13 First Steps beyond the Standard Model

13.1 Preliminaries

We will next extend our study of weak decays beyond the SM. The technology of OPE and renormalization group can easily be extended to any model containing heavy particles and new interactions. Starting with a NP scale Λ, which could still be of order of a few TeV but also much larger, like 100 TeV, one can consecutively integrate out heavy particles and construct a series of effective theories until one reaches the electroweak scale. Below it only gluons, photon and SM quarks, and leptons except for the top quark appear as dynamical degrees of freedom. Subsequently the known RG evolution down to scales relevant for a given decay is performed. The resulting effective Hamiltonian at these low-energy scales has the following general structure:

$$\mathcal{H}_{\rm eff} = \sum_i C_i O_i^{\rm SM} + \sum_j C_j^{\rm NP} O_j^{\rm NP}, \tag{13.1}$$

where

- $O_i^{\rm SM}$ are local operators present in the SM, and $O_j^{\rm NP}$ are new operators having typically new Dirac structures, in particular scalar-scalar and tensor-tensor operators. They will play an important role soon.
- C_i and $C_j^{\rm NP}$ are the Wilson coefficients of these operators. But whereas $C_j^{\rm NP}$ are only nonvanishing in the presence of NP,

$$\boxed{C_i = C_i^{\rm SM} + \Delta_i^{\rm NP}.} \tag{13.2}$$

  This means that in the presence of NP the Wilson coefficients of SM operators that we calculated in previous chapters are generally modified.

The amplitude for a transition $M \to F$ with M being a decaying meson and F any final state, including $\bar{M}$ in the case of $M - \bar{M}$ mixing, can now be written as follows

$$\boxed{\mathcal{A}(M \to F) = \sum_i C_i \langle F|O_i^{\rm SM}|M\rangle + \sum_j C_j^{\rm NP}\langle F|O_j^{\rm NP}|M\rangle.} \tag{13.3}$$

The coefficients C_i and $C_j^{\rm NP}$ can again be calculated in the RG-improved perturbation theory, although generally the analysis is much more complicated than in the SM because

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of the presence of new operators that mix under renormalization. We will discuss this aspect in the next chapter, in particular in the context of the so-called SM effective theory.

In this chapter we will concentrate our discussion on the operators $O_j^{\rm NP}$. We have already encountered in previous chapters those contributing to leptonic and semileptonic decays and discussed their properties. In this chapter we will discuss exclusively operators contributing to nonleptonic decays and transitions like

$$K^0 - \bar{K}^0, \qquad B_{s,d} - \bar{B}_{s,d}, \qquad K \to \pi\pi. \tag{13.4}$$

We will not only classify new operators contributing to these transitions. We will also summarize what is known about the relevant hadronic matrix elements both from the DQCD approach and LQCD. DQCD is not useful for the calculation of nonleptonic B decays as this would require, in the process of meson evolution, the inclusion of the contributions from many particles beyond pseudoscalar and vector mesons with many free couplings. Whether LQCD, without any input from the QCDF approach, will one day help us in describing these decays is an open question that we will not address in this book. The analyses of nonleptonic decays are presently dominated by QCDF approach presented in Section 7.4 and isospin, SU(3), and U-spin global symmetries as discussed in particular in Chapter 8.

13.2 $\Delta F = 2$ Transitions

13.2.1 Operators

We begin by listing the operators that can contribute to $\Delta F = 2$ observables in any extension of the SM. For the $K^0 - \bar{K}^0$ system the operators in the so-called BMU-basis [744] are given as follows

$$Q_1^{\rm VLL} = (\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma^\mu P_L d), \tag{13.5a}$$
$$Q_1^{\rm VRR} = (\bar{s}\gamma_\mu P_R d)(\bar{s}\gamma^\mu P_R d), \tag{13.5b}$$
$$Q_1^{\rm LR} = (\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma^\mu P_R d), \tag{13.5c}$$
$$Q_2^{\rm LR} = (\bar{s}P_L d)(\bar{s}P_R d), \tag{13.5d}$$

$$Q_1^{\rm SLL} = (\bar{s}P_L d)(\bar{s}P_L d), \tag{13.6a}$$
$$Q_1^{\rm SRR} = (\bar{s}P_R d)(\bar{s}P_R d), \tag{13.6b}$$
$$Q_2^{\rm SLL} = -(\bar{s}\sigma_{\mu\nu} P_L d)(\bar{s}\sigma^{\mu\nu} P_L d), \tag{13.6c}$$
$$Q_2^{\rm SRR} = -(\bar{s}\sigma_{\mu\nu} P_R d)(\bar{s}\sigma^{\mu\nu} P_R d), \tag{13.6d}$$

where

$$P_{R,L} = \frac{1}{2}(1 \pm \gamma_5), \qquad \sigma_{\mu\nu} = i\frac{1}{2}[\gamma_\mu, \gamma_\nu], \tag{13.7}$$

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and we suppressed color indices as they are summed up in each factor. For instance, $\bar{s}\gamma_\mu P_L d$ stands for $\bar{s}_\alpha \gamma_\mu P_L d_\alpha$ and similarly for other factors. This basis dealing exclusively with products of color singlet currents and densities is very useful for calculations in DQCD as the matrix elements in the large N limit can be obtained directly without using Fierz identities.

For $B_q^0 - \bar{B}_q^0$ mixing we have correspondingly:

$$Q_1^{\mathrm{VLL}} = (\bar{b}\gamma_\mu P_L q)(\bar{b}\gamma^\mu P_L q), \tag{13.8a}$$
$$Q_1^{\mathrm{VRR}} = (\bar{b}\gamma_\mu P_R q)(\bar{b}\gamma^\mu P_R q), \tag{13.8b}$$
$$Q_1^{\mathrm{LR}} = (\bar{b}\gamma_\mu P_L q)(\bar{b}\gamma^\mu P_R q), \tag{13.8c}$$
$$Q_2^{\mathrm{LR}} = (\bar{b}P_L q)(\bar{b}P_R q), \tag{13.8d}$$

$$Q_1^{\mathrm{SLL}} = (\bar{b}P_L q)(\bar{b}P_L q), \tag{13.9a}$$
$$Q_1^{\mathrm{SRR}} = (\bar{b}P_R q)(\bar{b}P_R q), \tag{13.9b}$$
$$Q_2^{\mathrm{SLL}} = -(\bar{b}\sigma_{\mu\nu} P_L q)(\bar{b}\sigma^{\mu\nu} P_L q), \tag{13.9c}$$
$$Q_2^{\mathrm{SRR}} = -(\bar{b}\sigma_{\mu\nu} P_R q)(\bar{b}\sigma^{\mu\nu} P_R q). \tag{13.9d}$$

In the SM only the operator Q_1^{VLL} is present. The remaining operators contribute in various extensions of the SM. Even if these extensions will be introduced in later chapters, we think it is useful to expose these different cases already here. In particular

- Q_1^{VRR} is generated by gauge bosons with only right-handed (RH) couplings to quarks as is the case of W' in LR symmetric models. Similar to Q_1^{VLL}, it does not mix with other operators under renormalization. The anomalous dimensions of Q_1^{VRR} and Q_1^{VLL} are equal to each other as QCD does not care about the sign of γ_5. But the initial conditions for RG evolution of their Wilson coefficients are in general different. In particular one of them could vanish.
- $Q_{1,2}^{\mathrm{LR}}$ are generated by gauge interactions with both LH and RH couplings to quarks, which is the case of box diagrams with both W and W' exchanges in LR symmetric models or tree-level Kaluza–Klein gluon exchanges within models with extra dimensions. Also in models with tree-level Z and Z' exchanges these operators can be generated. As discussed in the next chapter also renormalization group evolution from high-energy scales down to the electroweak scale that involves top Yukawa coupling can generate such operators in the presence of right-handed quark currents. Q_1^{LR} and Q_2^{LR} mix under renormalization, and their RG group evolution is governed by a 2×2 anomalous dimension matrix. Some of the entries in this matrix are much larger than the anomalous dimension of Q_1^{VLL} and consequently RG effects in this sector are much larger than in the SM. Also hadronic matrix elements are larger, in particular in the K meson system.
- $Q_{1,2}^{\mathrm{SLL}}$ and $Q_{1,2}^{\mathrm{SRR}}$ are generated through tree-level scalar exchanges, and their RG evolution is governed by another 2×2 anomalous dimension matrix, which is the same for these two sets of operators. Also in this sector RG effects and hadronic matrix elements are

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larger than in the SM. Moreover, in models with scalars having both LH and RH scalar couplings also the operators $Q_{1,2}^{\rm LR}$ can contribute.

The anomalous dimensions of new operators including two-loops have been calculated in [744, 745], and a detailed renormalization group analysis with many analytical expressions can be found in [746].

In the literature another basis, the so-called SUSY basis, introduced in [745], is often used. It involves the operators

$$O_1, \quad O_2, \quad O_3, \quad O_4, \quad O_5, \tag{13.10}$$

which are given in the case of $K^0 - \bar{K}^0$ system as follows

$$\begin{aligned} O_1 &= \bar{s}^\alpha \gamma_\mu P_L d^\alpha \; \bar{s}^\beta \gamma^\mu P_L d^\beta, \\ O_2 &= (\bar{s}^\alpha P_L d^\alpha)\; (\bar{s}^\beta P_L d^\beta), \\ O_3 &= (\bar{s}^\alpha P_L d^\beta)\; (\bar{s}^\beta P_L d^\alpha), \\ O_4 &= (\bar{s}^\alpha P_L d^\alpha)\; (\bar{s}^\beta P_R d^\beta), \\ O_5 &= (\bar{s}^\alpha P_L d^\beta)\; (\bar{s}^\beta P_R d^\alpha), \end{aligned} \tag{13.11}$$

and for B_q systems with $q = d, s$ by

$$\begin{aligned} O_1 &= (\bar{b}^\alpha \gamma_\mu P_L q^\alpha)\; (\bar{b}^\beta \gamma^\mu P_L q^\beta), \\ O_2 &= (\bar{b}^\alpha P_L q^\alpha)\; (\bar{b}^\beta P_L q^\beta), \\ O_3 &= (\bar{b}^\alpha P_L q^\beta)\; (\bar{b}^\beta P_L q^\alpha), \\ O_4 &= (\bar{b}^\alpha P_L q^\alpha)\; (\bar{b}^\beta P_R q^\beta), \\ O_5 &= (\bar{b}^\alpha P_L q^\beta)\; (\bar{b}^\beta P_R q^\alpha). \end{aligned} \tag{13.12}$$

13.2.2 Relation between BMU and SUSY Bases

The operators in BMU and SUSY bases are related through

$$\boxed{Q_1^{\rm VLL} = O_1, \quad Q_1^{\rm LR} = -2O_5, \quad Q_2^{\rm LR} = O_4, \quad Q_1^{\rm SLL} = O_2, \quad Q_2^{\rm SLL} = 4O_2 + 8O_3.} \tag{13.13}$$

The structure of one-loop anomalous dimension matrices of the new operators differs significantly from the one of current-current operators. In particular some entries are much larger, which in turn implies much larger RG effects. They are given in the BMU basis of [744] as follows

$$\hat{\gamma}^{(0)\rm LR} = \begin{pmatrix} \frac{6}{N} & 12 \\ 0 & -6N + \frac{6}{N} \end{pmatrix}, \tag{13.14}$$

$$\hat{\gamma}^{(0)\rm SLL} = \begin{pmatrix} -6N + 6 + \frac{6}{N} & \frac{1}{2} - \frac{1}{N} \\ -24 - \frac{48}{N} & 2N + 6 - \frac{2}{N} \end{pmatrix}. \tag{13.15}$$

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The two operator systems do not mix under renormalization with each other. The corresponding two-loop results can be found in [744].

Using the relations between the two bases just given one finds the one-loop anomalous dimension matrices in the SUSY basis

$$\hat{\gamma}^{(0)}(O_2, O_3) = \begin{pmatrix} -6N + 8 + \frac{2}{N} & 4 - \frac{8}{N} \\ 4N - 4 - \frac{8}{N} & 2N + 4 + \frac{2}{N} \end{pmatrix}, \tag{13.16}$$

$$\hat{\gamma}^{(0)}(O_4, O_5) = \begin{pmatrix} -6N + \frac{6}{N} & 0 \\ -6 & \frac{6}{N} \end{pmatrix}. \tag{13.17}$$

The corresponding two-loop results can be found in [744, 745].

13.2.3 Hadronic Matrix Elements

In some papers the matrix elements of operators are given in terms of B_i parameters with $B_i = 1$ in the vacuum insertion approximation (VIA). This turns out to be useful for getting the insight from DQCD in the values of B_i in the case of $K^0 - \bar{K}^0$ mixing obtained by LQCD. But in the case of $B^0_{s,d} - \bar{B}^0_{s,d}$ mixing, where there is no reliable analytic method to calculate the B_i, we think that LQCD should provide numerical results for hadronic matrix elements instead of B_i as this avoids additional errors generated in extracting B_i and having directly the matrix elements simplifies the phenomenology.

The expressions for the matrix elements $\langle \bar{K}^0 | Q_i(\mu) | K^0 \rangle \equiv \langle Q_i(\mu) \rangle$ contributing to $K^0 - \bar{K}^0$ mixing are given in the BMU basis as follows

$$\langle Q_1^{\mathrm{VLL}}(\mu) \rangle = \frac{1}{3} m_K F_K^2 B_1^{\mathrm{VLL}}(\mu), \tag{13.18}$$

$$\langle Q_1^{\mathrm{LR}}(\mu) \rangle = -\frac{1}{6} R(\mu)\, m_K F_K^2 B_1^{\mathrm{LR}}(\mu), \tag{13.19}$$

$$\langle Q_2^{\mathrm{LR}}(\mu) \rangle = \frac{1}{4} R(\mu)\, m_K F_K^2 B_2^{\mathrm{LR}}(\mu), \tag{13.20}$$

$$\langle Q_1^{\mathrm{SLL}}(\mu) \rangle = -\frac{5}{24} R(\mu)\, m_K F_K^2 B_1^{\mathrm{SLL}}(\mu), \tag{13.21}$$

$$\langle Q_2^{\mathrm{SLL}}(\mu) \rangle = -\frac{1}{2} R(\mu)\, m_K F_K^2 B_2^{\mathrm{SLL}}(\mu), \tag{13.22}$$

where

$$R(\mu) = \left(\frac{m_K}{m_s(\mu) + m_d(\mu)} \right)^2 = \frac{r^2(\mu)}{4 m_K^2} \tag{13.23}$$

refers to the generic factorized evolution of any density-density operator with μ. F_K is the K-meson decay constant. We have met $r(\mu)$ already in Section 7.2, see (7.15).

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In the SUSY basis one has

$$\langle \bar{K}^0|O_1(\mu)|K^0\rangle = \frac{1}{3} m_K F_K^2 B_1(\mu), \tag{13.24}$$

$$\langle \bar{K}^0|O_2(\mu)|K^0\rangle = -\frac{5}{24} R(\mu)\, m_K F_K^2 B_2(\mu), \tag{13.25}$$

$$\langle \bar{K}^0|O_3(\mu)|K^0\rangle = \frac{1}{24} R(\mu)\, m_K F_K^2 B_3(\mu), \tag{13.26}$$

$$\langle \bar{K}^0|O_4(\mu)|K^0\rangle = \frac{1}{4} R(\mu)\, m_K F_K^2 B_4(\mu), \tag{13.27}$$

$$\langle \bar{K}^0|O_5(\mu)|K^0\rangle = \frac{1}{12} R(\mu)\, m_K F_K^2 B_5(\mu). \tag{13.28}$$

The B_i parameters in these two bases are then related through

$$\begin{aligned} B_1^{\rm VLL}(\mu) &= B_1(\mu), \\ B_1^{\rm LR}(\mu) &= B_5(\mu), \qquad & B_2^{\rm LR}(\mu) &= B_4(\mu), \\ B_1^{\rm SLL}(\mu) &= B_2(\mu), \qquad & B_2^{\rm SLL}(\mu) &= \frac{5}{3} B_2(\mu) - \frac{2}{3} B_3(\mu). \end{aligned} \tag{13.29}$$

We warn the reader that we use here the normalizations of states as in the rest of this book. This normalization differs from the one used by LQCD collaborations and also in DQCD, where additional factors of $2m_K$ and $2m_{B_q}$ in the hadronic matrix elements given here are present. This difference is then compensated by other factors as discussed after (8.37) so that phenomenological results are unchanged. It is very easy to remember this difference. The dimensions of matrix elements used by us are GeV^3, while those in LQCD are GeV^4.

The matrix elements of all these operators are evaluated by LQCD at scales corresponding roughly to the scale of the decaying hadron, although in the case of K-meson decays, to improve the matching with the Wilson coefficients, the lattice calculations are performed these days at scales $\mu \approx (2\text{–}3)$ GeV. The values of all matrix elements obtained by various lattice groups are averaged by FLAG, and their updates can be found there.

Here we would like only to discuss some strategies for using them, which possibly could turn out to be useful for some readers. The following presentation will also give an idea about the pattern of sizes of different matrix elements. Now, LQCD results for the matrix elements in question should be combined with the Wilson coefficients of these operators evaluated at the same scales and in the same renormalization scheme, usually NDR scheme. However, for a transparent study of NP contributions that are generated at very high scales it is useful, starting from the values calculated by LQCD at low scales, to evaluate these matrix elements at NP scales. This can be done by means of RG methods discussed in previous chapters, and the corresponding NLO analytic formulas to achieve this goal can be found in [746]. What follows is the illustration of the size of such evolved matrix elements as function of the scale at which NP is integrated out. These results have been obtained in [211], using lattice results known at that time. But the pattern has not been changed since then as one can check by comparing the relevant references to LQCD quoted in that paper with more recent lattice results in [747, 748]. The matrix elements of operators with L replaced by R are equal to the ones given in the following tables simply because parity is conserved in QCD.

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Table 13.1 Hadronic matrix elements $\langle Q_i^a(\mu_t)\rangle$ in units of GeV^3 at $m_t(m_t)$.

	$\langle Q_1^{\rm LR}(m_t)\rangle$	$\langle Q_2^{\rm LR}(m_t)\rangle$	$\langle Q_1^{\rm SLL}(m_t)\rangle$	$\langle Q_2^{\rm SLL}(m_t)\rangle$
$K^0-\bar K^0$	−0.11	0.18	−0.064	−0.107
$B_d^0-\bar B_d^0$	−0.21	0.27	−0.095	−0.191
$B_s^0-\bar B_s^0$	−0.30	0.40	−0.14	−0.29

In the case of tree-level SM Z and SM Higgs exchanges, absent in the SM but generated by NP as discussed at length in Chapter 15, we evaluate the matrix elements at $m_t(m_t)$. The inclusion of NLO QCD corrections allows us to choose any scale of $O(m_t)$ without changing physical results. The values of hadronic matrix elements at $m_t(m_t)$ in the $\overline{\rm MS}$-NDR scheme are given in Table 13.1.

For higher scales other effects, like top Yukawa effects and electroweak corrections, discussed in Chapter 14, could become relevant. Still it is instructive to present here the results for the matrix elements evaluated at high scales when these corrections are not included. The reason is that the latter effects are model dependent, while the effects presented here represent a sole effect of QCD. We follow here [211].

The matrix elements depend on the renormalization scheme, but this scheme dependence is canceled by the one present in Wilson coefficients. Here, using the technology of [746, 749] one finds then renormalization scheme independent matrix elements, which are relevant for tree-level Z' exchanges discussed in detail in Section 15.4

$$\langle \hat Q_1^{\rm VLL}(M_{Z'})\rangle^{sd} = \langle Q_1^{\rm VLL}(M_{Z'})\rangle^{sd}\left(1+\frac{11}{3}\frac{\alpha_s(M_{Z'})}{4\pi}\right), \tag{13.30}$$

$$\langle \hat Q_1^{\rm LR}(M_{Z'})\rangle^{sd} = \langle Q_1^{\rm LR}(M_{Z'})\rangle^{sd}\left(1-\frac{1}{6}\frac{\alpha_s(M_{Z'})}{4\pi}\right) - \frac{\alpha_s(M_{Z'})}{4\pi}\langle Q_2^{\rm LR}(M_{Z'})\rangle^{sd}. \tag{13.31}$$

$\langle Q_1^{\rm VLL}(M_{Z'})\rangle^{sd}$ and $\langle Q_{1,2}^{\rm LR}(M_{Z'})\rangle^{sd}$ are the matrix elements evaluated at $\mu = M_{Z'}$ in the $\overline{\rm MS}$-NDR scheme, and the presence of $O(\alpha_s)$ corrections removes the scheme dependence. $\alpha_s(M_{Z'})$ is the value of the strong coupling at $M_{Z'}$. The corresponding formulas for B_q mesons are obtained by simply changing sd to bq without changing α_s corrections.

In Table 13.2 we give the values of the matrix elements in (13.30) and (13.31) for the three meson systems considered and for different values of $M_{Z'}$. For simplicity we choose the renormalization scale to be $M_{Z'}$, but any scale of this order would give the same results for the physical quantities up to NNLO QCD corrections that are negligible at these high scales.

In the case of heavy tree-level colorless scalar exchanges, where other operators are dominant, it is useful to define the following renormalization scheme independent matrix elements [749]

$$\langle \hat Q_1^{\rm SLL}(M_H)\rangle^{bq} = \langle Q_1^{\rm SLL}(M_H)\rangle^{bq}\left(1+\frac{9}{2}\frac{\alpha_s(M_H)}{4\pi}\right) + \frac{1}{8}\frac{\alpha_s(M_H)}{4\pi}\langle Q_2^{\rm SLL}(M_H)\rangle^{bq}, \tag{13.32}$$

$$\langle \hat Q_2^{\rm LR}(M_H)\rangle^{bq} = \langle Q_2^{\rm LR}(M_H)\rangle^{bq}\left(1-\frac{\alpha_s(M_H)}{4\pi}\right) - \frac{3}{2}\frac{\alpha_s(M_H)}{4\pi}\langle Q_1^{\rm LR}(M_H)\rangle^{bq}. \tag{13.33}$$

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Table 13.2 Central values of the scheme-independent hadronic matrix elements evaluated at different values of $M_{Z'}$. $\langle \hat{Q}_1^{\rm VLL} \rangle^{ij}$ and $\langle \hat{Q}_1^{\rm LR} \rangle^{ij}$ are in units of GeV^3.

$M_{Z'}$	5 TeV	10 TeV	20 TeV	50 TeV	100 TeV	200 TeV
$\langle \hat{Q}_1^{\rm VLL}(M_{Z'}) \rangle^{sd}$	0.00158	0.00156	0.00153	0.00150	0.00148	0.00146
$\langle \hat{Q}_1^{\rm LR}(M_{Z'}) \rangle^{sd}$	−0.183	−0.197	−0.211	−0.230	−0.244	−0.259
$\kappa_{sd}(M_{Z'})$	−115.46	−126.51	−137.84	−153.24	−165.20	−177.41
$\langle \hat{Q}_1^{\rm VLL}(M_{Z'}) \rangle^{bd}$	0.0423	0.0416	0.0409	0.0401	0.0395	0.0390
$\langle \hat{Q}_1^{\rm LR}(M_{Z'}) \rangle^{bd}$	−0.183	−0.195	−0.206	−0.222	−0.234	−0.246
$\kappa_{bd}(M_{Z'})$	−4.33	−4.68	−5.04	−5.53	−5.92	−6.30
$\langle \hat{Q}_1^{\rm VLL}(M_{Z'}) \rangle^{bs}$	0.0622	0.0611	0.0601	0.0589	0.0581	0.0573
$\langle \hat{Q}_1^{\rm LR}(M_{Z'}) \rangle^{bs}$	−0.268	−0.284	−0.301	−0.323	−0.340	−0.357
$\kappa_{bs}(M_{Z'})$	−4.31	−4.66	−5.01	−5.48	−5.85	−6.23

Table 13.3 Central values of the scheme-independent hadronic matrix elements evaluated at different values of M_H. $\langle \hat{Q}_1^{\rm SLL} \rangle^{ij}$ and $\langle \hat{Q}_2^{\rm LR} \rangle^{ij}$ are in units of GeV^3.

M_H	5 TeV	10 TeV	20 TeV	50 TeV	100 TeV	200 TeV
$\langle \hat{Q}_1^{\rm SLL}(M_H) \rangle^{sd}$	−0.089	−0.093	−0.096	−0.101	−0.105	−0.108
$\langle \hat{Q}_2^{\rm LR}(M_H) \rangle^{sd}$	0.291	0.312	0.334	0.362	0.384	0.405
$\tilde{\kappa}_{sd}(M_H)$	−3.27	−3.37	−3.46	−3.58	−3.66	−3.75
$\langle \hat{Q}_1^{\rm SLL}(M_H) \rangle^{bd}$	−0.095	−0.099	−0.103	−0.108	−0.112	−0.116
$\langle \hat{Q}_2^{\rm LR}(M_H) \rangle^{bd}$	0.245	0.262	0.280	0.304	0.322	0.340
$\tilde{\kappa}_{bd}(M_H)$	−2.57	−2.64	−2.72	−2.81	−2.88	−2.95
$\langle \hat{Q}_1^{\rm SLL}(M_H) \rangle^{bs}$	−0.140	−0.146	−0.152	−0.159	−0.164	−0.170
$\langle \hat{Q}_2^{\rm LR}(M_H) \rangle^{bs}$	0.348	0.373	0.399	0.432	0.458	0.484
$\tilde{\kappa}_{bs}(M_H)$	−2.48	−2.56	−2.63	−2.72	−2.79	−2.85

The operators $Q_{1,2}^{\rm LR}$ were already present in the case of Z' but now, as seen from (13.33), the operator $Q_2^{\rm LR}$ plays the dominant role. In Table 13.3 we give the central values of the renormalization scheme independent matrix elements in (13.32) and (13.33) for the three meson systems and for different values of M_H.

In both tables we show the values of

$$\kappa_{sd}(M_{Z'}) = \frac{\langle \hat{Q}_1^{\rm LR}(M_{Z'}) \rangle^{sd}}{\langle \hat{Q}_1^{\rm VLL}(M_{Z'}) \rangle^{sd}}, \qquad \tilde{\kappa}_{bq}(M_H) = \frac{\langle \hat{Q}_2^{\rm LR}(M_H) \rangle^{sd}}{\langle \hat{Q}_1^{\rm SLL}(M_H) \rangle^{sd}} \tag{13.34}$$

that allow to describe the general pattern of these results. Inspecting Tables 13.2 and 13.3 we observe the following pattern:

- The matrix elements of LR operators are in the $K^0 - \bar{K}^0$ system by two orders of magnitude larger than of the SM VLL operator, and this disparity increases with increasing scale. Comparing Tables 13.2 and 13.3, we observe that in particular the matrix elements of the operator $\hat{Q}_2^{\rm LR}$ are very large with significant impact on phenomenology as we will

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see in the BSM chapters of this book. This feature has been known already for decades [744, 745, 750, 751].

- The matrix elements of LR operators are in $B^0_{s,d} - \bar{B}^0_{s,d}$ systems also larger than the matrix element of the SM operator but only by factors of 4–6.
- The matrix elements of scalar operators SLL are significantly larger than the ones of the SM operator VLL but smaller than those of LR operators.

13.2.4 DQCD Insight into $K^0 - \bar{K}^0$ Hadronic Matrix Elements from LQCD

During the last years significant progress in the evaluation of $K^0 - \bar{K}^0$ matrix elements by ETM, SWME, and RBC-UKQCD lattice collaborations [295, 747, 752–754] has been made, and we have used their results in constructing the preceding tables.

Let us have a closer look at these results that differ significantly from those obtained in the VIA, which gives for the parameters B_i simply

$$B_1 = B_2 = B_3 = B_4 = B_5 = 1 \qquad \text{(VIA)}. \tag{13.35}$$

In VIA these parameters do not depend on the renormalization scale μ. Already this property of VIA, which is based on the factorization of matrix elements of four-quark operators into products of quark currents or quark densities, is problematic, as generally these parameters depend on μ.

Now the RBC-UKQCD collaboration working at $\mu = 3\,\text{GeV}$ finds [747, 753, 754]

$$B_1 = 0.523(9)(7), \qquad B_2 = 0.488(7)(17), \qquad B_3 = 0.743(14)(65), \tag{13.36}$$

and

$$B_4 = 0.920(12)(16), \qquad B_5 = 0.707(8)(44), \tag{13.37}$$

with the first error being statistical and the second systematic. Similar results are obtained by EMT and SWME collaborations, although the values for B_4 and B_5 from the ETM collaboration are visibly below the ones obtained by RBC-UKQCD collaboration: $B_4 = 0.78(4)(3)$ and $B_5 = 0.49(4)(1)$. Except for B_4 all values differ significantly from unity prohibiting the use of VIA.

To our knowledge no lattice group made an attempt to understand this pattern of values, probably because within LQCD, which works in this case at scale $\mu = 3\,\text{GeV}$, this pattern cannot be understood. But in Section 7.2.3 we have stressed that the QCD dynamics in $K \to \pi\pi$ decays at low scales, represented in the DQCD by meson evolution, is responsible for the dominant part of the $\Delta I = 1/2$ rule. Let us then see whether DQCD and meson evolution can also be helpful here.

Indeed, it has been demonstrated in [755] that the pattern of the values of the parameters B_i in (13.36) and (13.37) can be understood within DQCD approach through meson evolution from very low energy scales to scales $O(1\,\text{GeV})$ followed by the usual RG QCD evolution to scale $\mu = 3\,\text{GeV}$ at which the parameters B_i have been evaluated in lattice QCD.

The case of B_1 is well known, and we discussed it already in Section 7.2.4. In the large N limit one finds $B_1 = 3/4$ [230]. The meson evolution followed by quark-gluon evolution

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brings it into the ballpark of the lattice result in (13.36). In this particular case, as discussed in Section 7.2.4, one usually multiplies the result by the corresponding SD renormalization group factor to find the scale and renormalization scheme independent $\hat{B}_K = 0.73 \pm 0.02$ [239] in a very good agreement with the world average of lattice QCD calculations $\hat{B}_K = 0.766 \pm 0.010$ [48].

In the case of the BSM operators O_i with $i = 2 - 5$ the construction of scale independent $\hat{B}_i$ parameters, although possible, is not particularly useful because O_2 mixes under renormalization with O_3 and O_4 with O_5. As stated already earlier, this mixing is known at the NLO level [744, 745] and useful NLO expressions for the μ dependence of hadronic matrix elements and their Wilson coefficients can be found in [746]. But for our purposes here, LO approximation is sufficient.

Starting with the large N limit one finds [755]

$$B_2 = 1.20, \qquad B_3 = 3.0, \qquad B_4 = 1.0, \qquad B_5 = 0.2 \qquad \text{(large N limit)}. \tag{13.38}$$

These results differ significantly from lattice results but apply to $\mu = O(m_\pi)$, while the lattice results where obtained at $\mu = 3\,\text{GeV}$. It is therefore remarkable that the pattern

$$B_2 < B_5 \leq B_3 < B_4 \qquad (\mu = 3\,\text{GeV}) \quad \text{(LQCD)} \tag{13.39}$$

can indeed be eventually understood within DQCD, although there one finds first

$$B_5 < B_4 < B_2 < B_3 \qquad (\mu = O(m_\pi)) \quad \text{(DQCD)}. \tag{13.40}$$

As meson evolution with the inclusion of pseudoscalar mesons can be done only up to $\mu = 0.70\,\text{GeV}$, let us use the standard RG equations to find first lattice values for B_i at $\mu = 1\,\text{GeV}$, where perturbation theory is still reliable. From the central values in (13.36) and (13.37) one finds at $\mu = 1\,\text{GeV}$ [755].

$$B_2 = 0.608, \quad B_3 = 1.06, \quad B_4 = 0.920, \quad B_5 = 0.519 \quad \text{(Lattice QCD)}. \tag{13.41}$$

Using ETM values for B_4 and B_5, one would find $B_4 = 0.78$ and $B_5 = 0.24$.

We observe that B_2, B_3, and B_5, all moved toward their large N values in (13.38), while B_4 did not change in LO approximation. These results are already very encouraging but certainly not the explanation of the lattice results. Fortunately, the rest of the job is done by the meson evolution. Starting with the values in (13.38) and performing the meson evolution in the chiral limit one finds at order $1/N$ [755]

$$B_2(\Lambda) = 1.2 \left[1 - \frac{8}{3}\frac{M^2}{(4\pi F_K)^2}\right], \qquad B_3(\Lambda) = 3.0 \left[1 - \frac{16}{3}\frac{M^2}{(4\pi F_K)^2}\right], \tag{13.42}$$

$$B_4(\Lambda) = 1.0 \left[1 - \frac{4}{3}\frac{M^2}{(4\pi F_K)^2}\right], \qquad B_5(\Lambda) = 0.23 \left[1 + 4\frac{M^2}{(4\pi F_K)^2}\right], \tag{13.43}$$

where M is the cutoff of DQCD, which allows us to separate the nonfactorizable meson evolution from the quark-gluon one. The general trend already observed in the quark-gluon evolution is nicely outlined in the meson evolution with a strong suppression of B_2, an even stronger suppression of B_3, a smooth evolution of B_4, and a strong enhancement of B_5.

Consequently, for $M = 0.7\,\text{GeV}$, one finds

$$B_2 = 0.79, \quad B_3 = 0.96, \quad B_4 = 0.83, \quad B_5 = 0.30, \quad \text{(DQCD)}. \tag{13.44}$$

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We note also that the values for B_4 and B_5 are in between those from RBC-UKQCD and ETM collaborations, and we are looking forward to new improved lattice results for all four parameters to see how well DQCD reproduces the LQCD numbers in question.

In any case as the meson evolution has been performed in the chiral limit without the inclusion of vector meson contributions this result should be considered as not only satisfactory but also remarkable as our calculations involved only one parameter, the cutoff scale M, which in any case should be around 0.7 GeV if only pseudoscalar meson contributions are taken into account. It demonstrates the importance of the QCD dynamics at scales below 1 GeV and gives additional support to our claim that meson evolution is the dominant QCD dynamic responsible for the $\Delta I = 1/2$ rule and also for the ε'/ε anomaly. I am not aware of any analytical approach that could provide such insight in LQCD results in question.

13.3 $\Delta F = 1$ Nonleptonic Operators

13.3.1 Preliminaries

We now move to list all $\Delta F = 1$ operators, which can be present beyond the SM. They have been already considered in [744, 745]. There the two-loop anomalous dimensions of operators containing four different quark flavors have been calculated. In such a case, the only possible diagrams are the current-current ones. In addition in [744] also two-loop anomalous dimensions of operators resulting from penguin diagrams, that is containing two identical quarks, have been calculated. However, the discussions in these papers is very technical and not very transparent. We would like to improve on it here by following a recent analysis by Christoph Bobeth, Mikolaj Misiak, and myself. The aim of this analysis was to reduce the number of operators involved that mix into SM penguin operators as much as possible to simplify the structure of the RG evolution matrices.

To be specific, we will consider $\Delta S = 1$ operators. As shown in Table 13.4 the number of independent nonleptonic $\Delta S = 1$ operators with quark content $(sd) + (uu, dd, ss, cc, bb)$

Table 13.4 Number of nonleptonic $\Delta S = \Delta D = 1$ four-quark operators for given quark content in LEFT with N_f active quark flavors. The factor of 2 represents the existence of two sectors related by $P_L \leftrightarrow P_R$ interchange.

quark content	$N_f = 5$	$N_f = 4$	$N_f = 3$
$(sd)(bb)$	2×10	—	—
$(sd)(cc)$	2×10	2×10	—
$(sd)(uu)$	2×10	2×10	2×10
$(sd)(ss)$	2×5	2×5	2×5
$(sd)(dd)$	2×5	2×5	2×5
$\sum$	2×40	2×30	2×20

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is $2 \times 40 = 80$. We will next, starting with $N_f = 5$ and ending with $N_f = 3$, list operators corresponding to the numbers in Table 13.4.

13.3.2 $N_f = 5$

We will begin with the SM operators. We will use here the projection $P_{L,R}$, which differs by a factor of 4 from the usual $(V \pm A)$. This has to be remembered when calculating hadronic matrix elements. But this is what these days is done when considering BSM operators, and we can as well do it for SM operators. Despite this difference we will still keep the usual names for SM operators. With this warning no confusion should arise.

We have then two $(sd)(uu)$ current-current (CC)-type operators in the SM from tree-level exchange

$$\begin{aligned} Q_1 &= (\bar{s}^\alpha \gamma_\mu P_L u^\beta)(\bar{u}^\beta \gamma^\mu P_L d^\alpha) = Q_1^{\mathrm{VLL},u}, \\ Q_2 &= (\bar{s}^\alpha \gamma_\mu P_L u^\alpha)(\bar{u}^\beta \gamma^\mu P_L d^\beta) = Q_2^{\mathrm{VLL},u}, \end{aligned} \tag{13.45}$$

where the naming given also in [744] will be useful in what follows.

VLL, $u \Rightarrow 2$

The QCD and QED penguin-type diagrams with insertions of CC and P (penguin) operators into CC and P diagrams as already encountered in Section 6.4.3 give rise to mixing into the eight QCD and QED penguin operators, unless the involved Dirac structure does not permit it or vanishing traces of the Dirac structure and/or color structure appear. We have seen this explicitly in Section 6.4.3. They are P-type operators with left-handed $(\bar{s}\gamma_\mu P_L d)$ vector currents and are part of the SM operator basis

$$\begin{aligned} Q_3 &= (\bar{s}^\alpha \gamma_\mu P_L d^\alpha) \sum_{q=u,d,s,c,b} (\bar{q}^\beta \gamma^\mu P_L\, q^\beta), \\ Q_4 &= (\bar{s}^\alpha \gamma_\mu P_L d^\beta) \sum_{q=u,d,s,c,b} (\bar{q}^\beta \gamma^\mu P_L\, q^\alpha), \\ Q_5 &= (\bar{s}^\alpha \gamma_\mu P_L d^\alpha) \sum_{q=u,d,s,c,b} (\bar{q}^\beta \gamma^\mu P_R\, q^\beta), \\ Q_6 &= (\bar{s}^\alpha \gamma_\mu P_L d^\beta) \sum_{q=u,d,s,c,b} (\bar{q}^\beta \gamma^\mu P_R\, q^\alpha), \end{aligned} \tag{13.46}$$

$$\begin{aligned} Q_7 &= \frac{3}{2} (\bar{s}^\alpha \gamma_\mu P_L d^\alpha) \sum_{q=u,d,s,c,b} Q_q\, (\bar{q}^\beta \gamma^\mu P_R\, q^\beta), \\ Q_8 &= \frac{3}{2} (\bar{s}^\alpha \gamma_\mu P_L d^\beta) \sum_{q=u,d,s,c,b} Q_q\, (\bar{q}^\beta \gamma^\mu P_R\, q^\alpha), \\ Q_9 &= \frac{3}{2} (\bar{s}^\alpha \gamma_\mu P_L d^\alpha) \sum_{q=u,d,s,c,b} Q_q\, (\bar{q}^\beta \gamma^\mu P_L\, q^\beta), \\ Q_{10} &= \frac{3}{2} (\bar{s}^\alpha \gamma_\mu P_L d^\beta) \sum_{q=u,d,s,c,b} Q_q\, (\bar{q}^\beta \gamma^\mu P_L\, q^\alpha). \end{aligned} \tag{13.47}$$

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Their chirality-flipped counterparts are not present in the SM.

Penguins $\Rightarrow$ 8

Other $(sd)(QQ)$ operators with $Q = d, s$ are [744]

$$\begin{aligned} Q_{11} &= Q_1^{\mathrm{VLL},d+s} = (\bar{s}^\alpha \gamma_\mu P_L d^\alpha)\,[(\bar{d}^\beta \gamma^\mu P_L d^\beta) + (\bar{s}^\beta \gamma^\mu P_L s^\beta)], \\ Q_{12} &= Q_1^{\mathrm{VLR},d+s} = (\bar{s}^\alpha \gamma_\mu P_L d^\beta)\,[(\bar{d}^\beta \gamma^\mu P_R d^\alpha) + (\bar{s}^\beta \gamma^\mu P_R s^\alpha)], \\ Q_{13} &= Q_2^{\mathrm{VLR},d+s} = (\bar{s}^\alpha \gamma_\mu P_L d^\alpha)\,[(\bar{d}^\beta \gamma^\mu P_R d^\beta) + (\bar{s}^\beta \gamma^\mu P_R s^\beta)], \end{aligned} \tag{13.48}$$

and

$$\begin{aligned} Q_{14} &= Q_1^{\mathrm{VLL},d-s} = (\bar{s}^\alpha \gamma_\mu P_L d^\alpha)\,[(\bar{d}^\beta \gamma^\mu P_L d^\beta) - (\bar{s}^\beta \gamma^\mu P_L s^\beta)], \\ Q_{15} &= Q_1^{\mathrm{VLR},d-s} = (\bar{s}^\alpha \gamma_\mu P_L d^\beta)\,[(\bar{d}^\beta \gamma^\mu P_R d^\alpha) - (\bar{s}^\beta \gamma^\mu P_R s^\alpha)], \\ Q_{16} &= Q_2^{\mathrm{VLR},d-s} = (\bar{s}^\alpha \gamma_\mu P_L d^\alpha)\,[(\bar{d}^\beta \gamma^\mu P_R d^\beta) - (\bar{s}^\beta \gamma^\mu P_R s^\beta)]. \end{aligned} \tag{13.49}$$

Only $d + s$ operators mix into QCD- and QED-penguin operators $Q_{3,\ldots 10}$, whereas $d - s$ ones do not.

VLL, $d - s$, VLR, $d - s$, VLL, $d + s$, VLR, $d + s \Rightarrow 6$

Further $(sd)(uu)$ and $(sd)(cc)$ operators in analogy to $Q_{5,6}$ are

$$\begin{aligned} Q_{17} &= Q_1^{\mathrm{VLR},u-c} = (\bar{s}^\alpha \gamma_\mu P_L d^\beta)\,[(\bar{u}^\beta \gamma^\mu P_R u^\alpha) - (\bar{c}^\beta \gamma^\mu P_R c^\alpha)], \\ Q_{18} &= Q_2^{\mathrm{VLR},u-c} = (\bar{s}^\alpha \gamma_\mu P_L d^\alpha)\,[(\bar{u}^\beta \gamma^\mu P_R u^\beta) - (\bar{c}^\beta \gamma^\mu P_R c^\beta)], \end{aligned} \tag{13.50}$$

which do not mix into QCD- and QED-penguin operators $Q_{3,\ldots 10}$, but give threshold corrections when decoupling the c quark.

VLR, $u - c \Rightarrow 2$

The sector of scalar operators SLL for $(sd)(QQ)$ with $Q = u, c, b$ comprises

$$Q_1^{\mathrm{SLL},Q} = (\bar{s}^\alpha P_L d^\beta)\,(\bar{Q}^\beta P_L Q^\alpha), \tag{13.51}$$

$$Q_2^{\mathrm{SLL},Q} = (\bar{s}^\alpha P_L d^\alpha)\,(\bar{Q}^\beta P_L Q^\beta), \tag{13.52}$$

$$Q_3^{\mathrm{SLL},Q} = -(\bar{s}^\alpha \sigma_{\mu\nu} P_L d^\beta)\,(\bar{Q}^\beta \sigma^{\mu\nu} P_L Q^\alpha), \tag{13.53}$$

$$Q_4^{\mathrm{SLL},Q} = -(\bar{s}^\alpha \sigma_{\mu\nu} P_L d^\alpha)\,(\bar{Q}^\beta \sigma^{\mu\nu} P_L Q^\beta), \tag{13.54}$$

with the minus sign resulting from our definition of $\sigma_{\mu\nu}$, which differs by a factor i relative to the one in [744].

Due to Fierz symmetries there are only two operators $(sd)(QQ)$ with $Q = d, s$

$$Q_1^{\mathrm{SLL},Q} = (\bar{s}^\alpha P_L d^\beta)\,(\bar{Q}^\beta P_L Q^\alpha), \tag{13.55}$$

$$Q_2^{\mathrm{SLL},Q} = (\bar{s}^\alpha P_L d^\alpha)\,(\bar{Q}^\beta P_L Q^\beta). \tag{13.56}$$

SLL, Q $(Q = u, c, b)$ and SLL, Q $(Q = d, s) \Rightarrow 3 \times 4 + 2 \times 2 = 16$

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The remaining operators are [744]

$$Q_1^{\mathrm{SLR},Q} = (\bar{s}^\alpha P_L d^\beta)\,(\bar{Q}^\beta P_R\, Q^\alpha), \tag{13.57}$$

$$Q_2^{\mathrm{SLR},Q} = (\bar{s}^\alpha P_L d^\alpha)\,(\bar{Q}^\beta P_R\, Q^\beta), \tag{13.58}$$

with $Q = u, c, b$.

$$\boxed{\mathrm{SLR}, Q\ (Q = u, c, b) \Rightarrow 3 \times 2 = 6}$$

In total there are $8 + 2 + 6 + 2 + 16 + 6 = 40$ operators. Including chirality-flipped operators we end up with 80 operators. Their anomalous dimension matrices will be given in our future paper, and in view of many operators involved we will not list them here.

13.3.3 $N_f = 4$

The minimal basis for $N_f = 4$ contains 30 operators of a given chirality and their chirality-flipped counterparts. It can be found by dropping operators that are exclusively $(sd)(bb)$, which are 6: SLL,b (4) and SLR,b (2). Further, in QCD- and QED-penguin operators the sum runs now only over $q = u, d, s, c$ and in consequence some operators become linearly dependent

$$Q_{10} = Q_9 + Q_4 - Q_3 \tag{13.59}$$

$$Q_1^{\mathrm{VLL},d+s} = \frac{2}{3}(Q_3 - Q_9), \tag{13.60}$$

$$Q_1^{\mathrm{VLR},d+s} = \frac{2}{3}(Q_6 - Q_8), \tag{13.61}$$

$$Q_2^{\mathrm{VLR},d+s} = \frac{2}{3}(Q_5 - Q_7). \tag{13.62}$$

It seems convenient to discard these 4 linearly dependent operators, which also removes one source of mixing into QCD- and QED-penguin operators from the operators VLL,$d+s$ and VLR,$d+s$. This gives a minimal basis for $N_f = 4$.

The operators relevant for $K \to \pi\pi$ are $30 - 6 = 24$ because the SLL,c and SLR,c sectors do not mix into the other operators and have vanishing $K \to \pi\pi$ matrix elements.

13.3.4 $N_f = 3$

The minimal basis for $N_f = 3$ contains 20 operators of a given chirality and their chirality-flipped counterparts. One can start from $N_f = 4$ by dropping operators with exclusively $(sd)(cc)$, which are 6: SLL,c (4) and SLR,c (2). Further, in QCD- and QED-penguin operators the sum runs now only over $q = u, d, s$, and in consequence some operators become linearly dependent

$$Q_4 = Q_3 + Q_2 - Q_1, \tag{13.63}$$

$$Q_9 = \frac{1}{2}(3Q_1 - Q_1). \tag{13.64}$$

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By decoupling the c-quark also $Q_{1,2}^{\mathrm{VLR},u-c} \to Q_{1,2}^{\mathrm{VLR},u}$, which are linearly dependent on

$$Q_1^{\mathrm{VLR},u} = \frac{1}{3}(Q_6 + 2Q_8), \tag{13.65}$$

$$Q_2^{\mathrm{VLR},u} = \frac{1}{3}(Q_5 + 2Q_7). \tag{13.66}$$

Thus the operators Q_4, Q_9, $Q_{1,2}^{\mathrm{VLR},u}$ can be discarded from the list of linearly independent operators.

13.3.5 Summary of BSM Operators for $K \to \pi\pi$

As we have seen for $N_f = 3$, relevant for $K \to \pi\pi$ decays, there are thirteen BSM linearly independent four-quark operators that are also linearly independent from the SM ones, in particular none of them mixes into QCD- and QED-penguin operators $Q_{3,\ldots 10}$. Let us group them in classes as done in [246].

Class I

$$Q_1^{\mathrm{VLL},d-s} = (\bar{s}^\alpha \gamma_\mu P_L d^\alpha)\,[(\bar{d}^\beta \gamma^\mu P_L d^\beta) - (\bar{s}^\beta \gamma^\mu P_L s^\beta)], \tag{13.67}$$

Class II

$$Q_1^{\mathrm{SLR},u} = (\bar{s}^\alpha P_L d^\beta)\,(\bar{u}^\beta P_R\, u^\alpha), \tag{13.68}$$

$$Q_2^{\mathrm{SLR},u} = (\bar{s}^\alpha P_L d^\alpha)\,(\bar{u}^\beta P_R\, u^\beta), \tag{13.69}$$

$$Q_1^{\mathrm{VLR},d-s} = (\bar{s}^\alpha \gamma_\mu P_L d^\beta)\,[(\bar{d}^\beta \gamma^\mu P_R d^\alpha) - (\bar{s}^\beta \gamma^\mu P_R s^\alpha)], \tag{13.70}$$

$$Q_2^{\mathrm{VLR},d-s} = (\bar{s}^\alpha \gamma_\mu P_L d^\alpha)\,[(\bar{d}^\beta \gamma^\mu P_R d^\beta) - (\bar{s}^\beta \gamma^\mu P_R s^\beta)], \tag{13.71}$$

Class III

$$Q_1^{\mathrm{SLL},u} = (\bar{s}^\alpha P_L d^\beta)\,(\bar{u}^\beta P_L\, u^\alpha), \tag{13.72}$$

$$Q_2^{\mathrm{SLL},u} = (\bar{s}^\alpha P_L d^\alpha)\,(\bar{u}^\beta P_L\, u^\beta), \tag{13.73}$$

$$Q_3^{\mathrm{SLL},u} = -(\bar{s}^\alpha \sigma_{\mu\nu} P_L d^\beta)\,(\bar{u}^\beta \sigma^{\mu\nu} P_L\, u^\alpha), \tag{13.74}$$

$$Q_4^{\mathrm{SLL},u} = -(\bar{s}^\alpha \sigma_{\mu\nu} P_L d^\alpha)\,(\bar{u}^\beta \sigma^{\mu\nu} P_L\, u^\beta), \tag{13.75}$$

Class IV

$$Q_1^{\mathrm{SLL},d} = (\bar{s}^\alpha P_L d^\beta)\,(\bar{d}^\beta P_L\, d^\alpha), \tag{13.76}$$

$$Q_2^{\mathrm{SLL},d} = (\bar{s}^\alpha P_L d^\alpha)\,(\bar{d}^\beta P_L\, d^\beta), \tag{13.77}$$

$$Q_1^{\mathrm{SLL},s} = (\bar{s}^\alpha P_L d^\beta)\,(\bar{s}^\beta P_L\, s^\alpha), \tag{13.78}$$

$$Q_2^{\mathrm{SLL},s} = (\bar{s}^\alpha P_L d^\alpha)\,(\bar{s}^\beta P_L\, s^\beta). \tag{13.79}$$

The one-loop anomalous dimensions for all these BSM operators are then given in units of $\alpha_s/4\pi$) as follows [246, 744, 756]

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Class I

$$\gamma^{(0)}(Q_1^{\mathrm{VLL,d-s}}) = 4, \tag{13.80}$$

Class II

$$\hat{\gamma}^{(0)}(Q_1^{\mathrm{SLR,u}}, Q_2^{\mathrm{SLR,u}}) = \begin{pmatrix} \frac{6}{N} & -6 \\ 0 & -6N + \frac{6}{N} \end{pmatrix} = \begin{pmatrix} 2 & -6 \\ 0 & -16 \end{pmatrix}, \tag{13.81}$$

$$\hat{\gamma}^{(0)}(Q_1^{\mathrm{VLR,d-s}}, Q_2^{\mathrm{VLR,d-s}}) = \begin{pmatrix} -6N + \frac{6}{N} & 0 \\ -6 & \frac{6}{N} \end{pmatrix} = \begin{pmatrix} -16 & 0 \\ -6 & 2 \end{pmatrix}, \tag{13.82}$$

where N denotes the number of colors with $N = 3$.

Class III

In the basis $(Q_1^{\mathrm{SLL,u}}, Q_2^{\mathrm{SLL,u}}, Q_3^{\mathrm{SLL,u}}, Q_4^{\mathrm{SLL,u}})$ we have

$$\hat{\gamma}^{(0)\mathrm{SLL,u}} = \begin{pmatrix} \frac{6}{N} & -6 & \frac{N}{2} - \frac{1}{N} & \frac{1}{2} \\ 0 & -6N + \frac{6}{N} & 1 & -\frac{1}{N} \\ -\frac{48}{N} + 24N & 24 & -\frac{2}{N} - 4N & 6 \\ 48 & -\frac{48}{N} & 0 & 2N - \frac{2}{N} \end{pmatrix} \tag{13.83}$$

$$= \begin{pmatrix} 2 & -6 & 7/6 & 1/2 \\ 0 & -16 & 1 & -1/3 \\ 56 & 24 & -38/3 & 6 \\ 48 & -16 & 0 & 16/3 \end{pmatrix}. \tag{13.84}$$

Class IV

$$\hat{\gamma}^{(0)}(Q_1^{\mathrm{SLL,d}}, Q_2^{\mathrm{SLL,d}}) = \begin{pmatrix} 2N + 4 + \frac{2}{N} & 4N - 4 - \frac{8}{N} \\ 4 - \frac{8}{N} & -6N + 8 + \frac{2}{N} \end{pmatrix} \tag{13.85}$$

$$= \begin{pmatrix} 32/3 & 16/3 \\ 4/3 & -28/3 \end{pmatrix}, \tag{13.86}$$

with the same matrix for the operators $Q_{1,2}^{\mathrm{SLL,s}}$.

We observe that some entries in these matrices are large, implying strong RG effects. A detailed numerical analysis of these effects at the NLO level should be available in the hep-arxiv within 2020. The values of their $K \to \pi\pi$ matrix elements, evaluated in the chiral limit in the DQCD approach, are collected in [246].

IT IS TIME TO LEARN THE BSM LANGUAGE: SMEFT!

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