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AN ASPECT OF VARIABLE POPULATION POVERTY COMPARISONS: DOES ADDING A RICH PERSON TO A POPULATION REDUCE POVERTY?

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Poverty indexes are essential for monitoring poverty, setting targets for poverty reduction, and tracking progress on these goals. This paper suggests that further justification is necessary for using the main poverty indexes in the literature in any of these ways. It does so by arguing that poverty should not decline with the mere addition of a rich person to a population and showing that the standard indexes do not satisfy this axiom. It, then, suggests a way of modifying these indexes to avoid this problem.

1. INTRODUCTION

Poverty indexes are essential for monitoring poverty, setting targets for poverty reduction, and tracking progress on these goals (Cranfield *et al.*

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2007; UNDP 2007; Pogge 2010). No single paper could review, never mind evaluate, every proposed poverty measure in the literature. This paper suggests, however, that further justification is necessary for using the main poverty indexes in the literature in any of these ways (including those the World Bank makes easily available for empirical research).[1] It does so by arguing that poverty should not decline with the mere addition of a rich person to a population. It, then, demonstrates that the standard poverty indexes do not satisfy this *Weak Population Focus Axiom.* Weak Population Focus is similar to, but less demanding than, what some have called the *Poverty Focus* and *Non-Poverty-Invariance* axioms (Subramanian 2002; Paxton 2003).[2] So, it is significant that, unless the main poverty indexes in the literature are modified, they do not satisfy Weak Population Focus. In suggesting a modification of the standard indexes, this paper also isolates the reason why they fail to satisfy this axiom.[3]

Unlike most work on poverty indexes that focuses on income as a proxy for poverty, this paper will not presuppose an answer to the question: 'What is the proper basis for poverty measurement?' Poverty may, for instance, be deprivation in the space of welfare, capabilities, resources, or opportunities. To stay neutral on the proper basis for poverty measurement, this paper will refer to this basis as *need*. It will refer to whatever alleviates need or brings someone up higher above the poverty line as a *good*. Material goods, capabilities, opportunities, and welfare may be *goods* in this sense and they may alleviate need.

Like most work on axiomatic poverty measurement, however, this paper will assume that there are better and worse answers to questions such as: 'What is poverty?' 'How much poverty is there in a population?' and 'Which population is poorer?' There may be room for multiple answers to these questions. What counts as poverty may, for example, be different in some contexts (e.g. in the USA and Peru).[4] Still, some things do not count as appropriate answers to the questions above. On any good

[1] For the World Bank's indicators, see: http://data.worldbank.org/indicator

[2] It is the opposite of Non-poverty Growth (Kundu and Smith 1983).

[3] There are many articles on desirable properties of poverty indexes in fixed populations (Sen 1976; Donaldson and Weymark 1986; Zheng 1997; Xu and Osberg 1999; Subramanian 2006). This paper takes up, and hopefully adds impetus to, the recent move to consider poverty indexes in variable population contexts (Kundu and Smith 1983; Bossert 1990; Subramanian 2002, 2005a, 2005b; Chakravarty *et al*. 2006; Kanbur 2006; Kanbur and Mukherjee 2007; Hassoun 2010; Hassoun and Subramanian 2010).

[4] The first part of this paper will be less controversial if one assumes that poverty is absolute as opposed to relative. Poverty is relative (to some degree) if how much need someone in a population has is sensitive to how much other people have (Townsend 1985; Sen 1997). Whether poverty is relative or absolute may depend on the basis for poverty – whether it is subjective vs. objective welfare or resources etc. I owe thanks to an anonymous referee for suggesting this point.

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account, for instance, most of the people living in the slums of New Delhi must count as poor.[5]

Moreover, for simplicity, this paper will assume that we know each individual's poverty level and that full interpersonal comparisons of need are possible.[6] That is, it assumes that questions like 'How do individual identities and histories matter to their poverty?' and 'Is poverty absolute or relative?' have already been answered.[7]

Keeping these preliminaries in mind, the first part of this paper will consider methods for calculating poverty. It will argue that, when the main indexes in the literature are interpreted as providing a measure of the *poverty in a population*, these indexes violate an axiom or intuition that Hassoun and Subramanian (2012) have called:

(1) Weak Population Focus: Poverty in a population is not reduced by changes in the non-poor population which leave the distribution of goods amongst the poor unchanged.[8]

The next section will consider why the main indexes in the literature violate this axiom if they are interpreted as measures of *poverty in a population*. It will then explain how it is easy to modify these indexes to avoid this problem.

2. AXIOM FOR A PLAUSIBLE ACCOUNT OF POVERTY IN VARIABLE POPULATIONS

Before defending Weak Population Focus, it is important to distinguish between *poverty in a population* and related concepts like *a population's poverty*.[9] Considering two populations, X and Y, saying there is more

[5] I focus throughout on indexes that recognize the possibility of unambiguously poor and non-poor segments of a population. That is, I do not propose adopting Weak Population Focus for fuzzy measures of poverty and the like.

[6] For one way of making such comparisons see Hare (1981). Recent work on theory of mind may add to Hare's account by helping to explicate the process by which such comparisons are made. (See Goldman 1993; Gopnik and Wellman 1995.)

[7] An individual's poverty may be worse if that individual has been poor for a longer period of time or if the individual was richer (or poorer) in the past. It may also matter who is poor so that one of two people with identical histories could be poorer than another.

[8] In the economic literature on axiomatic poverty measurement it is customary to talk in terms of income so this generalization was not included in the initial version of this axiom, but it is appropriate to substitute 'goods' for 'income' here.

[9] I am incredibly thankful to an anonymous referee for suggesting this way of clarifying the distinction and the puzzle that follows.

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poverty in population X than in Y means:

(1) X has more poverty than Y (that is: there is more poverty present in X than in Y).

This is not necessarily equivalent to the following statement about these *populations' poverty*:

(2) X is poorer than Y (that is: X is less wealthy overall than Y).

If 'poorer' is the converse of 'richer' like, e.g. 'worse' is the inverse of 'better'), (2) is equivalent to:

(3) Y is richer than X.

But (3) is not equivalent to (1). Suppose X and Y only contain rich people but each person in Y is much richer than each person in X. Y is richer than X but it is not the case that X has more poverty than Y. Neither contains any poverty at all.

Another way of seeing the distinction is to consider the following claims:

(4) Merely adding rich people to a population can make a population richer.
(5) Merely adding rich people to a population cannot reduce poverty.
(6) Making a population less poor is the same as making it richer.
(7) Making a population less poor is the same as reducing poverty.

Each of these claims is plausible on its own but, together, they are inconsistent.

Combining (4) and (6) we get:

(8) Merely adding rich people to a population can make a population less poor.

Combining (8) and (7) we get:

(9) Merely adding rich people to a population can reduce poverty.

This contradicts:

(5) Merely adding rich people to a population cannot reduce poverty.

The problem is that 'less poor' is ambiguous – it may mean 'more rich' or 'contain(s) less poverty'. The first definition is only clearly appropriate

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when we are considering *a population's poverty*.[10] Judgements about *a population's poverty* entail that if population A is poorer than B, B is richer than A. Saying there is more *poverty in population* A than B means A has more poverty than B but does not necessarily entail B is richer than A. Rather, this paper will argue that this claim is false.

It is also possible to distinguish between the *poverty in a population* and the *average poverty*.[11] The later will presumably divide the *poverty in the population* by the population size. Though, there are many possible measures of both the *poverty in a population* and the *average poverty*.

Keeping these distinctions in mind, it is possible to put this paper's point this way: it may be precisely because people have not distinguished carefully between the *poverty in a population* and related concepts like *a population's poverty* or the *average poverty* that most poverty indexes entail that merely adding a rich person to a population reduces poverty. This may be appropriate where we are concerned about *a population's poverty* (and 'poorer' is the inverse of 'richer') or the *average poverty*. It is not clearly true that merely adding rich people to a population will reduce the *poverty in a population*. Rather, this paper will argue that merely adding rich people to a population will *not* reduce the *poverty in a population*.

More precisely, this paper suggests adopting Weak Population Focus on which the *poverty in a population* cannot decline with additions to the rich population that leave the number and needs of the poor unchanged (though *a population's poverty* or the *average poverty* may decline with additions to the rich population that leave the number and needs of the poor unchanged). The rich may, of course, do a lot to alleviate *poverty in a population*. They might voluntarily or involuntarily give money (or other things) to the poor or their money (etc.) might trickle down to the poor. But, *their mere existence* in a population does not reduce *poverty in a population*. To illustrate the import of this axiom consider the following population:

[10] Alternatively we can derive a contradiction this way:
Combining (5) and (7) we get:
(10) Merely adding rich people to a population cannot make a population less poor.
Combining (10) and (6) we get:
(9) Merely adding rich people to a population cannot make a population richer.
This contradicts:
(4) Merely adding rich people to a population can make a population richer.
The analysis of the problem here is that 'making a population less poor' is not clearly the same as 'reducing poverty'. The ambiguity still concerns the distinction between the *poverty in a population* and *a population's poverty*.

[11] For related work on inequality measurement see Rabinowicz (2003); Arrhenius (2013).

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FIGURE 1. (Colour online)

Here the (dotted) poverty line is at three units of good. The first person, P1, has one unit of good, so needs two units to reach the poverty line. The second person, P2, has two units of good, so needs one more unit to reach the poverty line. The last person, P3, has more than three units of good, so is not poor. The Weak Population Focus Axiom suggests that simply adding more people who can meet their needs to the population does not reduce its poverty. So, adding an additional person P4 who can meet his or her needs to the population does not reduce the *poverty in the population*.

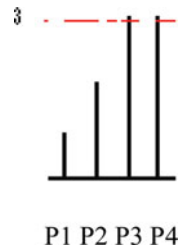


FIGURE 2. (Colour online)

Nor would poverty in this population decline if P4 were much richer or there were additional rich people in it.

If some of the rich become poor, the *poverty in a population* might change. Even if some poor become rich at the same time, one might say poverty in the population has increased. This might be so if the newly poor are poorer than the newly rich were when they were poor or if there are more poor people.

As Amartya Sen puts it, it is also important to distinguish between descriptive indexes which are concerned with 'the state of the poor' and indexes intended to measure other things like a population's 'potential ability to meet the challenge of poverty' (Sen 1981: 190). The *burden of poverty* might, for instance, be less if rich people are added to a population

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and nothing else changes.[12] That is, it might be much easier to alleviate poverty by redistributing some goods from the rich to the poor. There may also be some respects in which adding rich people to a population makes it *better* (*or worse*) than the initial population. The new population might be better able to support the arts or sciences. Still, the *mere addition* of P4 does not *in itself* reduce poverty in this population.[13]

The intuition supporting Weak Population Focus is along the same lines as the intuition Sen expresses in objecting to the idea 'that some increase in the income shortfall of the poor may be compensated by a sufficiently high rise in the income of the non-poor' (Sen 1981: 190). He says 'poverty is a characteristic of the poor, and a reduction of the incomes of the poor must increase the measure of poverty, no matter how much the incomes of the non-poor go up at the same time' (Sen 1981: 190). Similarly, if poverty is a characteristic of the poor, an increase in the number of rich people should not alone reduce *poverty in a population*. A change in the number of rich people does not affect the poor any more than if the incomes of the rich increase. That is, adequate measures of *poverty in a population* will satisfy Weak Population Focus.

One might, however, object to Weak Population Focus by noting that, even though a measure of the *poverty in a population* is not necessarily a measure of how it is best to fulfil needs, these things are related. Perhaps the best explanation for intuitions about how it is best to meet need is that *poverty in a population* decreases when the only change is that more people who can meet their needs are added to the population. Consider an argument for this conclusion inspired by David Miller's work on how we should fulfil need. Miller is concerned with distribution according to need and claims that it is important, in deciding whom to help, to consider not only P1's claims vs. P2 and P3's claims but P1's claims against P4's claims (e.g. to what they need). He believes it is important to consider 'the relative position of everyone' in determining whom we should aid in a population (Miller 1999: 219). Miller thinks it is better to help those in the middle of the stack when there are a greater number of rich people around. Perhaps this is because *poverty in the population* with more rich people is worse. This might support the intuition that it is best to 'equalize degrees of unmet need, which means distributing in favor of those in greater need until they are brought up to the same level as others' (Miller 1999: 74).

[12] One reason to think potential ability to alleviate poverty matters in some cases (even if it does not map onto the *poverty in a population*) is this: If one were behind a Rawlsian veil of ignorance and did not know who one would be in a population, one might reasonably choose to enter a population with a smaller proportion of poor people.

[13] The idea here is different from Derek Parfit's mere addition paradox discussed below (Parfit 1984).

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Miller suggests that his intuition about how to distribute according to need underlies some empirical evidence from experiments in social psychology. In one experiment, there were two students, one of whom needed extra money for books. Subjects had to decide how to split a set amount of money between them. Most subjects wanted to give the needy student enough to buy the textbooks before splitting the rest equally (Miller 1999: 74). In another experiment, intended to mimic John Rawls' original position, subjects had to choose the rules for remuneration for work they were to perform. Most subjects chose to maximize average income subject to a floor constraint.

The empirical evidence does not support Miller's intuition about how to distribute according to need so one cannot use this evidence to argue that there is less *poverty in a population* when the only change is that more people who can meet their needs are added to the population. The first experiment only provides evidence that sometimes people will try to help others meet their needs before distributing the remaining goods equally. The second experiment only provides evidence that most people want to provide a flat minimum for everyone (Frohlich and Oppenheimer 1992; Miller 1999: 79). The results just show that people are concerned about need, not that 'people will aim to equalize degrees of unmet need' (Miller 1999: 74).[14] The evidence does not support Miller's intuition about what distribution according to need requires (Hassoun 2008). Hence, one cannot argue that the best explanation for why people share Miller's intuition about how we should meet need is that *poverty in a population* decreases when the only change is that more people who can meet their needs are added to the population. Unless there is a better reason to reject Weak Population Focus, we should accept it.

3. PROBLEMS WITH THE EXISTING POVERTY INDEXES AND A SOLUTION

Many of the most common poverty indexes fail to satisfy the Weak Population Focus Axiom because these indexes normalize for population size by dividing by the total number of people in the population. These indexes are better interpreted as measures of the *average poverty*. Consider the simplest example – the Headcount Index (H) which measures the proportion of people below the poverty line.

$$H = q/n \tag{1}$$

n is the number of people in the population and q the number of people who have an amount of good y_i less than the amount necessary to

[14] At another point, Miller also suggests subjects are balancing a concern for giving people their just deserts against a concern for need fulfillment. But, it is not clear that the evidence supports this contention (Hassoun 2008).

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reach the poverty line z. The problem is that adding a person above the poverty line to the population only increments n and, so, poverty declines. Many other common poverty indexes fail to satisfy Weak Population Focus. The Appendix demonstrates that this is so for Sen's Index and the Sen-Shorrocks-Thon Index. Consider, here, just the Foster, Greer, Thorbecke (*FGT*) Index (Foster *et al.* 1984).

$$FGT = 1/n\left(\sum_{i=1}^{q} x_i^a\right) \tag{2}$$

x_i is the poverty gap of individual i that is $(z - y_i)/z$. Since $\sum_{i=1}^{q} x_i^a$ aggregates the (weighted) poverty gaps of the poor only, it is unchanged by the mere addition of a rich person to a population. Because the addition of a rich person increases n, however, poverty declines on the *FGT* when the only change is that a rich person is added to the population.

It, thus, seems that some justification is necessary for using the main poverty indexes in the literature in tracking poverty, setting poverty reduction targets, and so forth. If one is concerned to measure the *poverty in a population*, one should not use these indexes because they suggest poverty decreases with the mere addition of a rich person to a population. These indexes are better interpreted as providing measures of *a population's poverty* or the *average poverty.*

If the above indexes are interpreted as measuring *a population's poverty* or the *average poverty*, it is easy to see how to modify them to arrive at a measure of the *poverty in a population*: Do not divide by the total number of people in the population. So, for instance, H would just become q and the *FGT* would become the Aggregate Gap Index: $\sum_{i=1}^{q} x_i^a$. Modifying the standard poverty indexes in this way will ensure that they satisfy Weak Population Focus.

Moreover, there seems to be some reason to accept a version of the Aggregate Gap Index as a measure of the *poverty in a population*. As least this is so if researchers can arrive at the correct account of individuals' poverty. The Aggregate Gap Index (with $a = 1$) just adds up each individual's poverty. So it is plausible to hold that the Aggregate Gap Index yields the (total) *poverty in a population*.[15]

[15] If goods have declining marginal needs satisfaction that is already taken into account in calculating individuals' needs (recall that individuals' needs are fixed before any aggregation takes place). The marginal need-satisfaction of a good for a person is the difference an additional unit of that good will make to how much need a person has. The marginal need satisfaction a person gets from a unit of necessary good declines the more units of necessary good the person already has.

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4. CONCLUSION

This paper argued that the main poverty indexes in the literature may not be appropriate for tracking poverty, setting poverty reduction goals, and monitoring progress in meeting those targets. If the main poverty indexes in the literature are interpreted as measures of *poverty in a population* and are not modified, they unintuitively suggest that *poverty in a population* declines with the mere addition of rich people. So some justification for using these indexes is necessary, especially since it is easy to modify them so that they satisfy Weak Population Focus. Moreover, assuming that researchers can arrive at the proper basis for poverty measurement, there is some reason to accept the Aggregate Gap Index as a measure of *poverty in a population*; there is reason to believe it tells us the sum-total amount of *poverty in the population*. In any case, further work on variable population poverty comparisons is pressing and important.

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APPENDIX

Consider the Sen-Shorrocks-Thon Index (SST) and Sen's Index (S) Here is the SST:

$$\mathrm{P}^{SST}(a;z) = \sum_{i=1}^{q}((2(n-i))+1)/n^2)((z-x_i)/z) \tag{3}$$

Here is S:

$$P^S(a;z) =$$

$$P^S(a;z) = 2/((q+1)nz)\sum_{i=1}^{q}(q+1-i)(z-x_i) \tag{4}$$

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where a is a vector of goods, n is the number of people in the population, q is the number of poor individuals in the population, i is the order of the individuals in the distribution of goods – where n has the most, and x_i is the amount of good individual i has.

It is easy to demonstrate that the *SST* and *S* do not satisfy Weak Population Focus.[16] It will suffice to present a counter-example. Consider the following vectors of goods x and y such that y is derived from x by the addition of a non-poor person. Without loss of generality, supposing an exclusive definition of poverty (that is that those with 0 good do not count as poor), and in line with the statement above, let $x = (0, z)$ and $y = (0, z, z)$, where z denotes the poverty line.

Proposition 1. The Sen-Shorrocks-Thon index (*SST*) does not satisfy Weak Population Focus.

Proof.

We have:

$$P^{SST}((0,z);z) = (2^*1+1)/2^{2*}(z/z) + (2^*0+1)/2^{2*}(0/z) = 3/4$$

$$P^{SST}((0,z,z);z) = (2^*2+1)/3^{2*}(z/z) + 0 + 0 = 5/9$$

So according to the *SST*, poverty in $y < x$. This violates Weak Population Focus. •

Proposition 2. The Sen Index (*S*) does not satisfy Weak Population Focus.

Proof:

We have:

$$P^S((0,z);z) = 2/(2^*2^*z)^*(1^*(z)) = 1/(2z)^*z = 1/2$$

$$P^S((0,z,z);z) = 2/(2^*3^*z)^*(1^*z) = 1/(3z)^*z = 1/3$$

So according to *S*, poverty in $y < x$. This violates Weak Population Focus. •

[16] The formulation of the *S* and *SST* index come from Sen (1976) and Aguirregabiria (2006), respectively.

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