\lambda_i^2 u_i\sigma_i^2\gamma^2}{[\lambda_i u_i + \sigma_i\gamma p_i]^3} > 0, \tag{31}$$

for $i = 1, \ldots, N$, if the PFS exists. The cost function C is a strictly convex function with respect to p since the Hessian matrix is strictly positive-definite, i.e. $\nabla_{pp}C \succ 0$.

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TABLE 2. Parameters of the EPN with $N = 3$

Parameters	Values	Parameters	Values
$u_1,\ u_2,\ u_3$	$0.2, 0.2, 0.2$	$\lambda_1,\ \lambda_2,\ \lambda_3$	$50, 30, 10$ jobs/sec
$\delta_1,\ \delta_2,\ \delta_3$	$10, 8, 6$ EPs/sec	$w_1,\ w_2,\ w_3$	$100, 80, 50$ EPs/sec

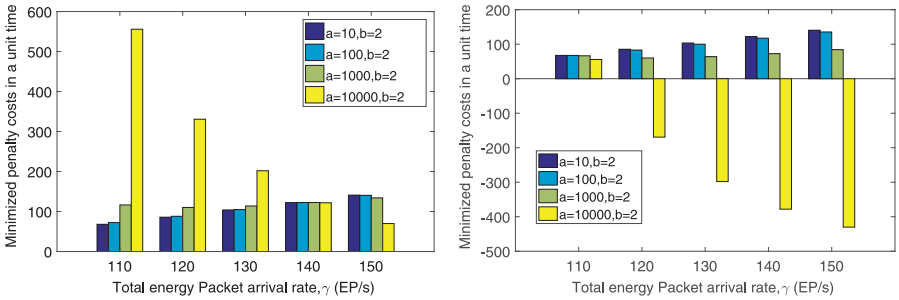


FIGURE 4. The minimized cost function with the different total energy packet arrival rates. The threshold is set to 0.05 (left) and 0.1 (right), respectively. The negative penalty costs are rewards.

The following proof is by contradiction. Assume p^a is a minimal point of the cost function and p^b is another minimal point of the cost function such that $C(p^a) = C(p^b)$. Since the cost function is strictly convex, we have

$$C(\alpha p^a + (1-\alpha)p^b) < \alpha C(p^a) + (1-\alpha)C(p^b) = C(p^a), \tag{32}$$

where $0 \le \alpha \le 1$ is a real number. This contradicts to the assumption that p^a is the minimal point of the cost function. Therefore, there exists a globally optimal solution of the composite cost function C if the PFS exits. ■

In order to illustrate the analytical results for the case of $N \ge 3$, we consider another set of numerical examples. Without loss of generality, we use three pairs of WS and ES nodes. The parameters are given in Table 2.

In Figure 4, the trends of the EPN system with $N = 3$ is very similar to that with $N = 2$. We omit the comment on the trends in this numerical example.

Now the second numerical example results are presented in Figure 5. Analogously, the lower bound in this example is 79.36 EPs/sec. According to (21), the numerical conditions:

$$0.4400 < p_1 < 1.0000, \quad 0.2640 < p_2 < 0.8800,$$
$$0.0896 < p_3 < 0.5600, \quad p_1 + p_2 + p_3 = 1,$$

must hold to guarantee $q_{c,i} < 1$ for all c, i. Then we calculate the values of the cost function C with all (p_1, p_2, p_3) and compare them to the optimal solution.

The theoretical optimal solution $(q^*_{1,1}, q^*_{1,2}, q^*_{1,3}) = (0.8557, 0.8212, 0.7281)$ gives energy distribution probability $(p^*_1, p^*_2, p^*_3) = (0.5327, 0.3358, 0.1315)$. Note that all p_i satisfies the numerical conditions and the constraint. It yields $C = 1016.8$ cents/sec, $E = 24.9407$ EPs/sec, and $W = 0.1467$ seconds.

COMMENT 3: *It is observed that the maximum values of W and E are* 7.3395 *seconds and* 26.2403 *EP/sec. Compared with the optimal solution, the overall average response time is*

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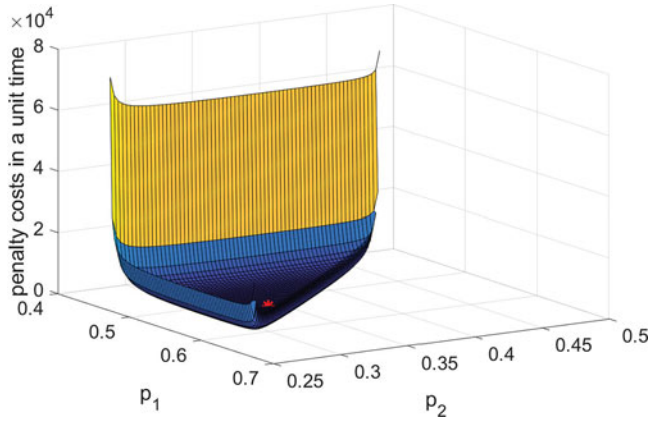


FIGURE **5.** The cost function C of the system with all (p_1, p_2) pairs. The red point designates the optimal solution. Note that the range p_i for all i is not $[0, 1]$ due to constraints. The threshold is set to $T_W = 0.05$. The a and b are set to 10000 and 2, respectively.

reduced by 7.1928 *seconds. We can also save* 1.2996 *EPs in every second. The observation is very similar to the case of* $N = 2$. *When* γ *is low, the penalty is reduced significantly by keeping* W *small.*

5. A SPECIAL CASE OF THE OPTIMIZATION PROBLEM

In the previous examples, we consider the case when the total EH rate is small. Here, we present a special case that $q_{1,i}$ is small (close to 0) for all i, which implies that the rate of total harvesting energy is sufficiently large. In this case, the function W can be approximated to

$$W = \frac{1}{\lambda^+} \sum_{i=1}^{N} \frac{q_{1,i}}{1 - q_{1,i}} \approx \frac{1}{\lambda^+} \sum_{i=1}^{N} q_{1,i}^2 + q_{1,i}, \tag{33}$$

if $q_{1,i}$ is small. Then the cost function becomes

$$C = b(\gamma - \lambda^+) - aTw + \sum_{i=1}^{N} \frac{a}{\lambda^+} q_{1,i}^2 + \left(\frac{a}{\lambda^+} + bu_i\lambda_i \right) q_{1,i}. \tag{34}$$

We know C is a monotonic increasing function in the range of $0 < q_{1,i} < 1$ for all i. Therefore, we can write a modified cost function based on Lagrangian multiplier method, which is

$$\hat{C} = \sum_{i=1}^{N} \frac{\lambda_i}{\lambda_i u_i + \sigma_i \gamma p_i} + \beta \left(\sum_{i=1}^{N} p_i - 1 \right), \tag{35}$$

where β is a real number. Let $\partial \hat{C} / \partial p_i = 0$. Then the optimal solution is

$$\hat{p}_i = \frac{\sqrt{((\lambda_i)/(\sigma_i))}}{\sum_{i=1}^{N} \sqrt{((\lambda_i)/(\sigma_i))}} \left(1 + \sum_{i=1}^{N} \frac{\lambda_i u_i}{\sigma_i \gamma} \right) - \frac{\lambda_i u_i}{\sigma_i \gamma}, \tag{36}$$

which fulfills the specific constraint, i.e. $\sum_{i=1}^{N} p_i = 1$.

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TABLE 3. Parameters of the EPN with large γ

Parameters	Values	Parameters	Values
γ	230 EPs/sec	$\lambda_1,\ \lambda_2,\ \lambda_3$	$5, 6, 5$ jobs/sec
$\delta_1,\ \delta_2,\ \delta_3$	$10, 10, 25$ EPs/sec	$w_1,\ w_2,\ w_3$	$100, 80, 65$ EPs/sec
$u_1,\ u_2,\ u_3$	$0.2, 0.2, 0.2$		

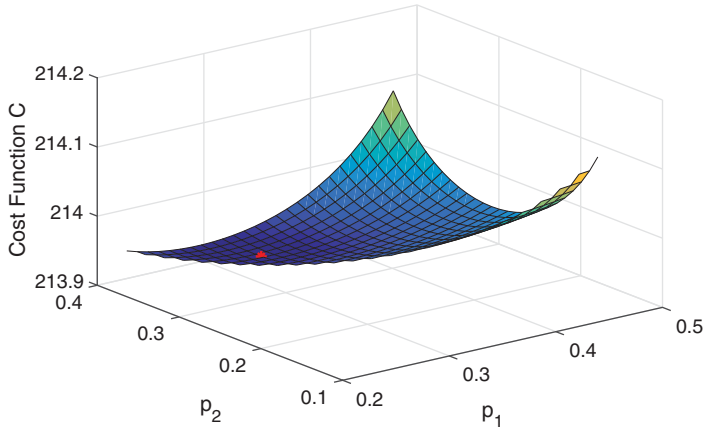


FIGURE 6. The cost function of the system with all (p_1, p_2) pairs. Here we let $a = 10$, $b = 1$, and the threshold $T_w = 0.05$. The red point designates the solution by solving (36). Note that the range p_i for all i is not $[0, 1]$ due to constraints.The theoretical solution given by (36) is $(\hat{p_1}, \hat{p_2}, \hat{p_3}) = (0.3100, 0.3429, 0.3471)$. It produces the cost function $\hat{C} = 213.9292$ cents/sec. At this point, we have $q_{1,1} = 0.0760$, $q_{1,2} = 0.0842$, and $q_{1,3} = 0.0852$. They are all close to zero. Moreover, the minimal cost function observed in Figure 6 is $C^* = 213.9290$ cents/sec and the error between C^* and $\hat{C}$ is 0.002.

Now we consider a numerical example with a sufficiently large γ to illustrate the analytically obtained optimal solution for this special case. The parameters of three pairs of WSs and ESs are shown in Table 3.

The numerical conditions

$$0.0191 < p_1 < 0.4783, \quad 0.0235 < p_2 < 0.3913,$$
$$0.0241 < p_3 < 0.3913, \quad p_1 + p_2 + p_3 = 1,$$

must hold to guarantee $q_{c,i} < 1$ for all c, i. The optimal solution is given by (36). Then We calculate the value of the cost function C with all (p_1, p_2, p_3) and compare the theoretical solution $\hat{C}$ given by $\hat{p}$ and the real minimal C^* observed in Figure 6. The results are shown in Figure 6 in which the x-axis and y-axis are p_1 and p_2.

COMMENT 4: *The theoretical solution has optimal* $\hat{W} = 0.0167$ *seconds and* $\hat{E} = 214.2622$ *EPs/sec. In the feasible region, the observed maximum values of* W *and* E *are* 0.0245 *seconds and* 214.3678 *EPs/sec, respectively. The solution reduces the overall average response time by* 0.00078 *seconds and saves* 0.1056 *EPs every second. Moreover, the minimal* W^* *and* E^* *observed in Figure 6 are* 0.0167 *seconds and* 214.2617 *EPs/sec. The difference between the theoretical solutions and the real minimal values are small. In the case of sufficient EH rate,*

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rewards may be given to encourage small W. However, the probability that one EP is sent to empty WSs increases. The penalty costs on the energy loss are significant. Moreover, in the example, we set $a = 10$ and $b = 1$ only for the presentation. Since these two factors are not in the optimal solution (36), the theoretical solutions, i.e. $\hat{W}$ and $\hat{E}$, should not be affected by these two factors. These two factors will scale only the error between $\hat{C}$ and C^*.

6. CONCLUSIONS

We have considered an EPN system where the number of jobs processed by a single EP follows a probability distribution. In this paper, we assume the probability distribution is geometric to help us analyze the system. We also have considered the energy loss due to both ES leakages and empty WSs where energy is still consumed even when they are idle.

In the energy distribution problem, neither EPs nor jobs can be moved. Locally, each WS consumes EPs received from the coupled ES to process jobs. In the problem, we proposed a cost function which computes the penalty costs caused by the overall average response time and the energy loss to evaluate the system performance. We also have investigated how to optimally distribute the flow of energy harvested by the EH unit among the ESs so that the cost function can be minimized. The problem has been solved analytically. The numerical examples and relevant comments are provided.

A special case has also been considered to make our research comprehensive. Here we make the total EH rate sufficiently large to investigate the system performance at the extreme condition. We modify the cost function to solve the optimal solution directly. The error between the theoretical solution and the real minimum value is small.

Future work will investigate the EPN system that jobs and EPs can be moved among WSs and ESs. We could use the same cost function to evaluate the system performance regarding different distribution problems.

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