J. Plasma Physics (2006), *vol.* 72, *part* 6, *pp.* 1061–1064. © 2006 Cambridge University Press
doi:10.1017/S002237780600523X Printed in the United Kingdom

Numerical simulation of inductive measurement of critical current density in HTS: high-performance method for solving shielding current equations

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(Received 11 August 2005 and accepted 29 November 2005)

Abstract. A numerical code has been developed for analyzing the time evolution of the shielding current density in the high-temperature superconductor (HTS) bulk. In the code, the shielding current equations are discretized by means of the element-free Galerkin method and the resulting nonlinear system is solved by use of the pseudo Newton method with a variable relaxation factor. By means of the code, the shielding current density is calculated in cases where the ac current flows in a coil placed just above the HTS. The voltage induced in the coil is evaluated as a function of time and its spectral analysis is performed. The results of computations show a qualitative agreement with Mawatari's experimental results.

1. Introduction

Since the critical current density is one of the most important parameters for applications of the high-temperature superconductor (HTS), a non-destructive and contactless method has been desired for measuring the critical current density. While applying the ac current $I_c(t) = I_0 \sin 2\pi f t$ in a coil placed just above the HTS bulk, Mawatari et al. observed the harmonic voltages induced in the coil [1]. As a result, they found that the square root of the third-harmonic voltage is roughly proportional to the coil current. On the basis of the result, they have developed the inductive method for measuring the critical current density of the HTS bulk.

The purpose of the present study is to develop a numerical code for analyzing the time evolution of the shielding current density in the HTS bulk and to numerically reproduce Mawatari's experimental results.

2. Governing equations

Let us assume that a HTS bulk of square cross section Ω and of thickness b is exposed not only to the dc homogeneous magnetic flux density $B_{dc}\boldsymbol{e}_z$ but to the magnetic flux density generated by the ac coil current $I_c(t)$ (see Fig. 1).

Under the multiple-thin-layer approximation, the state of the shielding current density can be expressed by the state vector, $\boldsymbol{S}(\boldsymbol{x}, t) \equiv [S_1(\boldsymbol{x}, t), \ldots, S_M(\boldsymbol{x}, t)]^T$

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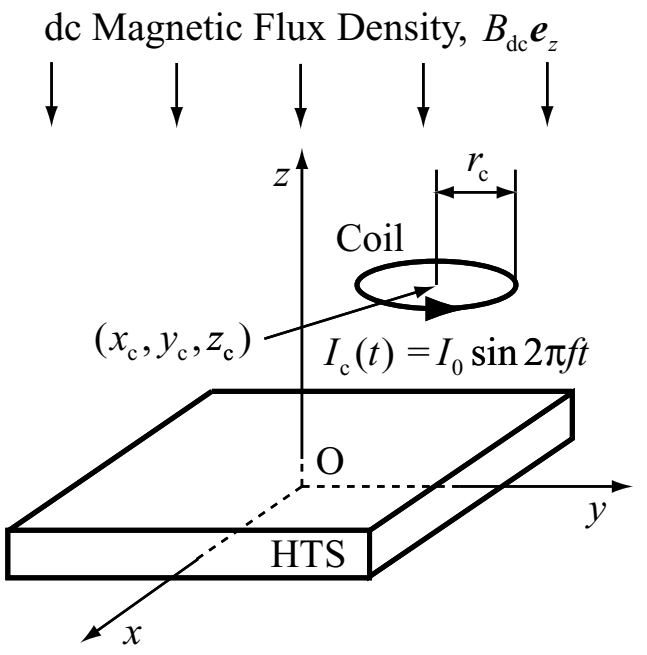


Figure 1. The schematic view of the inductive measurement for the critical current density.

where $S_p(\boldsymbol{x}, t)/\epsilon$ is the z-component of the current vector potential in the pth layer. In addition, M and ϵ are the total layer number and the layer thickness, respectively. In the following, $\boldsymbol{x}$ and $\boldsymbol{x}'$ denote the position vectors on the xy plane.

The time evolution of the state vector is governed by the following integro-differential equation [2,3]:

$$\mu_0 \frac{\partial}{\partial t}\left(\hat{Q} + \frac{1}{\epsilon}\hat{I}\right)\boldsymbol{S} + \frac{\partial}{\partial t}\boldsymbol{B} + \boldsymbol{E} = \boldsymbol{0}, \tag{2.1}$$

where the M-dimensional vectors, $\boldsymbol{B}(\boldsymbol{x}, t)$ and $\boldsymbol{E}(\boldsymbol{x}, t)$, originate from the applied magnetic field and the electric field, respectively. In addition, $\hat{I}$ denotes an identity operator and $\hat{Q}$ is the operator defined by

$$\hat{Q}\boldsymbol{S} \equiv \int_\Omega Q(|\boldsymbol{x} - \boldsymbol{x}'|)\boldsymbol{S}(\boldsymbol{x}', t)\, d\boldsymbol{x}'. \tag{2.2}$$

Here $Q(r)$ is the $M \times M$ matrix whose (i, j) element is given in [3]. The initial and the boundary conditions to (2.1) are assumed as follows: $\boldsymbol{S} = \boldsymbol{0}$ at $t = 0$ and on $\partial\Omega$. For the J–E constitutive relation, we adopt the flux-flow and flux-creep model [2,3,5].

Throughout the present study, values of parameters are fixed as follows: b = 2 mm, M = 4, B_{dc} = 2 T, f = 1 kHz, r_c = 2 mm and (x_c, y_c, z_c) = (0 mm, 0 mm, 3 mm). Furthermore, the side length $2a$ of Ω is fixed as $2a$ = 40 mm.

3. Numerical method

After applying the backward Euler method to (2.1), the resulting equation together with the boundary conditions constitutes the boundary-value problem at each

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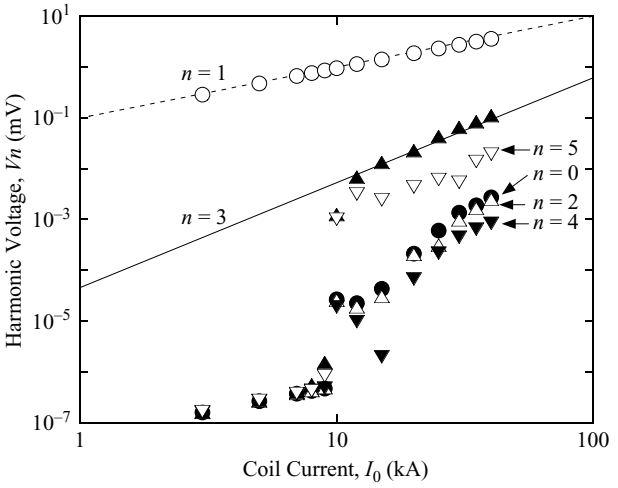


Figure 2. The harmonic voltage V_n as a function of the coil current I_0.

time step. From the standard manner of the element-free Galerkin method [4], the boundary-value problem can be discretized as the following nonlinear system [3]:

$$\boldsymbol{G}(\boldsymbol{s},\boldsymbol{\lambda}) \equiv \begin{pmatrix} W & B \\ B^T & O \end{pmatrix} \begin{pmatrix} \boldsymbol{s} \\ \boldsymbol{\lambda} \end{pmatrix} + \Delta t \begin{pmatrix} \boldsymbol{e}(\boldsymbol{s}) \\ \boldsymbol{0} \end{pmatrix} - \begin{pmatrix} \boldsymbol{u} \\ \boldsymbol{0} \end{pmatrix} = \boldsymbol{0}. \tag{3.1}$$

Here the nodal vectors, $\boldsymbol{s}$ and $\boldsymbol{e}(\boldsymbol{s})$, correspond to $\boldsymbol{S}$ and $\boldsymbol{E}$ at the nth time step, respectively. In addition, $\boldsymbol{u}$ is a nodal vector irrelevant to $\boldsymbol{s}$ and $\boldsymbol{\lambda}$ denotes the Lagrange multiplier. Moreover, the real symmetric matrix W is calculated from the Hermitian matrix $\hat{Q} + (1/\epsilon)\hat{I}$, whereas the matrix B is evaluated from the shape functions of the moving least-squares approximation [4]. Apparently, the nodal vectors, $\boldsymbol{s}$ and $\boldsymbol{\lambda}$, are the solution of the nonlinear system $\boldsymbol{G}(\boldsymbol{s},\boldsymbol{\lambda}) = \boldsymbol{0}$. In this way, the initial-boundary-value problem of (2.1) is reduced to the problem in which the nonlinear system $\boldsymbol{G}(\boldsymbol{s},\boldsymbol{\lambda}) = \boldsymbol{0}$ is solved at each time step.

As mentioned in our earlier work [2], the successive substitution method is inapplicable to the solution of the nonlinear system (3.1). The main reason for this is that both the relaxation factor and the time step size must be sufficiently small to obtain the converged solution. On the other hand, when the deaccelerated Newton method (DNM) is applied to the solution of the system, the recursive relation has several equilibrium points and the approximate solution moves only among them. In other words, the iteration of the DNM falls into a limit cycle so that the converged solution cannot be obtained. Indeed, the oscillation of the residual norm is shown in [5, Fig. 1]. For these reasons, the DNM has been modified so as to have a high performance in convergence.

In the modified method, the solution of the nonlinear system $\boldsymbol{G}(\boldsymbol{s}) = \boldsymbol{0}$ is iteratively determined by using the following three steps.

(1) The linear system $J(\boldsymbol{s}^{(k-1)})\delta\boldsymbol{s} = -\boldsymbol{G}(\boldsymbol{s}^{(k-1)})$ is solved to determine $\delta\boldsymbol{s}$.

(2) The minimum m of the non-negative integer, which satisfies $\|\boldsymbol{G}(\boldsymbol{s}^{(k-1)} + \beta^m\delta\boldsymbol{s})\| < \|\boldsymbol{G}(\boldsymbol{s}^{(k-1)})\|$, is determined.

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(3) The relaxation factor γ is evaluated by $\gamma = \beta^m$ and, subsequently, the approximate solution is corrected by use of the underrelaxation: $\boldsymbol{s}^{(k)} = \boldsymbol{s}^{(k-1)} + \gamma\delta\boldsymbol{s}$.

Here, the superscript (k) represents an iteration number label and β is a constant such that $0 < \beta \leqslant 1$. In addition, $J(\boldsymbol{s})$ is the Jacobian matrix of the vector function $\boldsymbol{G}(\boldsymbol{s})$. The above three steps are repeated until the approximate solution $\boldsymbol{s}^{(k)}$ converges to a certain vector. By means of the modified method, we can solve the nonlinear system (3.1) stably and accurately.

4. Numerical results

On the basis of the above method, we have developed the numerical code for analyzing the time evolution of the shielding current density. By means of the code, the shielding current density is calculated in cases where the ac current $I_c(t)$ flows in a coil placed just above the HTS bulk. The voltage induced in the coil is evaluated as a function of time and its spectral analysis is performed by using the fast Fourier transform (FFT) algorithm.

The dependence of the harmonic voltage on the coil current I_0 is shown in Fig. 2. The $n=3$ mode is abruptly excited above $I_0 \cong 12\,\mathrm{kA}$ until it becomes the second most dominant mode. Furthermore, the $n=3$ mode develops more rapidly than any other mode. Figure 2 further indicates that the square root of the third-harmonic voltage is roughly proportional to the coil current I_0 for $I_0 > 12$ kA. These tendencies agree well with the experimental results obtained by Mawatari et al. [1]

5. Conclusions

We have investigated the numerical method for analyzing the time evolution of the shielding current density in the HTS. After being discretized, the initial-boundary-value problem of the shielding current equations is reduced to the problem in which the nonlinear system $\boldsymbol{G}(\boldsymbol{s}) = \boldsymbol{0}$ is solved at every time step. A high-performance method for solving the system has been proposed and the method is implemented as numerical code for calculating the shielding current density. As an application of the code, the inductive method for measuring the critical current density has been numerically simulated. Conclusions obtained in the present study are summarized as follows. As the coil current is increased, the third-harmonic voltage is abruptly excited above a certain value of the current and it becomes the second most dominant mode. Furthermore, the square root of the third-harmonic voltage is roughly proportional to the coil current I_0. From these results, we conclude that Mawatari's experimental results have been reproduced numerically.

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