Laser and Particle Beams (2014), *32*, 277–284.
© Cambridge University Press, 2014 0263-0346/14 $20.00
doi:10.1017/S0263034614000111

Temporal and spatial expansion of a multi-dimensional model for electron acceleration in the bubble regime

J. THOMAS,[1] A. PUKHOV,[1,3] AND I.YU. KOSTYUKOV[2,3]

[1]Institut für Theoretische Physik I, Heinrich-Heine-Universität Düsseldorf, Düsseldorf, Germany
[2]Institute of Applied Physics, Russian Academy of Sciences, Nizhny Novgorod, Russia
[3]Lobachevsky State University of Nizhny Novgorod, Nizhny Novgorod, Russia

(Received 16 December 2013; Accepted 12 February 2014)

Abstract

An extended analytical model for particle dynamics in fields of a highly-nonlinear plasma wakefield (the bubble or blow out regime) is derived. A recently proposed piecewise model is generalized to include a time dependent bubble radius and full field solution in the acceleration direction. Incorporation of the cavity dynamics in the model is required to simulate the particle trapping properly. On the other hand, it is shown that the previously reported piecewise model does not reproduce the formation of a mono energetic peak in the particle spectrum. The mono energetic electron beams are recovered only when the full longitudinal field gradient is included in the model.

Keywords: Analytical model; Electron bubble acceleration; Semi-piecewise bubble-model; Wakefield

1. INTRODUCTION

When a plasma wakefield is driven to nonlinear amplitudes, it takes the form of an electronic cavity that is called "bubble" in the case of a short pulse laser driver (Pukhov & ter Vehn, 2002) or "blowout regime" in the case of dense highly energetic particle bunches (Rosenzweig *et al.*, 1991; Lotov, 2004). For the laser driven case, the pulse is shorter than the plasma wavelength and fits perfectly into the first half of the plasma period. The laser intensity is thought to be high enough that the created wakefield breaks after its first oscillation. In this regime, the wakefield takes the form of a distorted spherical cavity from which all electrons are banished and that moves with nearly the speed of light through the plasma. In the following, we refer this wakefield as "the bubble".

The physics of electron self-injection in a bubble is very complex. The modern theory of self-injection is still in an early stage as it cannot predict accurately quantities observed in experiments. In the last years, recent bubble models have been invented and most of them suggest their own criteria of electron trapping (Kostyukov *et al.*, 2009; 2010; kalmykov *et al.*, 2011; 2012; Yi *et al.*, 2011; 2013; Lu *et al.*, 2007; Pak *et al.*, 2010; Zeng *et al.*, 2012). The models in Kostyukov *et al.* (2009; 2010) e.g., suggest that plasma electrons are self-injected in the bubble when the normalized bubble radius is approximately more than the Lorentz gamma-factor of the bubble rear. A modification of this criterion is proposed in Thomas (2010). However, this modification has been seriously criticized in the comment of Corde *et al.* (2011) where mathematical correctness of the model presented in Kostyukov *et al.* (2009; 2010) has been demonstrated. The semi-analytical model in Kalmykov *et al.* (2009) suggest that electron self-injection occurs in the evolving bubble when the effective Hamiltonian becomes negative. This implies that the untrapped electrons cross the separatrix in the phase space, get into the bucket and become trapped. However, further analysis has shown (Yi *et al.*, 2011) that the plasma electrons can also efficiently accelerated outside of the bucket. In recent advanced versions, the effect of the laser pulse evolution (Kalmykov *et al.*, 2012) and the effect of the plasma sheath surrounding bubble (Yi *et al.*, 2013) are included. Lu *et al.* have deduced from PIC simulations a simple criterion that electron self-injection can occur for the bubble above a threshold radius (laser intensity) that has a weak dependence on the bubble velocity (Lu *et al.*, 2007). In recent papers (Pak *et al.*, 2010; Zeng *et al.*, 2012) the trapping condition is expressed through a wake pseudo-potential. The models we present in this paper are advanced versions of Kostyukov *et al.* (2010) that unify analytical descriptiveness and self-consistent treatment of self-injection physics.

Address correspondence and reprint requests to: J. Thomas, Institut für Theoretische Physik I, Heinrich-Heine-Universität Düsseldorf, Düsseldorf D-40225, Germany. E-mail: thomas@tpl.uni-duesseldorf.de

https://doi.org/10.1017/S0263034614000111 Published online by Cambridge University Press

In a pure analytical model, we can calculate the fields inside the bubble from Maxwell's equations for an empty sphere (Kostyukov *et al.*, 2004) and a sphere that takes into account the field enhancement at the bubble back due to electron sheet crossing (Bulanov *et al.*, 1997; Lu *et al.*, 2007). An even simpler model for the fields inside the bubble has been proposed recently: a static piecewise model (PWM) (Kostyukov *et al.*, 2010). This simplified model neglects any spatial or temporal field dependence, but due to its simplicity it is possible to calculate amplitude envelope functions to the transverse particle coordinates and momenta, and to analyze some self-injection physics. In contrast to recent experiments (Lu *et al.*, 2007; Wang *et al.*, 2013), the PWM misses the production of mono energetic electron beams. Thus, we are forced to modify the PWM to a more general case that remains analytically treatable and improves the self-injection physics found in the PWM.

Our first approach is to consider fields that are time dependent, but piecewise constant in space. In this case, our model is a sphere with an adiabatically growing radius $R(t)$ and a growing trapping cross-section. A physical justification for this approach is that now the interaction of the electron bunch with the bubble border is taken into account. Since the fields remain constant we still miss the production of mono energetic electron beams. Thus, in a second step we introduce a model that treats the right field gradient in the longitudinal direction. For this case indeed, we find energy spectra that are comparable to PIC simulations and a full gradient model.

Our work is based on the PWM. Thus, similar to Kostyukov *et al.* (2010), our bubble models neglect the interaction between the accelerated electrons and the laser pulse at the bubble front. We also assume a continuous plasma background density, and a reduced electron dynamics in the z-ρ-plane, where ρ is the perpendicular distance of an electron to the z-axis and z the propagation direction of the laser pulse. The motivation for all these assumptions are several observations in the phenomenological model of the bubble regime (Kostyukov *et al.*, 2004; 2009).

The full gradient potentials inside a bubble are given in Kostyukov *et al.* (2004) and Lu *et al.* (2006) In normalized variables, they are $A_\parallel = -\varphi = \Phi/2$, and $\mathbf{A}_\perp = 0$, where $\Phi(\mathbf{r}) = |\mathbf{r}|^2/4$ is the bubble potential. The normalization is done by $\mathbf{r} \to \mathbf{r}\omega_{pe}/c$ for length, $t \to t\omega_{pe}$ for time, $\mathbf{v} \to \mathbf{v}/c$ for velocity, $\mathbf{p} \to \mathbf{p}/(m_e c)$ for the kinetic momentum, and $\Phi \to e\Phi/(m_e c^2)$ for potentials. Here $\omega_{pe} = (4\pi e^2 n_0/m_e)^{1/2}$ is the electron plasma frequency, m_e is the electron mass, c is the velocity of light, and n_0 is the plasma background density. The equations of motion of the full gradient model in a co-moving frame of reference are Eq.(1)–(2) in Kostyukov *et al.* (2009). In the PWM, $\xi = z - Vt$ and ρ are replaced by

$$\xi = \mathrm{sgn}(\xi)F, \quad \rho = \mathrm{sgn}(\rho)F, \tag{1}$$

where F is the field-strength parameter of the PWM and V the group velocity of the laser pulse.

2. THE TIME-DEPENDENT PWM (TPWM)

One possibility to examine the interaction between the electron bunch and the bubble border is to consider an adiabatically slow bubble growth. This growth significantly affects the electron self-injection physics because the bubble moves back slower than the bubble (Kostyukov *et al.*, 2004; 2009; Whittum, 1992; Lu *et al.*, 2007; Kalmykov *et al.*, 2009; Yi *et al.*, 2011; Kalmykov *et al.*, 2011).

To adopt the idea of a growing bubble to the tPWM we make the same consideration as in Kalmykov *et al.* (2009) and Yi *et al.* (2011). Applied to the full gradient model (Kostyukov *et al.*, 2004), we find the sufficient trapping condition $\lim_{t\to\infty}\Phi(t) = -\infty$. Thus, a new approach for our wakefield potential in the tPWM is

$$\Phi(\mathbf{r}, t) = \frac{F}{2}(|\xi^-| + |\rho|) - \frac{R(t)^2}{4}, \tag{2}$$

with $R(t) = (1 + \epsilon t)R_0$, $\xi^- = z - V^- t$, $V^- = V - \epsilon R_0$, $V = \sqrt{1 - \gamma_0^{-2}}$, and the gamma-factor of the bubble front γ_0. In these coordinates, the bubble center rests and the equations of motion for the tPWM are

$$\frac{dp_\parallel}{dt} = -\frac{1}{4}\Omega^- + \frac{\mathrm{sgn}(\rho)Fp_\perp}{4\gamma}, \tag{3}$$

$$\frac{dp_\perp}{dt} = -\left(1 + \frac{p_\parallel}{\gamma}\right)\frac{\mathrm{sgn}(\rho)F}{4}, \tag{4}$$

$$\frac{d\xi^-}{dt} = \frac{p_\parallel}{\gamma} - V^-, \quad \frac{d\rho}{dt} = \frac{p_\perp}{\gamma}, \tag{5}$$

with $\Omega^- = (1 + V^-)\mathrm{sgn}(\xi^-)F + (1 + \epsilon t)R_0^2\epsilon$.

Based on the tPWM we perform two kinds of simulation. In the first part, we solve the equations of motion (3)–(5) for a single electron and discuss its trajectory, while we always assume the initial conditions $(\xi_0^-)^2 + \rho_0^2 = R_0^2$, $\mathbf{p}_0 = 0$, and ϵ, ξ_0^-, $\rho_0 > 0$. In the second part we place a grid of particles with zero momentum in front of the bubble as shown in Figure 1c and solve the equations of motion for every individual particle. The resulting energy spectra of trapped particles then give us a better insight in the self injection dynamics and acceleration mechanism in the tPWM.

If we compare a particle's trajectory in both the PWM (Figure 1a) and the tPWM (Figure 1b), we discover a new kind of trapping dynamic. The particle in the static model is lost because its trajectory crossed the bubble border. In the time-dependent model, this situation can be prevented as the border moves out. The amplitude function of the oscillation in the trajectory determines whether the electron will be trapped. So, Kostyukov *et al.* (2010) describe a way to calculate the envelope approximation functions $\tilde{\rho}(t)$ and $\tilde{p}_\perp(t)$ to the amplitude functions of $\rho(t)$ and $p_\perp(t)$. We follow their steps in the limit $\epsilon R_0^2 \ll 2F$ and introduce the times t_n by

https://doi.org/10.1017/S0263034614000111 Published online by Cambridge University Press

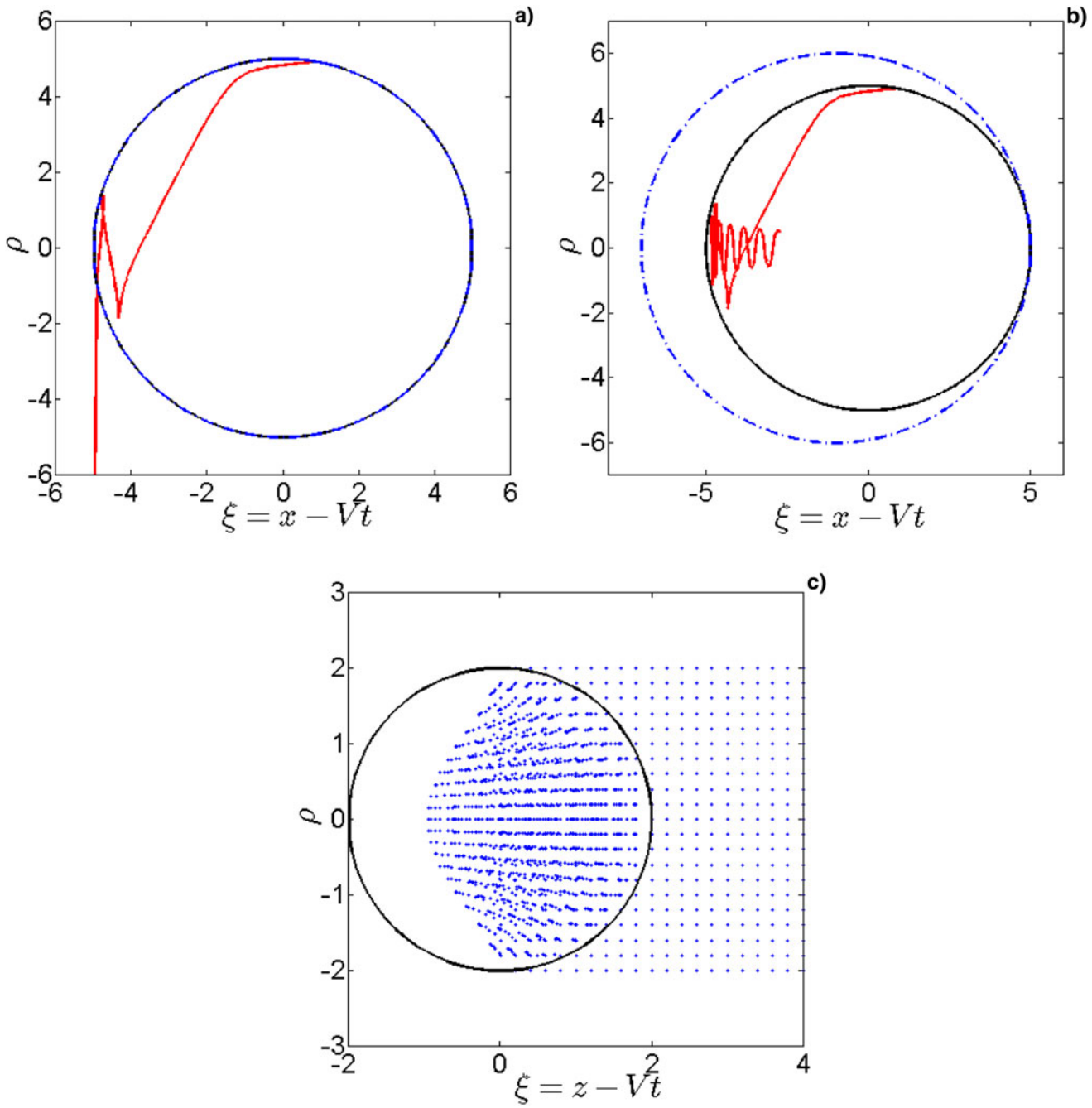


Fig. 1. (Color online) (**a**) Trajectory in the PWM, particle is not trapped. (**b**) Trajectory in the tPWM, particle is trapped. Both simulations are done with $R_0 = 5$, $F = 4$, $\rho_0 = 4.92$, and $\gamma_0 = 10$. The black solid circle has radius R_0. The blue dotted circle is the bubble after the simulation ends. (**c**): Simulation configuration: Bubble running into a grid of electrons.

$\rho(t_n) = 0$ for all $n \geq 1$. Then, with $n \gg 1$ and a guiding center approximation to $p_{\|}(t)$, we obtain the equation system

$$\tilde{p}_{||}(t) = \frac{1}{4}[(1+V^-)F - R_0^2\epsilon]t, \tag{6}$$

$$\tilde{p}_{\perp}(t) = 1.00\tilde{c}y_0^{2/3}\frac{F^{5/3}}{|\Omega^-(t)|}[(1+V^-)F - R_0^2\epsilon]^{1/3}t^{1/3}, \tag{7}$$

$$\tilde{\rho}(t) = 4.01\frac{\tilde{c}^2\,\rho_0^{4/3}F^{7/3}}{\Omega^-(t)^2[(1+V^-)F - R_0^2\epsilon]^{1/3}}t^{-1/3}, \tag{8}$$

while we use the constant $\tilde{c} = \tilde{c}(F, R_0, \gamma_0, \epsilon, \xi_0^-)$ to fit the curves to our simulations.

Two exemplary plots of the envelope functions $\tilde{p}_{\perp}(t)$ and $\tilde{p}_{\perp}(t)$ (red dotted lines) together with the simulated coordinate and momenta of a particle (blue solid lines) are shown in

https://doi.org/10.1017/S0263034614000111 Published online by Cambridge University Press

Figure 2. The analytical predictions are in good agreement with the simulations and converge very close to the actual enveloping functions.

To calculate the trapping conditions, we demand $|\mathbf{r}|^2 = (\xi^-)^2 + \rho^2 < R^2$ and show that for $n \gg 1$

$$\xi_0^- < -\frac{2}{F} + \frac{R_0 F^{3/4}\,\tilde{c}^{3/2}}{(F - R_0^2\epsilon)^{3/4}\gamma_0^{1/2}}\left(1.48 + 0.72\frac{R_0\epsilon\gamma_0^2 F}{2F - R_0^2\epsilon}\right), \qquad (9)$$

is the condition that must hold to trap electrons. From Eq.(9) and $\rho_0^2 = R(t)^2 - (\xi_0^-)^2$ we calculate the trapping cross section $\sigma = \pi(R^2 - \rho_{\min}^2)$ with $\rho_{\min} = \min\ \rho_0$. If we assume $\Delta R = R - \rho_{\min} = const$, we find that $\sigma(t)$ is linearly time-dependent. To confirm our estimation, we plot $\log(\sigma)$ and $\log(R_0)$ into one diagram (see Figure 2c). As we observe, the analytical prediction (blue line) follows the numerically found trapping cross sections (red circles) and stays especially for large R_0 in a 5% error range.

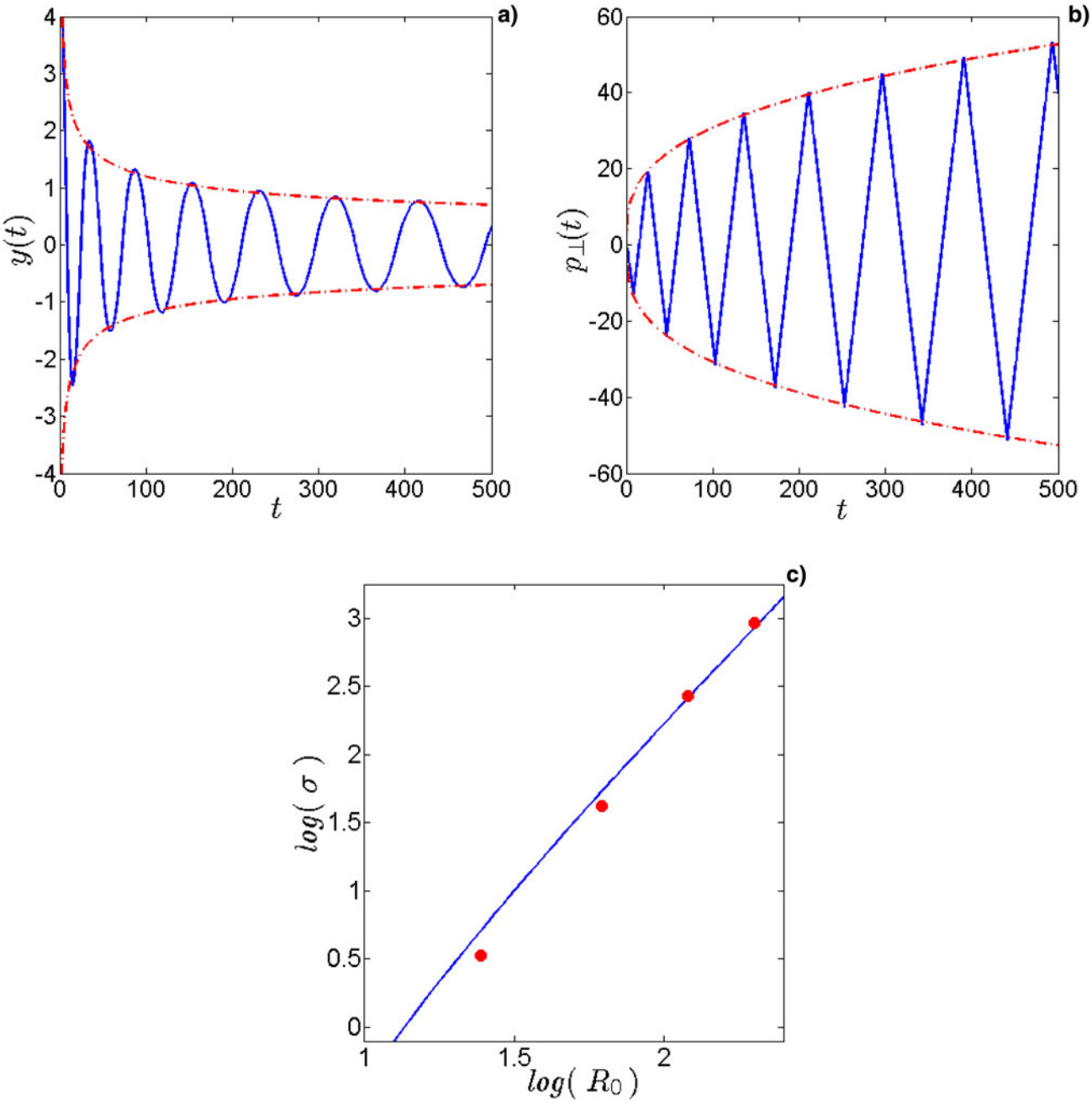


Fig. 2. (Color online) (**a, b**): Envelope approximation functions (red dotted lines) and numerical solutions (blue solid lines) to $\rho(t)$ and $p_\perp(t)$ in the tPWM. The simulation parameters are $F = 4$, $R_0 = 6$, $\gamma_0 = 10$, and $\varepsilon = 0.0004$. (**c**): Trapping cross-sections from analytical prediction (blue line) and numerical simulations (red circles) for $\gamma_0 = 10$, $\varepsilon = 0.0002$, and $F = 6$ in the tPWM.

https://doi.org/10.1017/S0263034614000111 Published online by Cambridge University Press

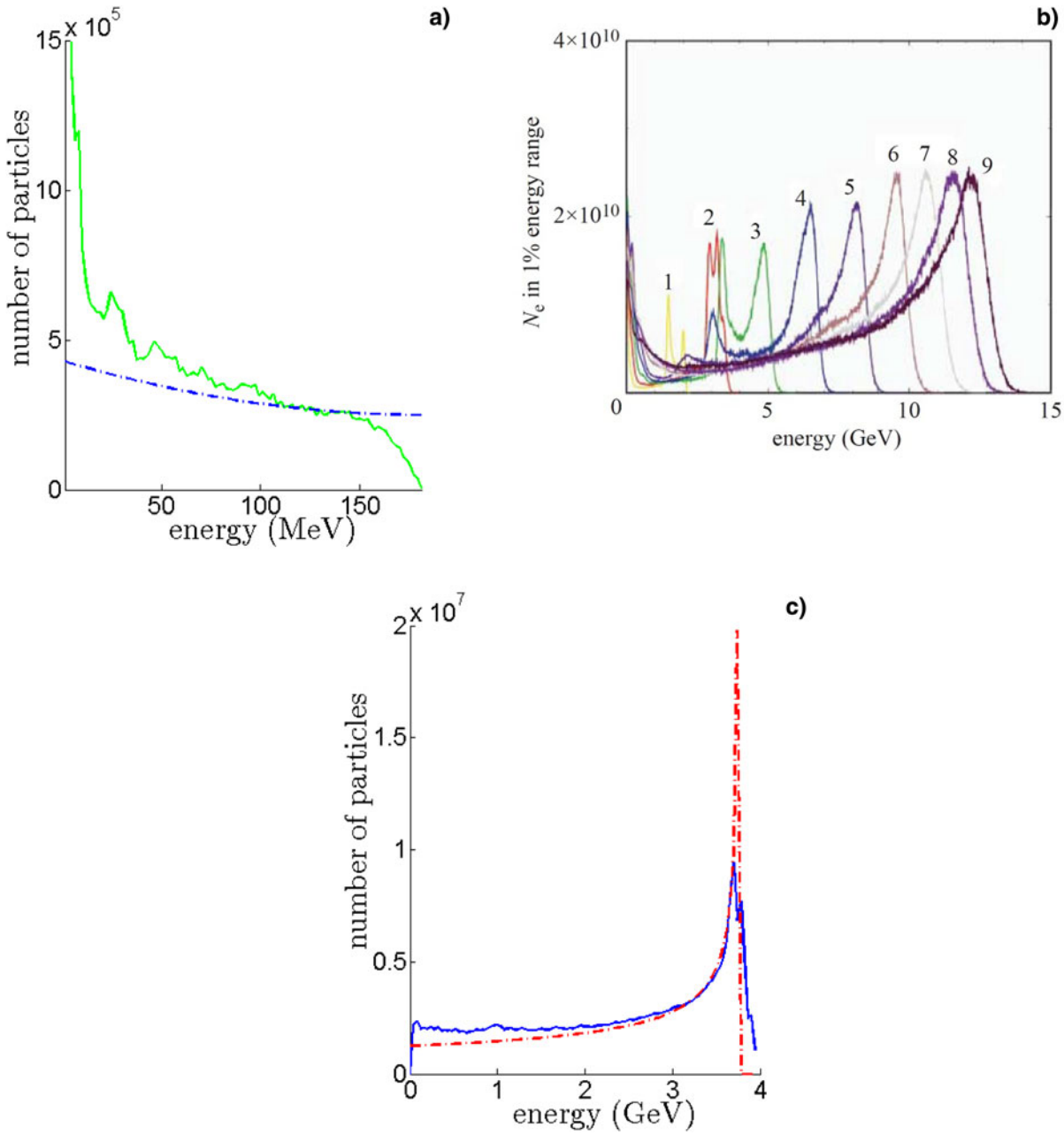


Fig. 3. (Color online) (**a**) Energy distribution for $F = 4$, $R_0 = 4$, $\varepsilon = 0.004$, and $\gamma_0 = 5$ in the tPWM. (**b**) PIC energy spectra taken from Figure 2 in Pukhov and Gordienko (2006). (**c**) Energy spectra in the hPWM for $\gamma_0 = 15$, $R_0 = 8$, and $F = 4$.

To analyze the energy spectra in the tPWM a grid of electrons with zero momentum is placed in front of the bubble (see Figure 1c). Afterward, we solve the equations of motion for every individual particle, i.e., we neglect the interaction of the test electrons. The electron spectra we receive after the electron bunch of trapped electrons has passed its acceleration length look like that in Figure 3a. Here, we don't observe any mono energetic electron beams but a quasi equal distribution of the particles' energy. Apparently, this contradicts pic simulations results as shown in Figure 3b and experiments (Hidding *et al.*, 2006; Mangles *et al.*, 2004; Faure *et al.*, 2004; Lu *et al.*, 2007; Wang *et al.*, 2013).

The monoenergetic spectra can be recovered in the full gradient model. Thus, in the following section we introduce another extension of the PWM that is still analytically treatable and preserves a production of mono energetic electron beams.

3. THE HALF-GRADIENT PWM (HPWM)

In the hPWM, we still replace the perpendicular field $E_\perp$ by the constant field-strength parameter F. In longitudinal

https://doi.org/10.1017/S0263034614000111 Published online by Cambridge University Press

direction, however, we take the full field gradient. The resulting wakefield potential is

$$\Phi(\mathbf{r}) = \frac{\xi^2 + 2F|\rho|}{4} - \frac{R_0^2}{4}. \tag{10}$$

Here, the coordinate $\xi = z - Vt$ always contains the group velocity of the generating laser pulse at the bubble front because the bubble radius is fixed.

If we repeat the simulations of the electron grid in the the hPWM, we get energy spectra similar to those shown in Figure 3. As we see, both PIC-simulated spectra in Figure 3b and our spectrum from a simulation with $F = 4$, $R_0 = 8$, and $\gamma_0 = 15$ exhibit the same structure. Since the hPWM is able to describe the production of monoenergetic electron beams, we calculate the equations of motion

$$\frac{dp_{||}}{dt} = -\Omega + \frac{p_\perp}{\gamma}\frac{\mathrm{sgn}(\rho)}{F}4, \tag{11}$$

$$\frac{dp_\perp}{dt} = -\left(1 + \frac{p_{||}}{\gamma}\right)\frac{\mathrm{sgn}(\rho)F}{4}, \tag{12}$$

$$\frac{d\xi}{dt} = \frac{p_{||}}{\gamma} - V, \quad \frac{d\rho}{dt} = \frac{p_\perp}{\gamma}, \tag{13}$$

with $\Omega = (1 + V)\xi/4$. A simple form for the envelope approximation functions to $\rho(t)$ and $p\perp(t)$ can be found, if we express them in terms of $\tau = t - t_{max}$, where $\xi(t_{max}) = 0$. For the calculation, we follow Kostyukov *et al.* (2010) again and assume $\gamma(t) \ll 2(n_{cr}/n_0)^2$ for all t for the electron energy. Then we have

$$\tilde{p}_{||}(\tau) \approx p_{max} - \frac{(1+V)}{16\gamma_0^2}\tau^2, \tag{14}$$

$$\tilde{p}_\perp(\tau) = \sqrt{\Lambda}(F\tilde{p}_{||}(\tau))^{1/3}, \tag{15}$$

$$\tilde{\rho}(\tau) = \Lambda(F\tilde{p}_{||}(\tau))^{-1/3}, \tag{16}$$

with $\Lambda = 2^{1/3}C(t_n)^2/8\Omega(t_n)^2$. As in the previous section, the constant $C(t_n)$ is used to fit our approximations to numerical simulations.

To compare the approximated functions to the actual amplitude envelope functions, we solve the equations of motion for a single electron with initial conditions $\xi_0^2 + \rho_0^2 = R_0^2$, $\mathbf{p}_0 = 0$, and ξ_0, $\rho_0 > 0$. An exemplary plot of Eqs.(15)–(16) together with numerical solutions is shown in Figure 4. The analytical predictions (red dotted lines) are in good agreement to the simulations (black solid lines) and converge very close to the actual enveloping functions.

Similar to Eqs.(9) the self-injection physics is described by the limiting condition

$$\rho_{\min} = F + \left(4 + F^2 - \frac{2P_0}{\gamma_0^2} - \frac{2\Lambda F^{2/3}}{P_0^{1/3}} + \frac{\Lambda^2}{(FP_0)^{2/3}}\right)^{1/2}, \tag{17}$$

together with

$$P_0^{5/3} - \frac{\gamma_0^2\Lambda F^{2/3}}{3}P_0^{1/3} + \frac{\gamma_0^2\Lambda^2 F^{-2/3}}{3} = 0. \tag{18}$$

The trapping cross-section $\sigma = \pi(R_0^2 - \rho_{\min}^2)$ is constant in

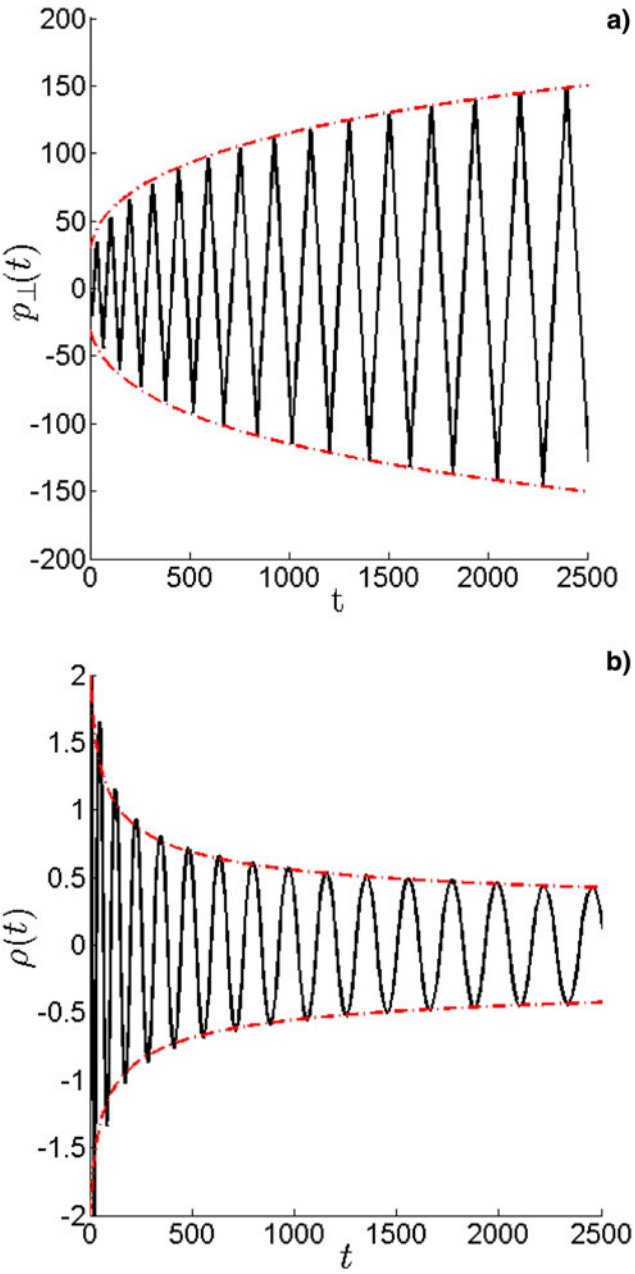


Fig. 4. (Color online) Envelope approximation functions (red dotted lines) and numerical solutions (blue solid lines) to $\rho(t)$ and $p_\perp(t)$ in the hPWM. The simulation parameters are $F = 5$, $R_0 = 10$, and $\gamma_0 = 20$.

https://doi.org/10.1017/S0263034614000111 Published online by Cambridge University Press

time since R_0 and ρ_{min} are. If we plot σ and R_0 into one diagram (see Figure 5a), we observe that for a large range of parameters the accuracy remains in a 1% range.

To get some analytical expressions for the limits of the bubble parameters, the so called trapping conditions, we start again with Eq.(17) and demand $\rho_{min} < R_0$. Then we find after some algebraic transformations

$$\gamma_0^2 < 2P_0(2FR_0 - R_0^2 + 4 - 2\Lambda F^{2/3}P_0^{-1/3} + \Lambda^2 F^{-2/3}P_0^{-2/3})^{-1}, \quad (19)$$

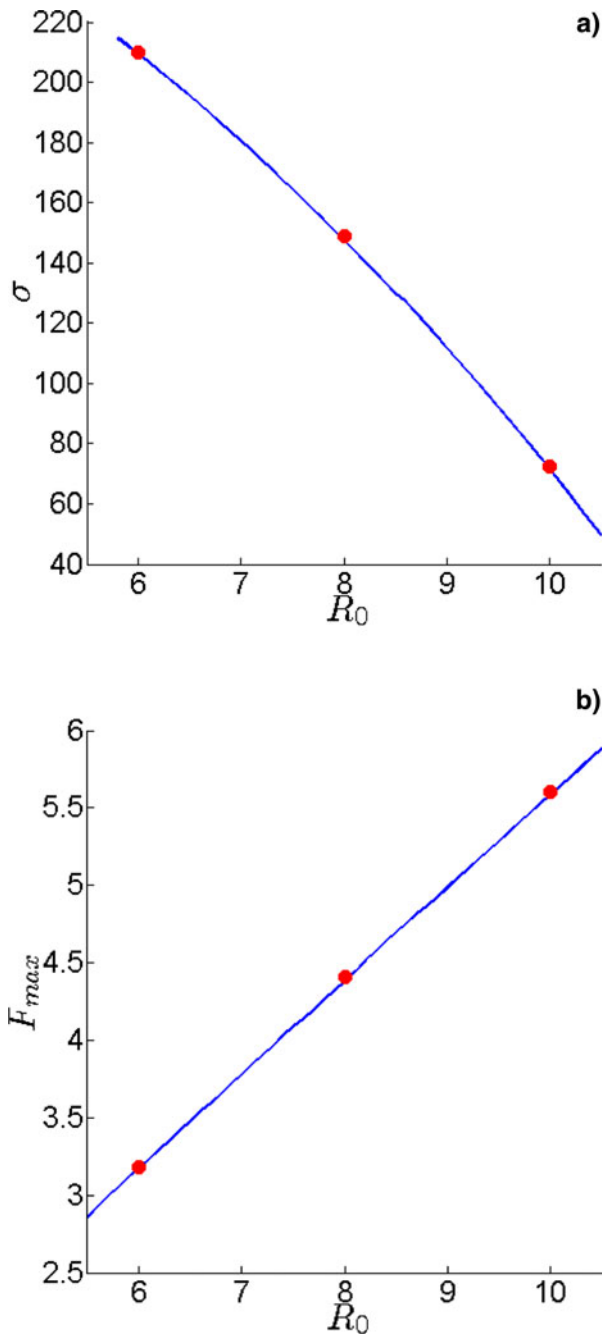


Fig. 5. (Color online) (**a**) Trapping cross-section for $F = R_0/2$. (**b**) Maximal applicable F. Both figures are simulated with $\gamma_0 = 15$ and belong to the hPWM.

as well as

$$F^{5/3} - \frac{\Lambda}{R_0 P_0^{1/3}} F^{4/3} + \left(\frac{2}{R_0} - \frac{R_0}{2} - \frac{P_0}{R_0\gamma_0^2}\right)F^{2/3} - \frac{\Lambda^2}{P_0^{2/3}} < 0. \quad (20)$$

These two equations give us all parameter limits for the hPWM. To demonstrate that these complicated-looking limits are indeed well calculated we plot the largest F for which still an electron is trapped against R_0 in a simulation with one electron (compare former sections) in Figure 5b. As we see the analytical found F_{max} (blue line) fits perfect to our simulations (red dots).

4. CONCLUSION

Summarizing our expansions of the original PWM we note two things. First, we see that it is possible to treat an enhanced interaction between the electron bunch and the bubble potentials. This in turn leads to a growing trapping cross-section and thus to a more efficient self-injection.

Energy spectra in our simulations show that no mono energetic electron beams are generated, if all field gradients are neglected. To fix this problem, we invent a hybrid model that includes the right fields in moving direction and replaces the transverse field gradients by a field strength parameter. In this modification, we observe the generation of mono energetic electron beams and are able to calculate trapping conditions analytically.

The next steps of our work will be to go back to the original full gradient model and to try to find the envelope approximation functions by other means than the introduction of a field strength parameter. If we succeed, we will have a fully-analytical model at hand that predicts trapping cross-sections and trapping conditions and that is directly comparable to computer simulations and experiments.

ACKNOWLEDGMENTS

This work has been supported by the Deutsche Forschungsgemeinschaft via GRK 1203 and SFB TR 18. This work has been supported by EU EuCARD-2 and ANAC2 -projects. This work has been supported by the Ministry of Education and Science of the Russian Federation No 14.B25.31.0008, by RFBR No 13-02-00886, by the Ministry of Science and Education of the Russian Federation, the Russian Federal Program "Scientific and scientific-pedagogical personnel of innovative Russia" (Agreement No 8835).

REFERENCES

Bulanov, S.V., Pegoraro, F., Pukhov, A.M. & Sakharov, A.S. (1997). Transverse-wake wave breaking. *Phys. Rev. Lett.* **78**, 4205–4208.

Corde, S., Stordeur, A. & Malka, V. (2011). Comment on scalings for radiation from plasma bubbles. *Phys plasmas* **18**, 034701.

Faure, J., Glinec, Y., Pukhov, A., Kiselev, S., Gordienko, S., Lefebvre, E., Rousseau, J.-P., Burgy, F. & Malka, V. (2004).

https://doi.org/10.1017/S0263034614000111 Published online by Cambridge University Press

A laser-plasma accelerator producing monoenergetic electron beams. *Nature* **431**, 541–544.

Hidding, B., Amthor, K.-U., Liesfeld, B., Schwoerer, H., Karsch, S., Geissler, M., Veisz, L., Schmid, K., Gallacher, J.G., Jamison, S.P., Jaroszynski, D., Pretzler, G. & Sauerbrey, R. (2006). Generation of quasi-monoenergetic electron bunches with 80-fs laser pulses. *Phys. Rev. Lett.* **96**, 105004.

Kalmykov, S., Yi, S.A., Khudik, V. & Shvets, G. (2009). Electron self-injection and trapping into an evolving plasma bubble. *Phys. Rev. Lett.* **103**, 135004.

Kalmykov, S.Y., Beck, A., Davoine, X., Lefebvre, E. & Shadwick, B.A. (2012). Laser plasma acceleration with a negatively chirped pulse: all-optical control over dark current In the blowout regime. *New J. Phys.* **14**, 033025.

Kalmykov, S.Y., Beck, A., Yi, S.A., Khudik, V.N., Downer, M.C., Lefebvre, E., Shadwick, B.A. & Umstadter, D.P. (2011). Electron self-injection into an evolving plasma bubble: Quasi-monoenergetic laser-plasma acceleration in the blowout regime. *Phys. Plasmas* **18**, 056704.

Kostyukov, I., Nerush, E., Pukhov, A. & Seredov, V. (2009). Electron self-injection in multidimensional relativistic-plasma wake fields. *Phys. Rev. Lett.* **103**, 175003.

Kostyukov, I., Nerush, E., Pukhov, A. & Seredov, V. (2010). A multidimensional theory for electron trapping by plasma wake generated in the bubble regime. *New J. Phys.* **12**, 045009.

Kostyukov, I., Pukhov, A. & Kiselev, S. (2004). Phenomenological theory of laser-plasma interaction in "bubble" regime. *Phys. Plasmas* **11**, 5256–5264.

Lotov, K.V. (2004). Blowout regimes of plasma wakefield acceleration. *Phys. Rev. E* **69**, 046405.

Lu, W. and Huang, C., Zhou, M., Tzoufras, M., Tsung, F.S., Morl, W. B. & Kat- Souleas, T. (2006). A nonlinear theory for multidimensional relativistic plasma wave wake-fields. *Phys. Plasma* **13**, 056709.

Lu, W., Tzoufras, M., Joshi, C., Tsung, F.S., Mori, W.B., Vieira, J., Fonseca, R.A. & Silva, L.O. (2007). Generating muliti-gev electron bunches using single stage laser wf ace. In a 3d nonlin. Regime. *Phys. Rev. ST Accel. Beams* **10**, **061301**.

Mangles, S.P.D., Murphy, C.D., Najmudin, Z., Thomas, A.G.R., Coller, J.L., Dangor, A.E., Divall, E.J., Foster, P.S., Gallacher, J.G., Hooker, S.M., Jaroszynski, D., Langley, A. J., Mori, W.B., Norreys, P.A., Tsung, F.S., Viskup, R., Walton, B. & Krushelnick, K. (2004). Monoenergetic beams of relativis- tic electrons from intense laser-plasma interaction. *Nature* **431**, 535–538.

Pak, A., Marsh, K.A., Martins, S.F., Lu, W., Mori, W.B. & Joshi, C. (2010). Injection and trapping of tunnel-ionized electrons into laser-produced wakes. *Phys. Rev. Lett.* **104**, 025003.

Pukhov, A. & Gordienko, S. (2006). Bubble regime of wake field acceleration: similarity theory and optical scalings. *Phil. Trans. R. Soc. A* **364**, 623–633.

Pukhov, A. & ter Vehn, J. Meyer (2002). Laser wake field acceleration: The highly non-linear broken-wave regime. *Appl. Phys. B* **74**, 355.

Rosenzweig, J.B., Breizman, B., Katsouleas, T. & Su, J.J. (1991). Acceleration and focusing of electrons in two-dimensional nonlinear plasma wake fields. *phys. Rev. A* Vol. **44** R6189R6192.

Thomas, A.G.R. (2010). Scalings for radiation from plasma bubbles. *Phys. Plasmas* **17**, 056708.

Wang, X., Zgadzaj, R., Fazel, N., Li, Z., Yi, S. A., Zhang, Xi, Henderson, W., Chang, Y.-Y., Korzekwa, R., Tsai, H.-E., Andn Pal, C.-H., Quevedo, H., Dyer, G., Gaul, E., Martinez, M., Bernstein, A.C., Borger, T., Spinks, M., Donovan, M., Khudik, V., Shvets, G., Ditmire, T. & Downer, M.C. (2013). Quasi-monoenergetic laser-plasma acceleration of electrons to 2 gev. *Nat. Com.* **4**, 1988.

Whittum, D.H. (1992). Electromagnetic instability of the ion-focused regime. *Phys. Fluids B* **4**, 730.

Yi, S.A., Khudik, V., Kalmykov, S.Y. & Shvets, G. (2011). Hamiltonian analysis of electron self-injection and acceleration into an evolving-plasma bubble. *Plasma phys, Control Fusion* **53**, 014012.

Yi, S.A., Khudik, V., Siemon, C. & Shvetsa, G. (2013). Analytic model of electromagnetic fields around a plasma bubble in the blow-out regime. *Phys. Plasmas* **20**, 013108.

Zeng, M., Davidson, A., Lu, W., An, W., Sheng, Z.-M. & Mori, W. B. (2012). Some observations on trapping in nonlinear multidimensional wakes. *AlP Conf. Proc.* **1507**, 351.

https://doi.org/10.1017/S0263034614000111 Published online by Cambridge University Press