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STRUCTURAL CHANGE AND A CONSTANT GROWTH PATH IN A THREE-SECTOR GROWTH MODEL WITH THREE FACTORS

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This study proposes the unified framework of a three-sector model with structural change where agriculture, manufacturing, and service sectors have different production technologies. All three sectors use the factors of capital, labor, and land as inputs. The constant-growth path (CGP), which is the trajectory along which the rental rate of capital remains constant, is used to reconcile the Kaldor and Kuznets facts, and plays a role in linking the wage rate and land rent in the three-factors model. Because the prices of agricultural goods and services are determined endogenously, the CGP condition is no longer the knife-edge condition. We find that the dynamic system along the CGP in the three-sector, three-factor model corresponds to the standard two-sector optimal growth model.

Keywords: Three-Sector GDP Function, Stone–Geary Utility Function, Constant-Growth Path, Kuznets Facts, Kaldor Facts

1. INTRODUCTION

The proportion of economic activity and employment accounted for by the service industry is increasing, with developed countries shifting toward a so-called "service economy." Thus, to analyze the dynamic behavior of the service price, we

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need to examine the causes of this structural change. Numerous factors have been proposed to explain the structural change to the service economy, including Engel's law, technological progress that saves labor, urbanization and depopulation, international trade, the progress of developing countries, financial development, consumption externalities, and upswings in labor force participation by woman, which increase the demand for various services, but decrease home production. The manufacturing industry is able to decrease its demand for labor because technological change makes it possible to substitute capital for labor. However, this does not apply to the service industry, because the quality of service decreases when it reduces labor;[1] Baumol (1967, 2012) refers to this as the *cost disease*. Thus, the price of services, measured in terms of manufactured goods, is destined to rise.

Kuznets (1966) emphasizes that economic growth involves structural changes from manufacturing to services. Most studies of economic growth have focused on the balanced-growth path (BGP) to explain the six stylized Kaldor facts. However, we need to deviate from the BGP if we are to explain the Kuznets facts. Dynamic macroeconomic theory requires that we reconcile Kaldor and Kuznets facts. Here, we are interested in the dynamic behavior of the service price, economic growth, and structural change. This study proposes the unified framework of a three-sector model with structural change, where the agriculture, manufacturing, and service sectors have different production technologies. Here, all three sectors use the factors of capital, labor, and land as inputs. The three factor prices consist of the rate of return on capital, the wage rate, and land rent and depend on the prices of agricultural goods and services. Both prices are measured in terms of manufactured goods.

Kongsamut et al. (2001) develop a model in which nonhomothetic preferences drive structural change. They propose a condition for the *constant-growth path* (CGP), which is a trajectory along which the real interest rate remains constant, in order to harmonize the Kaldor and Kuznets facts in the three-goods-and-two-factors model. Their CGP condition restricts the price of agricultural goods relative to services to the ratio of two exogenous preference parameters, namely the home production of services and the subsistence level of agricultural goods. In their model, the relative prices become the inverse of the relative productivities because the production functions for the three sectors are proportional to each other. As a result, the CGP condition states that the ratio of exogenous preference parameters must be equal to the productivity of services relative to that of agriculture, where the two productivities are given exogenously. Therefore, their CGP restriction has an undesirable property, because the CGP condition becomes the knife-edge condition linking the parameters of preferences and technologies directly. It is because there are two production factors and three goods in their model that it generates defects of its knife-edge CGP condition. In this case, the ratios of capital to labor for the three sectors are proportional to the ratios of the wage rates to the rental rates of capital. The price–unit cost equalization condition cannot, on its own, determine unique prices for agricultural goods and services. Therefore,

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the capital intensities for the three sectors are not determined uniquely in the three-goods model with two factors. However, the CGP condition can pin down one pair of two prices and, therefore, decide the unique capital intensities across sectors. Here, the CGP condition makes the ratio of the wage rate to the rental rate constant over time in the case of no technological progress. Therefore, the capital intensities across the three sectors are eventually given by parameters, and they become constant over time. Moreover, because the CGP requires that the rental rate be constant, this makes the wage rate constant over time, which possesses undesirable characteristics. When two production factors are considered in the three-sector model, these undesirable results occur when the production functions are identical in the three sectors (except for technologies), and when the production functions are different across the three sectors and a nonlinear production possibility frontier is assumed.

On the other hand, we find that in the three-goods and three-factors model, the ratios of capital to labor for the three sectors are proportional to the ratios of the wage rate to rental rate. However, the ratios of land to labor for the three sectors are proportional to the ratios of wage rate to land rent. Because the number of goods is equal to the number of factors, the price–unit cost equalization condition can determine the prices of agricultural goods and services uniquely. Therefore, the ratios of capital to labor or the ratios of land to labor are reallocated efficiently, depending on various ratios of factor prices.

This study tries to avoid the knife-edge result by adding the third production factor, namely land. For simplicity, we ignore Harrod-neutral technological progress, capital depreciation, population growth, and international trade. By considering three factors and a nonlinear production possibility frontier, the model determines the prices of agricultural goods and services endogenously. The reasons are as follows. The CGP condition requires that the constant discount rate must be equal to the rental rate of capital, which depends on the prices of both agricultural goods and services. Because we ignore capital depreciation, the rental rate of capital must be equal to the real interest rate by the nonarbitrage condition. Importantly, the constant real interest rate under the CGP requires the linkage between wage rate and land rent, where wage rate and land rent depend on the prices of agricultural goods and services. This wage-land rent restriction derived from the three-factor model plays an important role in the pricing of agricultural goods and services. This effect does not appear in the two-factor model because the CGP condition eventually makes the real interest rate and the wage rate constant over time. Even if all productivities for the three sectors are equal to one, our model can derive the prices of agricultural goods and services, unlike the model of Kongsamut et al. (2001). Because the prices of agricultural goods and services are derived as a solution of simultaneous equations, the three-factor model does not make the CGP condition a knife-edge condition.

Interestingly, the dynamic system in the model of Kongsamut et al. (2001) is identical to the one-sector growth model of Ramsey, Cass, and Koopmans. Here, we show that the dynamic system along the CGP in the three-sector model

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TABLE 1. Employment share by three sectors in the United States

	Agriculture	Manufacturing	Services
1800	68	18	13
1900	41	28	31
1950	15	34	50
2012	2	18	80

Source: Piketty (2014, p. 91).

with three factors reduces to the two-sector neoclassical optimal growth model of Uzawa (1964).[2] Crucially, it is sufficient for us to understand the dynamic behavior of the standard two-sector optimal growth model when considering the three-sector growth model with three factors which is consistent with the Kuznets and Kaldor facts.

The rest of this paper is organized as follows. Section 2 reviews the literature for economic growth with structural change. Section 3 presents the three-sector GDP function. Section 4 develops a unified framework of a three-sector and three-factor growth model with structural change. Section 5 discusses the mechanism of the three-sector dynamic model. Section 6 considers transitional dynamics and the dynamic stability in our model. Section 7 concludes this paper. Appendix A explains the model of three goods and two factors. Appendix B provides several computations in the three-good and three-factor model.

2. RESEARCH ON STRUCTURAL CHANGE AND ECONOMIC GROWTH

Many aspects of economic growth are far from the balanced benchmark implied by standard neoclassical growth models. Kuznets (1966) insists that there were changes in the industrial structure within which products were manufactured and resources were employed, first from agriculture to manufacturing, and then from manufacturing to services. The employment share in agriculture has declined and now stands at less than 5%, whereas more than 75% of U.S. workers are now employed in the service sector. As Lee and Wolpin (2006) and Buera and Kaboski (2012b) show, one of the most striking features of the last 20 years in advanced economies has been the rapid growth of the service sector. Piketty (2014) shows the employment share by sector by using long-term historical data.

There are changes in sectoral employment and production that include a significant nonbalanced component. Kongsamut et al. (2001) refer to these changes as "Kuznets facts." They contribute to providing a tractable model to reconcile the Kuznet facts with the Kaldor facts, which are expressed as the relative constancy of factor shares and interest rates over time. At the heart of their approach is Engel's law, which states that as a household's income increases, the fraction spent on agricultural goods tends to decline. Because consumers have Stone–Geary

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preferences over agricultural goods (a necessity), manufactured goods (with an income elasticity near unity), and services (a luxury), structural change is driven by sectoral differences in income elasticities. Kongsamut et al. (2001) find that the CGP is only possible if the parameters of preferences and technology both satisfy the knife-edge condition. However, there are several ways to improve upon the model of Kongsamut et al. (2001).[3] First, they assume that all three sectors have the same production function, although different productivities, and that the production possibility frontier is linear. In other words, their model has the same marginal rate of transformation and the same factor intensity across all three sectors. This is why the relative price of agricultural goods (services) is determined exogenously by the ratio of manufacturing productivity to agricultural (services) sector productivity. Second, their CGP condition becomes a knife-edge condition, that is, a joint restriction on preference and technology parameters.

There are two streams of research that focus on reasons for economic growth together with structural change. The first focuses on preference-related (demand-side) reasons, and the second on technology-related (supply-side) reasons. On the supply side, Baumol (1967) insists that production costs and sale prices of a stagnant sector increase indefinitely, a process known as *Baumol's cost disease*. Furthermore, labor moves in the direction of the stagnant sector. Baumol (2012) illustrates how the prices of services (in particular, health care and education) are increasing relative to the prices of goods. Acemoglu and Guerrieri (2008) emphasize the technological causes of unbalanced growth. They show that differences in factor proportions, with capital deepening, bring about an economy that experiences unbalanced growth at the sectoral level, even though aggregate growth is consistent with the Kaldor facts. On the demand side, Ngai and Pissarides (2007) consider a multisector model in which sectors experience different productivity growth rates but identical capital intensity. They show that an aggregate BGP exists only if the utility function is logarithmic in the consumption composite. Foellmi and Zweimuller (2008) study the endogenous growth driven by introduction of new goods into a hierarchic utility function. They derive nonlinear Engel curves for the various products in order to generate consumption cycles. Hori et al. (2012) construct a two-sector endogenous growth model and show that the presence of commodity-specific consumption externalities can be a source of structural change. They do not assume nonhomothetic preferences of individuals or sectoral differences in productivity growth. However, they do assume that consumers' preferences are homothetic in their private consumption, and that each sector employs AK technology with constant TFP. Then, they show that if commodity-specific consumption externalities are introduced, the utility function exhibits nonhomotheticity from a social perspective. Meckl (2002) presents a sectoral disaggregated version of an R&D-driven growth model. This model provides a generalized BGP that can explain the simultaneous existence of balanced growth in aggregate variables and changes in the sectoral composition of the economy. Buera and Kaboski (2012a, 2012b) consider households' choices between market services and home production and explain the growing demand for skill-intensive

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services. Conditions for industrialization have been studied by, among others, Murphy et al. (1989), Matsuyama (1992), Goodfriend and McDermott (1995), Laitner (2000), and Vollrath (2012). Other studies on structural changes include Echevarria (1997), Gollin et al. (2000), Caselli and Coleman (2001), Alvarez-Cuadrado and Poschke (2011), and Herrendorf et al. (2014). Echevarria (1997) and Hu and Mino (2014) consider a three-sector model in which the agriculture, manufacturing, and service industries use two factors, namely capital and labor. In their model, the three sectors each have different production technologies and different factor intensities. At this point, the production side of our model resembles those of the models by Echevarria (1997), although our model includes the third factor, land. Echevarria (1997) does not use the Stone–Geary utility function, nor does she try to reconcile the Kuznets and Kaldor facts. Importantly, we focus on demand-side and supply-side reasons for structural change, because changes in the prices of agricultural goods and services are the driving forces behind shifts in the composition of industrial production.

There have been empirical studies on structural change and economic growth. McMillan and Rodrik (2011) and McMillan et al. (2014) emphasize that structural change involving labor movement from a less productive sector to a more productive sector gives rise to economic growth, even if there is no productivity growth within sectors. They show empirically that growth-enhancing structural change occurs in Asia, whereas growth-reducing structural change, involving labor movement in the wrong direction, occurs in many Latin American and Sub-Saharan African countries. Fagerberg (2000) examines the impact of structural change on productivity growth in manufacturing by observing 39 countries between 1973 and 1990. In the first half of the twentieth century, rapid expansion in employment in industries based on new technologies (e.g., electricity and synthetic materials) brought about productivity growth. On the other hand, he reports that the correlation between productivity growth and structural change based on new technology has weakened in recent decades.

There have also been recent theoretical studies on structural change and international trade. Matsuyama (2009) points out the importance of structural change and globalization and examines the impact of international trade on structural change. Uy et al. (2013) construct a two-country, three-sector model based on Ricardian model, where the only production factor is labor. Hu and Mino (2014) study structural change in a small-country model with three goods and two factors. They consider that capital accumulation plays a leading role in the process of structural change and show that the trade pattern affects structural change. Mao and Yao (2012) consider a three-sector open economy where one sector produces nontraded goods and the other two produce tradable goods.

With regard to important empirical studies on multisector growth models, Takahashi et al. (2012) measure the capital intensities of the consumption good and investment good sectors, mainly in the postwar Japanese economy and in other OECD countries. They demonstrate that capital intensity does matter, and their empirical evidence strongly supports the common assumption that the

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consumption goods sector is more capital-intensive than the capital goods sector. Valentinyi and Herrendorf (2008) use an input–output table to measure factor income shares at the sector level. Their empirical research provided a breakthrough in terms of studying multisector models quantitatively. They show that capital income shares differ widely across sectors, and that factor income shares within a sector can be fairly different from the aggregate factor income share. The latter finding contrasts with that of Kongsamut et al. (2001), who use the economywide factor income share as an approximation to the factor income share within a sector. Therefore, based on the findings of Valentinyi and Herrendorf (2008), it is necessary that we develop a three-sector and three-factor model with different factor intensities across sectors.

In general, it is difficult to derive the Stolper–Samuelson theorem in a three-sector model. Muro(2013) derives the three-sector Cobb–Douglas GDP function with three factors and adopts the factor income shares found empirically by Valentinyi and Herrendorf (2008). Muro (2013) shows that an increase in the price of agricultural goods decreases the real interest rate and the wage rate, but increases land rent. On the other hand, an increase in the service price decreases the real interest rate, but increases the wage rate land rent. By introducing the prices of agricultural goods and services into the price–cost equalization conditions, we can compute the interest rate, the wage rate, and land rent.

3. THE MODEL OF THREE GOODS AND THREE FACTORS

The model of three goods and two factors is explained in Appendix A. Solving the model involves three key considerations. First, the optimal allocation of production factors requires the marginal rates of transformation (MRT) to be equal across the three sectors, which determines each capital–labor ratio between different sectors. Second, given the sale prices, the optimal allocation of consumption from the Euler equation determines the actual allocation of production factors, that is, the overall capital–labor ratio. This is valid for the case of three goods and two factors. Note that if the number of goods is equal to the number of factors, then the Euler equation does not depend on the overall capital–labor ratio, according to the two-sector and two-factor economic growth model. Third, the sale prices are determined by equalizing the value marginal products of capital (labor) across three sectors under the free mobility of production factors across sectors. The sale prices depend on the relative productivity and on the wage–rental ratio in the case of three goods and two factors. Because the capital–labor ratios in different sectors are functions of the wage–rental ratio in the case, the sale prices depend not only on the relative productivity but also on the capital–labor ratios in different sectors. Finally, because the Euler equation determines the overall capital–labor ratio in each case, the demand side pins down the sale prices and factor prices in each case. However, the case of three goods and three factors, as in this paper, and the case of two goods and two factors, as in Uzawa (1964), are different from the case

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of three goods and two factors, as in Echevarria (1997), Kongsamut et al. (2001), Ngai and Pissarides (2007), and Hu and Mino (2014).

3.1. The Production Sectors of Three Industries

Muro (2013) derives the three-sector Cobb–Douglas GDP function with three factors. Let us adopt it in this paper. There are three sectors in our economy, agriculture, manufacturing, and service. All sectors employ labor, capital, and land as production factors. We focus only on the case of imperfect specialization. Denote outputs in agriculture, manufacturing, and service sectors by Y_{At}, Y_{Mt}, and Y_{St}, respectively. The production functions of three sectors $j = \{A, M, S\}$ are given by $Y_{jt} = (K_{jt})^{\alpha_j}(L_{jt})^{\eta_j}(Z_{jt})^{1-\alpha_j-\eta_j}$, for $j = \{A, M, S\}$, where K_{At}, K_{Mt}, K_{St} are the demand for capital, L_{At}, L_{Mt}, L_{St} are labor, and Z_{At}, Z_{Mt}, Z_{St} are land in the agriculture, manufacturing, and service sector, respectively. $\alpha_A, \alpha_M, \alpha_S \in (0, 1)$ are capital income shares whereas $\eta_A, \eta_M, \eta_S \in (0, 1)$ are labor income shares of the agriculture, manufacturing, and service sectors, respectively.

Denote total capital, total labor, and total land by K_t, L_t, Z_t, respectively. Full employment is assumed as follows: $K_t = K_{At}+K_{Mt}+K_{St}$, $L_t = L_{At}+L_{Mt}+L_{St}$, and $Z_t = Z_{At} + Z_{Mt} + Z_{St}$. In nature, land may be considered to be fixed in a sector and difficult to divert to another sector; or it may take time to convert farmland into commercial land. But we assume that land utilization occurs across sectors instantaneously because we focus on the long-run analysis in this paper. Because we consider the case of three goods, three factors, and Cobb–Douglas technologies, factor prices are determined by solving the price–cost equalization equations. The unit cost functions in the sector $j = \{A, M, S\}$ are given by

$$\psi^j(r_t, w_t, \phi_t) = \frac{r_t^{\alpha_j} w_t^{\eta_j} \phi_t^{1-\alpha_j-\eta_j}}{\alpha_j^{\alpha_j}\eta_j^{\eta_j}(1-\alpha_j-\eta_j)^{1-\alpha_j-\eta_j}}, \quad \text{for} \quad j = \{A, M, S\},$$

where r_t, w_t, and ϕ_t are the rental rate of capital, the wage rate, and land rent, where factor prices are measured in terms of manufactured goods, respectively. By solving the price–cost equalization given by

$$\begin{bmatrix} P_{At} \\ 1 \\ P_{St} \end{bmatrix} = \begin{bmatrix} \psi^A(r_t, w_t, \phi_t) \\ \psi^M(r_t, w_t, \phi_t) \\ \psi^S(r_t, w_t, \phi_t) \end{bmatrix},$$

we obtain the unique factor prices $r_t = r(P_{At}, P_{St})$, $w_t = w(P_{At}, P_{St})$, and $\phi_t = \phi(P_{At}, P_{St})$, given the sale prices of agricultural goods and services, denoted by P_{At} and P_{St}, respectively. A convenient feature is that each factor price does not depend on the quantities of production factors, but only on the sale prices. Because the price ratio of agricultural goods to services is given by $\frac{\psi^A(r_t,w_t,\phi_t)}{\psi^M(r_t,w_t,\phi_t)}$,

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we obtain an equation as follows:

$$\frac{P_{At}}{P_{St}} = \left[\frac{\alpha_S^{\alpha_S}\eta_S^{\eta_S}(1-\alpha_S-\eta_S)^{1-\alpha_S-\eta_S}}{\alpha_A^{\alpha_A}\eta_A^{\eta_A}(1-\alpha_A-\eta_A)^{1-\alpha_A-\eta_A}}\right] r_t^{\alpha_A-\alpha_S} w_t^{\eta_A-\eta_S} \phi_t^{[(\alpha_S+\eta_S)-(\alpha_A+\eta_A)]}. \quad (1)$$

Recall that the rental rate $r_t = r(P_{At}, P_{St})$, the wage rate $w_t = w(P_{At}, P_{St})$, and the land rent $\phi_t = \phi(P_{At}, P_{St})$ depend on the prices of agricultural goods and services. Therefore, the equation (1) represents the relationship between the agricultural goods price (P_{At}) and the service price (P_{St}).

The GDP function chooses the most efficient point on the production possibility frontier, and it is defined to be a maximal sum of the value of production in three sectors, which is expressed as $Y(P_{At}, P_{St}, K_t, L_t, Z_t) = \min_{\{r_t, w_t, \phi_t\}}\{r_t K_t + w_t L_t + \phi_t Z_t\}$ subject to the price–unit cost equalization conditions. The GDP function measured in terms of manufactured goods is expressed as $Y(P_{At}, P_{St}, K_t, L_t, Z_t) = L_t y(P_{At}, P_{St}, k_t, z_t)$, where $y(P_{At}, P_{St}, k_t, z_t)$ is GDP per labor, $k_t \equiv \frac{K_t}{L_t}$ is capital per labor, and $z_t \equiv \frac{Z_t}{L_t}$ is land per labor.

Remark. According to Muro (2013), the three-sector Cobb–Douglas GDP function in terms of manufactured goods per labor is given by $y(P_{At}, P_{St}, k_t, z_t) = r(P_{At}, P_{St})k_t + w(P_{At}, P_{St}) + \phi(P_{At}, P_{St})z_t$, where the rental rate of capital, the wage rate, and land rent in terms of manufacturing goods are given by, respectively,

$$r(P_{At}, P_{St}) = \Omega_K (P_{At})^{\frac{\eta_M(1-\alpha_S)-\eta_S(1-\alpha_M)}{\varepsilon}} (P_{St})^{\frac{\eta_A(1-\alpha_M)-\eta_M(1-\alpha_A)}{\varepsilon}}, \quad (2)$$

$$w(P_{At}, P_{St}) = \Omega_L (P_{At})^{\frac{\alpha_S(1-\eta_M)-\alpha_M(1-\eta_S)}{\varepsilon}} (P_{St})^{\frac{\alpha_M(1-\eta_A)-\alpha_A(1-\eta_M)}{\varepsilon}}, \quad (3)$$

$$\phi(P_{At}, P_{St}) = \Omega_Z (P_{At})^{\frac{\alpha_M\eta_S-\eta_M\alpha_S}{\varepsilon}} (P_{St})^{\frac{\alpha_A\eta_M-\alpha_M\eta_A}{\varepsilon}}, \quad (4)$$

where P_{At} and P_{St} are prices of agricultural goods and services, respectively. Parameters α_A, α_M, and α_S are capital share in agriculture, manufacturing, and service sectors, respectively. Parameters η_A, η_M, and η_S are labor share in agriculture, manufacturing, and service sectors, respectively. $1-\alpha_A-\eta_A$, $1-\alpha_M-\eta_M$, and $1-\alpha_S-\eta_S$ are land share in agriculture, manufacturing, and service sectors, respectively. Here the composite factor income share parameter is defined by

$$\varepsilon \equiv \alpha_A\eta_M - \alpha_M\eta_A - \alpha_S(\eta_M - \eta_A) - \eta_S(\alpha_A - \alpha_M).$$

The composite parameters are given by

$$\Omega_K \equiv \alpha_A^{\frac{\alpha_A[\eta_M(1-\alpha_S)-\eta_S(1-\alpha_M)]}{\varepsilon}} \eta_A^{\frac{\eta_A[\eta_M(1-\alpha_S)-\eta_S(1-\alpha_M)]}{\varepsilon}} \alpha_M^{\frac{\alpha_M[\eta_S(1-\alpha_A)-\eta_A(1-\alpha_S)]}{\varepsilon}} \eta_M^{\frac{\eta_M[\eta_S(1-\alpha_A)-\eta_A(1-\alpha_S)]}{\varepsilon}}$$
$$\alpha_S^{\frac{\alpha_S[\eta_A(1-\alpha_M)-\eta_M(1-\alpha_A)]}{\varepsilon}} \eta_S^{\frac{\eta_S[\eta_A(1-\alpha_M)-\eta_M(1-\alpha_A)]}{\varepsilon}} (1-\alpha_A-\eta_A)^{\frac{(1-\alpha_A-\eta_A)[\eta_M(1-\alpha_S)-\eta_S(1-\alpha_M)]}{\varepsilon}}$$
$$(1-\alpha_M-\eta_M)^{\frac{(1-\alpha_M-\eta_M)[\eta_S(1-\alpha)-\eta_A(1-\alpha_S)]}{\varepsilon}} (1-\alpha_S-\eta_S)^{\frac{(1-\alpha_S-\eta_S)[\eta_A(1-\alpha_M)-\eta_M(1-\alpha_A)]}{\varepsilon}},$$

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$$\Omega_L \equiv \alpha_A^{\frac{\alpha_A[\alpha_S(1-\eta_M)-\alpha_M(1-\eta_S)]}{\varepsilon}} \eta_A^{\frac{\eta_A[\alpha_S(1-\eta_M)-\alpha_M(1-\eta_S)]}{\varepsilon}} \alpha_M^{\frac{\alpha_M[\alpha_A(1-\eta_S)-\alpha_S(1-\eta_A)]}{\varepsilon}} \eta_M^{\frac{\eta_M[\alpha_A(1-\eta_S)-\alpha_S(1-\eta)]}{\varepsilon}}$$
$$\alpha_S^{\frac{\alpha_S[\alpha_M(1-\eta_A)-\alpha_A(1-\eta_M)]}{\varepsilon}} \eta_S^{\frac{\eta_S[\alpha_M(1-\eta_A)-\alpha_A(1-\eta_M)]}{\varepsilon}} (1-\alpha_A-\eta_A)^{\frac{(1-\alpha_A-\eta_A)[\alpha_S(1-\eta_M)-\alpha_M(1-\eta_S)]}{\varepsilon}}$$
$$(1-\alpha_M-\eta_M)^{\frac{(1-\alpha_M-\eta_M)[\alpha_A(1-\eta_S)-\alpha_S(1-\eta_A)]}{\varepsilon}} (1-\alpha_S-\eta_S)^{\frac{(1-\alpha_S-\eta_S)[\alpha_M(1-\eta_A)-\alpha_A(1-\eta_M)]}{\varepsilon}},$$

$$\Omega_Z \equiv \alpha_A^{\frac{\alpha_A[\alpha_M\eta_S-\eta_M\alpha_S]}{\varepsilon}} \eta_A^{\frac{\eta_A[\alpha_M\eta_S-\eta_M\alpha_S]}{\varepsilon}} \alpha_M^{\frac{\alpha_M[\eta_A\alpha_S-\alpha_A\eta_S]}{\varepsilon}} \eta_M^{\frac{\eta_M[\eta_A\alpha_S-\alpha_A\eta_S]}{\varepsilon}}$$
$$\alpha_S^{\frac{\alpha_S[\alpha_A\eta_M-\alpha_M\eta]}{\varepsilon}} \eta_S^{\frac{\eta_S[\alpha_A\eta_M-\alpha_M\eta_A]}{\varepsilon}} (1-\alpha_A-\eta_A)^{\frac{(1-\alpha_A-\eta_A)[\alpha_M\eta_S-\eta_M\alpha_S]}{\varepsilon}}$$
$$(1-\alpha_M-\eta_M)^{\frac{(1-\alpha_M-\eta_M)[\eta_A\alpha_S-\alpha_A\eta_S]}{\varepsilon}} (1-\alpha_S-\eta_S)^{\frac{(1-\alpha_S-\eta_S)[\alpha_A\eta_M-\alpha_M\eta_A]}{\varepsilon}}.$$

3.2. Features in the Three-Sector and Three-Factor Model

Let us explain the pricing mechanism in the model with three goods and three factors. According to Shepard's lemma, the optimal demand for production factors in the industry $j = \{A, M, S\}$ is determined by

$$K_{jt} = Y_{jt}\left(\frac{r_t}{\alpha_j}\right)^{\alpha_j-1}\left(\frac{w_t}{\eta_j}\right)^{\eta_j}\left(\frac{\phi_t}{1-\alpha_j-\eta_j}\right)^{1-\alpha_j-\eta_j},$$

$$L_{jt} = Y_{jt}\left(\frac{r_t}{\alpha_j}\right)^{\alpha_j}\left(\frac{w_t}{\eta_j}\right)^{\eta_j-1}\left(\frac{\phi_t}{1-\alpha_j-\eta_j}\right)^{1-\alpha_j-\eta_j},$$

and

$$Z_{jt} = Y_{jt}\left(\frac{r_t}{\alpha_j}\right)^{\alpha_j}\left(\frac{w_t}{\eta_j}\right)^{\eta_j}\left(\frac{\phi_t}{1-\alpha_j-\eta_j}\right)^{-\alpha_j-\eta_j}.$$

Therefore, the ratio of capital to labor in the sector $j = \{A, M, S\}$ and the ratio of land to labor in the sector $j = \{A, M, S\}$ are given by, respectively,

$$k_{jt} = \frac{K_{jt}}{L_{jt}} = \left(\frac{\alpha_j}{\eta_j}\right)\left(\frac{w_t}{r_t}\right), \tag{5}$$

$$z_{jt} = \frac{Z_{jt}}{L_{jt}} = \left(\frac{1-\alpha_j-\eta_j}{\eta_j}\right)\left(\frac{w_t}{\phi_t}\right). \tag{6}$$

Therefore, we find that the ratios of capital to labor for the three sectors depend on the wage–rental ratio, whereas the ratios of land to labor for the three sectors depend on the ratio of the wage rate to land rent. Although total land and total labor are fixed, the ratios of land to labor across the three sectors are allocated efficiently. Using the unit cost function and the price–cost equalization conditions,

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the prices of agricultural goods and services are given by, respectively,

$$\frac{P_A}{P_M} = P_A = \left[\frac{(\alpha_M)^{\alpha_M}(\eta_M)^{\eta_M}(1-\alpha_M-\eta_M)^{1-\alpha_M-\eta_M}}{(\alpha_A)^{\alpha_A}(\eta_A)^{\eta_A}(1-\alpha_A-\eta_A)^{1-\alpha_A-\eta_A}}\right] \times \left(\frac{w_t}{r_t}\right)^{\alpha_M-\alpha_A}\left(\frac{w_t}{\phi_t}\right)^{\eta_A-\eta_M+\alpha_A-\alpha_M}, \tag{7}$$

$$\frac{P_S}{P_M} = P_S = \left[\frac{(\alpha_M)^{\alpha_M}(\eta_M)^{\eta_M}(1-\alpha_M-\eta_M)^{1-\alpha_M-\eta_M}}{(\alpha_S)^{\alpha_S}(\eta_S)^{\eta_S}(1-\alpha_S-\eta_S)^{1-\alpha_S-\eta_S}}\right] \times \left(\frac{w_t}{r_t}\right)^{\alpha_M-\alpha_S}\left(\frac{w_t}{\phi_t}\right)^{\eta_S-\eta_M+\alpha_S-\alpha_M}. \tag{8}$$

By solving these simultaneous equations, we obtain the wage–rental ratio and the wage–land rent ratio as follows:

$$\frac{w_t}{r_t}(P_{At}, P_{St}) = \left(\frac{P_{At}}{G_A}\right)^{\frac{\eta_S-\eta_M+\alpha_S-\alpha_M}{\sigma}}\left(\frac{P_{St}}{G_S}\right)^{\frac{-(\eta_A-\eta_M+\alpha_A-\alpha_M)}{\sigma}}, \tag{9}$$

$$\frac{w_t}{\phi_t}(P_{At}, P_{St}) = \left(\frac{P_{At}}{G_A}\right)^{\frac{-(\alpha_M-\alpha_S)}{\sigma}}\left(\frac{P_{St}}{G_S}\right)^{\frac{\alpha_M-\alpha_A}{\sigma}}, \tag{10}$$

where $G_A \equiv \frac{(\alpha_M)^{\alpha_M}(\eta_M)^{\eta_M}(1-\alpha_M-\eta_M)^{1-\alpha_M-\eta_M}}{(\alpha_A)^{\alpha_A}(\eta_A)^{\eta_A}(1-\alpha_A-\eta_A)^{1-\alpha_A-\eta_A}}$, $G_S \equiv \frac{(\alpha_M)^{\alpha_M}(\eta_M)^{\eta_M}(1-\alpha_M-\eta_M)^{1-\alpha_M-\eta_M}}{(\alpha_S)^{\alpha_S}(\eta_S)^{\eta_S}(1-\alpha_S-\eta_S)^{1-\alpha_S-\eta_S}}$, and $\sigma \equiv \alpha_M(\eta_S+\eta_S-\eta_A-\alpha_A)-\alpha_A(\eta_S-\eta_M+\alpha_S-\alpha_M)+\alpha_S(\eta_A-\eta_M+\alpha_A-\alpha_M)$. The wage–rental ratio and the wage–land rent ratio are functions that consist of the prices of agricultural goods and services. Importantly, the ratio of the price of agricultural goods and services is given by equation (1). Under the CGP condition, which requires a constant interest rate, we have $\frac{P_A}{P_S} = \frac{\bar{S}}{\bar{A}}$. Because the CGP condition and the Euler equation lead to $r(P_{At}, P_{St}) = \rho$, where ρ is the discount rate, we obtain the crucial simultaneous equations as follows:

$$\frac{P_{At}}{P_{St}} = \left[\frac{(\alpha_S)^{\alpha_S}(\eta_S)^{\eta_S}(1-\alpha_S-\eta_S)^{1-\alpha_S-\eta_S}}{(\alpha_A)^{\alpha_A}(\eta_A)^{\eta_A}(1-\alpha_A-\eta_A)^{1-\alpha_A-\eta_A}}\right] \times \left[\frac{w_t}{\rho}(P_{At}, P_{St})\right]^{\alpha_S-\alpha_A}\left[\frac{w_t}{\phi_t}(P_{At}, P_{St})\right]^{\eta_A-\eta_S+\alpha_A-\alpha_S},$$

$$P_{At}\bar{A} = P_{St}\bar{S}.$$

From these simultaneous equations, we can identify the prices of agricultural goods and services satisfying the CGP condition. Once P_{At} and P_{St} are determined, we can obtain the wage–rental ratio $\frac{w_t}{r_t}(P_{At}, P_{St})$ and the wage–land rent ratio $\frac{w_t}{\phi_t}(P_{At}, P_{St})$. Then the ratio of capital to labor and the ratio of land to labor in the sector $j = \{A, M, S\}$ are computed by $k_{jt} = \frac{K_{jt}}{L_{jt}} = (\frac{\alpha_j}{\eta_j})[\frac{w_t}{r_t}(P_{At}, P_{St})]$, $z_{jt} = \frac{Z_{jt}}{L_{jt}} = (\frac{1-\alpha_j-\eta_j}{\eta_j})[\frac{w_t}{\phi_t}(P_{At}, P_{St})]$.

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4. STRUCTURAL CHANGE AND ECONOMIC GROWTH MODEL WITH THREE GOODS AND THREE FACTORS

This paper constructs a three-sector growth model involving sweeping structural change that is consistent with the Kaldor and Kuznets facts. Echevarria (1997), Kongsamut et al. (2001), Ngai and Pissarides (2007), and Hu and Mino (2014) use two factors of capital and labor to produce output. Kongsamut et al. (2001) and Ngai and Pissarides (2007) assume that their production functions are the same in three sectors except for differences in productivities between the three sectors. The three-sector and three-factors model that has the nonlinear production possibility frontier would overcome the drawback of the model of Kongsamut et al. (2001), in which the condition for the constant growth path (CGP) becomes knife-edge. Let us develop a three-sector dynamic infinite horizon model with structural change.

4.1. Preferences

Changes in sectoral composition stem from differences in the income elasticities for the different goods. We assume that preferences are time-separable, and that the instantaneous Stone–Geary utility function is given by $u(A_t, M_t, S_t) = \iota \log(A_t - \bar{A}) + b \log M_t + (1-\iota-b) \log(S_t + \bar{S})$, where A_t, M_t, and S_t are agricultural goods, manufactured goods, and services, respectively. $\iota > 0$ and $b > 0$ are preference parameters weighted on agricultural goods and manufactured goods, respectively. Here $\bar{A} > 0$ is interpreted as the subsistence consumption level, and $\bar{S} > 0$ is home production of services. The elasticity of demand for agricultural goods and services with respect to income no longer is one because of the nonhomothetic utility function, which can explain Engel's law. Note that the income elasticity of demand is less than one for agricultural goods (a necessity), equal to one for manufactured goods, and greater than one for services (a luxury). We assume in this paper for simplicity that technological progress does not occur. We assume that the total labor force is one ($L_t = 1$), and therefore the labor market equilibrium $L_{At} + L_{Mt} + L_{St} = 1$ holds. We does not consider population growth. We assume that total land is also fixed. Land market equilibrium $Z_{At} + Z_{Mt} + Z_{St} = Z$ holds. Land per labor is expressed by $z_t = \frac{Z_t}{L_t} = \frac{Z}{1} = z$, and it is constant ($z_t = z$). Outputs of agricultural goods and of services can be used for consumption only. The output of manufactured goods (industry 2) can be consumed M_t or invested $\dot{K}_t$. We assume that the rate of capital depreciation is zero for simplicity. According to the U.S. input–output tables, the manufacturing and construction sectors produced more than 90% of investment during the period 1958–1987. The resource constraint in our economy is given by $M_t + \dot{K}_t + P_{At} A_t + P_{St} S_t = Y(P_{At}, P_{St}, K_t, L_t, Z_t)$, where $Y(P_{At}, P_{St}, K_t, L_t, Z_t)$ is the GDP function, K_t is aggregate capital, L_t is total labor, Z_t is total land, A_t is aggregate consumption of agricultural goods, M_t is aggregate consumption for manufactured goods, S_t is aggregate demand for services, $\dot{K}_t$ is the demand for investment, and P_{At} and P_{St} are the price of agricultural goods and services, respectively. Recall that $L_t y(P_{At}, P_{St}, k_t, z_t) = Y(P_{At}, P_{St}, K_t, L_t, Z_t)$ holds.

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4.2. The Optimization Problem

A representative agent chooses the demand for agricultural goods, manufactured goods, and services to maximize his lifetime utility:

$$\max \int_{t=0}^{\infty} e^{-\rho t}[\iota \log(A_t - \bar{A}) + b \log M_t + (1 - \iota - b) \log(S_t + \bar{S})]dt,$$

subject to the resource constraint

$$\dot{K}_t = Y(P_{At}, P_{St}, K_t, L_t, Z_t) - M_t - P_{At}A_t - P_{St}S_t,$$

and the initial conditions K_0, where $L_t = 1$ and $Z_t = zL_t$ are assumed. The discount rate is denoted by $\rho > 0$. The current-value Hamiltonian is given by $\mathcal{H} \equiv \iota \log(A_t - \bar{A}) + b \log M_t + (1 - \iota - b) \log(S_t + \bar{S}) + q_t [Y(P_{At}, P_{St}, K_t, L_t, Z_t) - M_t - P_{At}A_t - P_{St}S_t]$, where q_t is the current shadow value of capital stock. The necessary conditions for optimization are given by

$$\left[(A_t - \bar{A})^{\iota} M_t^b (S_t + \bar{S})^{1-\iota-b}\right]^{-1} \times (A_t - \bar{A})^{\iota} b M_t^{b-1} (S_t + \bar{S})^{1-\iota-b} = q_t, \tag{11}$$

$$\left[(A_t - \bar{A})^{\iota} M_t^b (S_t + \bar{S})^{1-\iota-b}\right]^{-1} \iota \times (A_t - \bar{A})^{\iota-1} M_t^b (S_t + \bar{S})^{1-\iota-b} = q_t P_{At}, \tag{12}$$

$$\left[(A_t - \bar{A})^{\iota} M_t^b (S_t + \bar{S})^{1-\iota-b}\right]^{-1} (1 - \iota - b) \times (A_t - \bar{A})^{\iota} M_t^b (S_t + \bar{S})^{-\iota-b} = q_t P_{St}, \tag{13}$$

$$\lim_{t \to \infty} e^{-\rho t} q_t K_t = 0 \quad \text{(Transversality Condition)}, \tag{14}$$

$$\dot{q}_t = \rho q_t - \frac{\partial \mathcal{H}}{\partial k_t} = q_t \left[\rho - \frac{\partial Y(P_{At}, P_{St}, K_t, L_t, Z_t)}{\partial K_t}\right] = q_t [\rho - r(P_{At}, P_{St})]. \tag{15}$$

The optimal allocations of consumption across industries are given by $\frac{(A_t - \bar{A})P_{At}}{\iota} = \frac{M_t}{b}$ and $\frac{(S_t + \bar{S})P_{St}}{1-\iota-b} = \frac{M_t}{b}$. The Euler equation is given by $\frac{\dot{M}_t}{M_t} = r(P_{At}, P_{St}) - \rho$. Note that with logarithmic utility, the Euler equation does not depend on the growth rates of the prices of agricultural goods and services. According to the envelope theorem of the GDP function, the market equilibrium of agricultural goods and services is given by $A_t = \frac{\partial y(P_{At}, P_{St}, k_t, z)}{\partial P_{At}}$ and $S_t = \frac{\partial y(P_{At}, P_{St}, k_t, z)}{\partial P_{St}}$, respectively. Here, the left-hand side is the demand for agricultural goods (services), whereas the right-hand side is the supply for agricultural goods (services). The supply for agricultural goods (services) increases with the price of agricultural goods (services). By differentiating the two equations for optimal allocation with respect to time, we obtain $\frac{\dot{M}_t}{M_t} = \frac{\dot{A}_t}{A_t - \bar{A}} + \frac{\dot{P}_{At}}{P_{At}}$ and $\frac{\dot{M}_t}{M_t} = \frac{\dot{S}_t}{S_t + \bar{S}} + \frac{\dot{P}_{St}}{P_{St}}$. By differentiating

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the two equations for market equilibrium of agricultural goods and services with respect to time, we obtain

$$\frac{\dot{A}_t}{A_t-\bar{A}}=\left(\frac{A_t}{A_t-\bar{A}}\right)\times\left[\frac{P_{At}\frac{\partial^2 y(P_{At},P_{St},k_t,z)}{\partial P_{At}\partial P_{At}}}{\frac{\partial y(P_{At},P_{St},k_t,z)}{\partial P_{At}}}\frac{\dot{P}_{At}}{P_{At}}+\frac{P_{St}\frac{\partial^2 y(P_{At},P_{St},k_t,z)}{\partial P_{At}\partial P_{St}}}{\frac{\partial y(P_{At},P_{St},k_t,z)}{\partial P_{St}}}\frac{\dot{P}_{St}}{P_{St}}+\frac{k_t\frac{\partial^2 y(P_{At},P_{St},k_t,z)}{\partial P_{At}\partial k_t}}{\frac{\partial y(P_{At},P_{St},k_t,z)}{\partial P_{At}}}\frac{\dot{k}_t}{k_t}\right], \tag{16}$$

$$\frac{\dot{S}_t}{S_t-\bar{S}}=\left(\frac{S_t}{S_t-\bar{S}}\right)\times\left[\frac{P_{At}\frac{\partial^2 y(P_{At},P_{St},k_t,z)}{\partial P_{St}\partial P_{At}}}{\frac{\partial y(P_{At},P_{St},k_t,z)}{\partial P_{St}}}\frac{\dot{P}_{At}}{P_{At}}+\frac{P_{St}\frac{\partial^2 y(P_{At},P_{St},k_t,z)}{\partial P_{St}\partial P_{St}}}{\frac{\partial y(P_{At},P_{St},k_t,z)}{\partial P_{St}}}\frac{\dot{P}_{St}}{P_{St}}+\frac{k_t\frac{\partial^2 y(P_{At},P_{St},k_t,z)}{\partial P_{St}\partial k_t}}{\frac{\partial y(P_{At},P_{St},k_t,z)}{\partial P_{St}}}\frac{\dot{k}_t}{k_t}\right]. \tag{17}$$

By substituting the equations $\frac{\dot{M}_t}{M_t}=\frac{\dot{A}_t}{A_t-\bar{A}}+\frac{\dot{P}_{At}}{P_{At}}$, $\frac{\dot{M}_t}{M_t}=\frac{\dot{S}_t}{S_t+\bar{S}}+\frac{\dot{P}_{St}}{P_{St}}$, (16), and (17) into the Euler equation, and then using the market equilibrium conditions for agricultural goods and services, we can obtain the dynamic efficiency condition, given by

$$-\frac{\dot{P}_{At}}{P_{At}}-\left[\frac{\frac{\partial y(P_{At},P_{St},k_t,z)}{\partial P_{At}}}{\frac{\partial y(P_{At},P_{St},k_t,z)}{\partial P_{At}}-\bar{A}}\right]\times\left[\frac{P_{At}\frac{\partial^2 y(P_{At},P_{St},k_t,z)}{\partial P_{At}\partial P_{At}}}{\frac{\partial y(P_{At},P_{St},k_t,z)}{\partial P_{At}}}\frac{\dot{P}_{At}}{P_{At}}+\frac{P_{St}\frac{\partial^2 y(P_{At},P_{St},k_t,z)}{\partial P_{At}\partial P_{St}}}{\frac{\partial y(P_{At},P_{St},k_t,z)}{\partial P_{St}}}\frac{\dot{P}_{St}}{P_{St}}+\frac{k_t\frac{\partial^2 y(P_{At},P_{St},k_t,z)}{\partial P_{At}\partial k_t}}{\frac{\partial y(P_{At},P_{St},k_t,z)}{\partial P_{At}}}\frac{\dot{k}_t}{k_t}\right]$$
$$=\rho-r(P_{At},P_{St}). \tag{18}$$

By dividing the resource constraint by labor, we obtain $M_t+\dot{k}_t=y(P_{At},P_{St},k_t,z)-P_{At}A_t-P_{St}S_t$. Therefore, the accumulation equation of capital stock per labor is given by

$$\dot{k}_t=y(P_{At},P_{St},k_t,z)-\left(\frac{\iota+b}{\iota}\right)P_{At}\frac{\partial y(P_{At},P_{St},k_t,z)}{\partial P_{At}}+\frac{\bar{A}b}{\iota}P_{At}-P_{St}\frac{\partial y(P_{At},P_{St},k_t,z)}{\partial P_{St}}, \tag{19}$$

where we use $\frac{(A_t-\bar{A})P_{At}}{\iota}=\frac{M_t}{b}$, and we apply the market equilibrium conditions of agricultural goods and services. Given the sale prices and factor prices, firms produce on the production possibility frontier and the marginal rates of transformation (MRT) are equalized across sectors, and a representative consumer maximizes his

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TABLE 2. Real, geometrically compounded return to U.S. stock reported by Siegel (1992)

Period	U.S. real interest rate
1802–1992	6.7
1871–1992	6.6
1802–1870	7.0
1871–1925	6.6
1926–1992	6.6

utility subject to budget constraint. The marginal rate of substitution (MRS) is equal to the MRT. All markets clear under perfect foresight. Because of perfect competitive markets, complete markets, no externality, and free mobility of factors across sectors, the competitive equilibrium for this economy coincides with the Pareto optimum solution.

This is a standard dynamic general equilibrium model. The dynamics is described by the Euler equation, the capital accumulation equation, the initial capital stock, and the transversality condition. This describes the evolution of the capital stock. And for a given capital stock (and labor and land), we have at each date a given production possibility frontier. The point on the PPF depends on relative prices. But the realized relative prices are determined by the demand structure and the market-clearing conditions.

4.3. The Condition for Constant Growth Path

The Kuznets facts say that there is structural change in the development process in which the agriculture industry declines and services increase. Therefore, the model with the balanced growth path (BGP) cannot explain the Kuznets facts. Siegel (1992) reports data for the real rate of return in the U.S. stock market. The stability of this real rate of return over different historical periods is observed, as shown in Table 2.

Because the income elasticity of demand is less than one for agricultural goods, and greater than one for services, there is no BGP in our model with nonhomothetic utility function if $\bar{A} \neq 0$ and $\bar{S} \neq 0$ hold. Let us consider the case where $\bar{A}$ and $\bar{S}$ are strictly positive.

DEFINITION 1. *A constant growth path (CGP) or a generalized balanced growth path (BGP) is a trajectory along which the rental rate of capital* $r(P_{At}, P_{St})$ *is constant over time.*

The Condition for CGP: We impose the condition of Kongsamut et al. (2001) for CGP as follows:

$$P_{At}\bar{A} = P_{St}\bar{S}. \tag{20}$$

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One interpretation of the condition for CGP is that each agent has a positive endowment of services $\bar{S}$ and a negative endowment of agricultural goods $-\bar{A}$. The condition for CGP requires that the market value of these endowments $P_{St}\bar{S} - P_{At}\bar{A}$ be equal to zero. As Kongsamut et al. (2001) show, the constant growth path (CGP) exists whenever the condition $P_{At}\bar{A} = P_{St}\bar{S}$ holds. If we impose the condition for the CGP, that is, $P_{At}\bar{A} = P_{St}\bar{S}$, we have $M_t + \dot{K}_t + P_{At}(A_t - \bar{A}) + P_{St}(S_t + \bar{S}) = \hat{Y}(P_{At}, P_{St}, K_t, L_t, Z_t)$. Both $A_t - \bar{A}$ and $S_t + \bar{S}$ grow at rate $g = 0$, and all the terms grow at a constant rate as follows: $\frac{\dot{A}_t}{A_t} = (\frac{A_t - \bar{A}}{A_t})(\frac{\dot{M}_t}{M_t} - \frac{\dot{P}_{At}}{P_{At}})$ and $\frac{\dot{S}_t}{S_t} = (\frac{S_t + \bar{S}}{S_t})(\frac{\dot{M}_t}{M_t} - \frac{\dot{P}_{St}}{P_{St}})$. The CGP features time-varying sectoral growth rates and employment shares in the three sectors. As in Kongsamut et al. (2001), different sectors grow at different rates. The growth rate of agricultural goods (services) depends on the growth rate of the price of agricultural goods (services) price as well as on the consumption level of agricultural goods (services) and on the growth rate of manufacturing goods, unlike Kongsamut et al. (2001). Here $\frac{\dot{M}_t}{M_t} = g = 0$, $\frac{\dot{P}_{At}}{P_{At}} = 0$, and $\frac{\dot{P}_{St}}{P_{St}} = 0$ hold in the long run. According to the Euler equation, the CGP requires $r(P_{At}, P_{St}) = \rho$. Using the condition for CGP, we obtain the price of agricultural goods and of services along the CGP. Recall the equation for the price ratio of agricultural goods to services given by (1): that is,

$$\frac{P_{At}}{P_{St}} = \left[\frac{\alpha_S^{\alpha_S}\eta_S^{\eta_S}(1-\alpha_S-\eta_S)^{1-\alpha_S-\eta_S}}{\alpha_A^{\alpha_A}\eta_A^{\eta_A}(1-\alpha_A-\eta_A)^{1-\alpha_A-\eta_A}}\right] \times (\rho)^{\alpha_A-\alpha_S} w(P_{At}, P_{St})^{\eta_A-\eta_S} \phi(P_{At}, P_{St})^{[(\alpha_S+\eta_S)-(\alpha_A+\eta_A)]}, \quad (\mathbf{21})$$

where $r(P_A, P_S) = \rho$ holds along the CGP, and the wage rate $w(P_{At}, P_{St})$ and the land rent $\phi(P_{At}, P_{St})$ are computed by (3) and (4), respectively. Therefore, by solving the two simultaneous equations, (21) and the CGP condition $P_{At}\bar{A} = P_{St}\bar{S}$ (20), the prices of agricultural goods and services are determined endogenously.

PROPOSITION 1. *Prices of both agricultural goods and services are determined endogenously in the three-sector by three-factor model with different factor intensity across sectors. Even if we set Hicks neutral productivities to be equal to one for all three sectors, we can obtain significant equilibrium prices of agricultural goods and services.*

Now that the interest rate is constant under the CGP, the wage–land rent restriction from equation (21) is crucial for the pricing mechanism of agricultural goods and services. In other words, we emphasize that the CGP condition plays a role in linking wage and land rent in the "three-by-three" model. This effect does not appear in the model of three goods and two factors because the CGP condition eventually makes not only the rental rate constant but also the wage rate constant over time (see Appendix A). Because the three-sectors and two-factors model with structural change makes two factor prices rigid, it is significant to introduce the third factor into the model. Recall that in the model of Kongsamut

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et al. (2001), the assumption of different Hicks neutral productivities across three sectors is indispensable for different prices.

4.4. The Constant Growth Path Dynamical System

The CGP dynamic system with an additional restriction on the CGP is given by the Euler equation (18), the capital accumulation equation (19), and the CGP condition (20). This dynamic system determines the time series of $\{P_{At}, P_{St}, k_t\}_{t=0}^{\infty}$. By substituting $P_{At} = (\frac{\bar{S}}{\bar{A}})P_{St}$ into the other dynamic equations, we can derive the dynamic system with CGP, which is written in Appendix B. We can describe the dynamic system using two dynamic equations and two variables (k_t, P_{St}) under the CGP. Our three-sector model with structural change exhibits a dynamics similar to the two-sector neoclassical growth model of Uzawa (1964).

4.5. The Constant Growth Path Equilibrium

The CGP equilibrium is given by $P_A^*\bar{A} = P_S^*\bar{S}$, $r(P_A^*, P_S^*) = \rho$, $y(P_A^*, P_S^*, k^*, z) = \left(\frac{\iota+b}{\iota}\right) P_A^* \frac{\partial y(P_A^*, P_S^*, k^*, z)}{\partial P_A^*} + P_S^* \frac{\partial y(P_A^*, P_S^*, k^*, z)}{\partial P_S^*}$, where $r(P_A^*, P_S^*) = \Omega_K \left(P_A^*\right)^{\frac{\eta_M(1-\alpha_S)-\eta_S(1-\alpha_M)}{\varepsilon}} \left(P_S^*\right)^{\frac{\eta_A(1-\alpha_M)-\eta_M(1-\alpha_A)}{\varepsilon}}$. Note that the growth rate of manufactured goods is zero in our model and the growth rate of prices is zero at the Euler equation along the CGP. By solving the two equations $P_A^*\bar{A} = P_S^*\bar{S}$ and

$$\Omega_K \left(P_A^*\right)^{\frac{\eta_M(1-\alpha_S)-\eta_S(1-\alpha_M)}{\varepsilon}} \left(P_S^*\right)^{\frac{\eta_A(1-\alpha_M)-\eta_M(1-\alpha_A)}{\varepsilon}} = \rho$$

for the two variables P_A^* and P_S^*, we can obtain the CGP equilibrium prices of agricultural goods and service as follows:

$$P_A^* = \left(\frac{\bar{A}}{\bar{S}}\right)^{\frac{\eta_A(1-\alpha_M)-\eta_M(1-\alpha_A)}{(1-\alpha_M)(\eta_S-\eta_A)+\eta_M(\alpha_S-\alpha_A)}} \left(\frac{\rho}{\Omega_K}\right)^{\frac{-\varepsilon}{(1-\alpha_M)(\eta_S-\eta_A)+\eta_M(\alpha_S-\alpha_A)}}, \tag{22}$$

$$P_S^* = \left(\frac{\bar{A}}{\bar{S}}\right)^{\frac{\eta_S(1-\alpha_M)-\eta_M(1-\alpha_S)}{(1-\alpha_M)(\eta_S-\eta_A)+\eta_M(\alpha_S-\alpha_A)}} \left(\frac{\rho}{\Omega_K}\right)^{\frac{-\varepsilon}{(1-\alpha_M)(\eta_S-\eta_A)+\eta_M(\alpha_S-\alpha_A)}}, \tag{23}$$

where $\varepsilon \equiv \alpha_A\eta_M - \alpha_M\eta_A - \alpha_S(\eta_M-\eta_A) - \eta_S(\alpha_A-\alpha_M)$. Solving the CGP condition (that is, $P_A^*\bar{A} = P_S^*\bar{S}$) and the constant rental rate of capital $r(P_A^*, P_S^*) = \rho$ dictates the CGP equilibrium prices P_A^* and P_S^* in the steady state. The prices of agricultural goods and services in the steady state are computed by (22) and (23). By substituting the CGP equilibrium prices P_A^* and P_S^* into the resource constraint along the CGP, that is, $y(P_A^*, P_S^*, k^*, z) = \left(\frac{\iota+b}{\iota}\right) P_A^* \frac{\partial y(P_A^*, P_S^*, k^*, z)}{\partial P_A^*} + P_S^* \frac{\partial y(P_A^*, P_S^*, k^*, z)}{\partial P_S^*}$, we can obtain the CGP capital stock per labor, which will be displayed in Appendix B.

PROPOSITION 2 (the CGP equilibrium). *Let us consider the structural change in a three-sector growth model using the three-sector Cobb–Douglas*

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GDP function and focus on the CGP equilibrium, which is consistent with the constant rental rate of capital and the existence of structural change. Denote the CGP equilibrium price of agricultural goods by P_A^*, the CGP equilibrium price of services by P_S^*, and capital stock per labor in the CGP equilibrium by k^*. Then the CGP equilibrium values—which satisfy Kaldor and Kuznets facts—are given by (22), (23), and (B.1).

The macroeconomic variables in the CGP equilibrium are also written in Appendix B.

5. DISCUSSION

Importantly, three production factors are indispensable for a CGP condition that does not have the characteristics of a knife-edge. Now, let us divagate from the structural change model and explain the dynamics of the three-sector growth model heuristically. Assume a representative agent model with homothetic preference for simplicity. The one-sector optimal growth model of Ramsey, Cass, and Koopmans has two dynamic equations, the Euler equation and capital accumulation equation for two variables, which are consumption c_t (jump variable) and capital stock per capita k_t (state variable), and therefore the model has a unique steady state value (c^*, k^*). Next, the two-sector optimal growth model of Uzawa (1964) also has two dynamic equations, the Euler equation and the capital accumulation equation for two variables, which are the relative price of consumption goods measured in terms of investment goods P_t (jump variable) and the capital stock per capita k_t (state variable), and therefore the model has a unique steady state value (P^*, k^*). Note that the relative price of consumption goods P_t can be expressed as the ratio of the wage rate to the rental rate $\omega_t \equiv \frac{w_t}{r_t}$, or vice versa, in the two-goods and two-factors model.

Next, let us consider the standard three-sector model with three factors (capital, labor, land) that produce three outputs (agricultural goods, manufacturing goods, and services). Let us ignore the subsistence level of agricultural goods and home production of service and therefore consider the case of homothetic preference. Consider the case where given fixed labor and fixed land, only capital stock accumulates over time. The representative agent maximizes

$$\max \int_{t=0}^{\infty} e^{-\rho t}[\iota \log A_t + b \log M_t + (1 - \iota - b)S_t]dt,$$

subject to the resource constraint $\dot{k}_t = y(P_{At}, P_{St}, k_t) - M_t - P_{At}A_t - P_{St}S_t$, where $y(P_{At}, P_{St}, k_t) = r(P_{At}, P_{St})k_t + w(P_{At}, P_{St}) + \phi(P_{At}, P_{St})z$ is the GDP function per labor. From the first-order conditions, we obtain the optimal allocation of consumption $\frac{M_t}{b} = \frac{P_{At}A_t}{\iota}$ and $\frac{M_t}{b} = \frac{P_{St}S_t}{1-\iota-b}$, the capital accumulation equation $\dot{k}_t = y(P_{At}, P_{St}, k_t) - \frac{M_t}{b}$, the Euler equation $\frac{\dot{M}_t}{M_t} = r(P_{At}, P_{St}) - \rho$, and the transversality condition. The market equilibrium conditions for agricultural goods

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and services imply $A_t = \frac{\partial y(P_{At}, P_{St}, k_t)}{\partial P_{At}}$ and $S_t = \frac{\partial y(P_{At}, P_{St}, k_t)}{\partial P_{St}}$. By using $\frac{M_t}{b} = P_{At}\frac{\partial y(P_{At}, P_{St}, k_t)}{\partial P_{At}}/\iota$ and eliminating M_t from two dynamic equations, we obtain the capital accumulation equation and the Euler equation,

$$\dot{k}_t = y(P_{At}, P_{St}, k_t) - \frac{P_{At}\frac{\partial y(P_{At}, P_{St}, k_t)}{\partial P_{At}}}{\iota}, \tag{24}$$

$$\frac{\dot{P}_{At}}{P_{At}} + \frac{\frac{\partial^2 y(P_{At}, P_{St}, k_t)}{\partial P_{At}^2}\dot{P}_{At} + \frac{\partial^2 y(P_{At}, P_{St}, k_t)}{\partial P_{At}\partial P_{St}}\dot{P}_{St} + \frac{\partial^2 y(P_{At}, P_{St}, k_t)}{\partial P_{At}\partial k_t}\dot{k}_t}{\frac{\partial y(P_{At}, P_{St}, k_t)}{\partial P_{At}}} = r(P_{At}, P_{St}) - \rho. \tag{25}$$

Therefore, the three-sector model has two dynamic equations for three variables, which are the price of agricultural goods P_{At} (jump variable), the price of services P_{St} (jump variable), and capital stock k_t (state variable). In the steady state, we have

$$y(P_A^*, P_S^*, k^*) = \frac{P_A^*\frac{\partial y(P_A^*, P_S^*, k^*)}{\partial P_A}}{\iota} \tag{26}$$

and

$$r(P_A^*, P_S^*) = \rho. \tag{27}$$

Therefore, the model cannot identify a unique steady state value (P_A^*, P_S^*, k^*). There may exist possible diverse combinations of pairs of the prices (P_A^*, P_S^*) in the steady state. In other words, the three-sector and three-factor model with one capital accumulation equation may fall short of one more equation. Thus, without an additional restriction, the price of agricultural goods and the price of services cannot be determined uniquely in the steady state. Fortunately, however, we need one more equation to explain the Kuznets and Kaldor facts. We are inspired by Kongsamut et al. (2001): what is called the condition for the CGP that provides the restriction on the movement of the two prices. Thus, the CGP condition itself is the means of settlement that enables our model to have unique prices of agricultural goods and of services (P_A^*, P_S^*) and unique capital stock per labor k^* in the long run, and to avoid the knife-edge condition. Next, consider the model of three goods and three factors that include labor, equipment (capital stock 1), and structures (capital stock 2). If two factors (which are equipment and structures) accumulate over time, then two accumulation equations and the Euler equations could determinate unique steady state values perfectly. Then the additional CGP condition is redundant. It is the framework of three goods and three factors where only one factor accumulates over time that is indispensable for explaining economic growth with structural change. Adding the restrictive CGP condition to the three-sector model with only one accumulation equation

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of capital succeeds in building a model consistent with the Kaldor and Kuznets facts.

6. TRANSITIONAL DYNAMICS AND STABILITY

Let us show that the dynamics of the three-goods and three-factors model along the CGP is the same at that of the standard two-sector optimal growth model. Because there is no externality in our economy and firms produce outputs efficiently on the PPF, the first welfare theorem applies and the competitive market equilibrium should coincide with the solution to the planner's problem. This is also true for the transitional dynamics. Transition dynamics is easy to characterize, because the model has the same stability properties as the two-sector neoclassical growth model of Uzawa (1964).

By substituting $P_{At}\bar{A} = P_{St}\bar{S}$, $A_t - \bar{A} = \frac{\iota M_t}{bP_{At}}$, and $S_t + \bar{S} = \frac{(1-\iota-b)M_t}{bP_{St}}$ into the utility function $\int_{t=0}^{\infty} e^{-\rho t}[\iota \log(A_t - \bar{A}) + b \log M_t + (1-\iota-b)\log(S_t + \bar{S})]dt$ and the resource constraint $\dot{K}_t = Y(P_{At}, P_{St}, K_t, L_t, Z_t) - M_t - P_{At}A_t - P_{St}S_t$, we can rewrite the planning problem as

$$\max \int_{t=0}^{\infty} e^{-\rho t}[\log M_t - (1-b)\log P_{St}]dt,$$

subject to the resource constraint $\dot{K}_t = Y(P_{At}, P_{St}, K_t, L_t, Z_t) - \frac{M_t}{b}$, and the initial condition K_0, and $L_t = 1$, $Z_t = zL_t$, and land per labor z is constant. By using the condition for CGP, $P_{At} = \left(\frac{\bar{S}}{\bar{A}}\right) P_{St}$, the three-sector Cobb–Douglas GDP function can be rewritten as

$$Y(P_{St}, K_t, L_t, Z_t) = r(P_{St})K_t + w(P_{St})L_t + \phi(P_{St})Z_t,$$

$$r(P_{St}) = \Omega_K \left(\frac{\bar{S}}{\bar{A}}\right)^{\frac{\eta_M(1-\alpha_S)-\eta_S(1-\alpha_M)}{\varepsilon}} P_{St}^{\frac{(1-\alpha_M)(\eta_A-\eta_S)+\eta_M(\alpha_A-\alpha_S)}{\varepsilon}},$$

$$w(P_{St}) = \Omega_L \left(\frac{\bar{S}}{\bar{A}}\right)^{\frac{\alpha_S(1-\eta_M)-\alpha_M(1-\eta_S)}{\varepsilon}} P_{St}^{\frac{(1-\eta_M)(\alpha_S-\alpha_A)+\alpha_M(\eta_S-\eta_A)}{\varepsilon}},$$

$$\phi(P_{St}) = \Omega_Z \left(\frac{\bar{S}}{\bar{A}}\right)^{\frac{\alpha_M\eta_S-\eta_M\alpha_S}{\varepsilon}} P_{St}^{\frac{\alpha_M(\eta_S-\eta_A)+\eta_M(\alpha_A-\alpha_S)}{\varepsilon}}.$$

$Y(P_{St}, K_t, L_t, Z_t)$ is the GDP in terms of manufactured goods. Note that the factor price depends only on the service price on the CGP. The GDP per labor, $y(P_{St}, k_t, z) = \frac{Y(P_{St}, K_t, L_t, Z_t)}{L_t}$, can be expressed as $y(P_{St}, k_t, z) = r(P_{St})k_t + w(P_{St}) + \phi(P_{St})z$. The current value Hamiltonian is given by $\mathbf{H} \equiv [\log M_t - (1-b)\log P_{St}] + \lambda_t \left[Y(P_{St}, K_t, L_t, Z_t) - \frac{M_t}{b}\right]$, where λ_t is the current shadow value of capital stock. By differentiating the current value Hamiltonian with respect to M_t and by equalizing the resulting expression to be zero, we can obtain

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$bM_t^{-1} = \lambda_t$. We assume that a social planner can choose the price of services P_{St} to maximize social welfare. By differentiating the Hamiltonian with respect to P_{St}, and by equalizing the resulting expression to be zero, we obtain $-(1-b)P_{St}^{-1} = \lambda_t \frac{\partial Y(P_{St},K_t,L_t,Z_t)}{\partial P_{St}}$. Therefore we have $M_t = \left(\frac{b}{1-b}\right) P_{St} \frac{\partial Y(P_{St},K_t,L_t,Z_t)}{\partial P_{St}}$. The Euler equation is given by $\frac{\dot{M}_t}{M_t} = r(p) - \rho$.

The resource constraint can be rewritten as $\dot{K}_t = Y(P_{St}, K_t, L_t, Z_t) - \left(\frac{1}{1-b}\right) P_{St} \frac{\partial Y(P_{St},K_t,L_t,Z_t)}{\partial P_{St}}$. The growth rate of the consumption of manufactured goods per labor $\frac{M_t}{L_t}$ is given by

$$\frac{\dot{M}_t}{M_t} = \frac{\dot{P}_{St}}{P_{St}} + \frac{P_{St}\frac{\partial^2 y(P_{St},k_t,z)}{\partial P_{St}^2}}{\frac{\partial y(P_{St},k_t,z)}{\partial P_{St}}}\frac{\dot{P}_{St}}{P_{St}} + \frac{k_t\frac{\partial^2 y(P_{St},k_t,z)}{\partial P_{St}\partial k_t}}{\frac{\partial y(P_{St},k_t,z)}{\partial P_{St}}}\frac{\dot{k}_t}{k_t}.$$

6.1. The CGP Dynamic System

The dynamic system in the three-sector and three-factor model along the CGP can be reduced to

$$\frac{\dot{P}_{St}}{P_{St}}\left[1 + \frac{P_{St}\frac{\partial^2 y(P_{St},k_t,z)}{\partial P_{St}^2}}{\frac{\partial y(P_{St},k_t,z)}{\partial P_{St}}}\right] = r(P_{St}) - \rho - \frac{k_t\frac{\partial^2 y(P_{St},k_t,z)}{\partial P_{St}\partial k_t}}{\frac{\partial y(P_{St},k_t,z)}{\partial P_{St}}}\frac{\dot{k}_t}{k_t}, \tag{28}$$

$$\dot{k}_t = y(P_{St}, k_t, z) - \left(\frac{1}{1-b}\right) P_{St}\frac{\partial y(P_{St}, k_t, z)}{\partial P_{St}}. \tag{29}$$

There are two dynamic equations and two variables of P_{St} and k_t. This dynamic system is fundamentally the same as the two-sector neoclassical optimal growth model of Uzawa (1964).

PROPOSITION 3. *The dynamic sysytem in the three-sector and three-factor growth model with structual change involving the constant growth path (CGP)—which is consistent with the Kaldor facts and Kuznets facts—is essentially the same as that in the two-sector neoclassical optimal growth model.*

The dynamic system in the model of Kongsamut et al. (2001) is identical to that in the one-sector Ramsey model. In contrast, the dynamic system in our model is similar to that in the two-sector model of Uzawa (1964). Crucially, it is sufficient for us to understand the dynamic behavior of the two-sector optimal growth model when we consider the three-sector growth model that is consistent with Kaldor and Kuznets facts. We can explain the dynamic behavior of structural change along the CGP in the three-sector model by applying the Stolper–Samuleson and the Rybczynski theorem. According to the Stolper–Samuleson theorem, when the increase in the service price (P_{St}) increases the interest rate $r(P_{St})$, then the saddle path becomes horizontal. This corresponds to the case of the left-hand side of

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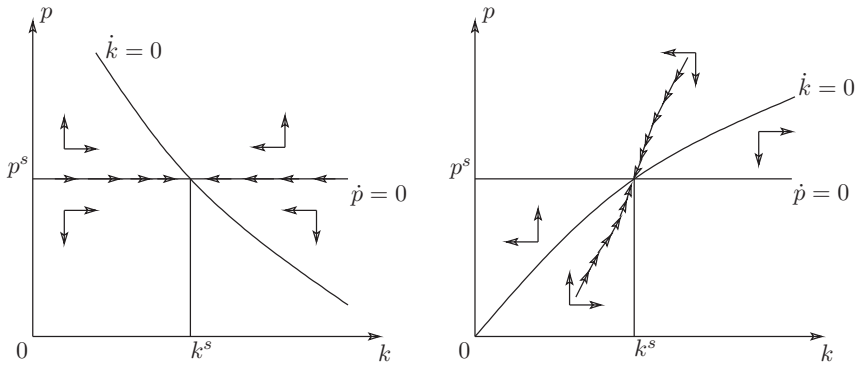


FIGURE 1. The phase diagram in the model of Uzawa (1964).

Figure 1. On the other hand, when the increase in the service price decreases the interest rate, the saddlepath becomes upward sloping. This corresponds to the case of the right-hand side of Figure 1.

Let us review the two-sector neoclassical growth model of Uzawa (1964), where the consumption goods are only consumed by households, and the investment goods are used only for capital accumulation. Suppose that total labor supply is constant over time and it is unitary. Industry 1 produces consumption goods and industry 2 produces investment goods using two factors of capital and labor that are mobile across sectors. The two-sector GDP function is given by $y(p, k) = r(p)k + w(p)$, where p is the relative price of consumption goods in terms of investment goods, $y(p, k)$ is the GDP in terms of investment goods, and $r(p)$ and $w(p)$ are the rate of return to capital and the wage rate, respectively. The optimization problem is given by $\max_{\{c_t\}_{t=0}^{\infty}} \int_{t=0}^{\infty} e^{-\rho t} \frac{c_t^{1-\theta}}{1-\theta} dt$ subject to $\dot{k} = y(p, k) - pc$ and given $k_0 > 0$, where c_t is consumption, ρ is the discount rate, and $\frac{1}{\theta}$ is an elasticity of intertemporal substitution. The Euler equation and capital accumulation equation in the two-sector growth model are given by, respectively,

$$\frac{\dot{p}}{p} = r(p) - \rho - \theta \frac{\dot{c}}{c}, \tag{30}$$

$$\dot{k} = y(p, k) - p \frac{\partial y(p, k)}{\partial p}. \tag{31}$$

where the market equilibrium of consumption goods is expressed as $c = \frac{\partial y(p,k)}{\partial p}$. The saddlepath is horizontal and therefore the relative price of consumption goods remains unchanged along the balanced growth path (BGP) when the consumption goods sector is capital-intensive, and then the interest rate increases with the relative price of consumption goods (the case of the left-hand side of Figure 1). On the other hand, the saddlepath is upward-sloping and therefore the relative price of consumption goods rises along the BGP in the process of development

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when the investment goods sector is capital-intensive, and then the interest rate decreases with the relative price of consumption goods (the case of the right-hand side of Figure 1). Interestingly, the dynamic system—(28) and (29)—in the three-sector model with CGP is fundamentally the same at that of Uzawa (1964) given by (30) and (31), provided that the log utility case ($\theta = 1$) is valid in our model, and that the relative price of consumption goods (p) in the model of Uzawa (1964) is interpreted as the service price in terms of manufacturing goods (P_{St}) in our model.

Finally, the dynamic stability of our model is explained in Appendix B.

PROPOSITION 4 (sufficent conditions for saddlepoint stability). *The dynamic system along the CGP in the three-goods and three-factors model always has saddlepoint stability if the conditions* $\{ \alpha_A > \alpha_S, \eta_A > \eta_S, \text{ and } \eta_S\alpha_A > \eta_A\alpha_S \}$ *or the conditions* $\{ \alpha_A < \alpha_S, \eta_A < \eta_S, \text{ and } \eta_S\alpha_A < \eta_A\alpha_S \}$ *hold.*

When the agricultural sector uses more capital ($\alpha_A > \alpha_S$) and more labor ($\eta_A > \eta_S$) than the service sector, and the capital share ratio of the agricultural sector relative to the service sector is greater than the labor share ratio ($\frac{\alpha_A}{\alpha_S} > \frac{\eta_A}{\eta_S}$), then the model always has saddlepoint stability, and vice versa.

7. CONCLUSION

This study emphasizes the challenge of reconciling the Kuznets and Kaldor facts, and examines the importance of the income and price channels of structural change. We focused on the dramatic sectoral dynamics that has taken place in agriculture, manufacturing, and services. It is important to develop a unified theoretical framework in order to understand the process of development. Furthermore, theoretical approaches are necessary to take advantage of the wealth of evidence collected by related empirical research.

We explained the relation between three industries and three factor prices in terms of two effects. The first is the effect of changes in sectoral composition driven by different income elasticities for agricultural goods, manufacturing, and services. The second is the effect of changes in the prices of agricultural goods and services. We emphasized that variations in the prices of agricultural goods and services should be considered in the model because the service sector has witnessed rapid growth and consistently rising prises. The dynamics of the price of services, treated as exogenous in the model of Kongsamut et al. (2001), is important for understanding the shift toward a service economy. However, the production share of the manufacturing sector is constant over time in our model and that of Kongsamut et al. (2001), although it should display an inverse-U shape over time because of the decline in manufacturing since the 1990s. Thus, it would be important to include this empirical evidence in studies [see Matsuyama (2009)].

The best-selling book *Capital in the 21st Century* [Piketty (2014)] challenges the Kaldor stylized facts by finding variations in the capital–income ratio and

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the capital income share over time. He insists that the elasticity of substitution between capital and labor may be between 1.3 and 1.6, because an increase in capital involves an increase in capital income share. What remains to be done may be a departure from the Cobb–Douglas production function. Moreover, home production should be reconsidered in the framework of structural change. Buera and Kaboski (2012a) established that economies experience rich product cycles between home and market production, including the marketization and, later, the demarketization of many services. Purchasing services in a market or producing at home is an important factor in economic development.

The economic growth model involving structural change requires a restriction on the wage rate and on land rent. Because the prices of agricultural goods and services are determined endogenously in our model, the condition for the CGP is no longer a knife-edge condition. Our three-sector model with different factor intensities across sectors has improved the ad hoc knife-edge condition of the CGP in the model of Kongsamut et al. (2001). Furthermore, our model overcomes the drawback in their model. We have shown that the dynamic system in our three-sector and three-factor model, with a CGP, is identical to that in the two-sector neoclassical growth model of Uzawa (1964). In contrast, the dynamic system of the model in Kongsamut et al. (2001) is identical to the one-sector growth model.

Herrendorf et al. (2015) show that structural transformation depends on sectoral differences in labor-augmenting technological progress rather than capital intensity, and substitutability between capital and labor. But our paper suggests that sectoral differences in the capital share and the substitutability also have important implications for changes in sectoral labor and in relative prices. The fusion between the excellent empirical study on factor income shares across sectors by Valentiny and Herrendorf (2008) and our theoretical contribution of a full three-sector model involving structural change must have a powerful impact on modern macroeconomics and development economics. By substituting the factor income shares in the United States for the three sectors, as found in the empirical study of Valentiny and Herrendorf (2008), into the three-sector Cobb–Douglas GDP function, we were able to determine the numerical equilibrium value along the CGP. Our three-sector and three-factor model incorporating structural change can be applied to DSGE models to analyze business cycles and structural change. However, the relation between financial development and structural change in the economic growth model should be studied, because credit market imperfections may be a factor in determining changes in the industrial structure.

NOTES

1. For example, the value of the service would become zero if the number of performers were reduced in a string quartet concert.

2. This paper is memory of Hirofumi Uzawa, who passed away in September 2014. Uzawa (1961) is a seminal paper that distinguishes consumption goods from investment goods in economic growth models.

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3. Note that the working version of Kongsamut et al. (1997) extends their model to introduce different sectoral production functions, where the three production sectors employ two factors (capital and labor). They extend their model to allow for different sectoral rates of technological progress so that the relative prices of different goods can vary over time. Thus, the growth rate of the price of agricultural goods (services) depends on the exogenous differentials in the growth rates of Harrod neutral technological progress in the manufacturing and agriculture (services) sectors. Nevertheless, their model still requires the knife-edge condition for the CGP.

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APPENDIX A: THE MODEL OF THREE GOODS AND TWO FACTORS

A.1. THE CASE OF THREE GOODS AND TWO FACTORS WITH THE SAME FACTOR INCOME SHARE

Let us review the production side of Kongsamut et al. (2001). We ignore Harrod-neutral technological progress and consider the case of the Cobb–Douglas production functions. Denote the output of agricultural, manufacturing, and services at time t by Y_{jt} for $j = \{A, M, S\}$.

K_{jt} and L_{jt} for $j = \{A, M, S\}$ are the capital and labor allocated to the three sectors. B_{jt} for $j = \{A, M, S\}$ is the Hicks-neutral technological level. Given the rental rate of capital (r_t) and the wage rate (w_t), firms minimize the cost $rK_j + wL_j$ for $j = \{A, M, S\}$ subject to the production function $Y_{jt} = B_{jt}K_{jt}^{\alpha}L_{jt}^{1-\alpha}$ for $j = \{A, M, S\}$, where the capital income share (α) is the same across three sectors. The elasticity of output with respect

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to factors is identical across three sectors. The demands for capital and labor are $K_{jt} = B_{jt}^{-1}\alpha^{1-\alpha}(1-\alpha)^{-(1-\alpha)}r_t^{\alpha-1}w_t^{1-\alpha}Y_{jt}$ for $j = \{A, M, S\}$, and $L_{jt} = B_{jt}^{-1}\alpha^{-\alpha}(1-\alpha)^{\alpha}r_t^{\alpha}w_t^{-\alpha}Y_{jt}$ for $j = \{A, M, S\}$, respectively. Therefore, the ratio of capital to labor of sector j is $k_{jt} \equiv \frac{K_{jt}}{L_{jt}} = (\frac{\alpha}{1-\alpha})(\frac{w_t}{r_t})$ for $j = \{A, M, S\}$. As a result, the production functions with identical capital income share imply that the capital–labor ratios across sectors must be equalized: $k_A = k_M = k_S$. Because the only differences across the three sectors are in productivity, the three isoquant curves are proportional and they have the same wage–rental ratio $(\frac{w_t}{r_t})$ for the optimal demand for two factors. Next, the unit cost function becomes $\psi^j(r_t, w_t) \equiv r_t\frac{K_{jt}}{Y_{jt}} + w_t\frac{L_{jt}}{Y_{jt}} = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}B_{jt}^{-1}r_t^{\alpha}w_t^{1-\alpha}$ for $j = \{A, M, S\}$. Denote the sales prices of agriculture, manufacturing, and services at time t by P_A, $P_M(= 1)$, P_S, where the manufacturing goods are assumed to be the numeraire. Because under the assumption of perfect competitive markets, the prices become equal to the unit costs, we have the price–cost equalization condition: $P_{jt} = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}B_{jt}^{-1}r_t^{\alpha}w_t^{1-\alpha}$ for $j = \{A, M, S\}$, where $P_M = 1$. Therefore, the relative price of agricultural goods in terms of manufacturing goods is $\frac{P_A}{P_M} = P_A = \frac{B_M}{B_A}$, and the relative service price is $\frac{P_S}{P_M} = P_S = \frac{B_M}{B_S}$. The relative prices are just the inverse of relative productivity: $\frac{P_{At}}{P_{St}} = \frac{B_S}{B_A}$. A CGP or a generalized BGP is defined to be a trajectory along which the rental rate of capital r_t is constant over time. The CGP condition of Kongsamut et al. (2001) is given by $P_{At}\bar{A} = P_{St}\bar{S}$, where $\bar{A}$ is the subsistence level of agricultural goods and $\bar{S}$ is home production of services. Therefore, because $\frac{P_{At}}{P_{St}} = \frac{\bar{S}}{\bar{A}}$ holds under the CGP condition, we obtain $\frac{B_S}{B_A} = \frac{\bar{S}}{\bar{A}}$. But this restriction is undesirable because it directly links preference and technology, which are given exogenously. This restriction is knife-edge condition. Both Kaldor and Kuznets facts are satisfied only when this restriction happens to hold by chance. One might think that allowing for different elasticities of output with respect to the factors would solve this issue. But even the case of three goods and two factors still cannot solve it. Let us demonstrate it in the following.

A.2. THE CASE OF THREE GOODS AND TWO FACTORS WITH DIFFERENT FACTOR INCOME SHARES

Let us relax the assumption of the same factor income share across sectors. See Hu and Mino (2014) in detail for this case of three goods and two factors with different factor income shares. Firms minimize the cost $rK_j + wL_j$ for $j = \{A, M, S\}$ subject to the production function $Y_{jt} = B_{jt}K_{jt}^{\alpha_j}L_{jt}^{1-\alpha_j}$ for $j = \{A, M, S\}$, where capital income shares (α_j) are different across the three sectors. The demands for capital and labor are $K_{jt} = B_{jt}^{-1}\alpha_j^{1-\alpha_j}(1-\alpha_j)^{-(1-\alpha_j)}r_t^{\alpha_j-1}w_t^{1-\alpha_j}Y_{jt}$ for $j = \{A, M, S\}$, and $L_{jt} = B_{jt}^{-1}\alpha_j^{-\alpha}(1-\alpha_j)^{\alpha_j}r_t^{\alpha_j}w_t^{-\alpha_j}Y_{jt}$ for $j = \{A, M, S\}$, respectively. The ratio of capital to labor in sector j is $k_{jt} \equiv \frac{K_{jt}}{L_{jt}} = (\frac{\alpha_j}{1-\alpha_j})(\frac{w_t}{r_t})$ for $j = \{A, M, S\}$. Therefore, the capital–labor ratios are not identical across sectors $(k_A \neq k_M \neq k_S)$, but are proportional to the ratio of wage to rental rate. Next, the unit cost function becomes $\psi^j(r_t, w_t) \equiv r_t\frac{K_{jt}}{Y_{jt}} + w_t\frac{L_{jt}}{Y_{jt}} = \alpha_j^{-\alpha_j}(1-\alpha_j)^{-(1-\alpha_j)}B_{jt}^{-1}r_t^{\alpha_j}w_t^{1-\alpha_j}$ for $j = \{A, M, S\}$. Under perfect competitive markets, we have the price–cost equalization condition $P_{jt} = \alpha_j^{-\alpha_j}(1-\alpha_j)^{-(1-\alpha_j)}B_{jt}^{-1}r_t^{\alpha_j}w_t^{1-\alpha_j}$ for $j = \{A, M, S\}$, where $P_M = 1$.

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Therefore the prices of agricultural goods and services are computed by, respectively,

$$\frac{P_{At}}{P_{Mt}} = P_{At} = \left[\frac{\alpha_A^{-\alpha_A}(1-\alpha_A)^{-(1-\alpha_A)}}{\alpha_M^{-\alpha_M}(1-\alpha_M)^{-(1-\alpha_M)}}\right]\left(\frac{B_M}{B_A}\right)\left(\frac{w_t}{r_t}\right)^{\alpha_M-\alpha_A}$$

and

$$\frac{P_{St}}{P_{Mt}} = P_{St} = \left[\frac{\alpha_S^{-\alpha_S}(1-\alpha_S)^{-(1-\alpha_S)}}{\alpha_M^{-\alpha_M}(1-\alpha_M)^{-(1-\alpha_M)}}\right]\left(\frac{B_M}{B_S}\right)\left(\frac{w_t}{r_t}\right)^{\alpha_M-\alpha_S}.$$

Rewritten, the wage–rental ratios are determined by

$$\frac{w_t}{r_t} = \left[\frac{\alpha_A^{-\alpha_A}(1-\alpha_A)^{-(1-\alpha_A)}}{\alpha_M^{-\alpha_M}(1-\alpha_M)^{-(1-\alpha_M)}}\right]^{\frac{-1}{\alpha_M-\alpha_A}}\left(\frac{B_M}{B_A}\right)^{\frac{-1}{\alpha_M-\alpha_A}}(P_{At})^{\frac{1}{\alpha_M-\alpha_A}}$$

and

$$\frac{w_t}{r_t} = \left[\frac{\alpha_S^{-\alpha_S}(1-\alpha_S)^{-(1-\alpha_S)}}{\alpha_M^{-\alpha_M}(1-\alpha_M)^{-(1-\alpha_M)}}\right]^{\frac{-1}{\alpha_M-\alpha_S}}\left(\frac{B_M}{B_S}\right)^{\frac{-1}{\alpha_M-\alpha_S}}(P_{St})^{\frac{1}{\alpha_M-\alpha_S}}.$$

Therefore, the prices of P_A and P_S would determine the wage–rental ratio. But we will show that the sale prices of P_A and P_S are not determined uniquely in the case of three goods and two factors. The relative price of agricultural goods to services is computed by

$$\frac{P_{At}}{P_{St}} = \left[\frac{\alpha_A^{-\alpha_A}(1-\alpha_A)^{-(1-\alpha_A)}}{\alpha_S^{-\alpha_S}(1-\alpha_S)^{-(1-\alpha_S)}}\right]\left(\frac{B_S}{B_A}\right)\left(\frac{w_t}{r_t}\right)^{\alpha_S-\alpha_A}.$$

Therefore, the price of agricultural goods relative to services depends not only on the inverse of relative productivity ($\frac{B_S}{B_A}$) but also on the wage–rental ratio. We have the wage–rental ratio

$$\frac{w_t}{r_t} = \left[\frac{\alpha_A^{-\alpha_A}(1-\alpha_A)^{-(1-\alpha_A)}}{\alpha_S^{-\alpha_S}(1-\alpha_S)^{-(1-\alpha_S)}}\right]^{\frac{-1}{\alpha_S-\alpha_A}}\left(\frac{B_S}{B_A}\right)^{\frac{-1}{\alpha_S-\alpha_A}}\left(\frac{P_{At}}{P_{St}}\right)^{\frac{1}{\alpha_S-\alpha_A}}.$$

The ratios of capital to labor of sectors $j = \{A, M, S\}$ are computed as

$$k_{jt} \equiv \frac{K_{jt}}{L_{jt}} = \left(\frac{\alpha_j}{1-\alpha_j}\right)\left(\frac{w_t}{r_t}\right) = \left(\frac{\alpha_j}{1-\alpha_j}\right)\left[\frac{\alpha_A^{-\alpha_A}(1-\alpha_A)^{-(1-\alpha_A)}}{\alpha_S^{-\alpha_S}(1-\alpha_S)^{-(1-\alpha_S)}}\right]^{\frac{-1}{\alpha_S-\alpha_A}}$$
$$\times\left(\frac{B_S}{B_A}\right)^{\frac{-1}{\alpha_S-\alpha_A}}\left(\frac{P_{At}}{P_{St}}\right)^{\frac{1}{\alpha_S-\alpha_A}}.$$

Again, the CGP condition requires that $\frac{P_{At}}{P_{St}} = \frac{\bar{S}}{\bar{A}}$. The CGP condition makes the wage–rental ratio constant:

$$\frac{w}{r} = \left[\frac{\alpha_A^{-\alpha_A}(1-\alpha_A)^{-(1-\alpha_A)}}{\alpha_S^{-\alpha_S}(1-\alpha_S)^{-(1-\alpha_S)}}\right]^{\frac{-1}{\alpha_S-\alpha_A}}\left(\frac{B_S}{B_A}\right)^{\frac{-1}{\alpha_S-\alpha_A}}\left(\frac{\bar{S}}{\bar{A}}\right)^{\frac{1}{\alpha_S-\alpha_A}}.$$

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Because the CGP requires a constant rental rate over time by definition, the wage rate itself becomes constant over time. The wage rate does not increase over time as the economy develops with structural change, if there is no improvement in productivities. Furthermore, the CGP condition makes the capital–labor ratios constant over time:

$$k_j \equiv \frac{K_j}{L_j} = \left(\frac{\alpha_j}{1-\alpha_j}\right)\left(\frac{w}{r}\right) = \left(\frac{\alpha_j}{1-\alpha_j}\right)\left[\frac{\alpha_A^{-\alpha_A}(1-\alpha_A)^{-(1-\alpha_A)}}{\alpha_S^{-\alpha_S}(1-\alpha_S)^{-(1-\alpha_S)}}\right]^{\frac{-1}{\alpha_S-\alpha_A}}$$
$$\times \left(\frac{B_S}{B_A}\right)^{\frac{-1}{\alpha_S-\alpha_A}}\left(\frac{\bar{S}}{\bar{A}}\right)^{\frac{1}{\alpha_S-\alpha_A}},$$

for $j = \{A, M, S\}$. Along the CGP, it is the ratios of home production of service to the subsistence levels of agricultural goods ($\frac{\bar{S}}{\bar{A}}$) and of technologies ($\frac{B_S}{B_A}$) that determine the capital–labor ratios. Under the CGP condition, the capital intensities in different sectors k_j for $j = \{A, M, S\}$ are invariant over time. Thus, exogenous preference parameters and technological levels make the capital–labor ratio constant over time, even if we consider the case of three goods and two factors with different factor income shares.

A.3. NONUNIQUENESS OF FACTOR PRICES IN THE CASE OF THREE GOODS AND TWO FACTORS

Next, let us set aside the CGP condition and explain the nonuniqueness of factor prices in the case of three goods and two factors. In the case of three goods and two factors with different capital income shares, the system of the price-cost equalization conditions given by

$$\begin{pmatrix} P_{At} \\ 1 \\ P_{St} \end{pmatrix} = \begin{bmatrix} \psi^A(r_t, w_t) \\ \psi^M(r_t, w_t) \\ \psi^S(r_t, w_t) \end{bmatrix}$$

does not have a unique solution. In other words, the system given by

$$\begin{pmatrix} P_{At} \\ 1 \\ P_{St} \end{pmatrix} = \begin{bmatrix} \alpha_A^{-\alpha_A}(1-\alpha_A)^{-(1-\alpha_A)} B_{At}^{-1} r_t^{\alpha_A} w_t^{1-\alpha_A} \\ \alpha_M^{-\alpha_M}(1-\alpha_M)^{-(1-\alpha_M)} B_{Mt}^{-1} r_t^{\alpha_M} w_t^{1-\alpha_M} \\ \alpha_S^{-\alpha_S}(1-\alpha_S)^{-(1-\alpha_S)} B_{St}^{-1} r_t^{\alpha_S} w_t^{1-\alpha_S} \end{bmatrix}$$

cannot identify the wage and rental rate uniquely, given the sale prices. The reason is as follows. By solving two equations of $P_A = \psi^A(r_t, w_t)$ and $1 = \psi^M(r_t, w_t)$ given P_A, we have $r_t = r^{AM}(P_A)$ and $w_t = w^{AM}(P_A)$. Next, by solving the two equations $P_S = \psi^S(r_t, w_t)$ and $1 = \psi^M(r_t, w_t)$, given P_S, we have $r_t = r^{SM}(P_S)$ and $w_t = w^{SM}(P_S)$. Therefore we obtain the factor prices as follows: $w^{AM}(P_A) = \Gamma_L^{AM} P_A^{\frac{\alpha_M}{\alpha_M-\alpha_A}}$, $w^{SM}(P_S) = \Gamma_L^{SM} P_S^{\frac{\alpha_M}{\alpha_M-\alpha_S}}$, $r^{AM}(P_A) = \Gamma_K^{AM} P_A^{\frac{\alpha_M-1}{\alpha_M-\alpha_A}}$, and $r^{SM}(P_S) = \Gamma_K^{SM} P_S^{\frac{\alpha_M-1}{\alpha_M-\alpha_S}}$, where Γ_L^{AM}, Γ_L^{SM}, Γ_K^{AM}, and Γ_K^{SM} are composite parameters that consist of capital income shares and productivities. Free mobility of factors across the three sectors requires the factor prices equalization

$$\begin{bmatrix} r^{AM}(P_A) = r^{SM}(P_S) \\ w^{AM}(P_A) = w^{SM}(P_S) \end{bmatrix}.$$

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These simultaneous equations of

$$\left[\begin{array}{l} \Gamma_K^{AM} P_A^{\frac{\alpha_M - 1}{\alpha_M - \alpha_A}} = \Gamma_K^{SM} P_S^{\frac{\alpha_M - 1}{\alpha_M - \alpha_S}} \\ \Gamma_L^{AM} P_A^{\frac{\alpha_M}{\alpha_M - \alpha_A}} = \Gamma_L^{SM} P_S^{\frac{\alpha_M}{\alpha_M - \alpha_S}} \end{array}\right]$$

cannot determine the unique sales prices P_A and P_S. Because the prices of P_A and P_S are up in the air, the factor prices (w_t and r_t) are not also determined uniquely. Because the prices of P_A and P_S are not determined uniquely, the wage–rental ratio ($\frac{w_t}{r_t}$) also cannot be determined uniquely. Therefore the undermined wage–rental ratio ($\frac{w_t}{r_t}$) cannot decide the price ratio of agricultural goods to services ($\frac{P_{At}}{P_{St}}$). Thus, this undetermined price ratio of agricultural goods to services ($\frac{P_{At}}{P_{St}}$) cannot identify the capital–labor ratio (k_{jt}). It is the CGP condition $\frac{P_{At}}{P_{St}} = \frac{\bar{S}}{\bar{A}}$ that plays a role as a device to determine uniquely the price ratio of agricultural goods to services ($\frac{P_{At}}{P_{St}}$), the wage–rental ratio ($\frac{w_t}{r_t}$), and the capital–labor ratio (k_{jt}), which are all determined exogenously. If the demand side through the Euler equation pins down the overall capital–labor ratio $k_t = \frac{K_t}{L_t}$, then capital intensities (k_j) for $j = \{A, M, S\}$ also are determined, the wage-rental ratio ($\frac{w_t}{r_t}$) is determined by the capital–labor ratio (k_{jt}), and then the wage–rental ratio determines the price ratio of agricultural goods to services ($\frac{P_{At}}{P_{St}}$). The wage–rental ratio determines the prices of P_{At} and P_{St}. Finally, the prices of P_{At} and P_{St} determine the factor prices of w_t and r_t. Therefore, when the number of goods is more than the number of factors, the factor prices depend not only on the sales price but also on the capital–labor ratio (k_j). Thus the demand side pins down the sales prices and factor prices in the case of three goods and two factors.

On the other hand, when the number of goods is equal to the number of factors, the factor prices depend only on the sales price and are determined uniquely for the given sales price. The case of two goods and two factors brings about price–cost equalization as follows:

$$\left(\begin{array}{c} P_t \\ 1 \end{array}\right) = \left[\begin{array}{l} \psi^1(r_t, w_t) \\ \psi^2(r_t, w_t) \end{array}\right],$$

where $\psi^i(r_t, w_t)$ is the unit cost function of sector $i = \{1, 2\}$ and P_t is the sales price of goods produced in sector 1. If sector 1 is the consumption goods sector and sector 2 is the investment goods sector, P_t expresses the price of consumption goods in terms of investment goods. Then $r = r(P_t)$ and $w = w(P_t)$ are determined uniquely, given the sales price. Factor prices depend only on the sales price, not on capital or labor. The same analogy is valid for the case of three goods and three factors. In the case, the price–cost equalization is as follows:

$$\left(\begin{array}{c} P_{At} \\ 1 \\ P_{St} \end{array}\right) = \left[\begin{array}{l} \psi^A(r_t, w_t, \phi_t) \\ \psi^M(r_t, w_t, \phi_t) \\ \psi^S(r_t, w_t, \phi_t) \end{array}\right],$$

where ϕ_t is land rent, $\psi^j(r_t, w_t, \phi_t)$ is the unit cost function of sector $j = \{A, M, S\}$, and P_{At} and P_{St} are the sales prices of goods produced in the agricultural sector and service sector, respectively. Then, by solving price–cost equalization with respect to the factor prices, given the sale prices of P_{At} and P_{St}, we obtain the unique factor prices $r = r(P_{At}, P_{St})$, $w_t = w(P_{At}, P_{St})$, and $\phi_t = \phi(P_{At}, P_{St})$. Thus, factor prices do not depend on the production factors capital, labor, and land.

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There exist essential differences between the case of three goods and two factors and the case of three goods and three factors. In the case of three goods and two factors, the factor prices of w_t and r_t depend on the production factors as well as on sales prices. Moreover, in the case, the Euler equation of consumption depends on the capital–labor ratio. On the other hand, in the case of two goods and two factors and the case of three goods and three factors, the Euler equation that gives price dynamics does not depend on the allocation of production factors or the capital–labor ratio. Recall that the wage–rental ratio $\omega_t \equiv \frac{w_t}{r_t}$ depends only on the price of consumption goods P_t, but $\omega_t \equiv \frac{w_t}{r_t}$ does not depend on the capital–labor ratio, in the two-sector optimal growth model of Uzawa (1964). In phase diagram, the $\dot{P}_t = 0$ locus or the $\dot{\omega}_t = 0$ locus becomes vertical line and does not depend on the capital–labor ratio in Uzawa (1964). Therefore, because the Euler equation in the case where the number of goods is equal to the number of factors depends only on the sales prices, the Euler equation pins down just sales prices. The case where the number of goods is equal to the number of factors is different to a large extent from the case where the number of goods is greater than the number of factors.

APPENDIX B: THE MODEL OF THREE GOODS AND THREE FACTORS

B.1. THE CGP DYNAMICAL SYSTEM

The dynamic system with CGP is described by two equations as follows:

$$
\begin{aligned}
&-\frac{\dot{P}_{St}}{P_{St}} - \frac{\dot{P}_{St}}{P_{St}}\left[\frac{\frac{\partial y(P_{At},P_{St},k_t,z)}{\partial P_{At}}}{\frac{\partial y(P_{At},P_{St},k_t,z)}{\partial P_{At}} - \bar{A}}\right]\left[\frac{P_{At}\frac{\partial^2 y(P_{At},P_{St},k_t,z)}{\partial P_{At}^2}}{\frac{\partial y(P_{At},P_{St},k_t,z)}{\partial P_{At}}} + \frac{P_{St}\frac{\partial^2 y(P_{At},P_{St},k_t,z)}{\partial P_{At}\partial P_{St}}}{\frac{\partial y(P_{At},P_{St},k_t,z)}{\partial P_{At}}}\right] \\
&-\frac{\dot{k}_t}{k_t}\left[\frac{\frac{\partial y(P_{At},P_{St},k_t,z)}{\partial P_{At}}}{\frac{\partial y(P_{At},P_{St},k_t,z)}{\partial P_{At}} - \bar{A}}\right]\frac{k_t\frac{\partial r(P_{At},P_{St})}{\partial P_{At}}}{\frac{\partial y(P_{At},P_{St},k_t,z)}{\partial P_{At}}} \\
&= \rho - \Omega_K\left(\frac{\bar{S}}{\bar{A}}\right)^{\frac{\eta_M(1-\alpha_S)-\eta_S(1-\alpha_M)}{\varepsilon}} P_{St}^{\frac{(1-\alpha_M)(\eta_A-\eta_S)+\eta_M(\alpha_A-\alpha_S)}{\varepsilon}},
\end{aligned}
$$

where P_{At} is evaluated at $P_{At} = \frac{\bar{S}}{\bar{A}}P_{St}$, and

$$
\begin{aligned}
\dot{k}_t &= \varepsilon^{-1}\left(\eta_S\left\{1-\alpha_A+\frac{b}{\iota}(1-\alpha_M)-(1-\alpha_S)\left[\eta_A+\frac{b}{\iota}\eta_M\right]\right\}\right) \\
&\times \Omega_K\left(\frac{\bar{S}}{\bar{A}}\right)^{\frac{\eta_M(1-\alpha_S)-\eta_S(1-\alpha_M)}{\varepsilon}} P_{St}^{\frac{(1-\alpha_M)(\eta_A-\eta_S)+\eta_M(\alpha_A-\alpha_S)}{\varepsilon}} k_t \\
&+ \varepsilon^{-1}\left((1-\eta_S)\left\{\alpha_A+\frac{b}{\iota}\alpha_M-\alpha_S\left[(1-\eta_A)+\frac{b}{\iota}(1-\eta_M)\right]\right\}\right) \\
&\times \Omega_L\left(\frac{\bar{S}}{\bar{A}}\right)^{\frac{\alpha_S(1-\eta_M)-\alpha_M(1-\eta_S)}{\varepsilon}} P_{St}^{\frac{(1-\eta_M)(\alpha_S-\alpha_A)+\alpha_M(\eta_S-\eta_A)}{\varepsilon}}
\end{aligned}
$$

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$$+\varepsilon^{-1}\left\{\alpha_S\left[\eta_A+\frac{b}{\iota}\eta_M-\eta_S\left(\alpha_A+\frac{b}{\iota}\alpha_M\right)\right]\right\}$$
$$\times\Omega_Z\left(\frac{\bar{S}}{\bar{A}}\right)^{\frac{\alpha_M\eta_S-\eta_M\alpha_S}{\varepsilon}}P_{St}^{\frac{\alpha_M(\eta_S-\eta_A)+\eta_M(\alpha_A-\alpha_S)}{\varepsilon}}z+\frac{b\bar{S}}{\iota}P_{St}.$$

B.2. THE PER CAPITA CAPITAL STOCK AT CGP EQUILIBRIUM

The capital per labor at CGP equilibrium is computed as

$$k^*=\Lambda^{-1}G_L\Omega_L\left(\frac{\bar{A}}{\bar{S}}\right)^{\frac{1-\eta_M-\alpha_M}{(1-\alpha_M)(\eta_S-\eta_A)+\eta_M(\alpha_S-\alpha_A)}}\left(\frac{\rho}{\Omega_K}\right)^{\frac{-\left[(1-\eta_M)(\alpha_S-\alpha_A)+\alpha_M(\eta_S-\eta_A)\right]}{(1-\alpha_M)(\eta_S-\eta_A)+\eta_M(\alpha_S-\alpha_A)}}$$
$$+\Lambda^{-1}G_Z\Omega_Z\left(\frac{\bar{A}}{\bar{S}}\right)^{\frac{-\eta_M}{(1-\alpha_M)(\eta_S-\eta_A)+\eta_M(\alpha_S-\alpha_A)}}\left(\frac{\rho}{\Omega_K}\right)^{\frac{-\alpha_M(\eta_S-\eta_A)+\eta_M(\alpha_S-\alpha_A)}{(1-\alpha_M)(\eta_S-\eta_A)+\eta_M(\alpha_S-\alpha_A)}}, \tag{B.1}$$

where

$$\Lambda\equiv-\left\{\eta_S\left[1-\alpha_A+\frac{b}{2}(1-\alpha_M)\right]-(1-\alpha_S)\left(\eta_A+\frac{b}{2}\eta_M\right)\right\}\rho,$$
$$G_L\equiv\left\{(1-\eta_S)\left(\alpha_A+\frac{b}{2}\alpha_M\right)-\alpha_S\left[1-\eta_A+\frac{b}{2}(1-\eta_M)\right]\right\},$$
$$G_Z\equiv\left[\alpha_S\left(\eta_A+\frac{b}{2}\eta_M\right)-\eta_S\left(\alpha_A+\frac{b}{2}\alpha_M\right)\right].$$

B.3. AGGREGATE VARIABLES AT CGP EQUILIBRIUM

Now that we know the CGP equilibrium values of P_A^*, P_S^*, and k^*, we can obtain macroeconomic variables in the CGP as follows:

$$r(P_A^*,P_S^*)=\Omega_K\left(P_A^*\right)^{\frac{\eta_M(1-\alpha_S)-\eta_S(1-\alpha_M)}{\varepsilon}}\left(P_S^*\right)^{\frac{\eta_A(1-\alpha_M)-\eta_M(1-\alpha_A)}{\varepsilon}},$$
$$w(P_A^*,P_S^*)=\Omega_L\left(P_A^*\right)^{\frac{\alpha_S(1-\eta_M)-\alpha_M(1-\eta_S)}{\varepsilon}}\left(P_S^*\right)^{\frac{\alpha_M(1-\eta_A)-\alpha_A(1-\eta_M)}{\varepsilon}},$$
$$\phi(P_A^*,P_S^*)=\Omega_Z\left(P_A^*\right)^{\frac{\alpha_M\eta_S-\eta_M\alpha_S}{\varepsilon}}\left(P_S^*\right)^{\frac{\alpha_A\eta_M-\alpha_M\eta_A}{\varepsilon}},$$
$$y(P_A^*,P_S^*,k^*,z^*)=r(P_A^*,P_S^*)k^*+w(P_A^*,P_S^*)+\phi(P_A^*,P_S^*)z^*,$$
$$Y^*=y(P_A^*,P_S^*,k^*,z^*)=r(P_A^*,P_S^*)K^*+w(P_A^*,P_S^*)+\phi(P_A^*,P_S^*)Z^*,$$
$$K^*=k^*,$$
$$A^*=\frac{\partial y(P_A^*,P_S^*,k^*,z^*)}{\partial P_A^*},$$
$$M^*=\frac{b(A^*-\bar{A})P_A^*}{\iota},$$
$$S^*=\frac{\partial y(P_A^*,P_S^*,k^*,z^*)}{\partial P_S^*}.$$

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B.4. DYNAMIC STABILITY

Let us examine the dynamic stability. Here, denote the service price by $P_S \equiv p$. The dynamical system around the CGP in the social planner problem is described as

$$\begin{pmatrix} \dot{p} \\ \dot{k} \end{pmatrix} = \begin{bmatrix} \left(1 + \frac{p^s y_{pp}}{y_p}\right)^{-1} p^s r_p(p^s), & 0 \\ \left(\frac{-b}{1-b}\right) y_p - \left(\frac{1}{1-b}\right) p^s y_{pp}, & \rho - \left(\frac{1}{1-b}\right) p^s r_p(p^s) \end{bmatrix} \begin{pmatrix} p - p^s \\ k - k^s \end{pmatrix},$$

where p^s and k^s are equilibrium values at the CGP in the social planner problem.

Then we obtain $\mathrm{DET} = (1+\frac{p^s y_{pp}}{y_p})^{-1} p^s r_p(p^s)[\rho-(\frac{1}{1-b})p^s r_p(p^s)]$. Because $(1+\frac{p^s y_{pp}}{y_p})^{-1}$ is always positive, if $p^s r_p(p^s)[r(p^s) - (\frac{1}{1-b})p^s r_p(p^s)] < 0$ holds, DET becomes negative.

Recall that $r(P_{St}) = \Omega_K(\frac{\bar{S}}{\bar{A}})^{[\eta_M(1-\alpha_S)-\eta_S(1-\alpha_M)]/\varepsilon} P_{St}^{[(1-\alpha_M)(\eta_A-\eta_S)+\eta_M(\alpha_A-\alpha_S)]/\varepsilon}$ and $\varepsilon \equiv \alpha_A\eta_M - \alpha_M\eta_A - \alpha_S(\eta_M - \eta_A) - \eta_S(\alpha_A - \alpha_M)$.

Because we have

$$p^s r_p(p^s) = \left(\frac{(1-\alpha_M)(\eta_A - \eta_S) + \eta_M(\alpha_A - \alpha_S)}{\varepsilon}\right) \times \Omega_K \left(\frac{\bar{S}}{\bar{A}}\right)^{\frac{\eta_M(1-\alpha_S)-\eta_S(1-\alpha_M)}{\varepsilon}} P_{St}^{\frac{(1-\alpha_M)(\eta_A-\eta_S)+\eta_M(\alpha_A-\alpha_S)}{\varepsilon}},$$

we obtain

$$r(p^s) - \left(\frac{1}{1-b}\right) p^s r_p(p^s) = \left\{1 - \left[\frac{(1-\alpha_M)(\eta_A - \eta_S) + \eta_M(\alpha_A - \alpha_S)}{(1-b)\varepsilon}\right]\right\} \times \Omega_K \left(\frac{\bar{S}}{\bar{A}}\right)^{\frac{\eta_M(1-\alpha_S)-\eta_S(1-\alpha_M)}{\varepsilon}} P_{St}^{\frac{(1-\alpha_M)(\eta_A-\eta_S)+\eta_M(\alpha_A-\alpha_S)}{\varepsilon}}.$$

If we have $[\frac{(1-\alpha_M)(\eta_A-\eta_S)+\eta_M(\alpha_A-\alpha_S)}{\varepsilon}](\frac{1}{1-b})[\frac{\eta_S(1-\alpha_A)-\eta_A(1-\alpha_S)-\varepsilon b}{\varepsilon}] < 0$, then DET is negative. Therefore, because $\frac{1}{(1-b)\varepsilon^2} > 0$ always holds, if we have $[(1-\alpha_M)(\eta_A - \eta_S) + \eta_M(\alpha_A - \alpha_S)][\eta_S(1-\alpha_A) - \eta_A(1-\alpha_S) - \varepsilon b] < 0$, then DET is negative. If the inequality

$$[(\eta_M(\alpha_A - \alpha_S) - \alpha_M(\eta_A - \eta_S)) + (\eta_A - \eta_S)] \times [-b(\eta_M(\alpha_A - \alpha_S) - \alpha_M(\eta_A - \eta_S)) - (\eta_A - \eta_S) - (1-b)(\eta_S\alpha_A - \eta_A\alpha_S)] < 0$$

holds, then DET is negative. As a result, if we have $\eta_M(\alpha_A - \alpha_S) - \alpha_M(\eta_A - \eta_S) > 0$, $(\eta_A - \eta_S) > 0$, and $(\eta_S\alpha_A - \eta_A\alpha_S) > 0$, then DET is negative. That is, if $\alpha_A > \alpha_S$, $\eta_A > \eta_S$, and $\eta_S\alpha_A > \eta_A\alpha_S$, then DET is negative. On the other hand, if we have $\eta_M(\alpha_A - \alpha_S) - \alpha_M(\eta_A - \eta_S) < 0$, $(\eta_A - \eta_S) < 0$, and $(\eta_S\alpha_A - \eta_A\alpha_S) < 0$, then DET is negative. That is, if $\alpha_A < \alpha_S$, $\eta_A < \eta_S$, and $\eta_S\alpha_A < \eta_A\alpha_S$, then DET is negative. These conditions are sufficient conditions for negative DET, but they are not necessary conditions. Because the system has one positive root and one negative root under these sufficient conditions, the dynamical system has saddlepoint stability. Note that even if parameters do not satisfy sufficient conditions, it is possible for the dynamic system to be saddlepoint stable.

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