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Loss-cone index in distribution function in a hot magnetized plasma

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Abstract. The components of the dielectric tensor for the distribution function given by Leubner and Schupfer have been obtained. The effect of the loss-cone index appearing in the particle distribution function in a hot magnetized plasma has been studied. A case study has been performed to calculate temporal growth rates of Bernstein waves using the distribution function given by Summers and Thorne and Leubner and Schupfer. The effect of the loss-cone index on growth rates is found to be quite different for the two distribution functions.

1. Introduction

Suprathermal particle populations are known to be a ubiquitous property of space plasma velocity distributions. Two major characteristics of non-thermal particle distributions are: (1) a significant percentage of suprathermal particles and (2) the presence of loss-cone populations. The generation of velocity–space distributions exhibiting pronounced energetic tails is usually interpreted as consequence of certain acceleration mechanisms. The high-energy tails seen in the spectra are accurately represented by a family of κ distributions. The second significant non-thermal feature of observed velocity distributions is related to the loss-cone effects inherent in any mirror configuration as the planetary magnetospheres. Summers and Thorne (1995) have summarized the variety of plasma processes associated with loss-cone distributions and have introduced a κ distribution function with a loss-cone feature given by

$$f_{\kappa\ell}(v_\parallel, v_\perp) = \frac{\Gamma(\kappa+\ell+1)}{\pi^{3/2}\theta_{\perp_\sigma}^2\theta_{\parallel_\sigma}\kappa^{\ell+3/2}\Gamma(\ell+1)\Gamma(\kappa-1/2)}\left(\frac{v_\perp}{\theta_{\perp_\sigma}}\right)^{2\ell} \times\left[1+\frac{v_\parallel^2}{\kappa\theta_{\parallel_\sigma}^2}+\frac{v_\perp^2}{\kappa\theta_{\perp_\sigma}^2}\right]^{-(\kappa+\ell+1)} \tag{1}$$

where Γ is the gamma function. The parameter ℓ (>0) is the loss-cone index and the parameter κ ($>\frac{3}{2}$) is the spectral index; ℓ is a measure of the angular size of the effective loss-cone region ($\partial f_\sigma^\kappa/\partial v_\perp > 0$), κ is a measure of the relative number of particles in the high-energy tail of the distribution, σ denotes the particle species and $\theta_{\perp_\sigma}$ and $\theta_{\parallel_\sigma}$ are the effective perpendicular and parallel thermal speeds,

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respectively, given by

$$\theta_{\perp_\sigma} = \left(\frac{2\kappa - 3}{\kappa}\right)^{1/2} (\ell + 1)^{-1/2} \left(\frac{\mathrm{T}_{\perp_\sigma}}{\mathrm{m}_\sigma}\right)^{1/2}, \tag{2}$$

$$\theta_{\parallel_\sigma} = \left(\frac{2\kappa - 3}{\kappa}\right)^{1/2} \left(\frac{\mathrm{T}_{\parallel_\sigma}}{\mathrm{m}_\sigma}\right)^{1/2}. \tag{3}$$

The temperature anisotropy

$$A_\sigma^\kappa = \frac{T_{\perp_\sigma}}{T_{\parallel_\sigma}} - 1 = (\ell + 1)\frac{\theta_{\perp_\sigma}^2}{\theta_{\parallel_\sigma}^2} - 1. \tag{4}$$

Dielectric tensor components have been given for distribution function (1) by Summers and Thorne (1995). More recently, Leubner and Schupfer (2001) have introduced a κ distribution with loss-cone feature given by

$$f_{\kappa\ell}(v_\parallel, v_\perp) = A(\kappa, \ell)\left(\frac{v_\perp}{v_{\mathrm{th}_\perp}}\right)^{2\ell} \left[1 + \frac{v_\parallel^2}{\kappa v_{\mathrm{th}_\parallel}^2} + \frac{v_\perp^2}{v_{\mathrm{th}_\perp}^2}\right]^{-(\kappa+1)} \tag{5}$$

where

$$A(\kappa, \ell) = \frac{1}{\pi^{3/2} v_{\mathrm{th}_\parallel} v_{\mathrm{th}_\perp}^2} \frac{\Gamma(\kappa + 1)}{\kappa^{\ell+3/2}\Gamma(\ell + 1)\Gamma(\kappa - \ell - 1/2)}, \tag{6}$$

$$v_{\mathrm{th}_\parallel} = (2k_\mathrm{B}T_\parallel/m)^{1/2}, \quad v_{\mathrm{th}_\perp} = (2k_\mathrm{B}T_\perp/m)^{1/2}. \tag{7}$$

The distribution (5) differs from the distribution (1) only in the manner in which the loss-cone index ℓ appears in the power of the $[1 + (v_\parallel^2/\kappa v_{\mathrm{th}_\parallel}^2) + (v_\perp^2/\kappa v_{\mathrm{th}_\perp}^2)]$ square bracket of (1) and (5). However, both distributions (1) and (5) reduce for $\ell = 0$ to the ordinary kappa distribution, for $\kappa \to \infty$ to the Dory–Guest–Harris type distribution and for both $\ell = 0$ and $\kappa \to \infty$ to the two-temperature Maxwellian distribution. Many observed plasma instabilities are governed by the loss-cone index ℓ and the value of κ in the distributions. For waves propagating parallel to the ambient magnetic field the loss-cone index does not affect the wave properties as it occurs only through the temperature anisotropy. For waves propagating at an angle to the magnetic field the loss-cone index does influence the instability. In particular for electrostatic waves the effect of the loss-cone index on the instability of plasma waves will be quite different for the distribution functions (1) and (5). In the present work we aim to study this effect. For completeness we have obtained the components of the dielectric tensor for distribution function (5). We apply the distribution functions (1) and (5) to obtain the growth of instability for (Bernstein wave) electrostatic waves.

In Sec. 2 we present the results for the components of the dielectric tensor using distribution function (5). In Sec. 3 the growth rates for electrostatic waves are described. Finally Sec. 4 gives the conclusions.

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2. Dispersion relations

The general dispersion relation for linear, oblique waves in a hot, magnetized plasma is given by (Clemmow and Dougherty 1969)

$$\begin{vmatrix} n_\parallel^2 + K_{xx} & K_{xy} & n_\perp n_\parallel + K_{xz} \\ K_{yx} & -n_\perp^2 - n_\parallel^2 + K_{yy} & K_{yz} \\ n_\perp n_\parallel + K_{zx} & K_{zy} & -n_\perp^2 + K_{zz} \end{vmatrix} = 0 \tag{8}$$

where K_{ij} are the elements of the dielectric tensor $\mathbf{K}$, and $n_\parallel$ and $n_\perp$ are the component refractive indices,

$$\mathbf{n} = \frac{c\mathbf{k}}{\omega}, \quad \mathbf{n}^2 = n_\parallel^2 + n_\perp^2, \tag{9}$$

$$n_\parallel = \frac{ck_\parallel}{\omega} = |\mathbf{n}| \cos\psi, \quad n_\perp = \frac{ck_\perp}{\omega} = |\mathbf{n}| \sin\psi,$$

with ω the wave frequency, c the velocity of light and ψ the angle made by the wave vector $\mathbf{k}$ with the Z (or magnetic field) direction. For given zeroth-order particle distribution functions $f_\sigma^0(v_\perp, v_\parallel)$, the elements of the dielectric tensor can be written in the form

$$\begin{pmatrix} K_{xx} - 1 \\ K_{zx} \\ K_{xy} \\ K_{yy} - 1 \\ K_{zy} \end{pmatrix} = \sum_\sigma \frac{\omega_{\mathrm{p}\sigma}^2}{\omega} \int v_\perp d^3v \sum_{n=-\infty}^{+\infty} \begin{pmatrix} \frac{n^2}{\mu^2} J_n^2 \\ \frac{n}{\mu} J_n^2 \frac{v_\parallel}{v_\perp} \\ \frac{in}{\mu} J_n J_n' \\ J_n'^2 \\ i J_n J_n' \frac{v_\parallel}{v_\perp} \end{pmatrix}$$

$$\times \frac{(1 - k_\parallel v_\parallel/\omega)(\partial f_\sigma^0/\partial v_\perp) + (k_\parallel v_\perp/\omega)(\partial f_\sigma^0/\partial v_\parallel)}{\omega - k_\parallel v_\parallel - n\Omega_\sigma}, \tag{10}$$

$$\begin{pmatrix} K_{xz} \\ K_{yz} \\ K_{zz} - 1 \end{pmatrix} = \sum_\sigma \frac{\omega_{\mathrm{p}\sigma}^2}{\omega} \int v_\perp d^3v \sum_{n=-\infty}^{+\infty} \begin{pmatrix} \frac{n}{\mu} J_n^2 \\ -i J_n J_n' \\ J_n^2 \frac{v_\parallel}{v_\perp} \end{pmatrix}$$

$$\times \frac{(n\Omega_\sigma/\omega)(v_\parallel/v_\perp)(\partial f_\sigma^0/\partial v_\perp) + (1 - n\Omega_\sigma/\omega)(\partial f_\sigma^0/\partial v_\parallel)}{\omega - k_\parallel v_\parallel - n\Omega_\sigma} \tag{11}$$

and

$$K_{yx} = -K_{xy}$$

where $J_n(\mu)$ is the Bessel function of the first kind of order n, $J_n'(\mu)$ is its derivative with respect to μ (Abramowitz and Stegun, 1970) and

$$\mu = \frac{k_\perp v_\perp}{\Omega_\sigma} \tag{12}$$

with the gyrofrequency Ω_σ and plasma frequency $\omega_{\mathrm{p}\sigma}$ given by

$$\Omega_\sigma = \frac{q_\sigma B_0}{m_\sigma c}, \quad \omega_{\mathrm{p}\sigma}^2 = \frac{4\pi n_{0\sigma} q_\sigma^2}{m_\sigma}, \tag{13}$$

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where q_σ, m_σ and $n_{0\sigma}$ are respectively particle charge, mass and number density of species σ, Ω_σ and μ contain the sign of the particle charge, and B_0 is ambient magnetic field.

If the distribution function f_σ^0 is an even function of $v_\parallel$, we further note that

$$K_{zx} = K_{xz}, \tag{14}$$

$$K_{zy} = -K_{yz}. \tag{15}$$

Substituting $f_\sigma^0 = f_{\kappa\ell}^\sigma$ as given in (5) into (10) and (11), we find the expression for K_{ij} as follows (Gradshteyn and Ryzhik 1980):

$$\begin{aligned}\begin{pmatrix} K_{xx}-1 \\ K_{xy} \\ K_{yy}-1 \end{pmatrix} = \sum_\sigma \frac{\omega_{\mathrm{p}\sigma}^2}{\omega^2} \frac{4}{(2\lambda)^{\ell+2}} \sum_{n=-\infty}^{+\infty} \begin{pmatrix} n^2 \\ in \\ 1 \end{pmatrix} \Bigg\{ \left(\frac{v_{\perp_{\mathrm{th}\sigma}}^2}{v_{\parallel_{\mathrm{th}\sigma}}^2} - 1 \right) \begin{pmatrix} T_1 \\ T_3 \\ T_5 \end{pmatrix} \\ + \left[\left(\frac{v_{\perp_{\mathrm{th}\sigma}}^2}{v_{\parallel_{\mathrm{th}\sigma}}^2} - 1 \right) \xi_{n\sigma} + \frac{\omega}{k_\parallel \theta_{\parallel_\sigma}} \right] \begin{pmatrix} T_2 \\ T_4 \\ T_6 \end{pmatrix} \\ + 2\lambda \left[\begin{pmatrix} T_7 \\ T_9 \\ T_{11} \end{pmatrix} + \left(\xi_{n\sigma} - \frac{\omega}{k_\parallel \theta_{\parallel_\sigma}} \right) \begin{pmatrix} T_8 \\ T_{10} \\ T_{12} \end{pmatrix} \right] \Bigg\}, \end{aligned} \tag{16}$$

$$\begin{aligned}\begin{pmatrix} K_{xz} \\ K_{yz} \\ K_{zz}-1 \end{pmatrix} = \sum_\sigma \frac{\omega_{\mathrm{p}\sigma}^2}{\omega^2} \frac{v_{\parallel_{\mathrm{th}\sigma}}}{v_{\perp_{\mathrm{th}\sigma}}} \sum_{n=-\infty}^{+\infty} \begin{pmatrix} 4n \in_\sigma /(2\lambda)^{\ell+3/2} \\ -4i \in_\sigma /(2\lambda)^{\ell+3/2} \\ \left(\frac{v_{\parallel_{\mathrm{th}\sigma}}}{v_{\perp_{\mathrm{th}\sigma}}} \right) \frac{2\xi_{n\sigma}}{(2\lambda)^{\ell+1}} \end{pmatrix} \\ \times \Bigg\{ \left[\left(\frac{v_{\perp_{\mathrm{th}\sigma}}^2}{v_{\parallel_{\mathrm{th}\sigma}}^2} - 1 \right) \xi_{n\sigma} + \frac{\omega}{k_\parallel \theta_{\parallel_\sigma}} \right] \left[\begin{pmatrix} T_1 \\ T_3 \\ T_1 \end{pmatrix} + \xi_{n\sigma} \begin{pmatrix} T_2 \\ T_4 \\ T_2 \end{pmatrix} \right] \\ + 2\lambda \left(\xi_{n\sigma} - \frac{\omega}{k_\parallel \theta_{\parallel_\sigma}} \right) \left[\begin{pmatrix} T_7 \\ T_9 \\ T_7 \end{pmatrix} + \xi_{n\sigma} \begin{pmatrix} T_8 \\ T_{10} \\ T_8 \end{pmatrix} \right] \Bigg\}, \end{aligned} \tag{17}$$

where

$$T_1 = \frac{\Gamma(\kappa+3/2)}{\kappa^{\ell+2}\Gamma(\ell+1)\Gamma(\kappa-\ell-1/2)} \int_0^\infty \frac{\bar{\mu}^{2\ell+1} J_n^2(\bar{\mu})\, d\bar{\mu}}{(1+\bar{\mu}^2/2\lambda\kappa)^{\kappa+3/2}}, \tag{18}$$

$$\begin{aligned} T_2 = {} & \frac{(\kappa+1)^{3/2}\Gamma(\kappa+1/2)}{\kappa^{\ell+5/2}\Gamma(\ell+1)\Gamma(\kappa-\ell-1/2)} \\ & \times \int_0^\infty \frac{\bar{\mu}^{2\ell+1} J_n^2(\bar{\mu})}{(1+\bar{\mu}^2/2\lambda\kappa)^{\kappa+2}} Z_{\kappa+1}^* \left(\left[\frac{(\kappa+1)/\kappa}{(1+\bar{\mu}^2/2\lambda\kappa)} \right]^{1/2} \xi_{n\sigma} \right) d\bar{\mu}, \end{aligned} \tag{19}$$

$$T_3 = \frac{\Gamma(\kappa+3/2)}{\kappa^{\ell+2}\Gamma(\ell+1)\Gamma(\kappa-1/2-\ell)} \int_0^\infty \frac{J_n(\bar{\mu}) J_n'(\bar{\mu})\, d\bar{\mu}}{(1+\bar{\mu}^2/2\lambda\kappa)^{(\kappa+3/2)}}, \tag{20}$$

$$\begin{aligned} T_4 = {} & \frac{(\kappa+1)^{3/2}\Gamma(\kappa+1/2)}{\kappa^{\ell+5/2}\Gamma(\ell+1)\Gamma(\kappa-1/2-\ell)} \\ & \times \int_0^\infty \frac{\bar{\mu}^{2\ell+2} J_n(\bar{\mu}) J_n'(\bar{\mu})}{(1+\bar{\mu}^2/2\lambda\kappa)^{(\kappa+2)}} Z_{\kappa+1}^* \left(\left[\frac{(\kappa+1)/\kappa}{1+\bar{\mu}^2/2\lambda\kappa} \right]^{1/2} \xi_{n\sigma} \right) d\bar{\mu}, \end{aligned} \tag{21}$$

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$$T_5 = \frac{\Gamma(\kappa+3/2)}{\kappa^{\ell+2}\Gamma(\ell+1)\Gamma(\kappa-1/2-\ell)} \int_0^\infty \frac{\bar{\mu}^{2\ell+3} J_n'^2(\bar{\mu})\, d\bar{\mu}}{(1+\bar{\mu}^2/2\lambda\kappa)^{(\kappa+3/2)}}, \tag{22}$$

$$T_6 = \frac{(\kappa+1)^{3/2}\Gamma(\kappa+1/2)}{\kappa^{\ell+5/2}\Gamma(\ell+1)\Gamma(\kappa-1/2-\ell)} \times \int_0^\infty \frac{\bar{\mu}^{2\ell+3} J_n'^2(\bar{\mu})}{(1+\bar{\mu}^2/2\lambda\kappa)^{(\kappa+2)}} Z^*_{\kappa+1}\left(\left[\frac{(\kappa+1)/\kappa}{1+\bar{\mu}^2/2\lambda\kappa}\right]^{1/2}\xi_{n\sigma}\right) d\bar{\mu}, \tag{23}$$

$$T_7 = \frac{\ell\Gamma(\kappa+1/2)}{\kappa^{\ell+1}\Gamma(\ell+1)\Gamma(\kappa-1/2-\ell)} \int_0^\infty \frac{\bar{\mu}^{2\ell-1} J_n^2(\bar{\mu})\, d\bar{\mu}}{(1+\bar{\mu}^2/2\lambda\kappa)^{(\kappa+1/2)}}, \tag{24}$$

$$T_8 = \frac{\ell\kappa^{3/2}\Gamma(\kappa-1/2)}{\kappa^{\ell+3/2}\Gamma(\ell+1)\Gamma(\kappa-\ell-1/2)} \times \int_0^\infty \frac{\bar{\mu}^{2\ell-1} J_n^2(\bar{\mu})}{(1+\bar{\mu}^2/2\lambda\kappa)^{(\kappa+1)}} Z^*_{\kappa}\left(\left[\frac{1}{1+\bar{\mu}^2/2\lambda\kappa}\right]^{1/2}\xi_{n\sigma}\right) d\bar{\mu}, \tag{25}$$

$$T_9 = \frac{\ell\Gamma(\kappa+1/2)}{\kappa^{\ell+1}\Gamma(\ell+1)\Gamma(\kappa-\ell-1/2)} \int_0^\infty \frac{\bar{\mu}^{2\ell} J_n(\bar{\mu}) J_n'(\bar{\mu})\, d\bar{\mu}}{(1+\bar{\mu}^2/2\lambda\kappa)^{(\kappa+1/2)}}, \tag{26}$$

$$T_{10} = \frac{\ell\kappa^{3/2}\Gamma(\kappa-1/2)}{\kappa^{\ell+3/2}\Gamma(\ell+1)\Gamma(\kappa-\ell-1/2)} \times \int_0^\infty \frac{\bar{\mu}^{2\ell} J_n'(\bar{\mu})}{(1+\bar{\mu}^2/2\lambda\kappa)^{(\kappa+1)}} Z^*_{\kappa}\left(\left[\frac{1}{1+\bar{\mu}^2/2\lambda\kappa}\right]^{1/2}\xi_{n\sigma}\right) d\bar{\mu}, \tag{27}$$

$$T_{11} = \frac{\ell\Gamma(\kappa+1/2)}{\kappa^{\ell+1}\Gamma(\ell+1)\Gamma(\kappa-\ell-1/2)} \int_0^\infty \frac{\bar{\mu}^{2\ell+1} J_n'^2(\bar{\mu})\, d\bar{\mu}}{(1+\bar{\mu}^2/2\lambda\kappa)^{(\kappa+1/2)}}, \tag{28}$$

$$T_{12} = \frac{\ell\kappa^{3/2}\Gamma(\kappa-1/2)}{\kappa^{\ell+3/2}\Gamma(\ell+1)\Gamma(\kappa-\ell-1/2)} \times \int_0^\infty \frac{\bar{\mu}^{2\ell+1} J_n'^2(\bar{\mu})}{(1+\bar{\mu}^2/2\lambda\kappa)^{(\kappa+1)}} Z^*_{\kappa}\left(\left[\frac{1}{1+\bar{\mu}^2/2\lambda\kappa}\right]^{1/2}\xi_{n\sigma}\right) d\bar{\mu}, \tag{29}$$

$$\bar{\mu} = \frac{k_\perp v_\perp}{|\Omega_\sigma|}, \tag{30}$$

$$\lambda = \frac{1}{2}\left(\frac{k_\perp v_{\mathrm{th}_{\perp\sigma}}}{\Omega_\sigma}\right)^2, \quad \xi_{n\sigma} = \frac{\omega - n\Omega_\sigma}{k_\parallel \theta_{\mathrm{th}_{\parallel\sigma}}}. \tag{31}$$

The functions Z^*_κ and $Z^*_{\kappa+1}$ are modified plasma dispersion functions introduced by Summers and Thorne (1991). We note that expressions (16) and (17) are the same as those obtained by Summers and Thorne (1995) for distribution function (1) with the replacement of $\theta_{\perp\sigma} \to v_{\mathrm{th}\perp\sigma}, \theta_{\parallel\sigma} \to v_{\mathrm{th}\parallel\sigma}$. However, the integrals given by (18)–(29) are different in two cases ((1) and (5)). For pure Maxwellian distribution $(\kappa \to \infty, \ell = 0), T_7 = T_8 = T_9 = T_{10} = T_{11} = T_{12} = 0$ and

$$T_1 = \lambda\Lambda_n, \tag{32}$$

$$T_2 = T_1 \mathrm{Z}(\xi_{n\sigma}), \tag{33}$$

$$T_3 = \lambda^2\Lambda_n', \tag{34}$$

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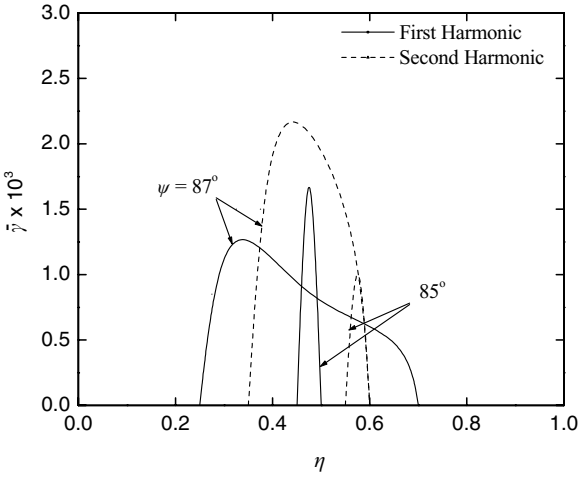


Figure 1. Normalized temporal growth rates $\bar{\gamma}(=\gamma/\Omega_{ce})$ of Bernstein waves for $\kappa = 2$ and $\ell = 1$. The frequency within each band is $\bar{\omega}_r(=\omega_r/\Omega_{ce}) = n + \eta$, where $n = 1, 2$ and η lies between 0 and 1 (Summers and Thorne (1995) model).

$$T_4 = T_3 Z(\xi_{n\sigma}), \tag{35}$$

$$T_5 = n^2\lambda\Lambda_n - 2\lambda^3\Lambda_n', \tag{36}$$

$$T_6 = T_5 Z(\xi_{n\sigma}), \tag{37}$$

where

$$\Lambda_n = \mathrm{e}^{-\lambda}\mathrm{I}_n(\lambda) \tag{38}$$

with Λ_n' the derivative of Λ_n with respect to λ and I_n the modified Bessel function of the first kind of order n. Z is the plasma dispersion function (Fried and Conte 1961).

3. Application to electrostatic waves

The general electrostatic dispersion relation can be obtained from (8) by applying the electrostatic approximation $(ck/\omega)^2 \gg K_{ij}$, the resulting equation being

$$k_\perp^2 K_{xx} + 2k_\perp k_\parallel K_{xz} + k_\parallel^2 K_{zz} = 0. \tag{39}$$

For a pure Maxwellian distribution, (39) gives the dispersion relation for Bernstein waves. In the present application of our results we consider a plasma consisting of a Maxwellian electron distribution (cold) and suprathermal electrons represented by a kappa distribution (hot). Thus the summation over species involved in (10) and (11) is taken over $\sigma = \mathrm{c}, \mathrm{h}$. As a representative case for magnetospheric plasma we use the plasma parameters: $n_\mathrm{c} = 0.04$ cm^{-3}, $n_\mathrm{h} = 0.01$ cm^{-3}, $T_{\mathrm{c}_\perp} = T_{\mathrm{c}_\parallel} = 40.0$ K, $T_{\mathrm{h}_\perp} = T_{\mathrm{h}_\parallel} = 1500\ ^0$K and $B_0 = 0.000\,16$ G in the case study. Equation (39) is then solved numerically for real and imaginary parts of $\omega = \omega_r + i\gamma$, with positive γ giving the growth of the instability. For distribution function (1) (ST model) we use the K_{ij} given in equations (35) and (36) of Summers and Thorne (1995). For distribution function (5) (LS model), K_{ij} is taken from (16) and (17) of the present work.

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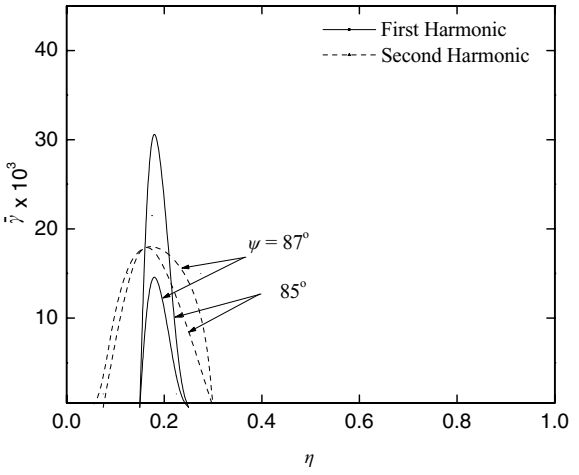


Figure 2. Same as Fig. 1 but for $\kappa = 2$ and $\ell = 3$ (ST model).

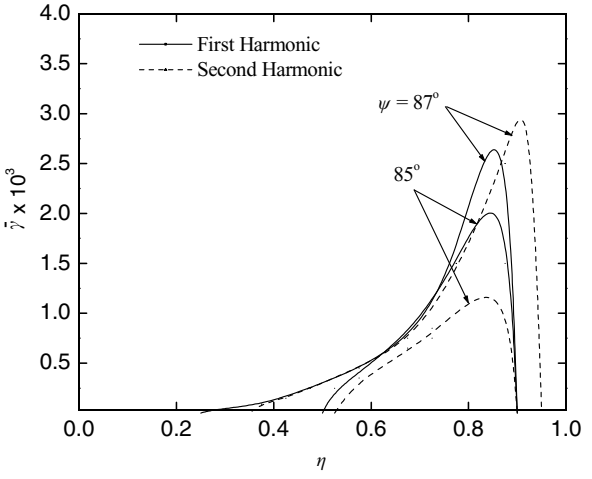


Figure 3. Normalized temporal growth rates of Bernstein waves for $\kappa = 3$ and $\ell = 1$ (Leubner and Schupfer (2001) model).

To study the effect of the loss-cone index ℓ on the instability we consider $\kappa = 2, \ell = 1$ and $\kappa = 2, \ell = 3$ in the ST model. For the LS model there is a constraint $\kappa - \ell - 1/2 > 0$. Therefore we have taken $\kappa = 3, \ell = 1$ and $\kappa = 3, \ell = 2$ for the LS model. Temporal growth rates for first and second harmonic bands are shown in Figs 1 and 2 for the ST model and in Figs 3 and 4 for the LS model. It may be noted from these figures that as the loss-cone index ℓ is increased, the temporal growth rate increases in the ST model. However in the LS model the growth rate decreases as ℓ is increased. Further, the band width at $\psi = 87^\circ$ in the ST model is larger in comparison with the band width for the case of the LS model. Also, in general, the frequency within the band shifts to a higher value in the LS model as compared to the ST model.

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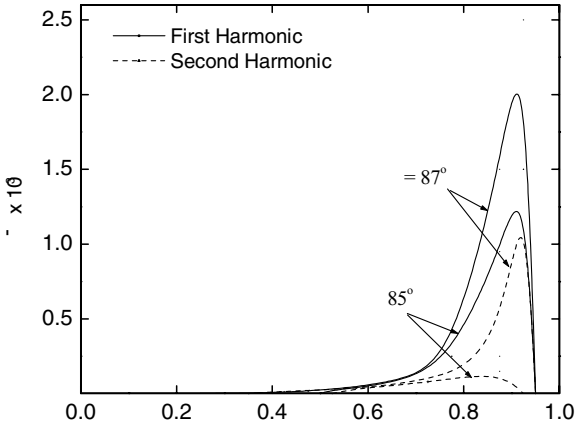


Figure 4. Same as Fig. 3 but for $\kappa = 3$ and $\ell = 2$ (LS model).

4. Conclusions

The effect of loss-cone index ℓ appearing in the distribution functions given by Summers and Thorne (1995) and Leubner and Schupfer (2001) has been studied. It is found that the temporal growth rate of Bernstein waves increases as the loss-cone index increases in the ST model. For the LS model the growth rate is found to decrease as ℓ increases. Further it is found that the band width is larger in the ST model as compared to the LS model and also the frequency within the band shifts to a higher value for the LS model in comparison with that for the ST model. These characteristics of Bernstein waves may be used as a diagnostic tool for inferring the nature of plasma distribution functions in the planetary magnetospheres.

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