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# Ambiguity Aversion and Underdiversification

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## Abstract

We examine asset allocation decisions under smooth ambiguity aversion when an investor has a prior degree of belief in the domestic capital asset pricing model (CAPM). Different from a Bayesian approach, the investor separately relies on the conditional distribution of returns and on the posterior over parameters to make decisions, rather than on the predictive distribution of returns that integrates priors and likelihood information. We find that in the perspective of U.S. investors, ambiguity aversion generates strong home bias in equity holdings, regardless of beliefs in the CAPM or risk aversion. Results become stronger under regime-switching investment opportunities.

## I. Introduction

Empirical evidence documents that U.S. investors tend to hold a substantially larger proportion of their portfolios in domestic stocks than what is suggested either by the weight of the U.S. market in the value-weighted world equity portfolio or by its sample return moments used in standard mean-variance calculations. Both mean-variance analysis and the composition of commonly used value-weighted world equity indices suggest that a U.S. investor should hold between approximately 50% and 60% in foreign stocks in her portfolio (see, e.g., Lewis (1999)). However, by all existing records and methodologies, foreign stock holdings of a U.S. investor remain as low as 10% (see, e.g., Ahearne, Griever, and Warnock (2004), Warnock (2002)).[1] Previous studies such as Pastor (2000) have

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[1]Bekaert and Wang (2009) extend this evidence based on U.S. data to a large worldwide sample of 27 countries, finding that, as reported since the 1990s, the home bias is a persistent (at most, slowly declining) phenomenon involving all countries. However, Bekaert and Wang document that when home bias indicators are adjusted by market size, Japan is the most home-biased developed country with a bias of close to 90%, the Netherlands is the least biased, emerging markets are more biased than developed markets on average, Europe is the least biased continent, and Asia the most biased continent. Therefore, after adjusting for the underlying scale of the countries involved, U.S. data may even underestimate the average, planetary extent of the phenomenon.

explored the home bias puzzle in a Bayesian framework where a U.S. investor holds uncertain beliefs over the domestic capital asset pricing model (CAPM). The major finding is that for the investor to persistently hold a significantly underdiversified portfolio, her confidence in the domestic CAPM must be extremely strong. In practice, the commonly observed degree of home bias in international equity holdings would rule out mispricing of foreign stocks larger than 2% (in annualized terms) in absolute value from naive domestic CAPM models. Because there is little evidence in the empirical literature supporting such a strong belief, the extent to which incorporating mispricing uncertainty concerning foreign stocks helps explain the observed home bias remains limited. Yet, understanding the sources of the home bias puzzle remains a priority to all finance scholars. The welfare costs of underdiversification in international stock and bond portfolios are conjectured to be massive; for instance, in the case of an average U.S. investor, Lewis (1999) estimates the costs of the forgone gains from diversification to be in the range of 20% to almost double that of lifetime (permanent) consumption.

In this article, we propose a new approach that takes into account uncertain beliefs over an asset pricing model (e.g., the CAPM), parameter uncertainty, and, more importantly, ambiguity aversion. We use the smooth ambiguity utility model proposed by Klibanoff, Marinacci, and Mukerji (KMM) (2005) to characterize ambiguity attitudes. Our quantitative analysis reveals that an ambiguity-averse U.S. investor with a moderate prior degree of belief in the domestic CAPM may rationally hold a substantially underdiversified portfolio relative to the standard mean-variance portfolio. The extent of such underdiversification can fully justify the home bias observed in the data. This result holds true regardless of the precise strength of an investor's prior belief about the mispricing of foreign stocks, that is, whether the level of mispricing uncertainty is high or low, and whether the investor believes a priori that the level of mispricing is negative or positive. In addition, we find that underdiversification in the investor's portfolio holdings cannot be obtained by simply increasing the degree of (constant, relative) risk aversion to somehow surrogate an increase in the aversion to ambiguity. As a result, in contrast to existing studies (e.g., Maenhout (2004), Liu (2011)), in our model the effect of ambiguity aversion is fundamentally different from that of risk aversion.

Different from the standard Bayesian approach, which fully relies on the predictive distribution of returns, our approach implies that the conditional distribution of returns cannot be integrated over the posterior of uncertain parameters to produce a predictive distribution. In other words, optimal portfolio choice separately depends on both the posterior of the parameters and the conditional distribution of returns, instead of a predictive distribution that amalgamates the two as prescribed by standard Bayesian statistics. This distinctive feature is implied by smooth ambiguity preferences, which distinguish the attitude toward pure risk from the attitude toward ambiguity and hence imply the separation of the parameters' posteriors from the conditional distribution of returns.[2]

Yet, even under smooth ambiguity preferences, investors are risk averse in that they dislike any mean-preserving spread of the end-of-period wealth given

[2]On the opposite, a Bayesian approach implicitly assumes that these two attitudes are intertwined and both governed by the curvature of a von Neumann–Morgenstern (vNM) (1944) utility function.

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a portfolio allocation at the beginning of the period. However, investors are also ambiguity averse in the sense that they are averse to any mean-preserving spread of conditional expected utility induced by the posterior distribution of the parameters. In fact, under smooth ambiguity preferences, a concave transformation is imposed on conditional expected utility values to reflect an inherent degree of pessimism. However, such pessimism may concern to an equal degree both domestic and foreign securities; that is, no differential of pessimism is exogenously imposed. The concave function results in a "irreducibility of compound lotteries" result. As a consequence, unlike the Bayesian approach, it is infeasible to hinge asset allocation decisions directly on the predictive distribution of returns: Under ambiguity aversion, investors effectively attach a higher weight on states with lower conditional expected utility values.

Several papers provide empirical evidence that stock markets often experience distinct regimes and show that accounting for regime switching in asset allocation decisions could be important to explain the home bias puzzle (see, e.g., Ang and Bekaert (2002), Guidolin and Timmermann (2008)). However, these papers do not explicitly incorporate parameter uncertainty and asset pricing model uncertainty. Guidolin and Timmermann (2008) use a Markov switching model with two regimes to characterize the time-series dynamics (predictability) in international stock markets and find that distinguishing regimes in asset allocation is helpful in explaining the home bias puzzle. However, at least for short investment horizons, in their paper a strong difference between the strength of underdiversification in the bear (when it is strong) and bull (when it is weaker) regimes remains. In this article, we extend our analysis to regime-switching investment opportunities, building on the Bayesian approach developed by Tu (2010). In particular, we embed smooth ambiguity preferences into a portfolio choice model that simultaneously takes into account parameter uncertainty, mispricing uncertainty, and regime switching. We find that with the addition of parameter uncertainty and asset pricing model uncertainty, the effect of regime switching on international asset allocation is stronger than what had been previously reported under expected utility maximization. Consistent with Guidolin and Timmermann (2008), investors prefer a substantially underdiversified portfolio in the bear regime, which features lower means and higher volatilities and correlations. Moreover, ambiguity aversion also results in a highly underdiversified portfolio in the bull regime. Based on a sensibly calibrated degree of ambiguity aversion, our model produces a weight in U.S. stocks that is close to standard empirical measures documented in the literature (e.g., Bekaert and Wang (2009), Tesar and Werner (1995)).

This article is related to a number of papers studying portfolio choice under ambiguity aversion. However, our focus on the mixed effects of aversion to parameter as well as model uncertainty on underdiversification is unique. One strand of literature has examined the impact of a preference for robustness (see, e.g., Maenhout (2004), (2006), Garlappi, Uppal, and Wang (2007), and Liu (2010)). Garlappi et al. (2007) and Liu (2011) analyze dynamic portfolio choice under incomplete information and ambiguity in the multiple priors framework. Campanale (2011) studies life-cycle portfolio choice when investors engage in learning under multiple priors. Chen, Ju, and Miao (2014) assume that investors have recursive smooth ambiguity preferences and are confronted with model

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uncertainty about return predictability. In a simple application to a portfolio problem with a single risky asset, they discuss the implications of ambiguity aversion for the economic value of return predictability. They find that an ambiguity-averse investor slants her beliefs toward the submodel that delivers the lowest continuation value so that both the myopic and the hedging demands implied by her strategy can be different from those implied by Bayesian strategies. However, all of these papers (except Garlappi et al. (2007)) assume that there is only one risky asset and that there is neither parameter nor asset pricing model uncertainty.

The closest study to ours, at least in terms of empirical objectives and underlying story line, is Uppal and Wang (2003), who study asset allocation with multiple risky assets in an independent and identically distributed (i.i.d.) setting where different levels of ambiguity are attached to the return distributions of domestic versus foreign assets and the parameters are assumed to be perfectly known. They find that differences in the level of ambiguity concerning alternative returns distributions are capable of generating international portfolios that are significantly underdiversified relative to those obtained in the standard mean-variance model. As an application, Uppal and Wang rely on this mechanism to address the home bias of U.S. investors. Our model differs from Uppal and Wang's in several respects. First, Uppal and Wang postulate that ambiguity arises as a result of model misspecification pertaining to the returns processes of the risky assets, whereas in our model ambiguity is captured by uncertainty directly concerning the assumed asset pricing model and the parameters of the distributions of returns. Second, to address the home bias puzzle, Uppal and Wang explicitly assume that a U.S. investor has more knowledge of domestic stock returns than of foreign stocks returns. This feature is achieved in the model by giving additional weight to the marginal distribution of U.S. returns beyond what is revealed by the joint distribution of all the stocks in the investor's portfolio. In contrast to Uppal and Wang, our model does not explicitly assume a different degree of ambiguity on U.S. versus foreign stocks. Third, to derive an analytical solution, Uppal and Wang assume a form of "homothetic robustness." In particular, in their model the degree of ambiguity must be scaled by some function of the investor's lifetime utility to preserve homogeneity of the value function. Even though "homothetic robustness" simplifies calculations, it is imposed ad hoc and fails to result directly from primitive assumptions concerning the vNM (1944) utility function and the smooth ambiguity-inducing transformation. In our article, the solution to the asset allocation problem is based on numerical methods and does not crucially hinge on homogeneity.

The remainder of this article is organized as follows: Section II introduces the smooth ambiguity preferences framework. Section III describes the investment opportunities set and the investor's portfolio choice problem. Section IV calibrates our model to the data and discusses quantitative results. Section V extends the analysis by incorporating regime switching. Section VI concludes. Our numerical algorithms and thought experiments for calibrating the degree of ambiguity aversion are summarized in the Internet Appendix.[3]

[3]The Internet Appendix is available at https://sites.google.com/site/liuhn2008/research/GL_app.pdf.

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## II. Smooth Ambiguity Preferences

Consider an investor with a 1-period investment horizon who allocates wealth between a risk-free asset and a portfolio of risky assets. The investor is assumed to make asset allocation decisions based on available information containing historical returns on all the investable assets and on her prior information. The investor is "small" in the sense that her individual portfolio decisions cannot influence the probability distribution of asset returns. In particular, let $W_t$ denote the investor's current wealth, and $\psi$ the proportion of her wealth invested in the risky portfolio. The investor's next-period wealth is given by:

$$W_{t+1} = W_t\left[(1-\psi)\exp(r_f)+\psi\boldsymbol{\omega}'\exp\left(\tilde{\mathbf{r}}^e_{t+1}+r_f\right)\right],$$

where $r_f$ is the log return on the risk-free asset, $\tilde{\mathbf{r}}^e_{t+1}$ is the vector of the next-period log returns on all the investable assets in excess of $r_f$, and $\boldsymbol{\omega}$ stands for a vector of weights in the risky portfolio (such that $\boldsymbol{\omega}'\boldsymbol{\iota}=1$, where $\boldsymbol{\iota}$ is an appropriately sized vector of ones).

In the smooth ambiguity utility model, the investor's preferences are characterized by the following functional that uses a double expectation operator:

$$(1)\quad V(W_{t+1}) \equiv \int_{\Theta}\phi\left(\int_{S}u(W_{t+1})d\pi_{\boldsymbol{\theta}}\right)d\mu(\boldsymbol{\theta}) \equiv \mathbb{E}_{\mu}\phi(\mathbb{E}_{\pi}u(W_{t+1})),$$

where $u(\cdot)$ is a standard vNM (1944) utility function, and $\phi(\cdot)$ is a monotonically increasing function. Due to parameter and model uncertainty, the investor suffers from subjective uncertainty about the probability distribution of asset returns. In a Bayesian framework, this uncertainty is captured by the multiplicity of the set $\Theta$, within which each element induces a probability distribution $\pi_{\boldsymbol{\theta}}$ over returns. An investor's subjective uncertainty is represented by the posterior $\mu(\boldsymbol{\theta})$ over the parameter space $\Theta$. As usual, the attitude toward pure, quantifiable risk is characterized by the shape of $u(\cdot)$: The investor is risk averse if and only if $u(\cdot)$ is concave. However, the utility function (1) features an additional form of uncertainty, commonly called ambiguity: The investor's attitude toward ambiguity is captured by the shape of the function $\phi(\cdot)$. In particular, a concave $\phi$ implies ambiguity aversion.

A key feature of the class of preferences defined in the utility function (1) is that it achieves a separation between ambiguity, identified as a characteristic of the investor's subjective beliefs, and ambiguity attitude, identified as a characteristic of the investor's tastes. Ambiguity is characterized by uncertainty about the parameter space $\Theta$, and ambiguity attitude by the shape of the function $\phi(\cdot)$. Ambiguity aversion is defined as an aversion to mean-preserving spreads in the distribution over expected utility values induced by the posterior $\mu(\boldsymbol{\theta})$. This distribution gives us probabilities of different evaluations assigned to a portfolio choice $\{\psi,\boldsymbol{\omega}\}$ under different probability measures induced by alternative elements in $\Theta$. In the ambiguity literature, this induced distribution is called a "second-order probability" distribution (see, e.g., Segal (1987), Segal and Spivak (1990), and Gollier (2011)).

An ambiguity-averse investor displays different attitudes toward risk and uncertainty embodied, respectively, in her first- and second-order probabilities. Intuitively, she prefers choices in which the outcomes are more robust to possible

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variation in second-order probabilities. In this article, we show that this preference for robustness (in a loose sense, not necessarily in the definition proposed by Hansen and Sargent (2007)) is able to generate important implications for international portfolio diversification that go well beyond what has been shown in the existing literature. As we show later, such empirical implications that may be useful to explain observed diversification behavior (or a lack thereof) derive from the fact that the smooth ambiguity utility model displays a property of *irreducibility of compound distributions*. That is, the model does not impose or implies a compound reduction involving the posterior distribution $\mu(\boldsymbol{\theta})$ and the conditionals $\pi_{\boldsymbol{\theta}}$s in support of $\mu(\boldsymbol{\theta})$. Only in the special case of $\phi(\cdot)$ assumed to be linear is such reduction feasible so that the investor displays ambiguity neutrality and becomes observationally equivalent to a subjective expected utility maximizer. Thus, this model includes a Bayesian investor who simply maximizes her expected utility as a special case when $\phi(\cdot)$ is linear; therefore, Bayesian portfolio analysis imposes reduction of the first- and second-order probabilities to a unique convolution, which implies that the attitudes toward pure risk and ambiguity (Knightian uncertainty, using the terminology of Epstein and Wang (1994)) become tightly intertwined by construction. In particular, a Bayesian investor will be characterized by the following expected utility functional:

$$\mathbb{E}_t(u(W_{t+1})) = \iint \dots \int_{\tilde{\mathbf{r}}^e_{t+1}} u(W_{t+1}) p(\tilde{\mathbf{r}}^e_{t+1}|\Phi_t) d\tilde{\mathbf{r}}^e_{t+1},$$

where $p(\tilde{\mathbf{r}}^e_{t+1}|\Phi_t)$ is the (joint, multivariate) predictive density of excess asset returns and $\Phi_t$ is the information set available to the portfolio optimizer.[4] In Bayesian portfolio analysis, the predictive density can be obtained by integrating the conditional density of excess asset returns with respect to the posterior distribution:

$$p(\tilde{\mathbf{r}}^e_{t+1}|\Phi_t) = \iint \dots \int_{\Theta} p(\tilde{\mathbf{r}}^e_{t+1}|\boldsymbol{\theta}, \Phi_t) p(\boldsymbol{\theta}|\Phi_t) d\boldsymbol{\theta}, \tag{2}$$

where $p(\tilde{\mathbf{r}}^e_{t+1}|\boldsymbol{\theta}, \Phi_t)$ is the density of returns conditional on the (vector of) parameters $\boldsymbol{\theta}$ and information up to time $t$, and $p(\boldsymbol{\theta}|\Phi_t)$ is the posterior density. The smooth ambiguity utility model precludes the reduction of compounded probabilities in equation (2), as the ambiguity-averse investor displays different attitudes toward the risk embodied in $p(\tilde{\mathbf{r}}^e_{t+1}|\boldsymbol{\theta}, \Phi_t)$ and the uncertainty captured by $p(\boldsymbol{\theta}|\Phi_t)$.[5]

[4] We assume that the investor has no informational advantage over domestic versus foreign stocks, thus ruling out information-based explanations of the home country bias (see Ahearne et al. (2004) for a discussion) arising from differential information flows (e.g., Brennan, Cao, Strong, and Xu (2005)), differential memory capacity constraints (e.g., van Nieuwerburgh and Veldkamp (2009)), or underdiversification caused by long-lasting private information (Liu, Peleg, and Subrahmanyam (2010)).

[5] An ambiguity-averse agent dislikes mean-preserving spreads applied to the distribution of conditional expected utility values induced by $p(\boldsymbol{\theta}|\Phi_t)$ and any selected portfolio choice $\{\psi, \boldsymbol{\omega}\}$. On the contrary, because a linear $\phi(\cdot)$ implies that in the utility function (1) $V(W_{t+1}) \propto \int_{\Theta} \int_{S} u(W_{t+1}) d\pi_{\boldsymbol{\theta}} d\mu(\boldsymbol{\theta}) = \int_{S} u(W_{t+1}) p(\tilde{\mathbf{r}}^e_{t+1}|\Phi_t) d(\tilde{\mathbf{r}}^e_{t+1})$, Bayesian investors will be neutral to mean-preserving spreads over expected utility values induced by the posteriors $\mu(\boldsymbol{\theta})$.

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## III. The Portfolio Choice Problem

We consider a problem in which there are $N+K$ risky assets, $K$ of which are benchmark portfolios, and $N$ of which are nonbenchmark assets. The benchmark portfolios mimic $K$ priced sources of risk in an asset pricing model, for instance, the CAPM. Our model specification for investable assets returns follows Pastor (2000): The multivariate conditional mean (regression) of the nonbenchmark returns as a function of the benchmark returns is given by:

$$\mathbf{r}^e = \mathbf{XB} + \mathbf{U}, \tag{3}$$

where $\mathbf{r}^e$ denotes a vector of excess log returns on nonbenchmark assets, $\mathbf{X} \equiv \begin{bmatrix} \iota_T & \mathbf{F}^T \end{bmatrix}$, $\iota_T$ denotes a $T \times 1$ vector of ones, $\mathbf{F}^T$ denotes a $T \times K$ matrix of excess log returns on the $K$ benchmarks, and the regression coefficient matrix is $\mathbf{B} \equiv \begin{bmatrix} \boldsymbol{\alpha} & \mathbf{B}_2 \end{bmatrix}'$ in which $\boldsymbol{\alpha}$ is $N \times 1$ and $\mathbf{B}_2$ is $N \times K$. If an asset pricing model is correctly specified, the nonbenchmark assets' vector of mispricings, $\boldsymbol{\alpha}$, equals zero. We assume that the rows of the disturbance matrix $\mathbf{U}$ are jointly distributed as i.i.d. normal variates, with a vector of zero means and positive definite $N \times N$ covariance matrix, $\boldsymbol{\Sigma}$. The benchmark returns are assumed to be i.i.d. $\mathbf{F}_t \sim N(\mathbf{E}_F, V_F)$ and independent of the rows of the disturbance matrix $\mathbf{U}$.

To allow for parameter uncertainty, the parameters $\{\mathbf{B}, \boldsymbol{\Sigma}, \mathbf{E}_F, \mathbf{V}_F\}$ are assumed to be unknown. Because our main focus is on mispricing uncertainty, the prior distribution is specified to be uninformative about all the parameters except the vector of mispricings, $\boldsymbol{\alpha}$. The regression coefficients in the multivariate regression equation (3) and the moments of the benchmark returns are assumed to be independent in the prior:

$$p(\mathbf{B}, \boldsymbol{\Sigma}, \mathbf{E}_F, \mathbf{V}_F) = p(\mathbf{B}, \boldsymbol{\Sigma}) p(\mathbf{E}_F, \mathbf{V}_F).$$

In particular, the priors on $\mathbf{B}$ and $\boldsymbol{\Sigma}$ are normal inverted Wishart:

$$\begin{pmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \end{pmatrix} \Big| \boldsymbol{\Sigma} \sim N\left( \begin{pmatrix} \bar{\boldsymbol{\alpha}} \\ \bar{\boldsymbol{\beta}} \end{pmatrix}, \begin{bmatrix} \frac{\sigma_\alpha^2}{s^2} \boldsymbol{\Sigma} & 0 \\ 0 & \boldsymbol{\Omega} \end{bmatrix} \right), \tag{4}$$

$$\boldsymbol{\Sigma} \sim IW(s^2(v - N - 1)\mathbf{I}_N, v), \quad v > N + 1, \tag{5}$$

where $\boldsymbol{\beta} \equiv \text{vec}(\mathbf{B}_2)$ (here "vec" is the operator that stacks the columns of a matrix into a vector), the mispricing vector $\boldsymbol{\alpha}$ and the nonbenchmark assets' betas in $\boldsymbol{\beta}$ are assumed to be independent in the prior, and $\bar{\boldsymbol{\alpha}}$ and $\bar{\boldsymbol{\beta}}$ are the prior means of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, respectively. The prior on $\boldsymbol{\beta}$ is made uninformative by assuming that $\boldsymbol{\Omega}$ is a diagonal matrix with extremely large diagonal terms. The prior on $\boldsymbol{\Sigma}$ is made uninformative by setting the degrees of freedom parameter $v$ to be a large number. In the prior expression (5), the properties of the inverted Wishart distribution imply that the prior mean of $\boldsymbol{\Sigma}$ is $s^2\mathbf{I}_N$. Thus, the marginal prior distribution of the mispricing vector $\boldsymbol{\alpha}$ has a mean of $\bar{\boldsymbol{\alpha}}$ and a covariance matrix of $s^2\mathbf{I}_N$.

In the specification described so far, the only informative prior is about the mispricing of the nonbenchmark assets, which can be allowed for by setting $\sigma_\alpha$ to

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small values. If the benchmark assets are zero-investment portfolios, a correctly specified asset pricing model implies that $\boldsymbol{\alpha}$ is a vector of zeros. When the investor believes a priori that possible values of $\boldsymbol{\alpha}$ center around zero (i.e., $\bar{\boldsymbol{\alpha}}$ is a vector of zeros), the magnitude of $\sigma_\alpha$ represents the investor's prior degree of belief in the pricing model. A large value of $\sigma_\alpha$ indicates a highly dispersed, and therefore weak, belief in the validity of the pricing model. As noted by Pastor (2000), the prior specification above also accommodates the case in which the prior mean of $\boldsymbol{\alpha}$ deviates from zero, $\bar{\boldsymbol{\alpha}} \neq \mathbf{0}$. In such a case, the value of $\sigma_\alpha$ reflects the investor's belief in the accuracy of the forecast of the mispricing parameters/indices. Finally, the prior on the benchmark moments $\mathbf{E}_F$ and $\mathbf{V}_F$ is a standard diffuse prior:

$$p(\mathbf{E}_F, \mathbf{V}_F) \quad \propto \quad |\mathbf{V}_F|^{-(K+1)/2}.$$

Note that the benchmark returns $\mathbf{F}$ are normal conditional on $\mathbf{E}_F$ and $\mathbf{V}_F$ and that the nonbenchmark returns $\mathbf{r}^e$ are normal conditional on $\{\mathbf{B}, \mathbf{F}, \boldsymbol{\Sigma}\}$. Thus, in what follows we take the set of all possible posterior values of $\{\boldsymbol{\alpha}, \boldsymbol{\beta}, \mathbf{F}, \boldsymbol{\Sigma}, \mathbf{E}_F, \mathbf{V}_F\}$ as the set $\Theta$ in the utility function (1). Given a sample of data of length $T$, the inner expectation in the utility function (1) can be rewritten as

$$\mathbb{E}_\pi[u(W_{T+1})|\boldsymbol{\theta}] \quad = \quad \int_{\mathbf{r}^e_{T+1}} \int_{\mathbf{F}_{T+1}} u(W_{T+1}) d\mathbf{F}_{T+1}(\mathbf{E}_F, \mathbf{V}_F) d\mathbf{r}^e_{T+1}(\mathbf{B}, \mathbf{F}, \boldsymbol{\Sigma}),$$

where $\boldsymbol{\theta} \equiv \{\boldsymbol{\alpha}, \boldsymbol{\beta}, \mathbf{F}, \boldsymbol{\Sigma}, \mathbf{E}_F, \mathbf{V}_F\}$, the second integral is taken over possible values of $\mathbf{F}_{T+1}$ given the posterior parameters $\mathbf{E}_F$ and $\mathbf{V}_F$, and the first integral is over possible values of $\mathbf{r}^e_{T+1}$ conditional on $\{\boldsymbol{\alpha}, \boldsymbol{\beta}, \mathbf{F}, \boldsymbol{\Sigma}\}$. $\tilde{\mathbf{r}}^e$ denotes the returns on all investable assets $\tilde{\mathbf{r}}^e \equiv \begin{bmatrix} \mathbf{r}^e & \mathbf{F} \end{bmatrix}$.

The investor chooses portfolio weights $\{\psi, \boldsymbol{\omega}\}$ to maximize her objective in equation (1). For concreteness, we select the vNM (1944) utility function $u(\cdot)$ to have a standard power, isoelastic form:

$$u(W) \quad = \quad \frac{W^{1-\gamma}}{1-\gamma}, \quad 0 < \gamma \neq 1,$$

where $\gamma$ is the coefficient of relative risk aversion. For analytical tractability, we adopt an atemporal version of the smooth ambiguity preferences proposed by Ju and Miao (2012),

$$V(W_{t+1}) \quad = \quad v^{-1}(\mathbb{E}_\mu\{v \circ u^{-1} \mathbb{E}_\pi[u(W_{t+1})]\}), \tag{6}$$

where $v$ is given by:

$$v(x) \quad = \quad \frac{x^{1-\eta}}{1-\eta}, \quad \eta \quad > \quad 0, \neq 1.$$

The parameter $\eta$ is interpreted as the coefficient of ambiguity aversion. The advantage of using this utility function is that it permits homogeneity and thus portfolio weights do not depend on the current wealth.[6] Note that because $\phi = v \circ u^{-1}$ by

[6] We thank the referee for bringing this point to our attention.

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definition, the utility function (6) is ordinally equivalent to the original smooth ambiguity utility function (1). Given the power–power form of preferences, the investor displays ambiguity aversion if and only if $\eta > \gamma$ (Ju and Miao (2012)).

The investor solves the following optimization problem:

$$\max_{\psi,\boldsymbol{\omega}} v^{-1}\left\{\int_{\Theta} v \circ u^{-1}\left(\int_{\mathbf{r}^e_{T+1}}\int_{\mathbf{F}_{T+1}} u(R_{p,T+1})\, d\mathbf{F}_{T+1}\, d\mathbf{r}^e_{T+1}\right) d\mu(\boldsymbol{\theta})\right\}, \tag{7}$$

where $\mu(\boldsymbol{\theta})$ is the joint posterior distribution of the parameters $\boldsymbol{\theta}$, and $R_{p,T+1}$ is the portfolio return and is given by

$$R_{p,T+1} = (1-\psi)\exp(r_f) + \psi\boldsymbol{\omega}'\exp(\tilde{\mathbf{r}}^e_{T+1} + r_f).$$

Because the optimization problem (7) does not admit an analytical solution, we use numerical methods to solve the problem. We first simulate $\{\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\Sigma}, \mathbf{E}_F, \mathbf{V}_F\}$ from their posterior densities and $\mathbf{F}$ from its predictive density. The inner expectation with respect to the joint normal densities that appear in the problem (7) is evaluated by numerical integration. Finally, we use a numerical optimization routine to solve for optimal portfolio weights. In the expected utility case, we rely on the standard Bayesian approach to solve the problem (see Pastor (2000)). In particular, we draw $\mathbf{F}_{T+1}$, and $\mathbf{r}^e_{T+1}$ using a Gibbs sampling procedure so that the optimal portfolio weights can then be found by numerically maximizing

$$\frac{1}{M}\sum_{m=1}^{M}\frac{\left[(1-\psi)\exp(r_f) + \psi\boldsymbol{\omega}'\exp\left(\tilde{\mathbf{r}}^e_{m,T+1} + r_f\right)\right]^{1-\gamma}}{1-\gamma},$$

where $M$ is the number of Gibbs sampler draws and $\tilde{\mathbf{r}}^e_m$ are simulated returns on all investable assets. For practical reasons, we constrain portfolio weights ($\psi$ and $\omega$) to be $\in [0\%, 100\%]$.

## IV. International Asset Allocation

In this section, we study the application of our methodology to a typical international asset allocation problem, in the perspective of a U.S. investor, similarly to Pastor (2000), among others. To gain intuition, we start by focusing on the simplest possible case of $K = N = 1$. $K = 1$ means that the only benchmark asset is the market portfolio.[7] Following Pastor, the proxy for the market portfolio throughout this article is the U.S. value-weighted (VWUS) portfolio. This choice is motivated by previous studies that have tested the global mean-variance efficiency of the U.S. market portfolio (see, e.g., Tesar and Werner (1995)). The monthly market returns are retrieved from the Center for Research in Security Prices (CRSP). We use the world-except-U.S. (WXUS) portfolio as a proxy for all the remaining nonbenchmark assets (equities), which implies $N = 1$. Monthly

[7] Our approach can also be applied to study asset allocation with multiple benchmark portfolios (e.g., the Fama–French (1993)–Carhart (1997) factors).

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returns on the WXUS portfolio are from Morgan Stanley Capital International (MSCI). The 1-month U.S. Treasury bill rate is used as the risk-free rate. The data span the sample period Jan. 1988–Dec. 2009.

It has been well documented that U.S. investors tend to hold domestic equities in a substantially larger proportion than what would be rationally implied by a standard mean-variance model or than the weight of U.S. stocks in the worldwide equity market portfolio. This is often referred to as the "home equity bias puzzle" (see French and Poterba (1991), Tesar and Werner (1995)). Previous studies (e.g., Pastor (2000)) have investigated international asset allocation using a Bayesian approach and found that a strong belief of U.S. investors in the global mean-variance efficiency of the U.S. market portfolio (i.e., the domestic CAPM) would be required to justify the substantial home bias observed in the data.

Our main goal is to examine the impact of ambiguity aversion on optimal weights in domestic and foreign stocks in a risky portfolio and to explore the potential of our model in explaining the home bias of U.S. investors. Equivalently, we attempt to understand i) the conditions that need to be imposed on investors' preferences for a standard, partial equilibrium asset allocation model to generate a significantly underdiversified portfolio that favors U.S. stocks over non-U.S. stocks, and ii) assuming that ambiguity may be part of such a specification of preferences, whether the required degree of ambiguity aversion is plausible given the experimental evidence (see, e.g., Camerer (1999)).

## A. Calibration of Parameters

Based on the sample period Jan. 1991–Dec. 2009, the regression of WXUS portfolio returns on VWUS portfolio returns yields the following results: $\hat{\alpha} = 0.14\%$ per month with a $t$-statistic of 0.77, $\hat{\beta} = 0.89$ with a $t$-statistic of 21.14, $\hat{\sigma}^2 = 8 \times 10^{-4}$, and $R^2 = 0.66$. The prior hyperparameters are specified as follows. The parameters $v$ and $\mathbf{\Omega}$ are set such that the priors on $\mathbf{\Sigma}$ and $\boldsymbol{\beta}$ are uninformative. We use the first 36 months of our data (Jan. 1988–Dec. 1990) to estimate the prior parameter $s^2$. This gives us a sample estimate of the residual variance of 0.002, $s^2 = 0.002$. We vary the values of $\bar{\alpha}$ and $\sigma_\alpha$ to study the comparative static results of various beliefs over the domestic CAPM as well as the expected level of mispricing.

Later in this article, Table 1 reports the optimal weights $\psi$ and the relative weights $\boldsymbol{\omega}$ in the WXUS and VWUS portfolios, respectively, when the prior belief about the expected mispricing $\bar{\alpha}$ takes the value of 0 as implied by the domestic CAPM. Tables 3 and 4 present instead results for $\bar{\alpha} = \pm 0.17\%$ per month when the investor expects some plausible degree of mispricing. The uncertainty about the level of mispricing, $\sigma_\alpha$, takes the following (monthly) values: 0, 0.001, 0.003, 0.006, 0.014, and 0.029. We also verify that a value of $\sigma_\alpha$ in excess of 0.029 leads to insignificant changes in optimal portfolio weights, everything else being equal.[8]

To study the comparative static effects of ambiguity and its interaction with mispricing uncertainty, we vary the degree of ambiguity aversion and the level of

[8] A small $\sigma_\alpha$ indicates a strong belief in the expected level of mispricing. Of course, the case $\sigma_\alpha = \bar{\alpha} = 0$ corresponds to complete confidence in the domestic CAPM.

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mispricing uncertainty. In particular, we set the coefficient of relative risk aversion, $\gamma$, at a typical value of 5 and consider $\eta = 60$ in the smooth ambiguity utility function. Furthermore, we assess a plausible range for $\eta$ based on the magnitude of the ambiguity premium in Ellsberg (1961)-type experiments. The Internet Appendix contains the details of these choices and explains how a parameterization of $\eta$ ranging between 20 and 80 will produce plausible premia. Camerer (1999) suggests that an ambiguity premium on the order of 10%–20% of the expected value of a bet in Ellsberg-type experiments is reasonable. For the power–power preferences specification adopted in this article, our numerical calculation shows that a 10% ambiguity premium, expressed as a percentage of the expected value of a bet, is associated with $\eta = 60$ for relatively small bets.[9] Thus, the value of $\eta$ used in the calibration is plausible.

### B. Implications for International Portfolio Diversification

Previous papers such as Pastor (2000) find that i) when the prior belief $\bar{\alpha} = 0$, the relative weight in foreign, non-U.S. stocks (WXUS) increases with $\sigma_\alpha$, up to some level to which they record asymptotic convergence of the weights, as $\sigma_\alpha \to \infty$, and ii) a strong belief in the global mean-variance efficiency of the U.S. market portfolio, that is, a remarkably low $\sigma_\alpha$, together with $\bar{\alpha} = 0$, is required to generate a substantially underdiversified international portfolio that is consistent with the actual allocation to foreign stocks observed in the equity holdings of U.S. investors. These results are replicated in Panel A of Table 1 under expected utility, that is, when there is no ambiguity aversion.[10] As $\sigma_\alpha$ increases, the investor has stronger doubts as to the validity of the domestic CAPM. In this case, she will rely more on the sample estimate of $\alpha$ to make asset allocation decisions. This results in a larger allocation to foreign stocks, given a positive sample estimate of $\alpha$ as found in our data. Therefore, in a Bayesian framework, a substantial international underdiversification would derive from the fact that investors, who are assumed to be Bayesian portfolio decision makers who are neutral to ambiguity, would perceive very little doubt about the validity of the domestic CAPM and hold a neutral prior on the sign of any mispricings ($\bar{\alpha} = 0$). For instance, when $\sigma_\alpha = 0$, a Bayesian investor would optimally hold 66% of her portfolio in stocks, of which 87% would go to U.S. stocks and only 13% to foreign stocks.

What is the impact of ambiguity aversion on portfolio choice? Panel B of Table 1 presents optimal and relative weights in WXUS and VWUS portfolios under ambiguity aversion. To assess how risk and ambiguity aversion may interact, we also report portfolio weights computed for $\gamma = 30$ in Panel C. Two results are noteworthy. The first result is related to the effect of ambiguity aversion on the size of the optimal risky portfolio. Does ambiguity aversion reduce investment in

[9] In the limit, as shown by KMM (2005), the ex ante welfare $V(W_{t+1})$ exhibits a kinked maxmin expected utility functional à la Gilboa and Schmeidler (1989) when $\eta \longrightarrow \infty$. Because the finance literature has examined the effects of ambiguity under max-min preferences (see, e.g., Epstein and Wang (1994), Garlappi et al. (2007)), in the smooth ambiguity framework any finite $\eta$ may be then considered to be realistic.

[10] The optimal weights computed under expected utility are slightly different from those in Pastor ((2000), Table 1) because we assume a power form for the vNM (1944) felicity function $u(\cdot)$, whereas Pastor uses a standard mean-variance approximation.

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TABLE 1
Optimal and Relative Portfolio Weights: $\bar{\alpha}=0$

Table 1 reports optimal and relative weights in the WXUS and VWUS portfolios when the prior mean of the mispricing $\alpha$ is zero ($\bar{\alpha}=0$). The WXUS portfolio is the world-except-U.S. portfolio provided by Morgan Stanley Capital International. The VWUS portfolio is the U.S. value-weighted market portfolio provided by the Center for Research in Security Prices. The optimal weights are computed for different monthly values of $\sigma_\alpha$. The relative weight is defined as the weight of a portfolio in the optimal risky portfolio. Both the optimal weight in the risky portfolio, $\psi$, and relative weights are constrained to be in [0%, 100%]. KMM stands for Klibanoff, Marinacci, and Mukerji (2005).

| $\sigma_\alpha$ | 0 | 0.001 | 0.003 | 0.006 | 0.014 | 0.029 |
|---|---|---|---|---|---|---|
| *Panel A. Expected Utility: $\gamma=5$* | | | | | | |
| *Optimal Risky Weight* | | | | | | |
| $\psi$ | 66.26 | 67.05 | 68.29 | 69.58 | 70.30 | 70.43 |
| *Relative Weights* | | | | | | |
| WXUS | 13.24 | 24.57 | 41.74 | 59.06 | 68.45 | 70.10 |
| VWUS | 86.76 | 75.43 | 58.26 | 40.94 | 31.55 | 29.90 |
| *Panel B. KMM Preferences: $\gamma=5$, $\eta=60$* | | | | | | |
| *Optimal Risky Weight* | | | | | | |
| $\psi$ | 69.12 | 69.38 | 69.79 | 70.22 | 70.46 | 70.51 |
| *Relative Weights* | | | | | | |
| WXUS | 8.45 | 8.82 | 9.38 | 9.96 | 10.28 | 10.34 |
| VWUS | 91.55 | 91.18 | 90.62 | 90.04 | 89.72 | 89.66 |
| *Panel C. Expected Utility: $\gamma=30$* | | | | | | |
| *Optimal Risky Weight* | | | | | | |
| $\psi$ | 11.03 | 11.17 | 11.37 | 11.59 | 11.71 | 11.73 |
| *Relative Weights* | | | | | | |
| WXUS | 13.24 | 24.57 | 41.73 | 59.04 | 68.42 | 70.08 |
| VWUS | 86.76 | 75.43 | 58.27 | 40.96 | 31.58 | 29.92 |

stocks relative to the risk-free asset? Not necessarily in our case. Comparing Panels A and B, we find that ambiguity aversion can even increase the sum of optimal weights in WXUS and VWUS portfolios, albeit mildly for $\eta=60$. For instance, when $\sigma_\alpha=0$, the total investment in risky assets increases from 66% to 69%. This result is in line with Gollier (2011), who studies a static portfolio choice problem, and with Chen et al. (2014), who investigate dynamic portfolio choice, both under ambiguity. In our model, an ambiguity-averse investor distorts her beliefs about the primitive parameters describing investment opportunities in a pessimistic way. Consistent with Gollier (2011) and Chen et al. (2014), a pessimistic distortion affecting the perceived distribution of returns does not necessarily lead to a monotonically declining demand for stocks.[11]

Second and most important, we observe that an ambiguity-averse U.S. investor optimally chooses an underdiversified portfolio that implies a substantial degree of home bias, regardless of her beliefs about the validity of the domestic CAPM. For example, an investor with $\gamma=5$, $\eta=60$, and a prior belief in the CAPM represented by $\sigma_\alpha=0.029$ per month should invest 90% of her risky portfolio in domestic stocks and 10% in foreign stocks. This is in contrast to the expected utility, Bayesian case in Panel A of Table 1, where an expected utility investor with $\gamma=5$ and $\sigma_\alpha=0.029$ allocates 30% of her risky investment in domestic stocks and 70% in foreign stocks; this latter allocation is in line both with

[11] In a way, this is a well-known result that holds under expected utility preferences and has been extended to the case of ambiguity aversion. For instance, Rothschild and Stiglitz (1971) analytically prove that an increase in the riskiness of an asset's payoffs may not decrease risk-averse investors' demand for this asset.

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standard mean-variance calculations (see, e.g., Lewis (1999)) and with a classical international CAPM estimate. For an ambiguity-averse investor, the optimal weight in the WXUS portfolio still increases as $\sigma_\alpha$ grows, but with a "slope" that is substantially smaller than in the expected utility case. Moreover, the optimal weight in the U.S. market portfolio remains almost unchanged across different values of $\sigma_\alpha$. For instance, the weight assigned to U.S. stocks moves from 92% when $\sigma_\alpha = 0$ to 90% when $\sigma_\alpha = 0.029$. Taken together, both patterns give rise to strong home bias in favor of U.S. stocks, no matter how dispersed the beliefs of an investor are about the pricing model.[12]

Another question that usually arises in the literature studying portfolio choice and ambiguity aversion is: Does ambiguity aversion just replaces aversion under a different label? Our quantitative analysis clearly illustrates that this is not the case. Panel C of Table 1 shows expected utility results for the case $\gamma = 30$. As is standard in the literature, it shows that a more risk-averse investor will invest less in stocks and more in the risk-free asset. However, the relative weight in domestic stocks remains approximately the same as in Panel A, regardless of the degree of risk aversion, that is, just increasing risk aversion cannot simply create home bias in equity portfolio decisions, whereas ambiguity-averse preferences can.

## C. Inspecting the Mechanism

To understand why an ambiguity-averse investor tends to hold an underdiversified portfolio, it is important to investigate how ambiguity aversion alters an investor's beliefs about the distribution of asset returns. Because it is impossible to derive analytical results, in this section we rely on numerical simulations to best illustrate the mechanism through which ambiguity attitudes generate the observed equity home bias by U.S. investors. The smooth ambiguity utility model introduced in Section II implies that an ambiguity-averse investor attaches more weight to states characterized by lower conditional expected utility and this reflects her pessimism. In our model, the value of her conditional expected utility depends on the parameters in $\boldsymbol{\theta} \equiv (\mathbf{B}, \mathbf{F}, \boldsymbol{\Sigma}, \mathbf{E}_F, \mathbf{V}_F)'$ and on the investor's portfolio decisions. Based on this reasoning, in the following, we check the regions of values for these key parameters that are associated with relatively low conditional expected utility values.

Table 2 summarizes the posterior means, minima, and maxima of the parameters in $\boldsymbol{\theta}$, and presents the same set of statistics computed with reference to values that are associated with conditional expected utilities that fall in the lowest decile of the empirical realized distribution of expected utility values. The empirical distribution of expected utility values also depends on the optimal portfolio weights obtained under smooth ambiguity preferences, and thus represent a sort of "fixed point." We first examine the case of perfect confidence in the domestic CAPM ($\bar{\alpha} = \sigma_\alpha = 0$), which is presented in Panel A. The results show that compared to the posterior statistics computed under a Bayesian approach, low expected value

[12] We have also experimented with alternative values of the ambiguity aversion parameter $\eta$ and found qualitatively similar results even with lower values of $\eta$. For instance, when $\eta = 15$, we obtain that an investor should invest 67% of her wealth in U.S. stocks, 22% in foreign stocks, and 11% in cash, which still implies a 75% weight assigned to domestic equities and a considerable home bias.

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TABLE 2
Posterior Statistics of Parameters

Panel A of Table 2 reports the posterior statistics of the parameters describing return distributions, where $\alpha$ is the mispricing of the nonbenchmark asset (world except U.S.), $\beta$ is the asset beta, $\sigma^2$ is the variance of the disturbance term in the capital asset pricing model regression, $F$ is the benchmark asset return, and $E_F$ and $V_F$ are the mean and variance of the benchmark returns. KMM stands for Klibanoff, Marinacci, and Mukerji (2005). Panel A also shows the posterior statistics of the parameters values that are associated with conditional expected utility values in the lowest 10 decile of the empirical distribution of expected utility values. The smooth ambiguity model is used to compute optimal portfolio weights, which give us expected utility conditional on the parameters. Panel B reports the same set of results for $\bar{\alpha}=0$ and $\sigma_\alpha=0.029$. All statistics are expressed in monthly and percentage terms except for $\beta$.

| Statistics | $\alpha$ | $\sigma^2$ | $\beta$ | $F$ | $E_F$ | $V_F$ |
|---|---|---|---|---|---|---|
| *Panel A. $\bar{\alpha}=0$, $\sigma_\alpha=0$* | | | | | | |
| *Posterior Statistics* | | | | | | |
| Mean | 0.00 | 0.07 | 0.90 | 0.54 | 0.55 | 0.19 |
| Min | 0.00 | 0.05 | 0.73 | −17.05 | −0.65 | 0.13 |
| Max | 0.00 | 0.10 | 1.07 | 19.02 | 1.75 | 0.30 |
| *Posterior Statistics for Low Conditional Expected Utility States under KMM* | | | | | | |
| Mean | 0.00 | 0.07 | 0.90 | −5.55 | 0.21 | 0.20 |
| Min | 0.00 | 0.05 | 0.76 | −17.05 | −0.65 | 0.14 |
| Max | 0.00 | 0.10 | 1.04 | 5.01 | 1.03 | 0.28 |
| *Panel B. $\bar{\alpha}=0$, $\sigma_\alpha=0.029$* | | | | | | |
| *Posterior Statistics* | | | | | | |
| Mean | 0.14 | 0.07 | 0.90 | 0.54 | 0.55 | 0.19 |
| Min | −0.58 | 0.05 | 0.72 | −17.05 | −0.65 | 0.13 |
| Max | 0.93 | 0.10 | 1.06 | 19.02 | 1.75 | 0.30 |
| *Posterior Statistics for Low Conditional Expected Utility States under KMM* | | | | | | |
| Mean | 0.13 | 0.07 | 0.90 | −6.00 | 0.25 | 0.20 |
| Min | −0.54 | 0.05 | 0.76 | −17.05 | −0.65 | 0.14 |
| Max | 0.71 | 0.10 | 1.05 | 3.71 | 1.28 | 0.28 |

states implied by the smooth ambiguity model are on average associated with low predicted benchmark returns ($F$), a low posterior mean of benchmark returns ($E_F$), and a marginally higher posterior for the assets' betas and the variance of the benchmark returns ($V_F$). For example, the mean values of $F$ and $E_F$ are $-6\%$ and $0.25\%$, less than their posterior mean counterparts ($0.54\%$ and $0.55\%$) obtained using the Bayesian approach. Moreover, when we allow for uncertainty about the validity of the CAPM (e.g., $\sigma_\alpha=0.029$), the posterior mean of $\alpha$ in low conditional expected value states declines to $0.13\%$, which is slightly lower than its Bayesian counterpart $0.14\%$ (Panel B). These results suggest that ambiguity aversion distorts the investor's beliefs about both domestic and foreign stocks returns in a pessimistic way, in which lower and more volatile benchmark returns are predicted and a lower positive mispricing of foreign stocks is expected when there is doubt about whether the domestic CAPM applies.

Because an ambiguity-averse decision maker overweights states characterized by low expected utility values, this feature of her preferences will crucially alter the relative investment allocations optimally assigned to domestic and foreign stocks. Moreover, our quantitative analysis in Table 2 reveals that the investor's pessimism over foreign stocks tends to dominate her pessimism over domestic stocks.[13] In particular, when the investor holds a belief in an asset pricing model,

[13]Because it is well known from expected utility theory that pessimistic biases in beliefs do not always reduce the demand for the risky asset (see Gollier (2011) and references therein), this also sheds light on the earlier finding that an increase in ambiguity aversion does not necessarily lead to a lower demand for stocks.

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low benchmark returns together with high systematic risk imply remarkably low conditional expected returns of the foreign, ex-U.S. portfolio, all else being equal. In other words, pessimism makes the investor rely more on states where the mean returns of foreign stocks are much lower than those of domestic stocks. This intuition is confirmed in Table 2. Thus, the investor steers her portfolio composition away from foreign stocks and toward domestic stocks, even though the overall sample estimate of $\alpha$ is positive. When an investor holds instead a strong belief in the domestic CAPM, her optimal portfolio is already underdiversified and therefore can naturally hedge against extremely low conditional mean returns of foreign stocks. In this case, ambiguity aversion does not affect the optimal risky portfolio much, as shown in Table 1.

When prior beliefs are centered around nonzero values of $\alpha$, in which case the investor nurses strong doubts about the average level of mispricing of foreign stocks, ambiguity aversion has a similar impact on portfolio allocations as in the case $\bar{\alpha}=0$. This can be seen in Tables 3 and 4, where the prior mean of $\alpha$ takes on two alternative values, $\pm 0.17\%$ per month, respectively. For $\bar{\alpha}=-0.17\%$ and $\sigma_\alpha=0$, the optimal asset allocation under expected utility ($\gamma=5$) involves a zero position in the foreign equity portfolio and 100% of risky investment in the domestic U.S. portfolio, due to the portfolio constraint as well as the maintained strong belief of a negative mispricing that would penalize foreign equity holdings. The aversion to ambiguity makes this asset allocation decision even more conservative: An investor with $\gamma=5$ and $\eta=60$ would like to hold a risky portfolio in which 94% of the optimal risky investment goes to domestic stocks and only 6% to foreign stocks. In this case, ambiguity-aversion-induced pessimism tends to increase the investment in foreign stocks because the initial position in the WXUS portfolio under expected utility would be negative in the absence of the portfolio constraint. By contrast, when the prior belief is centered around a positive $\alpha$, an expected utility investor who is ambiguity neutral would allocate a significant proportion of her wealth to foreign stocks even if $\sigma_\alpha$ is extremely small. Nevertheless, Panel C of Table 4 shows that the effect of ambiguity aversion clearly dominates that of a positive prior mean of $\alpha$, and that the investor with $\eta=60$ would still

TABLE 3

Optimal and Relative Portfolio Weights: $\bar{\alpha}=-0.0017$

Table 3 reports optimal and relative weights in the world-except-U.S. (WXUS) and U.S. value-weighted (VWUS) portfolios when the prior mean of the mispricing $\alpha$ is equal to −0.0017 (in monthly terms). The definition of quantities is the same as in Table 1. KMM stands for Klibanoff, Marinacci, and Mukerji (2005).

| $\sigma_\alpha$ | 0 | 0.001 | 0.003 | 0.006 | 0.014 | 0.029 |
|---|---|---|---|---|---|---|
| *Panel A. Expected Utility: $\gamma=5$* | | | | | | |
| *Optimal Risky Weight* | | | | | | |
| $\psi$ | 65.36 | 65.36 | 65.75 | 68.54 | 70.10 | 70.38 |
| *Relative Weights* | | | | | | |
| WXUS | 0.00 | 0.00 | 5.80 | 45.13 | 65.81 | 69.43 |
| VWUS | 100.00 | 100.00 | 94.20 | 54.87 | 34.19 | 30.57 |
| *Panel B. KMM Preferences: $\gamma=5$, $\eta=60$* | | | | | | |
| *Optimal Risky Weight* | | | | | | |
| $\psi$ | 67.48 | 68.06 | 68.95 | 69.88 | 70.40 | 70.49 |
| *Relative Weights* | | | | | | |
| WXUS | 6.13 | 6.95 | 8.21 | 9.49 | 10.19 | 10.31 |
| VWUS | 93.87 | 93.05 | 91.79 | 90.51 | 89.81 | 89.69 |

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TABLE 4
Optimal and Relative Portfolio Weights: $\bar{\alpha}=0.0017$

Table 4 reports optimal and relative weights in the world-except-U.S. (WXUS) and U.S. value-weighted (VWUS) portfolios when the prior mean of the mispricing $\alpha$ is equal to 0.0017 (in monthly terms). The definition of quantities is the same as in Table 1. KMM stands for Klibanoff, Marinacci, and Mukerji (2005).

| $\sigma_\alpha$ | 0 | 0.001 | 0.003 | 0.006 | 0.014 | 0.029 |
|---|---|---|---|---|---|---|
| *Panel A. Expected Utility: $\gamma=5$* | | | | | | |
| *Optimal Risky Weight* | | | | | | |
| $\psi$ | 71.17 | 71.04 | 70.83 | 70.62 | 70.50 | 70.48 |
| *Relative Weights* | | | | | | |
| WXUS | 79.45 | 77.78 | 75.22 | 72.55 | 71.05 | 70.78 |
| VWUS | 20.55 | 22.22 | 24.78 | 27.45 | 28.95 | 29.22 |
| *Panel B. KMM Preferences: $\gamma=5$, $\eta=60$* | | | | | | |
| *Optimal Risky Weight* | | | | | | |
| $\psi$ | 70.76 | 70.71 | 70.64 | 70.57 | 70.53 | 70.52 |
| *Relative Weights* | | | | | | |
| WXUS | 10.67 | 10.61 | 10.52 | 10.42 | 10.37 | 10.36 |
| VWUS | 89.33 | 89.39 | 89.48 | 89.58 | 89.63 | 89.64 |

like to hold a significantly underdiversified portfolio. For instance, even assuming $\sigma_\alpha=0.029$, an investor ought to hold only 10% of her risky portfolio in foreign equities.

## D. Real-Time Results

The results commented above are based on the full sample from Jan. 1988 to Dec. 2009. Because the end date of the sample falls in the midst of the recent financial crisis that has severely affected the world equity markets between 2008 and 2009, one may wonder whether any of our earlier results may be unduly driven by our use of full-sample estimates. To deal with this concern, we have alternatively implemented two recursive exercises, using both rolling and expanding window approaches, and computed the weights from every optimal monthly portfolio over our sample period. By doing so, we can examine the time-series evolution of the composition of the optimal risky portfolio of an investor and obtain a dynamic view of the comparative static effects of ambiguity aversion. We specify the investors' priors as $\bar{\alpha}=0$ and $\sigma_\alpha=0.029$ throughout the entire sample period.[14] Because our focus is on the recent financial crisis, the recursive sample initially spans 180 months from Jan. 1994 to Dec. 2007. Then the sample rolls over time such that all sample periods are 180 months long in the "rolling-sample" approach, and the sample expands over time through Dec. 2009 in the "expanding-sample" approach. For every portfolio formation month, we compute the optimal weights to the WXUS and VWUS portfolios.

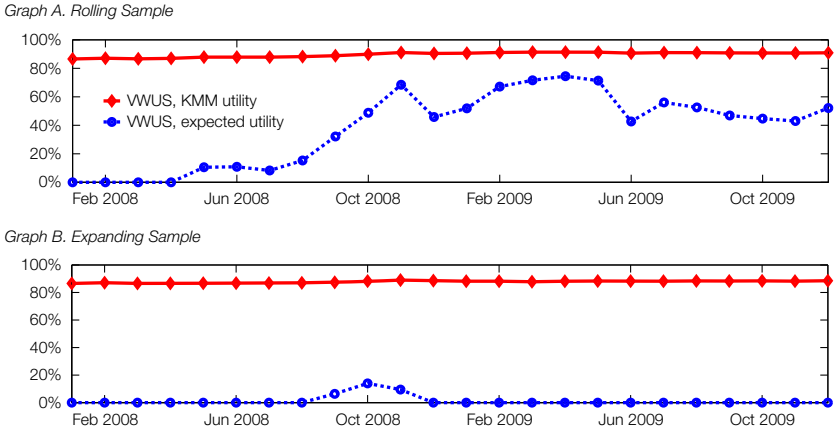
In Figure 1, Graph A plots the relative weights in the VWUS portfolio under expected utility and smooth ambiguity preferences during Jan. 2008–Dec. 2009. In this plot, under expected utility, the relative weight attached to the VWUS portfolio is unstable and varies from 0% to 75% (the circle-dashed line). The remarkably high level of the weight in foreign stocks in the beginning of 2008 is

[14] For $\sigma_\alpha=0$ or low levels of doubt about the domestic CAPM, the expected utility and the smooth ambiguity models generate similar weights, as seen in Table 1. Hence, we perform the exercise by focusing on the case $\sigma_\alpha=0.029$.

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FIGURE 1
Optimal Relative Percentage Weights in Domestic Stocks

Figure 1 plots the optimal relative percentage weights in domestic (U.S. value-weighted) stocks in a rolling-sample experiment (Graph A) and an expanding-sample experiment (Graph B). The sample spans Jan. 2008 to Dec. 2009. The experiments are for both an investor with expected utility and an investor with Klibanoff et al. (2005) (KMM)-type smooth ambiguity utility. The relative risk aversion parameter, $\gamma$, is assumed to be 5, and the ambiguity aversion parameter, $\eta$, is assumed to be 60. The priors of both investors are specified as $\bar{\alpha}=0$ and $\sigma_\alpha=0.029$.




primarily due to the high sample estimate of $\alpha$ before the crisis period. During the crisis, the sample estimate of $\alpha$ decreased in the rolling samples. Nevertheless, the average relative weight in foreign stocks remains higher than 50%. This observation of international diversification tilted toward foreign stocks has of course no correspondence in observed portfolio weights chosen by U.S. investors.

On the contrary, the KMM (2005) smooth ambiguity model generates consistently high and stable weights in domestic stocks (the diamond-solid line in Graph A of Figure 1), which appear to be consistent with the available empirical evidence on this phenomenon. Our results of a degree of underdiversification driven by ambiguity aversion becomes stronger in the expanding sample approach (see Graph B). Because the sample accumulates over time, the sample estimate of $\alpha$ decreases only mildly during the crisis, and thus the relative weight in foreign stocks stays persistently at a high level. Conversely, the impact of ambiguity aversion continues to dominate and the implied relative weights in the VWUS portfolio remain at notably high levels.

## E. Extension to Multiasset Problems

The results above are stark because they are derived within a simple 2-asset toy model in which the risky portfolio choice involves only U.S., domestic stocks versus foreign stocks. However, our analysis can be extended to multiple foreign stock index portfolios. We therefore consider an extension of the asset menu to four major foreign stock portfolios: Japan (JP), Pacific except Japan (PAXJP), United Kingdom (U.K.), and Europe except U.K. (EUXUK), besides the domestic, U.S. market. As before, the first 36 monthly observations are used to estimate the prior parameters. In particular, the parameter $s^2$ in specification (5) is set to the

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average of the diagonal elements of the sample estimate of the residual covariance matrix, which equals 0.0031. We estimate a multivariate regression of excess dollar returns on the PAXJP, JP, EUXUK, and U.K. portfolios onto the excess returns on the VWUS portfolio, using standard MSCI total return data from Jan. 1991 to Dec. 2007.[15] The regression estimates for PAXJP, JP, EUXUK, and U.K. are, respectively,

$$\hat{\boldsymbol{\alpha}} = \begin{bmatrix} 0.09\% & -0.35\% & 0.17\% & 0.03\% \end{bmatrix}' \text{ (per month)},$$
$$\hat{\boldsymbol{\beta}} = \begin{bmatrix} 0.88 & 0.57 & 0.86 & 0.70 \end{bmatrix}'.$$

Throughout the analysis, the prior means for $\boldsymbol{\alpha}$ are set to $\mathbf{0}$ for all foreign stock portfolios. Panel A of Table 5 reports the optimal and relative weights on all stock portfolios for an expected utility investor who is neutral to ambiguity. Similar to the 2-asset case, the investor needs to display a strong belief in the domestic CAPM to justify the low foreign equity holdings observed in the data. In particular, only when $\sigma_\alpha \leq 0.001$ do we obtain that the relative weight of domestic, U.S. stocks exceeds the 80%–85% typically found in the data. For higher values of $\sigma_\alpha$, such a weight declines toward 50%, which is more in line with the implications of a standard international CAPM (or even mean-variance) model.

Panel B of Table 5 shows that the effects of ambiguity aversion ($\gamma = 5$ and $\eta = 60$) are remarkable irrespective of the multiasset nature of the problem solved. Even if an ambiguity-averse investor is willing to invest in foreign stocks such as

TABLE 5
Optimal and Relative Portfolio Weights: Multiple Nonbenchmark Assets

Table 5 reports optimal and relative weights in international stock portfolios when the prior mean of the mispricing $\alpha$ is zero ($\bar{\alpha} = 0$). The PAXJP, JP, EUXUK, U.K., and U.S. portfolios are, respectively, the Pacific-except-Japan, Japan, Europe-except-U.K., U.K., and U.S. portfolios, for which data are provided by Morgan Stanley Capital International.

| $\sigma_\alpha$ | 0 | 0.001 | 0.003 | 0.006 | 0.014 | 0.029 |
|---|---|---|---|---|---|---|
| *Panel A. Expected Utility: $\gamma = 5$* | | | | | | |
| *Optimal Risky Weight* | | | | | | |
| $\psi$ | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| *Relative Weights* | | | | | | |
| PAXJP | 7.20 | 8.55 | 10.48 | 11.75 | 12.70 | 12.89 |
| JP | 7.16 | 3.88 | 0.00 | 0.00 | 0.00 | 0.00 |
| EUXUK | 1.82 | 5.72 | 12.73 | 21.43 | 27.97 | 29.32 |
| U.K. | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| U.S. | 83.82 | 81.84 | 76.78 | 66.82 | 59.33 | 57.79 |
| *Panel B. KMM Preferences: $\gamma = 5$, $\eta = 60$* | | | | | | |
| *Optimal Risky Weight* | | | | | | |
| $\psi$ | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| *Relative Weights* | | | | | | |
| PAXJP | 5.17 | 5.52 | 4.45 | 2.72 | 1.46 | 1.20 |
| JP | 3.77 | 0.52 | 0.00 | 0.00 | 0.00 | 0.00 |
| EUXUK | 1.01 | 2.98 | 4.81 | 7.16 | 8.89 | 9.25 |
| U.K. | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| U.S. | 90.06 | 90.98 | 90.74 | 90.12 | 89.65 | 89.55 |

[15] We work with a prefinancial crisis to emphasize the impact of ambiguity aversion on the extent of international underdiversification, as foreign stocks returned a particularly favorable performance during this period. Because stock returns are expressed in U.S. dollars, excess returns are computed with reference to the 1-month U.S. Treasury bill. Using unhedged, U.S. dollar returns seems to be typical of much literature on the home country bias (see, e.g., Ahearne et al. (2004), Tesar and Werner (1995)).

PAXJP and EUXUK, their weights in the optimal risky portfolio are substantially smaller than those under expected utility. Regardless of the choice of the "doubt parameter" $\sigma_\alpha$, the relative weight in U.S. stocks accounts for over 90%, consistent with our findings in the 2-asset case. Interestingly, ambiguity aversion fails to increase the overall spread of the low holdings (8%–10% at most) of foreign stocks among the available international portfolios.

## V. Portfolio Choice under Regime Switching

In this section, we extend our analysis to an alternative, more realistic statistical model of the dynamics of relations among benchmark and nonbenchmark asset returns that features regime shifts, that is, discontinuities/time instability in either the matrix of loadings $\mathbf{B}$ or the idiosyncratic risk matrix $\mathbf{\Sigma}$. As we discuss in the Introduction, previous studies (e.g., Guidolin and Timmermann (2007), (2008)) have shown that regime switching in investment opportunities is important in portfolio decisions when an investor has long horizons. Guidolin and Timmermann (2008) show that regime switching in itself may generate priced coskewness and cokurtosis factors in an extended 4-moment international CAPM that may go a long way toward explaining the observed home country bias in equity portfolios. However, in Guidolin and Timmermann (2008) the extent of this underdiversification remains considerably stronger in bear than in bull states, and it is questionable whether there is sufficient evidence of strong time variation in the extent of home equity bias in the data. Tu (2010) considers regime switching in a Bayesian framework that accommodates asset pricing model uncertainty as well as parameter uncertainty. Given that investment opportunities are time varying, both frameworks have the potential to generate not only a degree of international portfolio diversification comparable to the data, but also rich time variation in the extent of such underdiversification. Here, we embed smooth ambiguity preferences into the model proposed by Tu to study asset allocation under regime switching and ambiguity aversion. We have two goals in mind. First, because the previous literature has argued that regimes may yield substantial underdiversification in realistic asset pricing models with alternative Markov states, it is important to verify that the equally realistic assumption of ambiguity-averse investors will not cause this result to unravel. Second, we conjecture that the KMM (2005)-type smooth ambiguity has the potential to persistently raise the level of international underdiversification to the levels typically observed, in both bad and good states.

When regime-switching dynamics are taken into account, we consider the multivariate regression of the nonbenchmark returns on the benchmark returns to be

$$\mathbf{r}_t^e = \boldsymbol{\alpha}^{z_t} + \mathbf{B}_2^{z_t}\mathbf{X}_t + \mathbf{u}_t, \tag{8}$$

where $\boldsymbol{\alpha}^{z_t}$ is an $N \times 1$ vector, $\mathbf{B}_2^{z_t}$ is an $N \times K$ matrix of beta exposures to the benchmark factors/portfolios, and $\mathbf{u}_t$ is an $N \times 1$ disturbance vector with zero mean and nonsingular covariance matrix $\mathbf{\Sigma}^{z_t}$. Note that different from the simple, single-state regression framework in equation (3), the mispricing vector $\boldsymbol{\alpha}$, the nonbenchmark assets' beta matrix $\mathbf{B}_2$, and the covariance matrix $\mathbf{\Sigma}$ all depend

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on an unobservable state $z_t$. For simplicity, we assume the simplest possible state space, $Z=\{1,2\}$, and $z_t \in Z$. The transition probability matrix is given by:

$$\mathbf{\Pi} = \begin{pmatrix} P & 1-P \\ 1-Q & Q \end{pmatrix},$$

where $P=\Pr(z_t=1|z_{t-1}=1)$ and $Q=\Pr(z_t=2|z_{t-1}=2)$. It can be shown (see, e.g., Tu (2010)) that the above specification is equivalent to a Markov switching model where excess returns have multivariate normal distributions conditional on each of the two regimes.

To allow for mispricing uncertainty, we assume normal inverted Wishart priors:

$$\begin{aligned} \boldsymbol{\alpha}|\mathbf{\Sigma} &\sim N\left(\mathbf{0}, \frac{\sigma_\alpha^2}{(s^{z_t})^2}\mathbf{\Sigma}^{z_t}\right), \\ \mathbf{\Sigma} &\sim IW((s^{z_t})^2(v-N-1)\mathbf{I}_N, v), \quad v > N+1. \end{aligned}$$

The prior mean of $\mathbf{\Sigma}^{z_t}$ is therefore regime dependent and is given by $(s^{z_t})^2\mathbf{I}_N$. Similar to the case without changing regimes, the value of $(s^{z_t})^2$ is set to the average of the diagonal elements of the sample estimate of $\mathbf{\Sigma}^{z_t}$ in the quantitative analysis that follows. The parameter $\sigma_\alpha$ quantifies the level of mispricing uncertainty in this case. For simplicity, we assume that the prior mean of $\boldsymbol{\alpha}$ is equal to an $N \times 1$ vector of zeros. It follows that $\sigma_\alpha = 0$ corresponds to complete confidence in the domestic CAPM under regime switching. As usual, we impose an uninformative prior on $\mathbf{B}_2^{z_t}$ conditional on each state. The benchmark returns are normally distributed within each regime:

$$\mathbf{F}^{z_t} \sim N(\mathbf{E}_F^{z_t}, \mathbf{V}_F^{z_t}).$$

The priors on $E_F^{z_t}$ and $V_F^{z_t}$ are also diffuse:

$$p(\mathbf{E}_F, \mathbf{V}_F) \propto |\mathbf{V}_F|^{-(K+1)/2}.$$

Following Chib (1996), we let the priors on $P$ and $Q$ be a Dirichlet distribution. Further details on the numerical methods and the statistical framework can be found in the Internet Appendix.

## A. Calibration

We use a Gibbs sampling procedure developed by Tu (2010) to simulate the posterior distributions of the parameters $\{\boldsymbol{\alpha}^{z_t}, \mathbf{B}_2^{z_t}, \mathbf{\Sigma}^{z_t}, \mathbf{E}_F^{z_t}, \mathbf{V}_F^{z_t}\}$. Notice that the predictive distribution of the benchmark returns $\mathbf{F}$ is also regime dependent. Because the state $\{z_t\}$ is unobservable, we follow Chib (1996) and simulate the states $\{z_t\}_{t=1}^T$. The full-sample data are then grouped into two sets, according to the associated states. This procedure is repeated many times, and thus we can draw $\{\boldsymbol{\alpha}^{z_t}, \mathbf{B}_2^{z_t}, \mathbf{\Sigma}^{z_t}, \mathbf{E}_F^{z_t}, \mathbf{V}_F^{z_t}\}$ from their posterior densities and $\mathbf{F}^{z_t}$ from its predictive density in each regime.

To start the Gibbs sampling procedure, we need priors on the parameters. The corresponding hyperparameters are set at the corresponding maximum likelihood estimates of a multivariate regime-switching model of excess returns for both WXUS and VWUS portfolios (but in unreported tests, our results show robustness to other plausible values of the priors). The estimation results suggest

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that the prior beliefs on $P$ and $Q$ are centered around 0.983 and 0.974, respectively, which (after labeling the regimes as bull (regime 1) and bear (regime 2)) realistically suggests that bull states are more persistent (their average duration is 59 months) than bear states (approximately 39 months). In any event, the posterior distributions confirm overwhelming evidence of regimes in our asset return data, as indicated by a standard posterior odds ratio comparing 2- and single-state models. Monthly estimates of mean excess returns on the WXUS and U.S. portfolios are $\{1.59\%, 1.01\%\}$ in the bull regime (regime 1) and $\{0.22\%, 0.44\%\}$ in the bear regime (regime 2), respectively. Moreover, the monthly standard deviation estimates of the WXUS and U.S. portfolios are $\{3.16\%, 2.32\%\}$ in the bull regime and $\{5.61\%, 5.22\%\}$ in the bear regime, and the correlations between the two portfolios in the bull and bear regimes are 0.59 and 0.80, respectively.

## B. Underdiversification Results

As shown by previous studies (Ang and Bekaert (2002), Guidolin and Timmermann (2008), and Tu (2010)), optimal portfolio weights under regime switching differ from those implied by the single-regime models in important ways. Because the primitive parameters describing returns processes depend on regimes, the implied optimal portfolio weights are regime dependent. In addition, parameter uncertainty and mispricing uncertainty also affect the investor's portfolio choice in a Bayesian context (see in particular Tu (2010)). In this article we extend these results to an analysis of the effects of ambiguity aversion on the extent of portfolio underdiversification in each of the two regimes. We solve for the optimal portfolio weights numerically in each regime under expected utility and smooth ambiguity preferences.

Panel A of Table 6 shows the results for the benchmark expected utility case. We find that the weights in the bull regime are significantly different from those in the bear regime. This is because the posterior distributions of the key parameters are markedly different in the two states. However, characterizing the two regimes by looking at their asset allocation implications remains nontrivial as optimal portfolio decisions under power utility remain a highly nonlinear function of the posterior distributions of the parameters (via the implied predictive distributions). First, whereas in the bull regime a Bayesian investor would massively allocate her wealth to stocks, in the bear state the same investor would adopt a very cautious stance. For instance, in the case of $\sigma_\alpha = 0.003$, the optimal risky weight is 100% in the bull state and only 19% in the bear state, which seems appropriate as investment opportunities are favorable in the bull state but poor in the bear state (see Table 7). Moreover, the extent of international diversification is dramatically different in the two regimes. For $\sigma_\alpha = 0.003$, the relative weights in foreign and domestic stocks are 88% and 12%, respectively, in the bull regime, even under a low dispersion of beliefs in the domestic CAPM. However, in the bear regime, we observe a striking weight of 100% in domestic stocks.[16] Therefore,

[16] Interestingly, these results closely mimic those in Guidolin and Timmermann (2008) for the 2-state case and mean-variance preferences, when the investment horizon does not matter, and as such it can be taken to correspond to 1 month, as in our article. This is not surprising because over short horizons, expected power utility preferences are locally mean variance.

TABLE 6

Optimal and Relative Portfolio Weights under Regime Switching: Single Nonbenchmark Asset

Table 6 reports optimal and relative weights in the world-except-U.S. (WXUS) and U.S. value-weighted (VWUS) portfolios with regime switching, where the prior mean of the mispricing $\alpha$ is assumed to be zero in both regimes. KMM stands for Klibanoff, Marinacci, and Mukerji (2005).

| $\sigma_\alpha$ | 0 | 0.001 | 0.003 | 0.006 | 0.014 | 0.029 |
|---|---|---|---|---|---|---|
| *Panel A. Expected Utility: $\gamma = 5$* | | | | | | |
| *A.1. Bull Regime* | | | | | | |
| *Optimal Risky Weight* | | | | | | |
| $\psi$ | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| *Relative Weights* | | | | | | |
| WXUS | 0.00 | 32.88 | 87.92 | 100.00 | 100.00 | 100.00 |
| VWUS | 100.00 | 67.12 | 12.08 | 0.00 | 0.00 | 0.00 |
| *A.2. Bear Regime* | | | | | | |
| *Optimal Risky Weight* | | | | | | |
| $\psi$ | 21.57 | 18.47 | 19.23 | 18.74 | 19.44 | 19.72 |
| *Relative Weights* | | | | | | |
| WXUS | 15.57 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| VWUS | 84.43 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| *Panel B. KMM Preferences: $\gamma = 5$, $\eta = 60$* | | | | | | |
| *B.1. Bull Regime* | | | | | | |
| *Optimal Risky Weight* | | | | | | |
| $\psi$ | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| *Relative Weights* | | | | | | |
| WXUS | 6.33 | 10.39 | 16.58 | 24.00 | 27.99 | 28.52 |
| VWUS | 93.67 | 89.61 | 83.42 | 76.00 | 72.01 | 71.48 |
| *B.2. Bear Regime* | | | | | | |
| *Optimal Risky Weight* | | | | | | |
| $\psi$ | 21.81 | 20.10 | 19.06 | 18.36 | 18.84 | 18.98 |
| *Relative Weights* | | | | | | |
| WXUS | 9.00 | 7.43 | 6.02 | 3.26 | 2.05 | 1.89 |
| VWUS | 91.00 | 92.57 | 93.98 | 96.74 | 97.95 | 98.11 |

regime switching per se (i.e., under expected utility) yields very strong underdiversification in the bear regime but a much weaker bias toward domestic stocks in the bull regime, especially when the investor does not display strong confidence in the domestic CAPM. Implicitly then, an expected utility, short-horizon allocation model under regime switching implies considerable time variation in the extent of the bias. Also in this case, it takes considerable (one may argue, unreasonable) faith in the domestic CAPM (i.e., $\sigma_\alpha \leq 0.003$) for the extent of the home bias in the bull state to reach levels comparable to those found in the data.

To understand these results, we present in Table 7 means of the key parameters computed from the corresponding posterior and predictive densities. Panel A shows the results for the case with complete confidence in the CAPM ($\sigma_\alpha = 0$) and Panel B for the case with high mispricing uncertainty ($\sigma_\alpha = 0.029$). These statistics suggest that returns on the WXUS and VWUS portfolios perform worse in the bear regime than in the bull regime as far as their posterior distributions/properties are concerned. Specifically, in the bear regime posterior variances tend to be high whereas posterior mean returns are low. Thus, in the bear regime the investor will optimally invest less in the stock portfolios but more in the risk-free asset. When we allow for high mispricing uncertainty, there is a notable difference in the posterior means of $\alpha$ between the two regimes. In particular, the posterior mean

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TABLE 7

Posterior Statistics of Parameters: Regime-Switching Model

Panel A of Table 7 reports the posterior means of the parameters describing return distributions with regime switching, where the notation is the same as in Table 2. Panel A also shows the posterior means of the parameters that are associated with expected utility values in the lowest 10 decile of the empirical distribution of expected utility, conditioning on regimes. Panel B reports the same set of results for $\bar{\alpha}=0$ and $\sigma_\alpha=0.14$. All statistics are expressed in monthly and percentage terms except for $\beta$. KMM stands for Klibanoff, Marinacci, and Mukerji (2005).

| Parameters | $\alpha$ | $\sigma^2$ | $\beta$ | $F$ | $E_F$ | $V_F$ |
|---|---|---|---|---|---|---|
| *Panel A. $\sigma_\alpha=0$* | | | | | | |
| *Posterior Mean* | | | | | | |
| Bull regime | 0.00 | 0.07 | 0.91 | 1.04 | 1.04 | 0.06 |
| Bear regime | 0.00 | 0.11 | 0.88 | 0.17 | 0.18 | 0.29 |
| *Posterior Mean of Low Conditional Expected Value States (lower 10th percentile) under KMM* | | | | | | |
| Bull regime | 0.00 | 0.07 | 0.92 | −1.41 | 0.66 | 0.06 |
| Bear regime | 0.00 | 0.12 | 0.90 | −6.24 | −0.65 | 0.32 |
| *Panel B. $\sigma_\alpha=0.029$* | | | | | | |
| *Posterior Mean* | | | | | | |
| Bull regime | 0.74 | 0.06 | 0.77 | 1.03 | 1.05 | 0.06 |
| Bear regime | −0.22 | 0.11 | 0.88 | 0.15 | 0.16 | 0.30 |
| *Posterior Mean of Low Conditional Expected Value States (lower 10th percentile) under KMM* | | | | | | |
| Bull regime | 0.66 | 0.06 | 0.81 | −3.01 | 0.88 | 0.06 |
| Bear regime | −0.26 | 0.12 | 0.90 | −2.45 | −0.81 | 0.32 |

of $\alpha$ is estimated at 0.74% at a monthly frequency in the bull regime whereas it is −0.22% in the bear regime. Thus, the poor performance of the foreign stock portfolio in the bear regime leads to a remarkably underdiversified portfolio held by a typical U.S. investor, as observed in Table 6.

Turning to the effects of ambiguity aversion, Panel B of Table 6 presents optimal and relative weights for an investor with smooth ambiguity preferences. The results reveal that the effects of ambiguity aversion can be understood in a similar way as in the single-regime model. In the bull regime, the optimal risky portfolio is significantly underdiversified, even when mispricing uncertainty is high. For instance, when $\sigma_\alpha=0.003$, in the bull regime the relative weights for the ambiguity-averse investor are 17% in foreign stocks and 83% in domestic stocks, which differ substantially from the results under expected utility for the same level of mispricing uncertainty. Even in the presence of high mispricing uncertainty (e.g., $\sigma_\alpha=0.029$), the relative weight in domestic stocks is over 70% of the optimal risky portfolio. Interestingly, in the bear regime the complete home bias toward domestic stocks observed under expected utility does not obtain in the ambiguity aversion case: The ambiguity-averse investor still invests in the WXUS portfolio, albeit a small amount. This is because under expected utility the optimal weight in the WXUS portfolio would be negative without the nonnegative constraint. In such a case, adding ambiguity aversion to the problem would increase the investment in the foreign stock portfolio to a small and positive level. The mechanism driving the observed underdiversification in the regime-switching model is similar to the single-regime case, as shown in Panel B of Table 7 for a large $\sigma_\alpha$. For instance, in the bull regime low conditional expected utility states are associated with pessimistic beliefs over the distributions of both benchmark and nonbenchmark returns. However, when we allow for a dispersed prior belief in the domestic CAPM, pessimism over foreign stocks dominates and results in an underdiversified portfolio.

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Finally, we reexamine the case with multiple nonbenchmark assets.[17] We compute the optimal weights in PAXJP, JP, EUXUK, U.K., and VWUS in each regime. This asset menu is similar to the one investigated by Guidolin and Timmermann (2008). Table 8 reports results for an expected utility investor. Our findings regarding the impact of regime switching on the extent of international diversification are similar to the case of only one nonbenchmark asset. The bull and bear regimes feature significantly different international diversification patterns. However, under multiple nonbenchmark assets, we can further compute the composition of the optimal foreign stock portfolio. Panel A shows that in the bull regime with high mispricing uncertainty ($\sigma_\alpha = 0.014$ or higher), the investor prefers to hold more in PAXJP than other foreign stocks. This is because the posterior mean of PAXJP's $\alpha$ is the highest among all the nonbenchmark assets in the bull regime. In Panel A, the observed low weight in the U.S. stock portfolio, even when the Bayesian investor has complete confidence in the domestic CAPM ($\sigma_\alpha = 0$), is due to the risky portfolio constraint, $\psi \in [0\%, 100\%]$.[18] In addition, Panel B reveals that the effect of the bear regime on international diversification is robust to multiple nonbenchmark assets: In this case, even under a very low

TABLE 8

Optimal and Relative Weights under Regime Switching and Expected Utility: Multiple Nonbenchmark Assets

Table 8 reports optimal and relative weights in international stock portfolios under the assumption of regime switching and expected utility maximization. The PAXJP, JP, EUXUK, U.K., and U.S. portfolios are, respectively, the Pacific-except-Japan, Japan, Europe-except-U.K., U.K., and U.S. portfolios, for which data are provided by Morgan Stanley Capital International. The prior means of the mispricing $\alpha$ are zeros in both regimes. The optimal weights are computed conditional on each regime.

| | Expected Utility: $\gamma = 5$ | | | | | |
|---|---|---|---|---|---|---|
| $\sigma_\alpha$ | 0 | 0.001 | 0.003 | 0.006 | 0.014 | 0.029 |
| *Panel A. Bull Regime* | | | | | | |
| *Optimal Risky Weight* | | | | | | |
| $\psi$ | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| *Relative Weights* | | | | | | |
| PAXJP | 0.00 | 18.00 | 38.26 | 62.79 | 79.27 | 82.49 |
| JP | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| EUXUK | 70.92 | 78.55 | 61.74 | 37.21 | 20.73 | 17.51 |
| U.K. | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| U.S. | 29.08 | 3.45 | 0.00 | 0.00 | 0.00 | 0.00 |
| *Panel B. Bear Regime* | | | | | | |
| *Optimal Risky Weight* | | | | | | |
| $\psi$ | 42.17 | 36.86 | 36.61 | 36.40 | 36.40 | 36.40 |
| *Relative Weights* | | | | | | |
| PAXJP | 13.40 | 12.64 | 0.62 | 0.00 | 0.00 | 0.00 |
| JP | 11.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| EUXUK | 6.56 | 12.01 | 7.98 | 0.00 | 0.00 | 0.00 |
| U.K. | 4.46 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| U.S. | 63.61 | 75.35 | 91.40 | 100.00 | 100.00 | 100.00 |

[17] For the sake of brevity, we do not report the estimation results of the Markov switching model with multiple nonbenchmark assets. These results are available from the authors.

[18] Our numerical exercise shows that relaxing this constraint will generate a substantially more "home-biased" portfolio with a large relative weight in U.S. stocks, when $\sigma_\alpha$ is extremely small. But such a risky portfolio involves high leverage, which is largely in excess of 100% of the investor's initial wealth and thus may not be considered entirely realistic.

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degree of doubt about the domestic CAPM, the extent of home bias is extreme, that is, close or equal to 100%.

Table 9 shows that the impact of ambiguity aversion is considerable in the case of a richer and more realistic asset menu, especially in the bull regime. For instance, when $\sigma_\alpha = 0.003$ and in the bull regime, an ambiguity-averse investor will invest 85% of her initial wealth in U.S. stocks and 15% in Pacific (excluding Japanese) stocks. When the magnitude of mispricing uncertainty becomes large (e.g., $\sigma_\alpha = 0.029$), the relative weight in U.S. stocks is still over 70%. The resulting underdiversification is therefore remarkable. Moreover, under ambiguity aversion, the optimal portfolio in the bear state also features considerable underdiversification. Implicitly, a smooth ambiguity model implies not only that the home bias in equity allocations is large, but also that this will hardly fluctuate over bull and bear market phases, contrary to the implications of a Bayesian model under regime switching.

TABLE 9

Optimal and Relative Weights under Regime Switching and Smooth Ambiguity Preferences: Multiple Nonbenchmark Assets

Table 9 reports optimal and relative weights in international stock portfolios under the assumption of regime switching and smooth ambiguity preferences. The PAXJP, JP, EUXUK, U.K., and U.S. portfolios are, respectively, the Pacific-except-Japan, Japan, Europe-except-U.K., U.K., and U.S. portfolios, for which data are provided by Morgan Stanley Capital International. The prior means of the mispricing $\alpha$ are zeros in both regimes. The optimal weights are computed conditional on each regime. KMM stands for Klibanoff, Marinacci, and Mukerji (2005).

| | KMM Preferences: $\gamma = 5, \eta = 60$ | | | | | |
|---|---|---|---|---|---|---|
| $\sigma_\alpha$ | 0 | 0.001 | 0.003 | 0.006 | 0.014 | 0.029 |
| *Panel A. Bull Regime* | | | | | | |
| *Optimal Risky Weight* | | | | | | |
| $\psi$ | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| *Relative Weights* | | | | | | |
| PAXJP | 0.00 | 0.00 | 15.32 | 22.17 | 27.38 | 28.46 |
| JP | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| EUXUK | 10.05 | 11.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| U.K. | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| U.S. | 89.95 | 89.00 | 84.68 | 77.83 | 72.62 | 71.54 |
| *Panel B. Bear Regime* | | | | | | |
| *Optimal Risky Weight* | | | | | | |
| $\psi$ | 40.59 | 35.47 | 35.27 | 35.04 | 34.90 | 34.87 |
| *Relative Weights* | | | | | | |
| PAXJP | 2.16 | 7.95 | 2.04 | 0.00 | 0.00 | 0.00 |
| JP | 11.53 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| EUXUK | 0.00 | 1.47 | 6.82 | 8.23 | 7.83 | 7.75 |
| U.K. | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| U.S. | 86.31 | 90.58 | 91.14 | 91.77 | 92.17 | 92.25 |

## VI. Conclusion

In this article, we propose a new approach to explain the substantial amount of underdiversification in international portfolio decisions that has been reported by the empirical literature. We do this by incorporating a simple and yet powerful smooth ambiguity-aversion channel in a standard framework traditionally employed to examine asset allocation under parameter uncertainty and uncertain beliefs in an asset pricing model. Our approach is developed by embedding the KMM (2005) smooth ambiguity model in the Bayesian framework. Unlike in the standard Bayesian portfolio analysis, the ambiguity-averse investor in our model

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not only relies on the conditional distribution of returns and the posterior over the parameters in making asset allocation decisions, but she also places more weight on states with low conditional expected utility values when making decisions. We show that this feature of her preferences crucially alters the relative weights optimally assigned to different stocks.

We use this approach to study international asset allocation from the perspective of a U.S. investor who has a prior degree of belief in the global mean-variance efficiency of the U.S. market portfolio. We find that ambiguity aversion can lead to strong home bias in equity holdings, regardless of an investor's belief in the domestic CAPM. Moreover, we extend our analysis to regime-switching investment opportunities following Ang and Bekaert (2002), Guidolin and Timmermann (2008), and Tu (2010). We find that, consistent with the earlier literature, regime switching generates substantial underdiversification in the bear regime, when nonbenchmark and benchmark assets tend to have relatively low returns, and both volatilities and correlations are higher than on average. However, underdiversification is milder in the bull regime and tends to obtain only when a Bayesian investor places extremely strong faith in the conditional CAPM. On the contrary, the model with ambiguity aversion persistently generates international underdiversification that is typically observed in the data, in both bull and bear regimes and regardless of the degree of the investor's faith in the global mean-variance efficiency of the U.S. market portfolio.

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