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Adaptive impedance controller design for flexible-joint electrically-driven robots without computation of the regressor matrix

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SUMMARY
To the best of our knowledge, this is the first paper focus on the adaptive impedance control of robot manipulators with consideration of joint flexibility and actuator dynamics. Controller design for this problem is difficult because each joint of the robot has to be described by a fifth-order cascade differential equation. In this paper, a backstepping-like procedure incorporating the model reference adaptive control strategy is employed to construct the impedance controller. The function approximation technique is applied to estimate time-varying uncertainties in the system dynamics. The proposed control law is free from the calculation of the tedious regressor matrix, which is a significant simplification in implementation. Closed-loop stability and boundedness of internal signals are proved by the Lyapunov-like analysis with consideration of the function approximation error. Computer simulation results are presented to demonstrate the usefulness of the proposed scheme.

KEYWORDS: Adaptive impedance control; Flexible-joint electrically driven robot; FAT.

1. Introduction

Many industrial applications require robot manipulators to interact with the environment compliantly. Several control strategies have been suggested for this purpose. The well-known impedance control proposed by Hogan[1] is capable of giving a unified approach for controlling the robot in both the free space and constrained motion phase. Following the work of Hogan, several studies have been proposed.[2–7] However, most of these schemes ignore the effects coming from actuator dynamics and joint flexibility. As demonstrated in Good *et al*.[8] and Tarn *et al*.,[9] in order to construct tracking controllers for high-velocity movement or highly varying loads, actuator dynamics constitutes an important part of the complete robot dynamics. Besides, the effects of joint flexibility may cause instability or resonance in the robot system, especially for a force controlled robot.[10,11] Therefore, the effects of actuator dynamics and joint flexibility should be taken into account in the control of a robot interacting compliantly with the environment if high performance is required. However, under these conditions, the cascade structure in the robot dynamics increases the complexity of the system model greatly, and a fifth-order differential equation should be employed for describing a single link. Hence, the controller design problem becomes extremely difficult.

Some controllers for constrained robots with consideration of joint flexibility and actuator dynamics were presented in Krishnan.[12,13] However, these strategies are only valid for the case when exact knowledge of the robot dynamics is available. Several works can be found for constrained robots with consideration of model uncertainties incorporating actuator dynamics[14–22] or joint flexibility.[23–31] Chien and Huang[44] proposed a backstepping-like procedure incorporating the model reference adaptive control for robots with consideration of both actuator dynamics and joint flexibility for operating in the free space. To our best knowledge, no work has been reported for the adaptive control of rigid-link flexible-joint electrically driven (RLFJED) robots operating in the compliant motion applications.

In this paper, we would like to propose a function approximation technique (FAT)[32–46] based adaptive impedance controller for RLFJED robots. The design follows a backstepping-like procedure with the support of the model reference adaptive control to overcome the complex cascade structure in the robot dynamics, and the controller incorporates the impedance strategy to give a unified approach for controlling the robot in both the free space and constrained motion phase. The controller does not need to calculate the regressor matrix,[47,48] which is required in the conventional robot adaptive design. The FAT is employed to deal with the time-varying uncertainties, which plays an important role in the construction of the update laws. The closed-loop stability and boundedness of internal signals are proved by using the Lyapunov-like technique with consideration of the approximation error. In addition, the upper bounds of tracking errors in the transient state are also derived. Simulation results are presented to demonstrate the usefulness of the proposed scheme.

This paper is organized as follows. Section 2 derives the proposed adaptive impedance controller in detail. Section 3 presents simulation results of the control of a 2D constrained

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robot using the proposed controller. Section 4 concludes the paper.

2. Main Results

The dynamics of a RLFJED robot interacting with the environment can be described by

$$\mathbf{D}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{K}(\boldsymbol{\theta} - \mathbf{q}) - \mathbf{J}_a^{\mathrm{T}}\mathbf{F}_{ext}, \tag{1a}$$

$$\mathbf{J}\ddot{\boldsymbol{\theta}} + \mathbf{B}\dot{\boldsymbol{\theta}} + \mathbf{K}(\boldsymbol{\theta} - \mathbf{q}) = \mathbf{H}\mathbf{i}, \tag{1b}$$

$$\mathbf{L}\mathbf{i} + \mathbf{R}(\mathbf{i}, \dot{\boldsymbol{\theta}}) = \mathbf{u}, \tag{1c}$$

where $\mathbf{q} \in \Re^n$ is the vector of link angles, $\boldsymbol{\theta} \in \Re^n$ is the vector of actuator angles, $\mathbf{i} \in \Re^n$ is the motor armature current vector, $\mathbf{u} \in \Re^n$ is the vector of control input voltage, $\mathbf{D}(\mathbf{q})$ is the $n \times n$ inertia matrix, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$ is an n vector of centrifugal and Coriolis forces, and $\mathbf{g}(\mathbf{q})$ is the gravity vector. $\mathbf{J}_a(\mathbf{q}) \in \Re^{n\times n}$ is the Jacobian matrix, which is assumed to be nonsingular in its working space. $\mathbf{F}_{\text{ext}} \in \Re^n$ is the external force exerted by the end effector on the environment, and it is assumed to be precisely measurable by a wrist force sensor. $\mathbf{J}$, $\mathbf{B}$, and $\mathbf{K}$ are $n \times n$ constant diagonal matrices of the actuator inertia, damping, and joint stiffness, respectively. $\mathbf{H} \in \Re^{n\times n}$ is an invertible constant diagonal matrix characterizing the electro-mechanical conversion between the armature current and torque, $\mathbf{L} \in \Re^{n\times n}$ is a constant diagonal matrix of the electrical inductance, and $\mathbf{R}(\mathbf{i}, \dot{\boldsymbol{\theta}}) \in \Re^{n\times n}$ represents the effect of the electrical resistance and the motor back-EMF.

The mapping from the control input $\mathbf{u}$ in Eq. (1c) to the link trajectory $\mathbf{q}$ in Eq. (1a) is in a special cascade form. A backstepping-like procedure is employed to facilitate the construction of the control strategy. The procedure starts from designing a desired trajectory for the transmission torque $\boldsymbol{\tau}_d$ so that Eq. (1a) behaves like a given target impedance as defined in the traditional impedance control if $\boldsymbol{\tau}_t = \mathbf{K}(\boldsymbol{\theta} - \mathbf{q})$[49,50] in Eq. (1a) converges to $\boldsymbol{\tau}_{td}$, a desired $\boldsymbol{\tau}_t$. Then, a desired armature current $\mathbf{i}_d$ is designed such that Eq. (1b) can give proper performance. Finally, the control input is constructed in Eq. (1c) to ensure convergence of $\mathbf{i}$ to $\mathbf{i}_d$. In this design, we would like to consider the case when the precise forms of $\mathbf{D}$, $\mathbf{C}$, $\mathbf{g}$, $\mathbf{L}$, and $\mathbf{R}$ are not available and an adaptive impedance controller is to be designed without using the regressor matrix. Since the uncertainties are time-varying, traditional adaptive schemes do not apply. A FAT-based strategy is thus proposed to update the time-varying uncertainties so that an adaptive controller is realizable.

Define $\mathbf{J}_t = \mathbf{J}\mathbf{K}^{-1}$, $\mathbf{B}_t = \mathbf{B}\mathbf{K}^{-1}$, and $\bar{\mathbf{q}}(\dot{\mathbf{q}}, \ddot{\mathbf{q}}) = \mathbf{J}\ddot{\mathbf{q}} + \mathbf{B}\dot{\mathbf{q}}$ then Eqs. (1a) and (1b) can be represented as

$$\mathbf{D}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}_t - \mathbf{J}_a^{\mathrm{T}}\mathbf{F}_{ext}, \tag{2}$$

$$\mathbf{J}_t\ddot{\boldsymbol{\tau}}_t + \mathbf{B}_t\dot{\boldsymbol{\tau}}_t + \boldsymbol{\tau}_t = \mathbf{H}\mathbf{i} - \bar{\mathbf{q}}(\dot{\mathbf{q}}, \ddot{\mathbf{q}}). \tag{3}$$

It is often more convenient to describe the dynamics of the robot in the Cartesian space when interacting with the environment. Let $\mathbf{x} \in \Re^n$ be the position vector of the end effector in the Cartesian space and we may rewrite Eq. (2) as

$$\mathbf{D}_x(\mathbf{x})\ddot{\mathbf{x}} + \mathbf{C}_x(\mathbf{x}, \dot{\mathbf{x}})\dot{\mathbf{x}} + \mathbf{g}_x(\mathbf{x}) = \mathbf{J}_a^{-\mathrm{T}}\boldsymbol{\tau}_t - \mathbf{F}_{\text{ext}}, \tag{4}$$

where

$$\begin{aligned}\mathbf{D}_x(\mathbf{x}) &= \mathbf{J}_a^{-\mathrm{T}}\mathbf{D}(\mathbf{q})\mathbf{J}_a^{-1},\\ \mathbf{C}_x(\mathbf{x}, \dot{\mathbf{x}}) &= \mathbf{J}_a^{-\mathrm{T}}\big[\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{D}(\mathbf{q})\mathbf{J}_a^{-1}\dot{\mathbf{J}}_a\big]\mathbf{J}_a^{-1},\\ \mathbf{g}_x(\mathbf{x}) &= \mathbf{J}_a^{-\mathrm{T}}\mathbf{g}(\mathbf{q}).\end{aligned} \tag{5}$$

In the traditional impedance control, all system parameters are given and a controller is designed so that the closed-loop system behaves like the target impedance

$$\mathbf{M}_i(\ddot{\mathbf{x}} - \ddot{\mathbf{x}}_d) + \mathbf{B}_i(\dot{\mathbf{x}} - \dot{\mathbf{x}}_d) + \mathbf{K}_i(\mathbf{x} - \mathbf{x}_d) = -\mathbf{F}_{\text{ext}}, \tag{6}$$

where $\mathbf{x}_d \in \Re^n$ is the desired trajectory of the end effector in the Cartesian space, and $\mathbf{M}_i \in \Re^{n\times n}$, $\mathbf{B}_i \in \Re^{n\times n}$, and $\mathbf{K}_i \in \Re^{n\times n}$ are diagonal matrices representing the desired apparent inertia, damping, and stiffness, respectively. Equation (6) implies that, in the free space tracking phase, i.e., $\mathbf{F}_{ext} = \mathbf{0}$, the system trajectory converges to the desired trajectory asymptotically. On the other hand, in the constrained motion phase, it represents a stable second-order LTI system driven by the external force. Conceptually, we may regard the impedance controller as a model reference controller. The target impedance plays the role of the reference model and the impedance controller drives the robot to follow the dynamics of the reference model in both the free space tracking and constrained motion phases. Since some of the system parameters are assumed to be unavailable in this paper, the traditional impedance controller is not feasible. Instead of Eq. (6), let us consider a new target impedance

$$\mathbf{M}_i(\ddot{\mathbf{x}}_i - \ddot{\mathbf{x}}_d) + \mathbf{B}_i(\dot{\mathbf{x}}_i - \dot{\mathbf{x}}_d) + \mathbf{K}_i(\mathbf{x}_i - \mathbf{x}_d) = -\mathbf{F}_{\text{ext}}, \tag{7}$$

where $\mathbf{x}_i \in \Re^n$ is the state vector of the target impedance (7). If an adaptive impedance controller can be designed such that $\mathbf{x} \to \mathbf{x}_i$ asymptotically, then the new target impedance (7) converges to Eq. (6) as desired.

Define signal vectors $\mathbf{s} = \dot{\mathbf{e}} + \boldsymbol{\Lambda}\mathbf{e}$ and $\mathbf{v} = \dot{\mathbf{x}}_i - \boldsymbol{\Lambda}\mathbf{e}$, where $\mathbf{e} = \mathbf{x} - \mathbf{x}_i$ is the state error, and $\boldsymbol{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n)$ with $\lambda_i > 0$ for all $i = 1, \ldots n$. Rewrite Eq. (4) in the form

$$\mathbf{D}_x\dot{\mathbf{s}} + \mathbf{C}_x\mathbf{s} + \mathbf{g}_x + \mathbf{D}_x\dot{\mathbf{v}} + \mathbf{C}_x\mathbf{v} = \mathbf{J}_a^{-\mathrm{T}}\boldsymbol{\tau}_t - \mathbf{F}_{\text{ext}}. \tag{8}$$

2.1. Controller design for known robot

Suppose $\mathbf{D}_x$, $\mathbf{C}_x$, and $\mathbf{g}_x$ are known, and we may design a proper control law such that $\boldsymbol{\tau}$ follows the trajectory below

$$\boldsymbol{\tau}_t = \mathbf{J}_a^{\mathrm{T}}(\mathbf{g}_x + \mathbf{D}_x\dot{\mathbf{v}} + \mathbf{C}_x\mathbf{v} - \mathbf{K}_d\mathbf{s} + \mathbf{F}_{\text{ext}}), \tag{9}$$

where $\boldsymbol{K}_d$ is a positive definite matrix. Substituting Eq. (9) into Eq. (8), the closed-loop dynamics becomes $\mathbf{D}_x\dot{\mathbf{s}} + \mathbf{C}_x\mathbf{s} + \mathbf{K}_d\mathbf{s} = \mathbf{0}$. Define a Lyapunov-like function candidate $V = \frac{1}{2}\mathbf{s}^{\mathrm{T}}\mathbf{D}_x\mathbf{s}$. Its time derivative along the trajectory of system dynamics can be computed as $\dot{V}_1 = -\mathbf{s}^{\mathrm{T}}\mathbf{K}_d\mathbf{s} \le 0$. It is easy to prove that $\mathbf{s}$ is uniformly bounded and square integrable, and $\dot{\mathbf{s}}$ is also uniformly bounded. Hence, $\mathbf{s} \to \mathbf{0}$ as $t \to \infty$, or we may say that $\mathbf{e} \to \mathbf{0}$ as $t \to \infty$. To make the actual $\boldsymbol{\tau}_t$ converge to the perfect $\boldsymbol{\tau}_t$ in Eq. (9), let us consider the

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reference model

$$\mathbf{J}_r\ddot{\boldsymbol{\tau}}_r + \mathbf{B}_r\dot{\boldsymbol{\tau}}_r + \mathbf{K}_r\boldsymbol{\tau}_r = \mathbf{J}_r\ddot{\boldsymbol{\tau}}_{td} + \mathbf{B}_r\dot{\boldsymbol{\tau}}_{td} + \mathbf{K}_r\boldsymbol{\tau}_{td}, \quad (10)$$

where $\boldsymbol{\tau}_r \in \Re^n$ is the state vector of the reference model and $\boldsymbol{\tau}_{td} \in \Re^n$ is the vector of desired states that is equivalent to the perfect $\boldsymbol{\tau}_t$ in Eq. (9). Matrices $\mathbf{J}_r \in \Re^{n\times n}$, $\mathbf{B}_r \in \Re^{n\times n}$, and $\mathbf{K}_r \in \Re^{n\times n}$ are selected such that $\boldsymbol{\tau}_r \to \boldsymbol{\tau}_{td}$ exponentially. Define $\bar{\boldsymbol{\tau}}_{td}(\dot{\boldsymbol{\tau}}_{td}, \ddot{\boldsymbol{\tau}}_{td}) = \mathbf{K}_r^{-1}(\mathbf{B}_r\dot{\boldsymbol{\tau}}_{td} + \mathbf{J}_r\ddot{\boldsymbol{\tau}}_{td})$ to be a function of $\ddot{\mathbf{x}}$, $\dot{\mathbf{F}}_{\text{ext}}$, and $\ddot{\mathbf{F}}_{\text{ext}}$. Here, we may rewrite Eqs. (3) and (10) in the state space form as

$$\dot{\mathbf{x}}_p = \mathbf{A}_p\mathbf{x}_p + \mathbf{B}_p\mathbf{H}\mathbf{i} - \mathbf{B}_p\bar{\mathbf{q}}, \quad (11)$$

$$\dot{\mathbf{x}}_m = \mathbf{A}_m\mathbf{x}_m + \mathbf{B}_m(\boldsymbol{\tau}_{td} + \bar{\boldsymbol{\tau}}_{td}), \quad (12)$$

where $\mathbf{x}_p = [\,\boldsymbol{\tau}_t^{\mathrm{T}}\ \dot{\boldsymbol{\tau}}_t^{\mathrm{T}}]^{\mathrm{T}} \in \Re^{2n}$ and $\mathbf{x}_m = [\,\boldsymbol{\tau}_r^{\mathrm{T}}\ \dot{\boldsymbol{\tau}}_r^{\mathrm{T}}\,]^{\mathrm{T}} \in \Re^{2n}$ are augmented state vectors. $\mathbf{A}_p = \left[\begin{smallmatrix} \mathbf{0} & \mathbf{I}_{n\times n} \\ -\mathbf{J}_t^{-1} & -\mathbf{J}_t^{-1}\mathbf{B}_t \end{smallmatrix}\right] \in \Re^{2n\times 2n}$ and $\mathbf{A}_m = \left[\begin{smallmatrix} \mathbf{0} & \mathbf{I}_{n\times n} \\ -\mathbf{J}_r^{-1}\mathbf{K}_r & -\mathbf{J}_r^{-1}\mathbf{B}_r \end{smallmatrix}\right] \in \Re^{2n\times 2n}$ are augmented system matrices. $\mathbf{B}_p = \left[\begin{smallmatrix} \mathbf{0} \\ \mathbf{J}_t^{-1} \end{smallmatrix}\right] \in \Re^{2n\times n}$ and $\mathbf{B}_m = \left[\begin{smallmatrix} \mathbf{0} \\ \mathbf{J}_r^{-1}\mathbf{K}_r \end{smallmatrix}\right] \in \Re^{2n\times n}$ are augmented input gain matrices, and the pair $(\mathbf{A}_m, \mathbf{B}_m)$ is controllable. Let $\boldsymbol{\tau}_t = \mathbf{C}_p\mathbf{x}_p$ and $\boldsymbol{\tau}_r = \mathbf{C}_m\mathbf{x}_m$ be the output signal vectors for Eqs. (11) and (12), respectively, where $\mathbf{C}_p = \mathbf{C}_m = [\,\mathbf{I}_{n\times n}\ \mathbf{0}\,] \in \Re^{n\times 2n}$. Note that the pair $(\mathbf{A}_m, \mathbf{C}_m)$ is observable and the transfer function $\mathbf{C}_m(s\mathbf{I} - \mathbf{A}_m)^{-1}\mathbf{B}_m$ is strictly positive real. Since all system parameters are assumed to be available at the present stage, we may select a perfect current trajectory in the form[51]

$$\mathbf{i} = \mathbf{H}^{-1}[\boldsymbol{\Theta}\mathbf{x}_p + \boldsymbol{\Phi}\boldsymbol{\tau}_{td} + \mathbf{h}(\bar{\boldsymbol{\tau}}_{td}, \bar{\mathbf{q}})], \quad (13)$$

where $\boldsymbol{\Theta} \in \Re^{n\times 2n}$ and $\boldsymbol{\Phi} \in \Re^{n\times n}$ satisfy $\mathbf{A}_p + \mathbf{B}_p\boldsymbol{\Theta} = \mathbf{A}_m$ and $\mathbf{B}_p\boldsymbol{\Phi} = \mathbf{B}_m$, respectively, and $\mathbf{h}(\bar{\boldsymbol{\tau}}_{td}, \bar{\mathbf{q}}) = \boldsymbol{\Phi}\bar{\boldsymbol{\tau}}_{td} + \bar{\mathbf{q}}$ that is a function of $\ddot{\mathbf{q}}$, $\ddot{\boldsymbol{\theta}}$, $\dot{\mathbf{F}}_{\text{ext}}$, and $\ddot{\mathbf{F}}_{\text{ext}}$. Substituting Eq. (13) into Eq. (11) and after some rearrangements, we may have the system dynamics

$$\dot{\mathbf{x}}_p = \mathbf{A}_m\mathbf{x}_p + \mathbf{B}_m(\boldsymbol{\tau}_{td} + \bar{\boldsymbol{\tau}}_{td}). \quad (14)$$

Define $\mathbf{e}_m = \mathbf{x}_p - \mathbf{x}_m$ and we may have the error dynamics from Eqs. (12) and (14)

$$\dot{\mathbf{e}}_m = \mathbf{A}_m\mathbf{e}_m. \quad (15)$$

Let $\mathbf{e}_\tau = \boldsymbol{\tau}_t - \boldsymbol{\tau}_r$ be the output vector of Eq. (15), and it can be rewritten as

$$\mathbf{e}_\tau = \mathbf{C}_m\mathbf{e}_m. \quad (16)$$

Since $\mathbf{A}_m$ is stable, Eq. (15) implies $\mathbf{e}_m \to \mathbf{0}$ as $t \to \infty$. This further gives $\mathbf{e}_\tau \to \mathbf{0}$ as $t \to \infty$. To ensure that the actual current $\mathbf{i}$ is able to converge to the perfect $\mathbf{i}$ in Eq. (13), let us select the control input in Eq. (1c) as

$$\mathbf{u} = \mathbf{L}\mathbf{i}_d + \mathbf{R}(\mathbf{i}, \dot{\boldsymbol{\theta}}) - \mathbf{K}_c\mathbf{e}_i, \quad (17)$$

where $\mathbf{e}_i = \mathbf{i} - \mathbf{i}_d$ is the current error, $\mathbf{i}_d \in \Re^n$ is the desired current that is equivalent to the perfect current trajectory $\mathbf{i}$ in Eq. (13), and $\mathbf{K}_c \in \Re^{n\times n}$ is a positive definite matrix. Substituting Eq. (17) into Eq. (1c), the closed-loop dynamics becomes $\mathbf{L}\dot{\mathbf{e}}_i + \mathbf{K}_c\mathbf{e}_i = \mathbf{0}$. With this dynamics, it is easy to prove that $\mathbf{i} \to \mathbf{i}_d$ as $t \to \infty$ with proper selection of $\mathbf{K}_c$.

In summary, if all parameters in the RLFJED robot (1) are available, the desired transmission torque (9), desired current (13), and control input (17) can give asymptotic convergence tracking performance in the free space and the closed-loop system behaves like the target impedance in the compliant motion phase. It has to be noted that implementation of the present scheme requires the information of $\ddot{\mathbf{q}}$, $\ddot{\boldsymbol{\theta}}$, $\dot{\mathbf{F}}_{\text{ext}}$, and $\ddot{\mathbf{F}}_{\text{ext}}$ that are impractical in most industrial applications.

2.2. Controller design for uncertain robot

Suppose $\mathbf{D}_x$, $\mathbf{C}_x$, $\mathbf{g}_x$, $\mathbf{L}$, and $\mathbf{R}(\mathbf{i}, \dot{\boldsymbol{\theta}})$ are not available, and $\ddot{\mathbf{q}}$, $\ddot{\boldsymbol{\theta}}$, $\dot{\mathbf{F}}_{\text{ext}}$, and $\ddot{\mathbf{F}}_{\text{ext}}$ are not easy to measure, we would like to design a desired transmission torque $\boldsymbol{\tau}_{td}$ so that a proper controller $\mathbf{u}$ can be constructed to have $\boldsymbol{\tau}_t \to \boldsymbol{\tau}_{td}$. Instead of Eq. (9), let us design a new desired transmission torque as

$$\boldsymbol{\tau}_{td} = \mathbf{J}_a^{\mathrm{T}}(\hat{\mathbf{g}}_x + \hat{\mathbf{D}}_x\dot{\mathbf{v}} + \hat{\mathbf{C}}_x\mathbf{v} - \mathbf{K}_d\mathbf{s} + \mathbf{F}_{\text{ext}}), \quad (18)$$

where $\hat{\mathbf{D}}_x$, $\hat{\mathbf{C}}_x$, and $\hat{\mathbf{g}}_x$ are estimates of $\boldsymbol{D}_x$, $\boldsymbol{C}_x$, and $\boldsymbol{g}_x$, respectively. Substituting Eq. (18) into Eq. (8), we may have the closed-loop dynamics

$$\mathbf{D}_x\dot{\mathbf{s}} + \mathbf{C}_x\mathbf{s} + \mathbf{K}_d\mathbf{s} = \mathbf{J}_a^{-\mathrm{T}}(\boldsymbol{\tau}_t - \boldsymbol{\tau}_{td}) - (\mathbf{D}_x - \hat{\mathbf{D}}_x)\dot{\mathbf{v}} - (\mathbf{C}_x - \hat{\mathbf{C}}_x)\mathbf{v} - (\mathbf{g}_x - \hat{\mathbf{g}}_x). \quad (19)$$

If a proper controller and update laws for $\hat{\mathbf{D}}_x$, $\hat{\mathbf{C}}_x$, and $\hat{\mathbf{g}}_x$ can be designed, we may have $\boldsymbol{\tau}_t \to \boldsymbol{\tau}_{td}$, $\hat{\mathbf{D}}_x \to \mathbf{D}_x$, $\hat{\mathbf{C}}_x \to \mathbf{C}_x$, and $\hat{\mathbf{g}}_x \to \mathbf{g}_x$ so that Eq. (18) can give desired performance. Let us have a modify version of the desired current $\mathbf{i}_d$ as

$$\mathbf{i}_d = \mathbf{H}^{-1}(\boldsymbol{\Theta}x_p + \boldsymbol{\Phi}\tau_{td} + \hat{\mathbf{h}}), \quad (20)$$

where $\hat{\mathbf{h}}$ is an estimate of $\mathbf{h}$. Hence, the system dynamics can be derived as

$$\dot{\mathbf{x}}_p = \mathbf{A}_m\dot{\mathbf{x}}_p + \mathbf{B}_m\boldsymbol{\tau}_{td} + \mathbf{B}_p\mathbf{H}(\mathbf{i} - \mathbf{i}_d) - \mathbf{B}_p(\mathbf{h} - \hat{\mathbf{h}}). \quad (21)$$

Together with Eq. (12), we may have the error dynamics

$$\dot{\mathbf{e}}_m = \mathbf{A}_m\mathbf{e}_m + \mathbf{B}_p[\mathbf{H}\mathbf{e}_i - (\mathbf{h} - \hat{\mathbf{h}})], \quad (22)$$

$$\mathbf{e}_\tau = \mathbf{C}_m\mathbf{e}_m. \quad (23)$$

If we may design an appropriate controller and a set of update laws such that $\mathbf{i} \to \mathbf{i}_d$ and $\hat{\mathbf{h}} \to \mathbf{h}$, then Eq. (22) implies $\mathbf{e}_m \to \mathbf{0}$ as $t \to \infty$. This further implies $\mathbf{e}_\tau \to \mathbf{0}$ as $t \to \infty$. Instead of Eq. (17), let us select the control input in Eq. (1c) as

$$\mathbf{u} = \hat{\mathbf{f}} - \mathbf{K}_c\mathbf{e}_i, \quad (24)$$

where $\hat{\mathbf{f}}$ is an estimate of $\mathbf{f}(\mathbf{i}_d, \mathbf{i}, \dot{\boldsymbol{\theta}}) = \mathbf{L}\mathbf{i}_d + \mathbf{R}(\mathbf{i}, \dot{\boldsymbol{\theta}})$. Substituting Eq. (24) into Eq. (1c), we may have the system dynamics

$$\mathbf{L}\dot{\mathbf{e}}_i + \mathbf{K}_c\mathbf{e}_i = -(\mathbf{f} - \hat{\mathbf{f}}). \quad (25)$$

If an appropriate update law for $\hat{\mathbf{f}}$ can be selected, we may have $\mathbf{i} \to \mathbf{i}_d$. Since $\mathbf{D}_x$, $\mathbf{C}_x$, $\mathbf{g}_x$, $\mathbf{h}$, and $\mathbf{f}$ are functions

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of time, traditional adaptive controllers are not directly applicable. To design the update laws, let us apply the function approximation representation[32–46]

$$\begin{aligned}&\mathbf{D}_x = \mathbf{W}_{\mathbf{D}_x}^{\mathrm{T}}\mathbf{Z}_{\mathbf{D}_x} + \varepsilon_{\mathbf{D}_x}, \quad \mathbf{C}_x = \mathbf{W}_{\mathbf{C}_x}^{\mathrm{T}}\mathbf{Z}_{\mathbf{C}_x} + \varepsilon_{\mathbf{C}_x},\\ &\mathbf{g}_x = \mathbf{W}_{\mathbf{g}_x}^{\mathrm{T}}\mathbf{Z}_{\mathbf{g}_x} + \varepsilon_{\mathbf{g}_x}, \quad \mathbf{h} = \mathbf{W}_{\mathbf{h}}^{\mathrm{T}}\mathbf{Z}_{\mathbf{h}} + \varepsilon_{\mathbf{h}}, \quad \mathbf{f} = \mathbf{W}_{\mathbf{f}}^{\mathrm{T}}\mathbf{Z}_{\mathbf{f}} + \varepsilon_{\mathbf{f}},\end{aligned} \tag{26a}$$

where $\mathbf{W}_{\mathbf{D}_x} \in \Re^{n^2\beta_D \times n}$, $\mathbf{W}_{\mathbf{C}_x} \in \Re^{n^2\beta_C \times n}$, $\mathbf{W}_{\mathbf{g}_x} \in \Re^{n\beta_g \times n}$, $\mathbf{W}_{\mathbf{h}} \in \Re^{n\beta_h \times n}$, and $\mathbf{W}_{\mathbf{f}} \in \Re^{n\beta_f \times n}$ are weighting matrices; $\mathbf{Z}_{\mathbf{D}_x} \in \Re^{n^2\beta_D \times n}$, $\mathbf{Z}_{\mathbf{C}_x} \in \Re^{n^2\beta_C \times n}$, $\mathbf{Z}_{\mathbf{g}_x} \in \Re^{n\beta_g \times 1}$, $\mathbf{Z}_{\mathbf{h}} \in \Re^{n\beta_h \times 1}$, and $\mathbf{Z}_{\mathbf{f}} \in \Re^{n\beta_f \times 1}$ are matrices of basis functions; and $\boldsymbol{\varepsilon}_{(\cdot)}$ are approximation error matrices. The number $\beta_{(\cdot)}$ represents the number of basis functions used. Using the same set of basis functions, the corresponding estimates can also be represented as

$$\begin{aligned}&\hat{\mathbf{D}}_x = \hat{\mathbf{W}}_{\mathbf{D}_x}^{\mathrm{T}}\mathbf{Z}_{\mathbf{D}_x}, \quad \hat{\mathbf{C}}_x = \hat{\mathbf{W}}_{\mathbf{C}_x}^{\mathrm{T}}\mathbf{Z}_{\mathbf{C}_x}, \quad \hat{\mathbf{g}}_x = \hat{\mathbf{W}}_{\mathbf{g}_x}^{\mathrm{T}}\mathbf{Z}_{\mathbf{g}_x},\\ &\hat{\mathbf{h}} = \hat{\mathbf{W}}_{\mathbf{h}}^{\mathrm{T}}\mathbf{Z}_{\mathbf{h}}, \quad \hat{\mathbf{f}} = \hat{\mathbf{W}}_{\mathbf{f}}^{\mathrm{T}}\mathbf{Z}_{\mathbf{f}}.\end{aligned} \tag{26b}$$

Define $\tilde{\mathbf{W}}_{(\cdot)} = \mathbf{W}_{(\cdot)} - \hat{\mathbf{W}}_{(\cdot)}$, then Eqs. (19), (22), and (25) become

$$\begin{aligned}\mathbf{D}_x\dot{\mathbf{s}} + \mathbf{C}_x\mathbf{s} + \mathbf{K}_d\mathbf{s} &= \mathbf{J}_a^{-\mathrm{T}}(\boldsymbol{\tau}_t - \boldsymbol{\tau}_{td}) - \tilde{\mathbf{W}}_{\mathbf{D}_x}^{\mathrm{T}}\mathbf{Z}_{\mathbf{D}_x}\dot{\mathbf{v}}\\ &\quad -\tilde{\mathbf{W}}_{\mathbf{C}_x}^{\mathrm{T}}\mathbf{Z}_{\mathbf{C}_x}\mathbf{v} - \tilde{\mathbf{W}}_{\mathbf{g}_x}^{\mathrm{T}}\mathbf{Z}_{\mathbf{g}_x} + \varepsilon_1,\end{aligned} \tag{27}$$

$$\dot{\mathbf{e}}_m = \mathbf{A}_m\mathbf{e}_m - \mathbf{B}_p\tilde{\mathbf{W}}_{\mathbf{h}}^{\mathrm{T}}\mathbf{Z}_{\mathbf{h}} + \mathbf{B}_p\mathbf{H}\mathbf{e}_i + \mathbf{B}_p\varepsilon_2, \tag{28}$$

$$\mathbf{L}\dot{\mathbf{e}}_i + \mathbf{K}_c\mathbf{e}_i = -\tilde{\mathbf{W}}_{\mathbf{f}}^{\mathrm{T}}\mathbf{Z}_{\mathbf{f}} + \varepsilon_3, \tag{29}$$

where $\boldsymbol{\varepsilon}_1 = \boldsymbol{\varepsilon}_1(\boldsymbol{\varepsilon}_{\mathbf{D}}, \boldsymbol{\varepsilon}_{\mathbf{C}}, \boldsymbol{\varepsilon}_{\mathbf{g}}, \mathbf{s}, \ddot{\mathbf{x}}_i)$, $\boldsymbol{\varepsilon}_2 = \boldsymbol{\varepsilon}_2(\boldsymbol{\varepsilon}_{\mathbf{h}}, \mathbf{e}_m)$, and $\boldsymbol{\varepsilon}_3 = \boldsymbol{\varepsilon}_3(\boldsymbol{\varepsilon}_{\mathbf{f}}, \mathbf{e}_i)$ are lumped approximation errors. Since $\mathbf{W}_{(\cdot)}$ are constant matrices, their update laws can be easily found by proper selection of the Lyapunov-like function. Let us consider a candidate

$$\begin{aligned}&V(\mathbf{s}, \mathbf{e}_m, \mathbf{e}_i, \tilde{\mathbf{W}}_{\mathbf{D}_x}, \tilde{\mathbf{W}}_{\mathbf{C}_x}, \tilde{\mathbf{W}}_{\mathbf{g}_x}, \tilde{\mathbf{W}}_{\mathbf{h}}, \tilde{\mathbf{W}}_{\mathbf{f}})\\ &= \tfrac{1}{2}\big[\mathbf{s}^{\mathrm{T}}\mathbf{D}_x\mathbf{s} + 2\mathbf{e}_m^{\mathrm{T}}\mathbf{P}_t\mathbf{e}_m + \mathbf{e}_i^{\mathrm{T}}\mathbf{L}\mathbf{e}_i + Tr(\tilde{\mathbf{W}}_{\mathbf{D}_x}^{\mathrm{T}}\mathbf{Q}_{\mathbf{D}_x}\tilde{\mathbf{W}}_{\mathbf{D}_x}\\ &\quad + \tilde{\mathbf{W}}_{\mathbf{C}_x}^{\mathrm{T}}\mathbf{Q}_{\mathbf{C}_x}\tilde{\mathbf{W}}_{\mathbf{C}_x} + \tilde{\mathbf{W}}_{\mathbf{g}_x}^{\mathrm{T}}\mathbf{Q}_{\mathbf{g}_x}\tilde{\mathbf{W}}_{\mathbf{g}_x}\\ &\quad + \tilde{\mathbf{W}}_{\mathbf{h}}^{\mathrm{T}}\mathbf{Q}_{\mathbf{h}}\tilde{\mathbf{W}}_{\mathbf{h}} + \tilde{\mathbf{W}}_{\mathbf{f}}^{\mathrm{T}}\mathbf{Q}_{\mathbf{f}}\tilde{\mathbf{W}}_{\mathbf{f}})\big],\end{aligned} \tag{30}$$

where $\mathbf{P}_t = \mathbf{P}_t^{\mathrm{T}} \in \Re^{2n\times 2n}$ is a positive definite matrix satisfying the Lyapunov equation $\mathbf{A}_m^{\mathrm{T}}\mathbf{P}_t + \mathbf{P}_t\mathbf{A}_m = -\mathbf{C}_m^{\mathrm{T}}\mathbf{C}_m$. The matrices $\mathbf{Q}_{\mathbf{D}_x} \in \Re^{n^2\beta_{\mathbf{D}}\times n^2\beta_{\mathbf{D}}}$, $\mathbf{Q}_{\mathbf{C}_x} \in \Re^{n^2\beta_C\times n^2\beta_C}$, $\mathbf{Q}_{\mathbf{g}_x} \in \Re^{n\beta_g\times n\beta_g}$, $\mathbf{Q}_{\mathbf{h}} \in \Re^{n\beta_h\times n\beta_h}$, and $\mathbf{Q}_{\mathbf{f}} \in \Re^{n\beta_f\times n\beta_f}$ are positive definite. The notation $Tr(.)$ denotes the trace operation of matrices. The time derivative of V along the trajectory of Eqs. (27), (28), and (29) can be computed as

$$\begin{aligned}\dot{V} &= -\mathbf{s}^{\mathrm{T}}\mathbf{K}_d\mathbf{s} + \mathbf{s}^{\mathrm{T}}\mathbf{J}_a^{-\mathrm{T}}\mathbf{e}_\tau - \mathbf{e}_\tau^{\mathrm{T}}\mathbf{e}_\tau + \mathbf{e}_m^{\mathrm{T}}\mathbf{P}_t\mathbf{B}_p\mathbf{H}\mathbf{e}_i - \mathbf{e}_i^{\mathrm{T}}\mathbf{K}_c\mathbf{e}_i\\ &\quad + \mathbf{s}^{\mathrm{T}}\boldsymbol{\varepsilon}_1 + \mathbf{e}_m^{\mathrm{T}}\mathbf{P}_t\mathbf{B}_p\boldsymbol{\varepsilon}_2 + \mathbf{e}_i^{\mathrm{T}}\boldsymbol{\varepsilon}_3 - Tr[\tilde{\mathbf{W}}_{\mathbf{D}_x}^{\mathrm{T}}(\mathbf{Z}_{\mathbf{D}_x}\dot{\mathbf{v}}\mathbf{s}^{\mathrm{T}}\\ &\quad + \mathbf{Q}_{\mathbf{D}_x}\dot{\hat{\mathbf{W}}}_{\mathbf{D}_x}) + \tilde{\mathbf{W}}_{\mathbf{C}_x}^{\mathrm{T}}(\mathbf{Z}_{\mathbf{C}_x}\mathbf{v}\mathbf{s}^{\mathrm{T}} + \mathbf{Q}_{\mathbf{C}_x}\dot{\hat{\mathbf{W}}}_{\mathbf{C}_x})]\\ &\quad - Tr[\tilde{\mathbf{W}}_{\mathbf{g}_x}^{\mathrm{T}}(\mathbf{Z}_{\mathbf{g}_x}\mathbf{s}^{\mathrm{T}} + \mathbf{Q}_{\mathbf{g}_x}\dot{\hat{\mathbf{W}}}_{\mathbf{g}_x}) + \tilde{\mathbf{W}}_{\mathbf{h}}^{\mathrm{T}}(\mathbf{Z}_{\mathbf{h}}\mathbf{e}_m^{\mathrm{T}}\mathbf{P}_t\mathbf{B}_p\\ &\quad + \mathbf{Q}_{\mathbf{h}}\dot{\hat{\mathbf{W}}}_{\mathbf{h}}) + \tilde{\mathbf{W}}_{\mathbf{f}}^{\mathrm{T}}(\mathbf{Z}_{\mathbf{f}}\mathbf{e}_i^{\mathrm{T}} + \mathbf{Q}_{\mathbf{f}}\dot{\hat{\mathbf{W}}}_{\mathbf{f}})].\end{aligned} \tag{31}$$

According to the Kalman–Yakubovic Lemma, we have $\mathbf{P}_t\mathbf{B}_p = \mathbf{C}_m^{\mathrm{T}}$ by picking $\mathbf{B}_m = \mathbf{B}_p$.[52] The update laws can be selected as

$$\dot{\hat{\mathbf{W}}}_{\mathbf{D}_x} = -\mathbf{Q}_{\mathbf{D}_x}^{-1}\big(\mathbf{Z}_{\mathbf{D}_x}\dot{\mathbf{v}}\mathbf{s}^{\mathrm{T}} + \sigma_{\mathbf{D}_x}\hat{\mathbf{W}}_{\mathbf{D}_x}\big), \tag{32a}$$

$$\dot{\hat{\mathbf{W}}}_{\mathbf{C}_x} = -\mathbf{Q}_{\mathbf{C}_x}^{-1}\big(\mathbf{Z}_{\mathbf{C}_x}\mathbf{v}\mathbf{s}^{\mathrm{T}} + \sigma_{\mathbf{C}_x}\hat{\mathbf{W}}_{\mathbf{C}_x}\big), \tag{32b}$$

$$\dot{\hat{\mathbf{W}}}_{\mathbf{g}_x} = -\mathbf{Q}_{\mathbf{g}_x}^{-1}\big(\mathbf{Z}_{\mathbf{g}_x}\mathbf{s}^{\mathrm{T}} + \sigma_{\mathbf{g}_x}\hat{\mathbf{W}}_{\mathbf{g}_x}\big), \tag{32c}$$

$$\dot{\hat{\mathbf{W}}}_{\mathbf{h}} = -\mathbf{Q}_{\mathbf{h}}^{-1}\big(\mathbf{Z}_{\mathbf{h}}\mathbf{e}_\tau^{\mathrm{T}} + \sigma_{\mathbf{h}}\hat{\mathbf{W}}_{\mathbf{h}}\big), \tag{32d}$$

$$\dot{\hat{\mathbf{W}}}_{\mathbf{f}} = -\mathbf{Q}_{\mathbf{f}}^{-1}\big(\mathbf{Z}_{\mathbf{f}}\mathbf{e}_i^{\mathrm{T}} + \sigma_{\mathbf{f}}\hat{\mathbf{W}}_{\mathbf{f}}\big), \tag{32e}$$

where $\sigma_{(\cdot)}$ are positive numbers. Hence, Eq. (31) becomes

$$\begin{aligned}\dot{V} &= -\xi^{\mathrm{T}}\mathbf{Q}\xi + \xi^{\mathrm{T}}\boldsymbol{\varepsilon} + \sigma_{\mathbf{D}_x}Tr\big(\tilde{\mathbf{W}}_{\mathbf{D}_x}^{\mathrm{T}}\hat{\mathbf{W}}_{\mathbf{D}_x}\big) + \sigma_{\mathbf{C}_x}Tr\big(\tilde{\mathbf{W}}_{\mathbf{C}_x}^{\mathrm{T}}\hat{\mathbf{W}}_{\mathbf{C}_x}\big)\\ &\quad + \sigma_{\mathbf{g}_x}Tr\big(\tilde{\mathbf{W}}_{\mathbf{g}_x}^{\mathrm{T}}\hat{\mathbf{W}}_{\mathbf{g}_x}\big) + \sigma_{\mathbf{h}}Tr\big(\tilde{\mathbf{W}}_{\mathbf{h}}^{\mathrm{T}}\hat{\mathbf{W}}_{\mathbf{h}}\big) + \sigma_{\mathbf{f}}Tr\big(\tilde{\mathbf{W}}_{\mathbf{f}}^{\mathrm{T}}\hat{\mathbf{W}}_{\mathbf{f}}\big),\end{aligned} \tag{33}$$

where $\xi = [\,\mathbf{s}^{\mathrm{T}}\ \mathbf{e}_\tau^{\mathrm{T}}\ \mathbf{e}_i^{\mathrm{T}}\,]^{\mathrm{T}}$, $\boldsymbol{\varepsilon} = [\,\boldsymbol{\varepsilon}_1^{\mathrm{T}}\ \boldsymbol{\varepsilon}_2^{\mathrm{T}}\ \boldsymbol{\varepsilon}_3^{\mathrm{T}}\,]^{\mathrm{T}}$, and

$$\mathbf{Q} = \begin{bmatrix}\mathbf{K}_d & -\frac{1}{2}\mathbf{J}_a^{-\mathrm{T}} & \mathbf{0}\\ -\frac{1}{2}\mathbf{J}_a^{-\mathrm{T}} & \mathbf{I}_{n\times n} & -\frac{1}{2}\mathbf{H}\\ \mathbf{0} & -\frac{1}{2}\mathbf{H} & \mathbf{K}_c\end{bmatrix}$$

is positive definite due to proper selections of $\boldsymbol{K}_d$ and $\boldsymbol{K}_c$. Owing to the existence of ε, the definiteness of $\dot{V}$ cannot be determined. However, the right-hand side of Eq. (33) can be divided into two parts to derive following inequalities:

$$-\xi^{\mathrm{T}}\mathbf{Q}\xi + \xi^{\mathrm{T}}\boldsymbol{\varepsilon} \le -\frac{1}{2}\left(\lambda_{\min}(\mathbf{Q})\,\|\xi\|^2 - \frac{1}{\lambda_{\min}(\mathbf{Q})}\,\|\boldsymbol{\varepsilon}\|^2\right), \tag{34a}$$

$$Tr\big(\tilde{\mathbf{W}}_{(\cdot)}^{\mathrm{T}}\hat{\mathbf{W}}_{(\cdot)}\big) \le \frac{1}{2}Tr\big(\mathbf{W}_{(\cdot)}^{\mathrm{T}}\mathbf{W}_{(\cdot)}\big) - \frac{1}{2}Tr\big(\tilde{\mathbf{W}}_{(\cdot)}^{\mathrm{T}}\tilde{\mathbf{W}}_{(\cdot)}\big). \tag{34b}$$

According to Eq. (30), we have

$$\begin{aligned}V &\le \tfrac{1}{2}\big[\lambda_{\max}(\mathbf{A})\,\|\xi\|^2 + \lambda_{\max}(\mathbf{Q}_{\mathbf{D}_x})Tr\big(\tilde{\mathbf{W}}_{\mathbf{D}_x}^{\mathrm{T}}\tilde{\mathbf{W}}_{\mathbf{D}_x}\big)\\ &\quad + \lambda_{\max}(\mathbf{Q}_{\mathbf{C}_x})Tr\big(\tilde{\mathbf{W}}_{\mathbf{C}_x}^{\mathrm{T}}\tilde{\mathbf{W}}_{\mathbf{C}_x}\big) + \lambda_{\max}(\mathbf{Q}_{\mathbf{g}_x})Tr\big(\tilde{\mathbf{W}}_{\mathbf{g}_x}^{\mathrm{T}}\tilde{\mathbf{W}}_{\mathbf{g}_x}\big)\\ &\quad + \lambda_{\max}(\mathbf{Q}_{\mathbf{h}})Tr\big(\tilde{\mathbf{W}}_{\mathbf{h}}^{\mathrm{T}}\tilde{\mathbf{W}}_{\mathbf{h}}\big) + \lambda_{\max}(\mathbf{Q}_{\mathbf{f}})Tr\big(\tilde{\mathbf{W}}_{\mathbf{f}}^{\mathrm{T}}\tilde{\mathbf{W}}_{\mathbf{f}}\big)\big],\end{aligned} \tag{35}$$

where

$$\mathbf{A} = \begin{bmatrix}\mathbf{D}_x & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & 2\mathbf{C}_m^{\mathrm{T}}\mathbf{P}_t\mathbf{C}_m & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{L}\end{bmatrix}.$$

With Eqs. (34) and (35), Eq. (33) can be further written as

$$\begin{aligned}\dot{V} &\le -\alpha V + \frac{1}{2}\Bigg\{[\alpha\lambda_{\max}(\mathbf{A}) - \lambda_{\min}(\mathbf{Q})]\,\|\xi\|^2\\ &\quad + [\alpha\lambda_{\max}(\mathbf{Q}_{\mathbf{D}_x}) - \sigma_{\mathbf{D}_x}]Tr\big(\tilde{\mathbf{W}}_{\mathbf{D}_x}^{\mathrm{T}}\tilde{\mathbf{W}}_{\mathbf{D}_x}\big) + [\alpha\lambda_{\max}(\mathbf{Q}_{\mathbf{C}_x})\end{aligned}$$

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$$
\begin{aligned}
&- \sigma_{\mathbf{C}_x}]Tr\big(\tilde{\mathbf{W}}_{\mathbf{C}_x}^{\mathrm{T}}\tilde{\mathbf{W}}_{\mathbf{C}_x}\big)+[\alpha\lambda_{\max}(\mathbf{Q}_{\mathbf{g}_x})-\sigma_{\mathbf{g}_x}]Tr\big(\tilde{\mathbf{W}}_{\mathbf{g}_x}^{\mathrm{T}}\tilde{\mathbf{W}}_{\mathbf{g}_x}\big)\\
&+[\alpha\lambda_{\max}(\mathbf{Q}_{\mathbf{h}})-\sigma_{\mathbf{h}}]Tr\big(\tilde{\mathbf{W}}_{\mathbf{h}}^{\mathrm{T}}\tilde{\mathbf{W}}_{\mathbf{h}}\big)\\
&+[\alpha\lambda_{\max}(\mathbf{Q}_{\mathbf{f}})-\sigma_{\mathbf{f}}]\,Tr\big(\tilde{\mathbf{W}}_{\mathbf{f}}^{\mathrm{T}}\tilde{\mathbf{W}}_{\mathbf{f}}\big)\\
&+\frac{1}{\lambda_{\min}(\mathbf{Q})}\|\boldsymbol{\varepsilon}\|^2+\sigma_{\mathbf{D}_x}Tr\big(\mathbf{W}_{\mathbf{D}_x}^{\mathrm{T}}\mathbf{W}_{\mathbf{D}_x}\big)\\
&+\sigma_{\mathbf{C}_x}Tr\big(\mathbf{W}_{\mathbf{C}_x}^{\mathrm{T}}\mathbf{W}_{\mathbf{C}_x}\big)+\sigma_{\mathbf{g}_x}Tr\big(\mathbf{W}_{\mathbf{g}_x}^{\mathrm{T}}\mathbf{W}_{\mathbf{g}_x}\big)\\
&+\sigma_{\mathbf{h}}Tr\big(\mathbf{W}_{\mathbf{h}}^{\mathrm{T}}\mathbf{W}_{\mathbf{h}}\big)+\sigma_{\mathbf{f}}Tr\big(\mathbf{W}_{\mathbf{f}}^{\mathrm{T}}\mathbf{W}_{\mathbf{f}}\big)\Bigg\}. \qquad (36)
\end{aligned}
$$

Although $\mathbf{D}$ and $\mathbf{L}$ are unknown, we know that $\exists\overline{D}$ and $\underline{D}$ s.t. $\underline{D}\le\|\mathbf{D}\|\le\overline{D}$, $\exists\overline{L}$ and $\underline{L}$ s.t. $\underline{L}\le\|\mathbf{L}\|\le\overline{L}$, and $\exists\overline{\eta}_A$, $\underline{\eta}_A>0$ s.t. $\lambda_{\max}(\mathbf{A})\le\overline{\eta}_A$ and $\lambda_{\min}(\mathbf{A})\ge\underline{\eta}_A$.[53] Picking $\alpha\le\min\{\frac{\lambda_{\min}(\mathbf{Q})}{\lambda_{\max}(\mathbf{A})},\frac{\sigma_{\mathbf{D}_x}}{\lambda_{\max}(\mathbf{Q}_{\mathbf{D}_x})},\frac{\sigma_{\mathbf{C}_x}}{\lambda_{\max}(\mathbf{Q}_{\mathbf{C}_x})},\frac{\sigma_{\mathbf{g}_x}}{\lambda_{\max}(\mathbf{Q}_{\mathbf{g}_x})},\frac{\sigma_{\mathbf{h}}}{\lambda_{\max}(\mathbf{Q}_{\mathbf{h}})},\frac{\sigma_{\mathbf{f}}}{\lambda_{\max}(\mathbf{Q}_{\mathbf{f}})}\}$, then we have

$$
\begin{aligned}
\dot{V}\le&-\alpha V+\frac{1}{2\lambda_{\min}(\mathbf{Q})}\|\boldsymbol{\varepsilon}\|^2+\frac{1}{2}\big[\sigma_{\mathbf{D}_x}Tr\big(\mathbf{W}_{\mathbf{D}_x}^{\mathrm{T}}\mathbf{W}_{\mathbf{D}_x}\big)\\
&+\sigma_{\mathbf{C}_x}Tr\big(\mathbf{W}_{\mathbf{C}_x}^{\mathrm{T}}\mathbf{W}_{\mathbf{C}_x}\big)+\sigma_{\mathbf{g}_x}Tr\big(\mathbf{W}_{\mathbf{g}_x}^{\mathrm{T}}\mathbf{W}_{\mathbf{g}_x}\big)\\
&+\sigma_{\mathbf{h}}Tr\big(\mathbf{W}_{\mathbf{h}}^{\mathrm{T}}\mathbf{W}_{\mathbf{h}}\big)\\
&+\sigma_{\mathbf{f}}Tr\big(\mathbf{W}_{\mathbf{f}}^{\mathrm{T}}\mathbf{W}_{\mathbf{f}}\big)\big] \qquad (37)
\end{aligned}
$$

Hence, $\dot{V}<0$ whenever

$$
\begin{aligned}
&(\mathbf{s},\mathbf{e}_\tau,\mathbf{e}_i,\tilde{\mathbf{W}}_{\mathbf{D}_x},\tilde{\mathbf{W}}_{\mathbf{C}_x},\tilde{\mathbf{W}}_{\mathbf{g}_x},\tilde{\mathbf{W}}_{\mathbf{h}},\tilde{\mathbf{W}}_{\mathbf{f}})\\
&\in\Bigg\{(\mathbf{s},\mathbf{e}_\tau,\mathbf{e}_i,\tilde{\mathbf{W}}_{\mathbf{D}_x},\tilde{\mathbf{W}}_{\mathbf{C}_x},\tilde{\mathbf{W}}_{\mathbf{g}_x},\tilde{\mathbf{W}}_{\mathbf{h}},\tilde{\mathbf{W}}_{\mathbf{f}})\,|\,V\\
&>\frac{1}{2\alpha\lambda_{\min}(\mathbf{Q})}\sup_{\tau\ge t_0}\|\boldsymbol{\varepsilon}(\tau)\|^2+\frac{1}{2\alpha}\big[\sigma_{\mathbf{D}_x}Tr\big(\mathbf{W}_{\mathbf{D}_x}^{\mathrm{T}}\mathbf{W}_{\mathbf{D}_x}\big)\\
&+\sigma_{\mathbf{C}_x}Tr\big(\mathbf{W}_{\mathbf{C}_x}^{\mathrm{T}}\mathbf{W}_{\mathbf{C}_x}\big)+\sigma_{\mathbf{g}_x}Tr\big(\mathbf{W}_{\mathbf{g}_x}^{\mathrm{T}}\mathbf{W}_{\mathbf{g}_x}\big)\\
&+\sigma_{\mathbf{h}}Tr\big(\mathbf{W}_{\mathbf{h}}^{\mathrm{T}}\mathbf{W}_{\mathbf{h}}\big)+\sigma_{\mathbf{f}}Tr\big(\mathbf{W}_{\mathbf{f}}^{\mathrm{T}}\mathbf{W}_{\mathbf{f}}\big)\big]\Bigg\}.
\end{aligned}
$$

This further concludes that $\mathbf{s}$, $\mathbf{e}_\tau$, $\mathbf{e}_i$, $\tilde{\mathbf{W}}_{\mathbf{D}_x}$, $\tilde{\mathbf{W}}_{\mathbf{C}_x}$, $\tilde{\mathbf{W}}_{\mathbf{g}_x}$, $\tilde{\mathbf{W}}_{\mathbf{h}}$, and $\tilde{\mathbf{W}}_{\mathbf{f}}$ are uniformly ultimately bounded (u.u.b.). The implementation of the desired transmission torque (18), desired current (20), control input (24), and update law (32) does not need to calculate the regressor matrix that is required in most adaptive designs for robot manipulators. Moreover, we do not need the information of the time derivatives of $\mathbf{F}_{ext}$ nor do we need to measure the joint accelerations. The convergence of the parameters, however, can be proved to depend on the persistent excitation condition of the input.

The above derivation only demonstrates the boundedness of the closed-loop system, but in practical applications the transient performance is also of great importance. For further development, we may apply the comparison lemma[53] to Eq. (37) to have the upper bound for V:

$$
\begin{aligned}
V(t)\le&\,\mathrm{e}^{-\alpha(t-t_0)}V(t_0)+\frac{1}{2\alpha\lambda_{\min}(\mathbf{Q})}\sup_{t_0<\tau<t}\|\boldsymbol{\varepsilon}(\tau)\|^2\\
&+\frac{1}{2\alpha}\big[\sigma_{\mathbf{D}_x}Tr\big(\mathbf{W}_{\mathbf{D}_x}^{\mathrm{T}}\mathbf{W}_{\mathbf{D}_x}\big)+\sigma_{\mathbf{C}_x}Tr\big(\mathbf{W}_{\mathbf{C}_x}^{\mathrm{T}}\mathbf{W}_{\mathbf{C}_x}\big)\\
&+\sigma_{\mathbf{g}_x}Tr\big(\mathbf{W}_{\mathbf{g}_x}^{\mathrm{T}}\mathbf{W}_{\mathbf{g}_x}\big)+\sigma_{\mathbf{h}}Tr\big(\mathbf{W}_{\mathbf{h}}^{\mathrm{T}}\mathbf{W}_{\mathbf{h}}\big)+\sigma_{\mathbf{f}}Tr\big(\mathbf{W}_{\mathbf{f}}^{\mathrm{T}}\mathbf{W}_{\mathbf{f}}\big)\big]. \qquad (38)
\end{aligned}
$$

From Eq. (30), we obtain

$$
\begin{aligned}
V\ge&\frac{1}{2}\big[\lambda_{\min}(\mathbf{A})\,\|\boldsymbol{\xi}\|^2+\lambda_{\min}(\mathbf{Q}_{\mathbf{D}_x})Tr\big(\tilde{\mathbf{W}}_{\mathbf{D}_x}^{\mathrm{T}}\tilde{\mathbf{W}}_{\mathbf{D}_x}\big)\\
&+\lambda_{\min}(\mathbf{Q}_{\mathbf{C}_x})Tr\big(\tilde{\mathbf{W}}_{\mathbf{C}_x}^{\mathrm{T}}\tilde{\mathbf{W}}_{\mathbf{C}_x}\big)+\lambda_{\min}(\mathbf{Q}_{\mathbf{g}_x})Tr\big(\tilde{\mathbf{W}}_{\mathbf{g}_x}^{\mathrm{T}}\tilde{\mathbf{W}}_{\mathbf{g}_x}\big)\\
&+\lambda_{\min}(\mathbf{Q}_{\mathbf{h}})Tr\big(\tilde{\mathbf{W}}_{\mathbf{h}}^{\mathrm{T}}\tilde{\mathbf{W}}_{\mathbf{h}}\big)+\lambda_{\min}(\mathbf{Q}_{\mathbf{f}})Tr\big(\tilde{\mathbf{W}}_{\mathbf{f}}^{\mathrm{T}}\tilde{\mathbf{W}}_{\mathbf{f}}\big)\big]. \qquad (39)
\end{aligned}
$$

Thus, the bound of ξ for $t\ge t_0$ can be derived from Eqs. (38) and (39) as

$$
\begin{aligned}
\|\xi\|^2\le&\frac{1}{\underline{\eta}_A}\Bigg\{2\mathrm{e}^{-\alpha(t-t_0)}V(t_0)+\frac{1}{\alpha}\Bigg[\frac{1}{\lambda_{\min}(\mathbf{Q})}\sup_{t_0<\tau<t}\|\boldsymbol{\varepsilon}\|^2\\
&+\sigma_{\mathbf{D}_x}Tr\big(\mathbf{W}_{\mathbf{D}_x}^{\mathrm{T}}\mathbf{W}_{\mathbf{D}_x}\big)+\sigma_{\mathbf{C}_x}Tr\big(\mathbf{W}_{\mathbf{C}_x}^{\mathrm{T}}\mathbf{W}_{\mathbf{C}_x}\big)\\
&+\sigma_{\mathbf{g}_x}Tr\big(\mathbf{W}_{\mathbf{g}_x}^{\mathrm{T}}\mathbf{W}_{\mathbf{g}_x}\big)+\sigma_{\mathbf{h}}Tr\big(\mathbf{W}_{\mathbf{h}}^{\mathrm{T}}\mathbf{W}_{\mathbf{h}}\big)\\
&+\sigma_{\mathbf{f}}Tr\big(\mathbf{W}_{\mathbf{f}}^{\mathrm{T}}\mathbf{W}_{\mathbf{f}}\big)\Bigg]-\lambda_{\min}(\mathbf{Q}_{\mathbf{D}_x})Tr\big(\tilde{\mathbf{W}}_{\mathbf{D}_x}^{\mathrm{T}}\tilde{\mathbf{W}}_{\mathbf{D}_x}\big)\\
&-\lambda_{\min}(\mathbf{Q}_{\mathbf{C}_x})Tr\big(\tilde{\mathbf{W}}_{\mathbf{C}_x}^{\mathrm{T}}\tilde{\mathbf{W}}_{\mathbf{C}_x}\big)-\lambda_{\min}(\mathbf{Q}_{\mathbf{g}_x})Tr\big(\tilde{\mathbf{W}}_{\mathbf{g}_x}^{\mathrm{T}}\tilde{\mathbf{W}}_{\mathbf{g}_x}\big)\\
&-\lambda_{\min}(\mathbf{Q}_{\mathbf{h}})Tr\big(\tilde{\mathbf{W}}_{\mathbf{h}}^{\mathrm{T}}\tilde{\mathbf{W}}_{\mathbf{h}}\big)-\lambda_{\min}(\mathbf{Q}_{\mathbf{f}})Tr\big(\tilde{\mathbf{W}}_{\mathbf{f}}^{\mathrm{T}}\tilde{\mathbf{W}}_{\mathbf{f}}\big)\Bigg\}. \qquad (40)
\end{aligned}
$$

From the derivations above, we can conclude that the proposed design is able to give bounded tracking with guaranteed transient performance. The following theorem is a summary of the above results.

Theorem 1: Consider the RLFJED robot (1) with unknown parameters $\mathbf{D}$, $\mathbf{C}$, $\mathbf{g}$, $\mathbf{L}$, and $\mathbf{R}$, then desired transmission torque (18), desired current (20), control input (24), and update law (32) ensure that:

(i) Error signals $\mathbf{s}$, $\mathbf{e}_\tau$, $\mathbf{e}_i$, $\tilde{\mathbf{W}}_{\mathbf{D}_x}$, $\tilde{\mathbf{W}}_{\mathbf{C}_x}$, $\tilde{\mathbf{W}}_{\mathbf{g}_x}$, $\tilde{\mathbf{W}}_{\mathbf{h}}$, and $\tilde{\mathbf{W}}_{\mathbf{f}}$ are u.u.b.

(ii) The bound of the tracking error vectors for $t\ge t_0$ can be derived as the form of Eq. (40), if the Lyapunov-like function candidates are chosen as Eq. (30).

Remark 1: The term with $\sigma_{(\cdot)}$ in Eq. (32) is to modify the update law to robustify the closed-loop system for the effect of the approximation error.[34] Suppose a sufficient number of basis functions $\beta_{(\cdot)}$ is selected so that the approximation error can be neglected then we may have $\sigma_{(\cdot)}=0$, and Eq. (33) becomes

$$
\dot{V}=-\xi^{\mathrm{T}}\mathbf{Q}\xi\le 0. \qquad (41)
$$

It is easy to prove that $\mathbf{s}$, $\mathbf{e}_\tau$, and $\mathbf{e}_i$ are also square integrable. From Eqs. (27), (28), and (29), $\dot{\mathbf{s}}$, $\dot{\mathbf{e}}_\tau$, and $\dot{\mathbf{e}}_i$ are bounded; as a result, asymptotic convergence of $\mathbf{s}$, $\mathbf{e}_\tau$, and $\mathbf{e}_i$ can easily be shown by Barbalat's lemma. This further implies that $\mathbf{i}\to\mathbf{i}_d$, $\boldsymbol{\tau}\to\boldsymbol{\tau}_d$, and $\mathbf{x}\to\mathbf{x}_i$ even though $\mathbf{D}_x$, $\mathbf{C}_x$, $\mathbf{g}_x$, $\mathbf{h}$, and $\mathbf{f}$ are all unknown.

Remark 2: Suppose $\boldsymbol{\varepsilon}_1$, $\boldsymbol{\varepsilon}_2$, and $\boldsymbol{\varepsilon}_3$ cannot be ignored but their variation bounds are available,[33,34] i.e., there exists positive constants δ_1, δ_2, and δ_3 such that $\|\boldsymbol{\varepsilon}_1\|\le\delta_1$, $\|\boldsymbol{\varepsilon}_2\|\le\delta_2$, and $\|\boldsymbol{\varepsilon}_3\|\le\delta_3$. To cover the effect of these bounded approximation errors, the desired transmission torque (18), the desired current (20), and the control input (24) are modified to be

$$
\boldsymbol{\tau}_{td}=\mathbf{J}_a^{\mathrm{T}}(\mathbf{F}_{\mathrm{ext}}+\hat{\mathbf{g}}_x+\hat{\mathbf{D}}_x\dot{\mathbf{v}}+\hat{\mathbf{C}}_x\mathbf{v}-\mathbf{K}_d\mathbf{s}+\boldsymbol{\tau}_{\mathrm{robust\,1}}), \qquad (42)
$$

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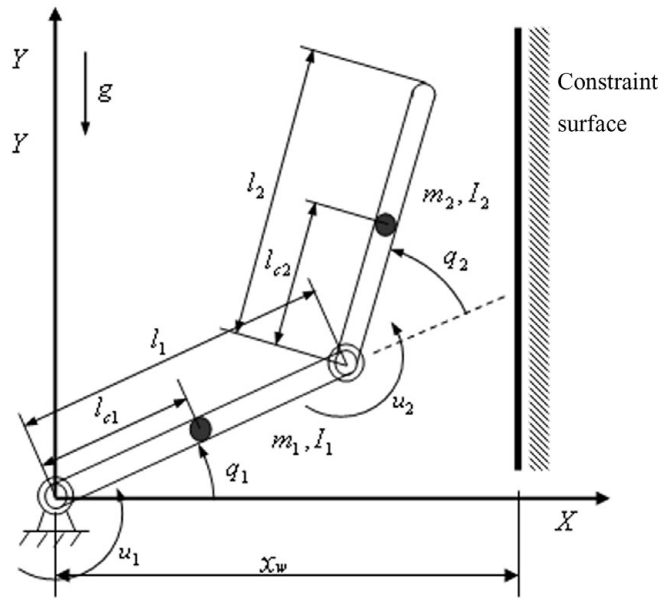


Fig. 1. 2 DOF constrained robot.

$$\mathbf{i}_d = \mathbf{H}^{-1}(\boldsymbol{\Theta} x_p + \boldsymbol{\Phi}\tau_d + \hat{\mathbf{h}} + \boldsymbol{\tau}_{\text{robust}\,2}), \tag{43}$$

$$\mathbf{u} = \hat{\mathbf{f}} - \mathbf{K}_c\mathbf{e}_i + \boldsymbol{\tau}_{\text{robust}3}, \quad \mathbf{e}_i = \mathbf{i} - \mathbf{i}_d, \tag{44}$$

where $\boldsymbol{\tau}_{\text{robust}\,1}$, $\boldsymbol{\tau}_{\text{robust}\,2}$, and $\boldsymbol{\tau}_{\text{robust}\,3}$ are robust terms to be designed. Let us consider the Lyapunov-like function candidate (30) and the update law (32) again. The time derivative of V can be computed as

$$\dot{V} = -\xi^{\mathrm{T}}\mathbf{Q}\xi + \boldsymbol{\delta}_1\|\mathbf{s}\| + \boldsymbol{\delta}_2\|\mathbf{e}_\tau\| + \boldsymbol{\delta}_3\|\mathbf{e}_i\| + \mathbf{s}^{\mathrm{T}}\boldsymbol{\tau}_{\text{robust}\,1} + \mathbf{e}_\tau^{\mathrm{T}}\boldsymbol{\tau}_{\text{robust}\,2} + \mathbf{e}_i^{\mathrm{T}}\boldsymbol{\tau}_{\text{robust}\,3}. \tag{45}$$

By picking $\boldsymbol{\tau}_{\text{robust}\,1} = -\boldsymbol{\delta}_1[\,\text{sgn}(s_1)\cdots\text{sgn}(s_n)\,]^{\mathrm{T}}$, where $s_k, k = 1, ..., n$ is the kth element of $\mathbf{s}$, $\boldsymbol{\tau}_{\text{robust}\,2} = -\boldsymbol{\delta}_2[\,\text{sgn}(e_{\tau_1})\cdots\text{sgn}(e_{\tau_n})\,]^{\mathrm{T}}$, where e_{τ_k}, $k = 1, ..., 2n$ is the kth element of $\boldsymbol{e}_\tau$, and $\boldsymbol{\tau}_{\text{robust}\,3} = -\boldsymbol{\delta}_3[\,\text{sgn}(e_{i_1})\cdots\text{sgn}(e_{i_n})\,]^{\mathrm{T}}$, where e_{i_k}, $k = 1, ..., n$ is the kth element of $\boldsymbol{e}_i$, we may have $\dot{V} \le 0$, and asymptotic convergence of the state error can be concluded by Barbalat's lemma.

3. Simulation Study

Consider a 2 DOF constrained robot (Fig. 1) represented by the differential equation (1). The quantities m_i, l_i, l_{ci}, and I_i are mass, length, gravity center distance, and inertia of link i, respectively. Actual values of these parameters in the simulation are[44] $m_1 = 0.5$ kg, $m_2 = 0.5$ kg, $l_1 = l_2 = 0.75$ m, $l_{c1} = l_{c2} = 0.375$ m, $I_1 = 0.09375\text{kgm}^2$, and $I_2 = 0.046975kgm^2$. The actuator inertia, damping, and joint stiffness are $\mathbf{J} = \text{diag}(0.02,0.01)$ (kg m^2), $\mathbf{B} = \text{diag}(5,4)$(N m s/rad), and $\mathbf{K}_s = \text{diag}(100,100)$(N m/rad), respectively. The motor parameters are chosen to be $\mathbf{L} = \text{diag}(0.025,0.025)(H)$, $\mathbf{H} = \text{diag}(10,10)$ (N m/A). Considering the cases of

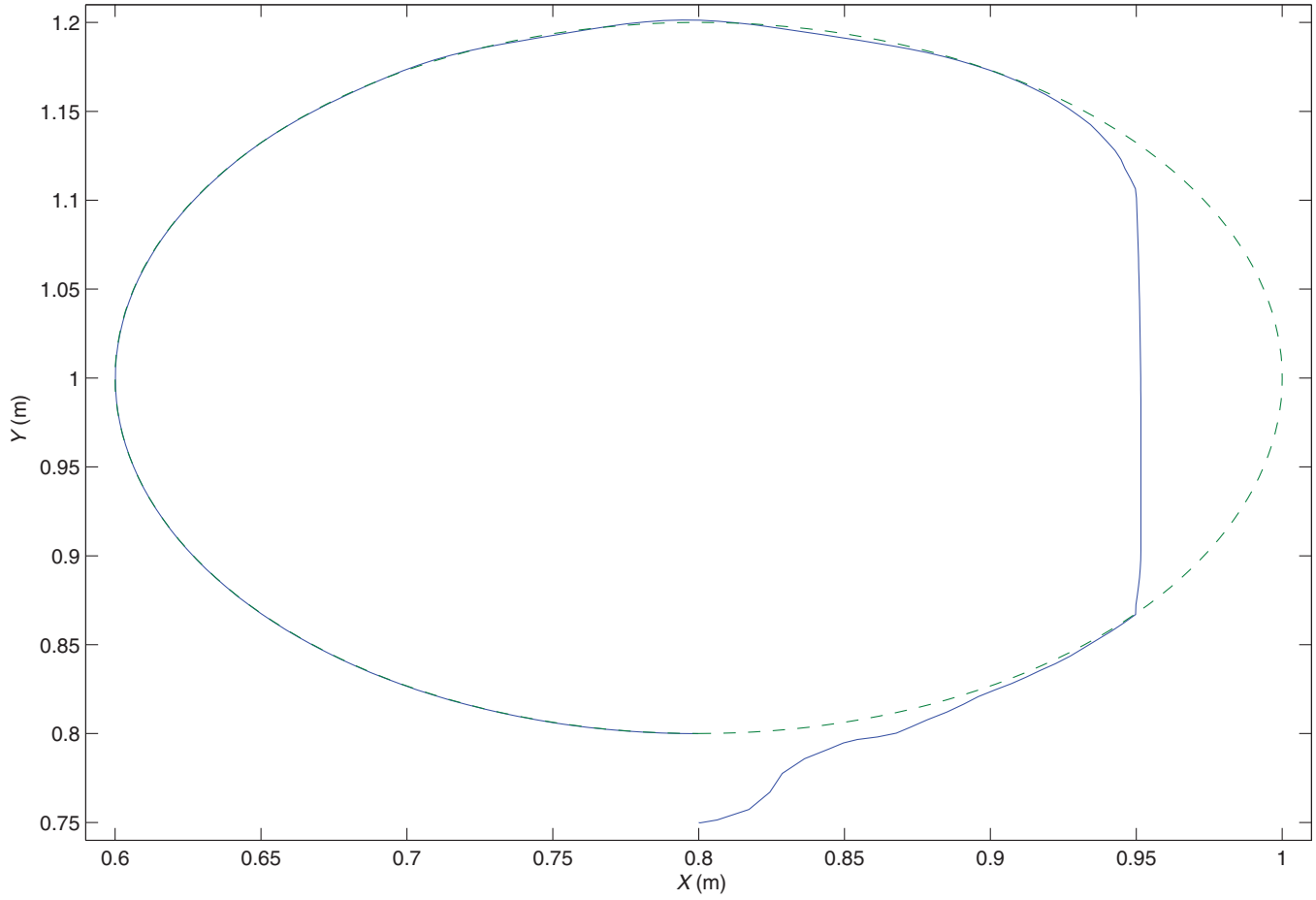


Fig. 2. (Colour Online) Tracking performance of end point in the Cartesian space (- actual; - - - desired). The tracking error is very small and the end point smoothly contacts the surface although we do not know precise dynamics of the robot.

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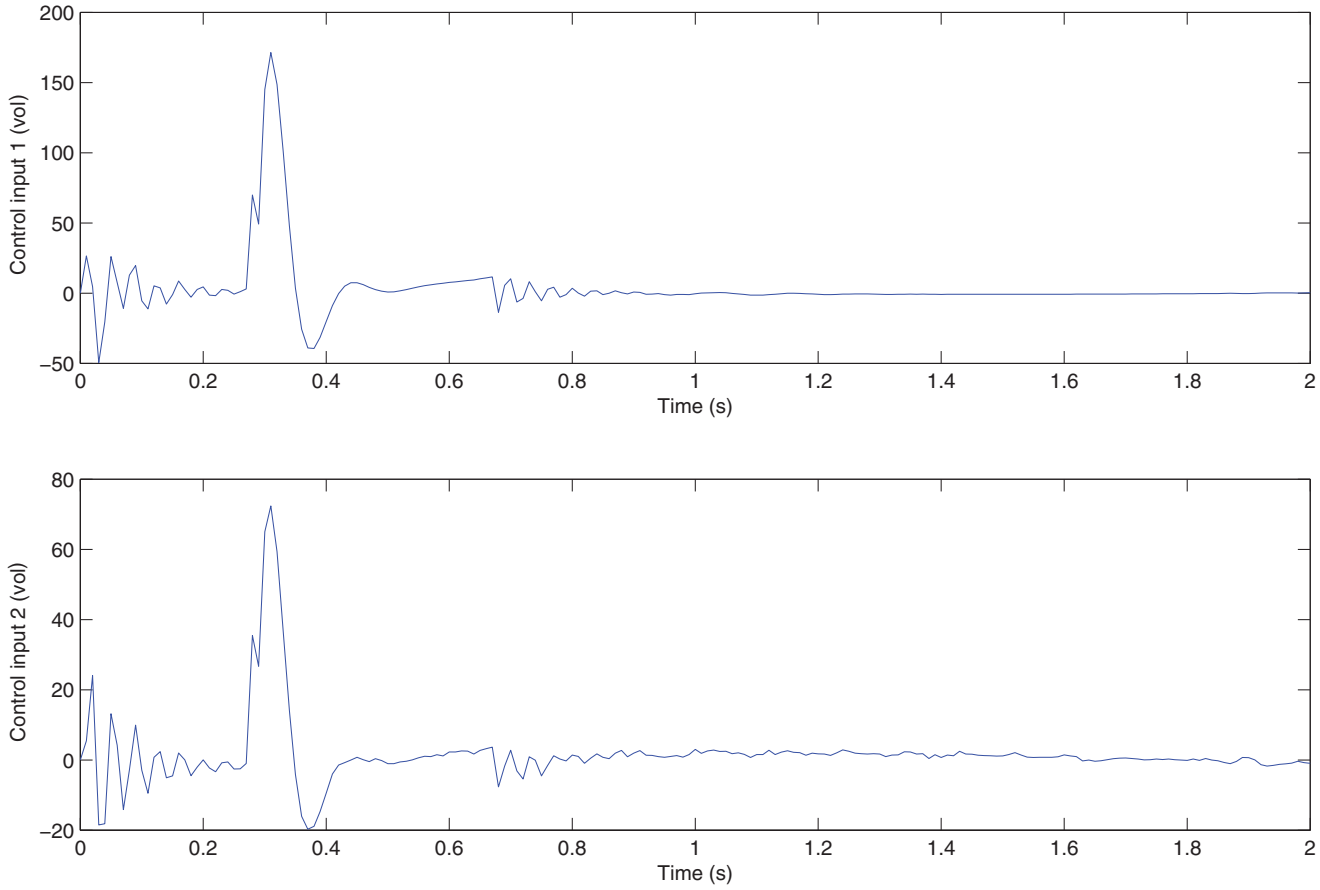


Fig. 3. (Colour Online) Actuator input.

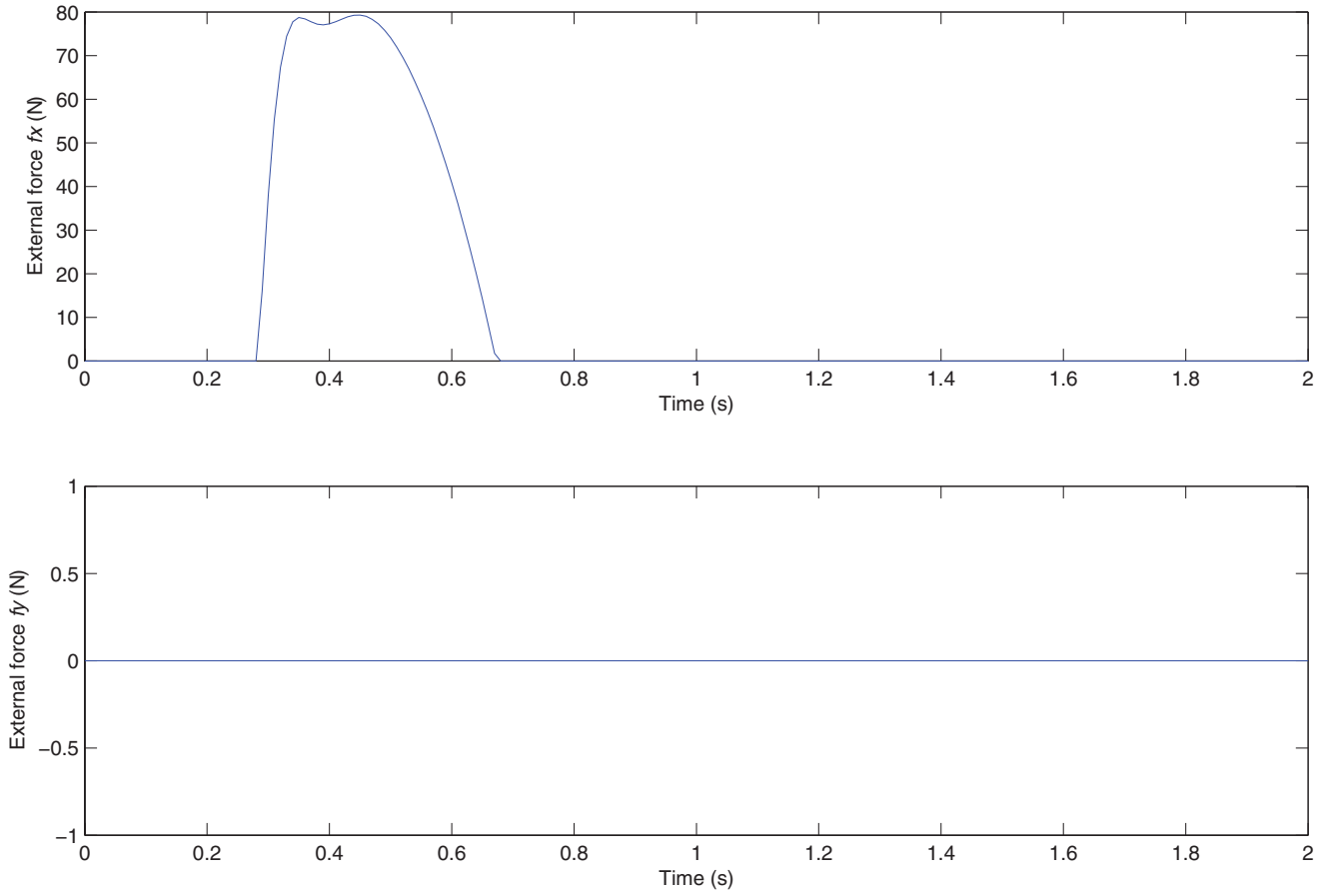


Fig. 4. (Colour Online) External forces.

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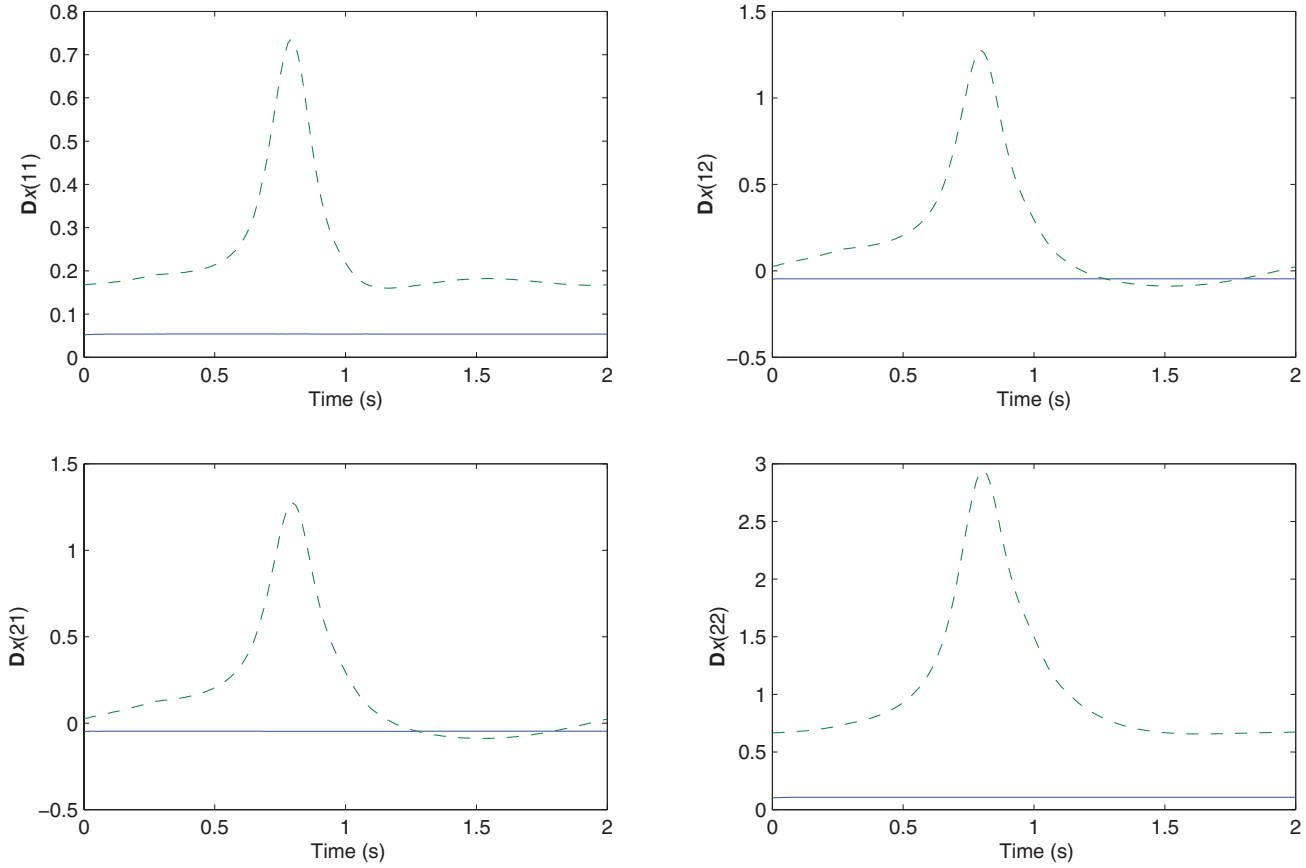


Fig. 5. (Colour Online) Approximation of $\mathbf{D}_x$ (- estimate; - - - real). Although the estimated values do not converge to the true values, they are bounded and small.

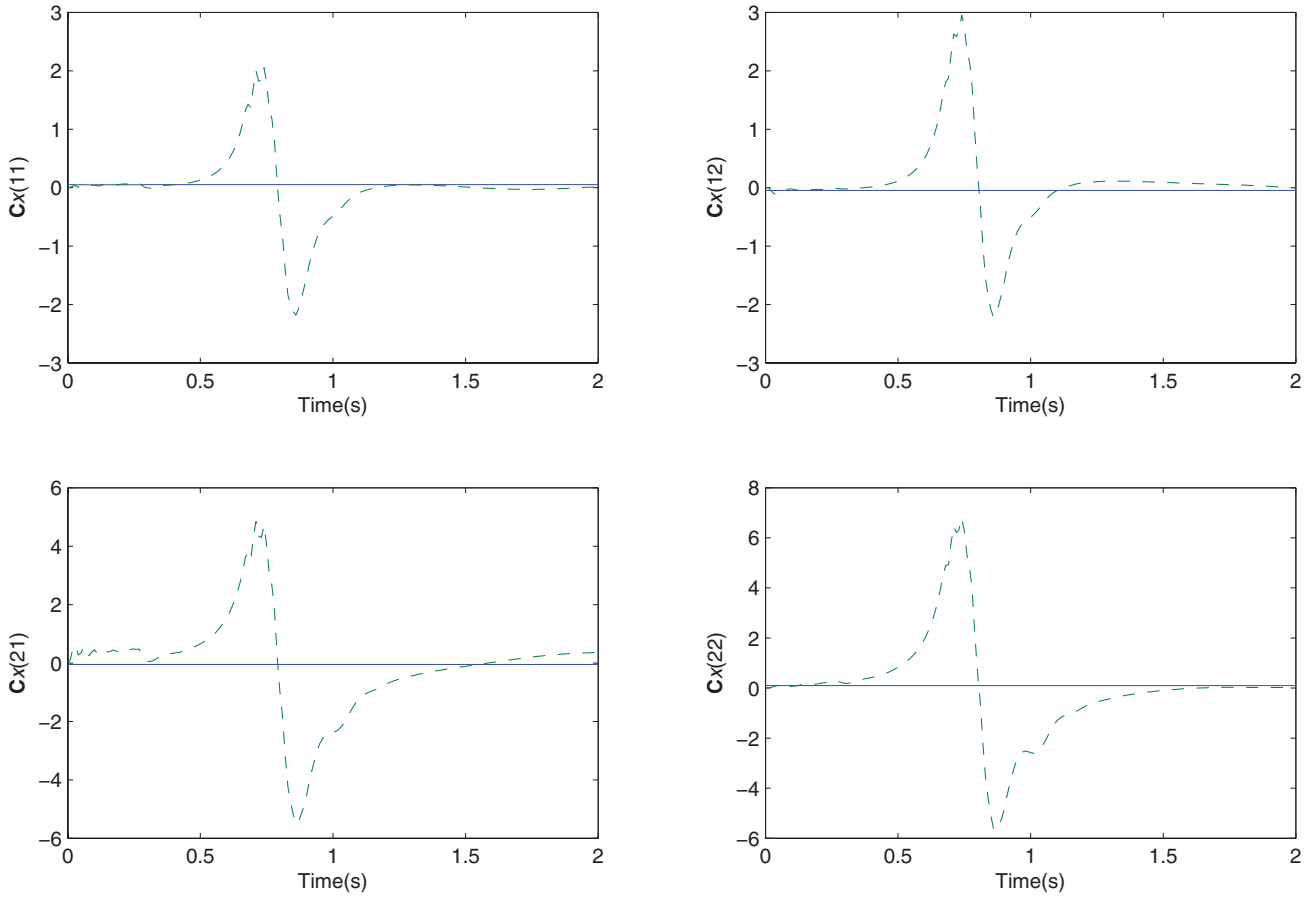


Fig. 6. (Colour Online) Approximation of $\mathbf{C}_x$ matrix(- estimate; - - - real).

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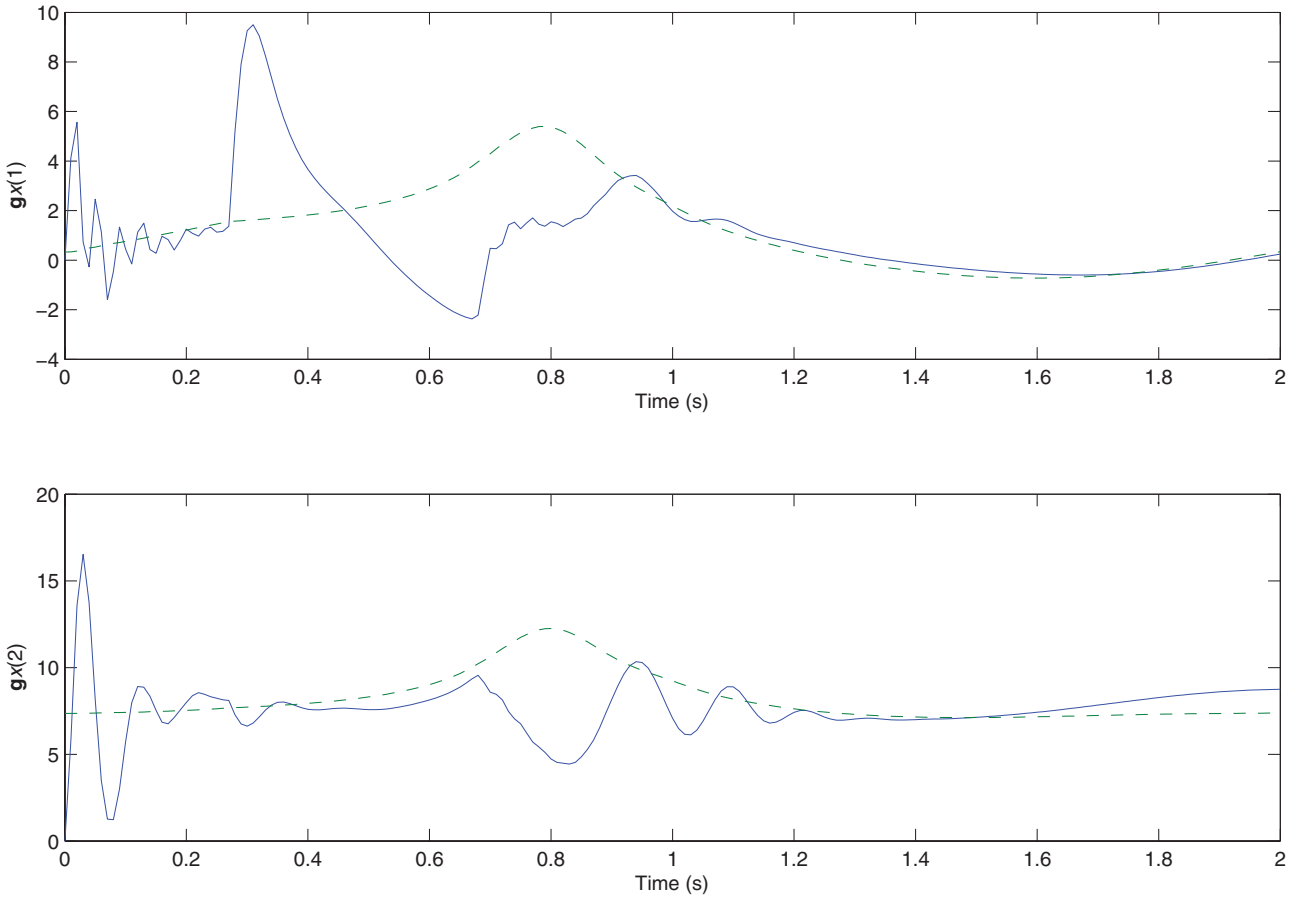


Fig. 7. (Colour Online) Approximation of vector $\mathbf{g}_x$ (- estimate; - - - real).

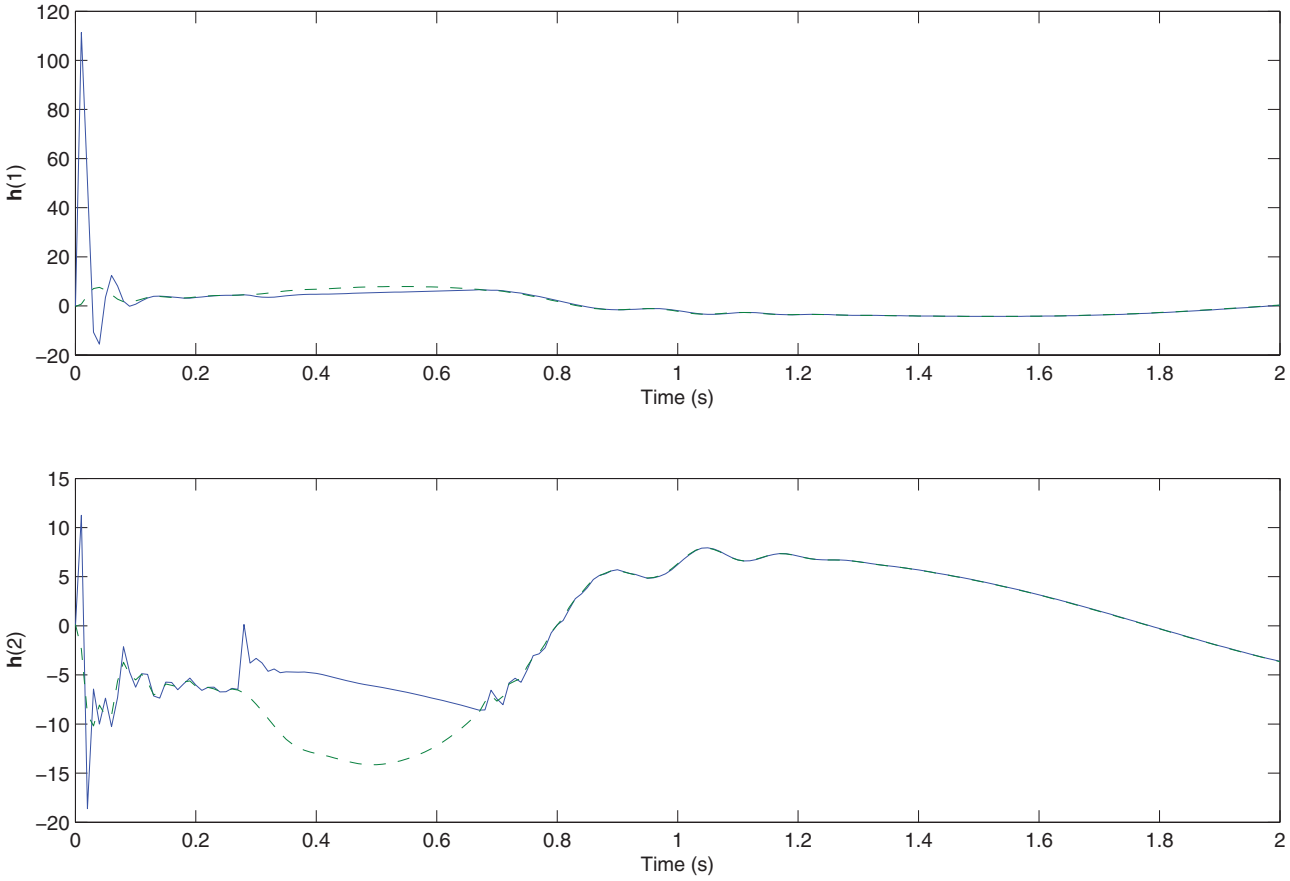


Fig. 8. (Colour Online) Approximation of vector **h** (- estimate; - - - real).

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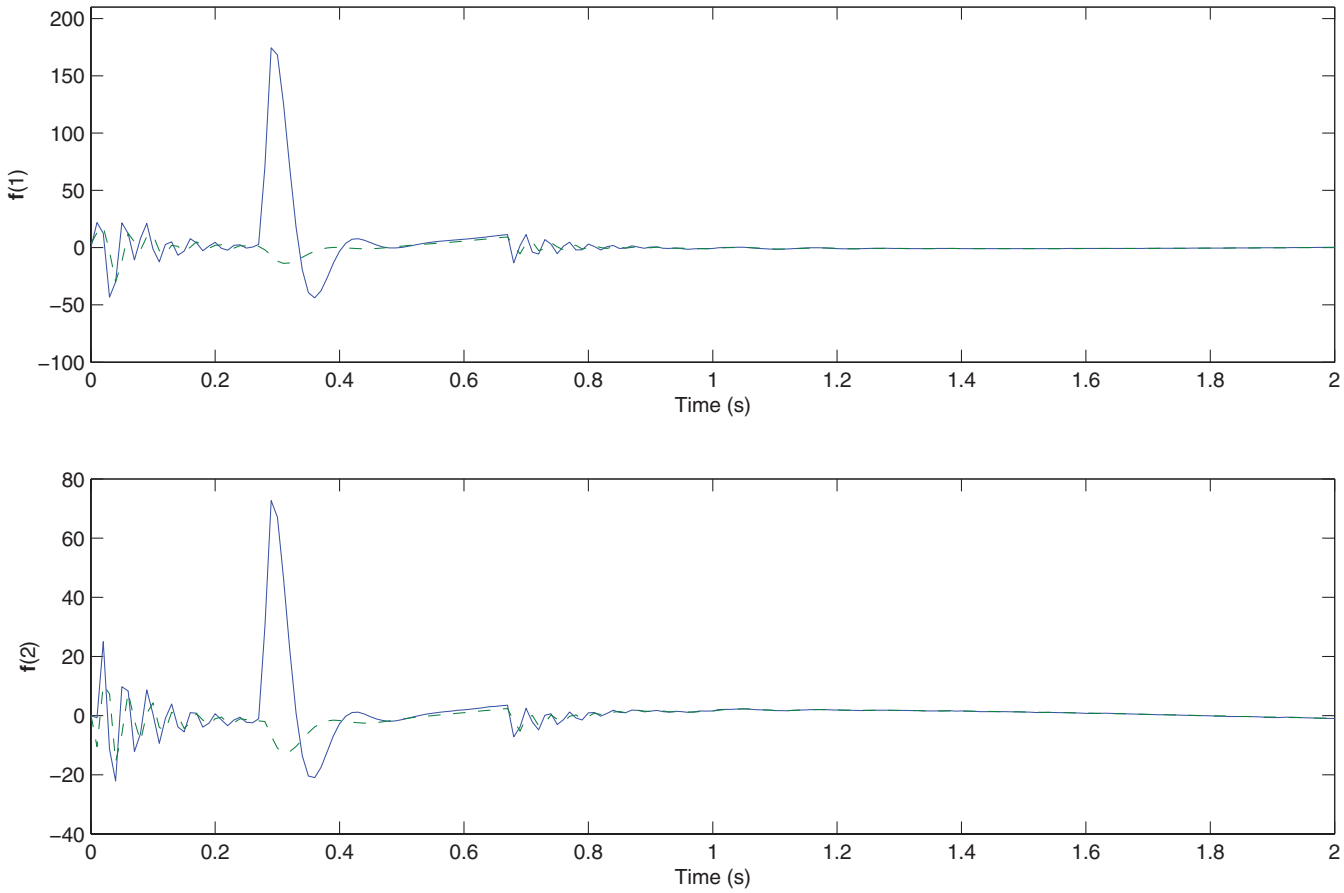


Fig. 9. (Colour Online) Approximation of vector **f** (- estimate; - - - real).

high-velocity movement, we would like the end point to track a 0.2 m radius circle centered at (0.8 m, 1.0 m) in 2 s without knowing its precise model. A smooth constraint surface is locates at $x = 0.95$ m so that the end point will contact with the surface for a significant period of time. Initial conditions of the end point, target impedance states, and actuator angles are $\mathbf{x} = \mathbf{x}_i = [\,0.8\ 0.75\ 0\ 0\,]^{\mathrm{T}}$, and $\boldsymbol{\theta} = [\,0.0022\ 1.5019\ 0\ 0\,]^{\mathrm{T}}$, respectively. The initial value of the reference model state vector is $\boldsymbol{\tau}_r(0) = [\,8.1\ 1.4\ 0\ 0\,]^{\mathrm{T}}$ that is the same as the initial value of the desired reference input τ_d. The initial condition of the motor armature currents is $\mathbf{i}(0) = [\,16.2\ 1.4\,]^{\mathrm{T}}$ that is the same as the initial value of the desired reference current $\boldsymbol{i}_d$. The controller gains are selected as $\mathbf{K}_d = \mathrm{diag}(50, 50)$, $\boldsymbol{\Lambda} = \mathrm{diag}(20, 20)$, and $\mathbf{K}_c = \mathrm{diag}(200, 200)$. Matrices in the target impedance are designed to be $\mathbf{M}_i = \mathrm{diag}(0.5, 0.5)$, $\mathbf{B}_i = \mathrm{diag}(100, 100)$, and $\mathbf{K}_i = \mathrm{diag}(1500, 1500)$. Each element of $\mathbf{D}_x$, $\mathbf{C}_x$, $\mathbf{g}_x$, $\mathbf{h}$, and $\mathbf{f}$ is approximated by the first 11 terms of the Fourier series. The constraint surface is smooth and can be modeled as a linear spring[54] $f_{\mathrm{ext}} = k_w(x - x_w)$. f_{ext} is the force acting on the surface, $k_w = 5000$ N/m is the environmental stiffness, x is the coordinate of the end point in the X-direction, and $x_w = 0.95$ m is the position of the surface. Hence, the external force vector in Eq. (1) becomes $\mathbf{F}_{\mathrm{ext}} = [\,f_{\mathrm{ext}}\ 0\,]^{\mathrm{T}}$. The simulation results are shown in Figs. 2–9. Figure 2 shows the tracking performance of the end point in the Cartesian space. It is observed that the end point compliantly contacts the constraint surface and the tracking error is very small in the free space. Computation of the complex regressor is avoided in this strategy that greatly simplifies the design and implementation of the control law. The control voltages to the two motors are reasonable that can be verified in Fig. 3. The spike at about 0.3 s is a result of the contact of the end point and the constraint surface. The external forces exerted on the end point during the constraint motion phase are shown in Fig. 4. Figures 5–9 are the performance of function approximation for $\mathbf{D}_x$, $\mathbf{C}_x$, $\mathbf{g}_x$, $\mathbf{h}$, and $\mathbf{f}$, respectively. Since the reference input does not satisfy the persistent excitation condition, some estimates do not converge to their actual values but remain bounded as desired.

4. Conclusions

An adaptive impedance controller is proposed for constrained RLFJED robots containing time-varying uncertainties. A backstepping-like procedure is developed to deal with the cascade structure in its dynamic equations. The FAT is employed to cope with the time-varying uncertainties. The closed-loop stability is proved by using the Lyapunov-like analysis. The realization of the proposed controller does not need to calculate the regressor that is required in most adaptive designs for robot manipulators. Simulation results justify the performance of the proposed controller in fast tracking operations although most of the robot parameters are not available. In addition, the information of time derivative of the external force and joint acceleration is not needed in the controller realization.

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