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A redesigned DCAL controller without velocity measurements: theory and demonstration*

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SUMMARY

A link position tracking controller is formulated for an n-link, rigid, revolute, serially-connected robot. The controller generates torque commands to the individual robot links based on adaptive estimates of the system parameters and measurements of only link positions. A filtering technique, based on the link position signal, is used to alleviate the need for velocity measurements. A complete development of the controller is presented along with a proof of semiglobal asymptotic link position-velocity tracking performance. Experimental validation of the proposed controller on the Integrated Motion Inc. (IMI) two-link direct drive robot is also presented. Several extensions to the basic controller are described that consider the use of fixed parameter estimates.

KEYWORDS: DCAL controller; Tracking control; Velocity estimation; Robots.

1 INTRODUCTION

A need common to many robot applications is the accurate following of a specific path by the end-effector of the robot, which translates to accurate positioning of the individual links that constitute the robot. Since the cost of implementing a controller for achieving link position tracking would typically include the cost of motion sensors, this paper addresses the problem of link position trajectory following for a multiple revolute-link rigid robot under the constraint of minimizing the sensor count (i.e., eliminate the need for link velocity measurements). The work here is suited for a direct drive robot system that is calibrated to accept torque commands to the individual links. Therefore, the final result will be a controller which generates torque commands to the individual links and ensures link position tracking.

Over the last ten years, many researchers have attacked the robot control problem from the perspective that the structure of the manipulator dynamics is well-defined; however, the physical parameters, representing mass, inertia, and friction, are often considered as unknown, constant, parametric disturbances. Along this line reasoning, many researchers have developed adaptive nonlinear controllers which include: i) a nonlinear feedforward term which is adjusted on-line by means of an adaptive update law and ii) a linear feedback term which usually requires measurement of link position and velocity. A detailed review of adaptive robot control work is provided in reference 3. Some of the adaptive techniques which are relevant to the proposed controller are quoted in references 4–14. Of particular interest with regard to the work presented in this paper, is the adaptive controller presented in reference 6. Specifically, the desired compensation adaptation law (DCAL)[6] is introduced to reduce on-line computations of the update law by allowing *a priori* calculation of the regressor matrix. Many other adaptive control results provide valuable contributions to the field; however, those listed above are the most relevant to the work presented in this paper.

An alternate approach, which contains a complete mathematical development, suggests the use of nonlinear observers to produce an estimate of unmeasurable quantities, particularly velocity. Often these observers contain dynamics which attempt to mimic the behavior of the physical system to a certain extent. For example in reference 15, Nicosia *et al.* design an exact knowledge based observer-controller which yielded an asymptotic stability result for the link position tracking error. In addition to reference 15, experimental work was presented to verify the feasibility of using high gain observers in conjunction with several different control techniques[16]. In references 17 and 18, exact knowledge based controllers were presented which utilized variable structure observers.[19] For the compensation of system uncertainty, Canudas de Wit *et al*. developed variable structure model-based observers for the design of adaptive[22] and robust[21] controllers. In addition, Erlic *et al.* designed reduced-order observers for the use in an exact knowledge-based controller[22] and an adaptive controller[23] which were shown to have applications for impedance control in reference 24. In reference 25, Qu *et al.* illustrated that an uniform ultimate bounded stability

* *Preliminary* versions of this paper were presented at conferences.[1,2]

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result can be obtained with a linear high gain observer in conjunction with a linear controller. Zhu *et al.*[26] presented a variable structure controller which utilized a model-based observer with fixed parameter estimates. Yuan[27] *et al.* used a filter on the position signal to create a new signal which is used to design a robust controller with feedforward terms with fixed parameter estimates and a linear feedback term. In Berghuis[28] *et al.* used a passivity approach to develop a controller-observer for robot motion control using only position measurements and in reference 29 formulated a robust controller for robot motion control based only on position measurements. Kelly *et al.*[30] used a filter to provide velocity estimates for a set-point robot controller with L. Hsu *et al.*[31] proposed an input/output variable structure model-reference adaptive control scheme. Colbaugh[32] *et al.* forwarded the design of an output feedback adaptive position regulation scheme. Recently, Kaneko and Horowitz[33] utilized a second-order filter to modify the DCAL control algorithm such that link velocity measurements are not required. Roughly speaking, the approaches given here and in reference 33 yield the same type of stability result using significantly different control structures.

The adaptive controller proposed here represents an amalgamation of the DCAL[6] and filtering techniques.[21,22] Specifically, a velocity error generating filter, based on link position measurements, is proposed to replace link velocity measurements. A Lyapunov-type analysis along with a judicious definition of the tracking error terms are then used to find the appropriate modifications to the adaptive control law. We then illustrate how the control law ensures semi-global[34] asymptotic link position-velocity tracking. The tracking performance is semi-global in the sense that a controller gain must be increased to cover a set of initial conditions. (Note this condition is explicitly defined in the theorem statement). The new adaptive control law represents several enhancements to previous control schemes. First, it combines elements of the DCAL and the velocity error generating filter to reduce computational overhead and measurement of link velocity. Second, the controller and the velocity error generating filter are both comprised of smooth functions, as opposed to variable structure type terms. Finally, the tracking performance analysis is straightforward as compared to some of the previous adaptive partial state feedback control results.

This paper is organized to logically follow the development of the proposed control. In Section 2, the notion of link position tracking is defined, the mechanical dynamics of a n-link, rigid, revolute robot are given, and useful properties based on the structure of the robot equation are given. Section 3 summarizes the proposed link position tracking controller. The performance of the controller, in the context of link position tracking, is then addressed in Section 4. Experimental validation of the proposed controller is provided in Section 5. The extension of the basic controller to include fixed parameter estimates is described in Section 6. The details of the tracking performance analysis are contained in Appendix A and Appendix B.

2 PROBLEM FORMULATION

This section lays the foundation for the later control development. First, we present the governing equation which describes the mechanical dynamics of a n-link revolute robot. Next, some useful properties related to the structure of the robot equation are presented. Finally, the goal of the controller to be designed is given in the context of a concise definition of link position tracking.

2.1 Robot equation

The mathematical model for the mechanical dynamics of a n-link, revolute, rigid, direct drive robot is given as[6]

$$\tau = M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} + G(q) + F_d\dot{q} \tag{1}$$

where $q \in \Re^n$ is the vector of link angles. The link angles will be considered as the output of the mechanical system. The matrices $M(q) \in \Re^{n\times n}$, $V_m(q, \dot{q}) \in \Re^{n\times n}$, $G(q) \in \Re^n$, and $F_d \in \Re^{n\times n}$ account for the system dynamics as they describe inertia, Coriolis-centripetal, gravity, and friction forces, respectively. The vector $\tau \in \Re^n$ contains the torque input to each link and is considered as the input to the mechanical system. The controller designed in the next section will be based on inexact knowledge of the constant parameters that are found in the system matrices $M(q)$, $V_m(q, \dot{q})$, $G(q)$, and F_d, it will be assumed that upper and lower bounds are known for each of these constant parameters.

2.2 Properties of the robot equation

The structure of the robot equation yields the following useful properties which will be used later in the development.

Property 1: Boundedness of the Inertia Matrix[35] The inertia matrix of the n-link revolute robot, $M(q)$, is known to be positive definite and symmetric. These properties are summarized by the useful form $m_1 \|x\|^2 \leq x^T M(q)x \leq m_2 \|x\|^2 = \|M(q)\|_{i2} \|x\|^2$, where m_1 and m_2 are positive constants, $\|\cdot\|_{i2}$ represents the 2 induced matrix norm,* and $\|\cdot\|$ is the Euclidean norm of an arbitrary vector $x \in \Re^n$.

Property 2: Skewed Symmetry[35] The following difference of the mechanical system matrices is skew-symmetric $\frac{1}{2}\dot{M}(q) - V_m(q, \dot{q})$, where $\dot{M}(q)$ is the element-wise time derivative of the inertia matrix. The skew-symmetry property can be written in the following form for later use $x^T(\frac{1}{2}\dot{M}(q) - V_m(q, \dot{q}))x = 0$, for an arbitrary vector $x \in \Re^n$.

Property 3: Boundedness of the Coriolis-centripetal Matrix[15] An upper bound can be placed on the induced ∞-norm of the Coriolis-centripetal matrix by $\|V_m(q, \dot{q})\|_{i\infty} < \zeta_c \|\dot{q}\|$, where ζ_c is a positive scalar constant.

Property 4: Linear in Parameters [6] The mechanical

* The induced matrix norms used above are based on the value of the joint position that causes the norm to be maximized.

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system in (1) is linear in the parameters in the sense $Y_d(q_d, \dot{q}_d, \ddot{q}_d)\Theta = M(q_d)\ddot{q}_d + V_m(q_d, \dot{q}_d)\dot{q}_d + G(q_d) + F_d\dot{q}_d$, where the vector $\Theta \in \Re^p$ contains the unknown, constant system parameters, $q_d \in \Re^n$, $\dot{q}_d \in \Re^n$, and $\ddot{q}_d \in \Re^n$ represent the desired link position, velocity and acceleration trajectories, respectively, and $Y_d(q_d, \dot{q}_d, \ddot{q}_d) \in \Re^{n \times p}$, called the regressor matrix, contains known functions related to the structure of the mechanical system.

2.3 Control objective

Briefly stated, the control objective is to design the torque input to provide for link position tracking based on uncertain system parameter knowledge and link position measurements only. The goal of the proposed control is to force the link positions to follow a desired trajectory, q_d; the closeness of this tracking is quantified in the link position tracking error defined by

$$e = q_d - q. \tag{2}$$

A suitable control will act to ensure that e is "small". The desired trajectory is restricted by the following bounds on q_d and its first three time derivatives

$$\|q_d\| < \zeta_{d1}, \quad \|\dot{q}_d\| < \zeta_{d2}, \quad \|\ddot{q}_d\| < \zeta_{d3},$$

and

$$\|\dddot{q}_d\| < \zeta_{dA} \tag{3}$$

where the ζ_{di}'s are known positive constants.

As part of fulfilling the link position tracking objective, the design of a proposed controller is to be predicted on uncertain knowledge of the parameters which are contained in the robot equation. In order to compensate for this lack of exact parametric knowledge, an adaptation law will be developed which will provide on-line parameter estimates. The difference between the actual and estimated parameters is described by

$$\tilde{\Theta} = \Theta - \hat{\Theta} \tag{4}$$

where $\hat{\Theta} \in \Re^p$ is a dynamic estimate of Θ. In addition to the assumption of uncertain parametric knowledge, a second constraint placed on the proposed controller is that it must achieve link position tracking based solely on measurement of link position. This presents a significant design challenge as knowledge of the link velocity, $\dot{q}$, is normally included in many position control designs. Here, knowledge of the velocity will be inferred through filtering of link position measurements.

3 LINK POSITION TRACKING CONTROLLER

The final form of the link position tracking controller formulated to achieve link position tracking in the robot equation is now presented. Full understanding of the proposed controller can be achieved by following the design details provided in Appendix A and Appendix B. That is, the controller posed here represents the culmination of an iterative design process. The design proceeds by forming dynamic equations which describe certain error terms in the closed loop system. These dynamic error equations are then analyzed using a Lyapunov-like approach in order to attempt to determine their behavior. Based on the Lyapunov-like analysis, the control can then be modified to yield a desired result. The controller given here is the final product of this iterative design process. A block diagram of the controller is shown in Figure 1.

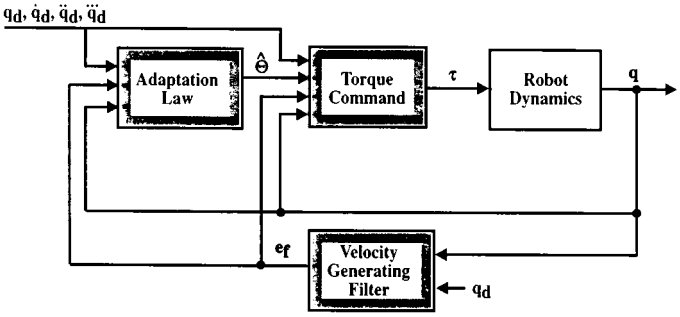


Fig. 1. Controller Block Diagram.

3.1 Velocity error generating filter

In order to compensate for the lack of direct measurements of the velocity signal, the link position tracking error is filtered using the following dynamic filter

$$\dot{p} = -(k+1)p + (k^2+1)e \tag{5}$$

$$e_f = -ke + p \tag{6}$$

where the variable $e_f \in \Re^n$ is the output of the filter, and $p \in \Re^n$ is used as an internal filter variable. The positive scalar constant k will be labeled as a controller gain due to its role in the stability analysis of Appendix B. The filter accepts the link position tracking error, e, as its input and produces the output signal e_f, which will be used to "capture" the behavior of $\dot{e}$. Therefore, as a surrogate to measurements of $\dot{e}$ (and therefore $\dot{q}$) the signal e_f will be used. Note that the form of (5) and (6) allows the filter to generate a signal that is related to $\dot{e}$ but yet can be physically implemented since it depends only on the measurable link position q.

Remark 1. In order to support the constraints on the controller gains set forth in Theorem 1 the initial condition on the internal filter variable is set to $p(0) = ke(0)$; hence, $e_f(0) = 0$.

3.2 Torque input and update law

To achieve the posiiton tracking objective, the following torque input controller is proposed

$$\tau = Y_d\hat{\Theta} - ke_f + e \tag{7}$$

where k and e_f were introduced in (5) and (6), Y_d was defined in Property 4, and $\hat{\Theta}$ is the dynamic estimate of Θ where Θ was defined in Property 4. To simplify the stability analysis, k is now explicitly defined in terms of bounding coefficients and a nonlinear damping gain[36] as follows

$$k = \frac{1}{m_1}(\alpha_o^2 k_N + \alpha_1^2 k_N + 1), \tag{8}$$

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where k_N is a positive, nonlinear damping gain, and α_o, α_1 are positive "bounding" constants. The constants α_o and α_1 are explicitly defined as

$$\alpha_o = 3 \cdot \max\{(\zeta_c\zeta_{d2} + 2m_2 + \zeta_3),\ (m_2 + \zeta_3),\ (\zeta_0 + \zeta_3 + \zeta_c\zeta_{d2})\} \quad (9)$$

and

$$\alpha_1 = 2 \cdot \max\{(\zeta_1 + \zeta_2 + \zeta_c),\ (\zeta_1 + \zeta_c),\ (\zeta_1),\ \ (6\zeta_1 + 4\zeta_c)\} \quad (10)$$

where ζ_{d2} is defined in (3), m_2 is defined in Property 1, ζ_c is generated as in Property 3, and the ζ_i's are positive bounding constants, which only depend on bounds on the desired trajectory and bounds on the parametric quantities of (1) (as similarly formulated in reference 6). The controller of (7) generates a vector of input torques, τ, based on the generated velocity signal e_f, the link position tracking error e, and the parameter update law provided by

$$\hat{\Theta} = \Gamma \int_0^t \left(Y_d^T(\sigma)(e(\sigma) + e_f(\sigma)) - \frac{d}{d\sigma}\{Y_d^T(\sigma)\}e(\sigma) \right) d\sigma + \Gamma Y_d^T e \quad (11)$$

where the form of the regressor matrix, Y_d, was outlined in Property 4. The diagonal gain matrix $\Gamma \in \Re^{p\times p}$ in (11) contains positive gains along the diagonal.

Remark 2. The parameter update law in (11) is based on the need to have $\hat{\Theta}$ in the form

$$\hat{\Theta} = \Gamma \int_0^t Y_d^t(\sigma)(\dot{e}(\sigma) + e(\sigma) + e_f(\sigma))\, d\sigma$$

as delineated by equations (14) and (49). The above form however would require measurement of $\dot{e}(t)$ which would in turn contain the unmeasurable link velocity. To circumvent this problem, the integral is segregated into measurable and unmeasurable terms as follows

$$\hat{\Theta} = \Gamma \int_0^t (Y_d^T(\sigma)(e(\sigma) + e_f(\sigma)))\, d\sigma + \Gamma \int_0^t Y_d^T(\sigma)\dot{e}(\sigma)\, d\sigma.$$

The first integral can be directly implemented and the second integral is now rewritten using *Integration by Parts* (as found in most Calculus textbooks) to yield the update law given in reference 11. Note that this last operation introduces the constraint that $Y_d(q_d, \dot{q}_d, \ddot{q}_d)$ must be differentiable with respect to time.

4 MAIN RESULT

A theorem delineating the link position tracking performance of the proposed controller is presented. The theorem is based on a Lyapunov-type analysis (see Appendix B) applied to the error systems developed in Appendix A.

Theorem 1

For the link position tracking controller and adaptation law given by (7) and (11) used in conjunction with the velocity error generating filter given by (5) and (6), if the control gain k_N of (8) is chosen such that

$$k_N > \frac{\lambda_2}{\lambda_1}\|x(0)\|^2 + 1, \quad (12)$$

where $x \in \Re^{3n+p}$, $\eta \in \Re^n$, $\lambda_1 \in \Re^1$, and $\lambda_2 \in \Re^1$, are defined as follows

$$x = [e^T \quad e_f^T \quad \eta^T \quad \tilde{\Theta}^T]^T, \quad (13)$$

$$\eta = \dot{e} + e + e_f, \quad \lambda_1 = \tfrac{1}{2}\min\{1, m_1, \lambda_{\min}\{\Gamma^{-1}\}\}, \quad (14)$$

$$\lambda_2 = \tfrac{1}{2}\max\{1, m_2, \lambda_{\max}\{\Gamma^{-1}\}\}, \quad (15)$$

then

$$\lim_{t\to\infty} z(t) = 0 \quad (16)$$

where

$$z = [e^T \quad e_f^T \quad \eta^T]^T \in \Re^{3n}. \quad (17)$$

Note that $\lambda_{\min}\{\cdot\}$ and $\lambda_{\max}\{\cdot\}$ are used to denote the minimum eignevalue and maximum eigenvalue in the expressions of (15).

Remark 3. The primary conclusion of Theorem 1 is that the proposed controller produces semi-global [34] asymptotic link position tracking (*i.e.*, the link position tracking error is driven to zero). Note that the gain k_N of (8) can be made arbitrarily large to encompass any set of initial conditions as illustrated by (12). It is obvious from the form of (12) and (13) that we must have *a priori* information regarding the bounds of the unknown parameters and the initial link velocities to facilitate the calculation of k_N. In addition, an important result that must be included with this theorem is that all system variables remain bounded, see Appendix B for details.

Remark 4. Note that theterm η of (14) is not physically measurable since it contains the link velocities; however, it can be seen from the form of (7) and (11) that the calculation of η is not required for implementation. However, the term η contains information utilized in the tracking performance analysis given in the appendices.

Remark 5. An extension of Theorem 1 can be made to describe link velocity tracking. The equality in equation (14) can be used to form the following inequality

$$\|\dot{e}\| = \|\dot{q}_d - \dot{q}\| \le \|\eta\| + \|e_f\| + \|e\|. \quad (18)$$

Since the crux of Theorem 1 is that the terms on the right-hand side of the inequality are driven to zero (see (16) and (17)), we can conclude that

$$\lim_{t\to\infty} \dot{e}(t) = 0 \quad (19)$$

or in other words the controller ensures asymptotic link velocity tracking.

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Remark 6. An interesting aside to the developments presented here lies in the similarity between the filter used to generate the velocity information and a continuous time approximation of the backwards difference algorithm. That is, the Laplace domain transfer function for producing a velocity tracking error estimate from the position tracking error by a backwards difference approach could be obtained using the bilinear transform as $T_{BackDiff}(s) = \dfrac{2s}{Ts+2}$ (where T is the sample time) while the Laplace domain transfer function for the velocity error generating filter in (5) and (6) is $T_{Filter}(s) = -\dfrac{ks+(k-1)}{s+(k+1)}$. The similarity in the magnitude of the frequency responses of these two transfer functions suggest that the velocity error generating filter acts much like the discrete differentiator over a range of frequencies. This similarity would also seem to suggest that since the velocity error generating filter dynamics can be incorporated into the closed-loop tracking analysis that this same tracking result might be extrapolated to the backwards difference approach, i.e., that the proven use of this velocity error generating filter validates to some degree the use of a backwards difference algorithm.

5 EXPERIMENTAL RESULTS

The proposed control scheme was implemented on a 2-link manipulator arm built by Integrated Motion Inc. (IMI). The robot arm feature links that are directly actuated by switched-reluctance motors, which are controlled through NSK torque controlled amplifiers. The equations which constitute the proposed controller are downloaded to a Texas Instruments TMS320C30 real-time DSP computer board mounted in a host PC. The DSP board will then generate the required torque signals for the amplifiers (every 200 μsec). A functional diagram of the control hardware is shown in Figure 2.

The 2-link IMI arm has the following dynamic equation[37]

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = \begin{bmatrix}(p_1+2p_3c_2)\ddot{q}_1+(p_2+p_3c_2)\ddot{q}_2-p_3s_2\dot{q}_1\dot{q}_2\\ -p_3s_2(\dot{q}_1+\dot{q}_2)\dot{q}_2+f_{d1}\dot{q}_1\\ (p_2+p_3c_2)\ddot{q}_1+p_2\ddot{q}_2+p_3s_2\dot{q}_1\dot{q}_1+f_{d2}\dot{q}_2\end{bmatrix}$$

where $p_1 = 3.473\ \text{kg}\cdot\text{m}^2$, $p_2 = 0.193\ \text{kg}\cdot\text{m}^2$, $p_3 = 0.242\ \text{kg}\cdot\text{m}^2$, $f_{d1} = 5.3\ \text{Nm}\cdot\text{sec}$, $f_{d1} = 1.1\ \text{Nm}\cdot\text{sec}$, c_2 denotes $\cos(q_2)$, and s_2 denotes $\sin(q_2)$. The above

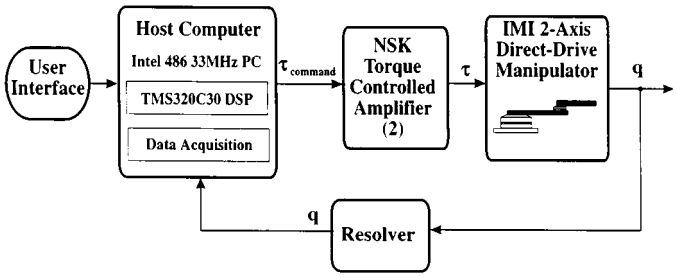


Fig. 2. Block Diagram of Control Hardware.

two-link robot equation mandates an unknown parameter vector in the following form

$$\Theta = [p_1 \quad p_2 \quad p_3 \quad f_{d1} \quad f_{d2}]^T.$$

The experiment was performed using the following desired trajectories for the links

$$q_{d1} = q_{d2} = 0.7\sin(2t)(1-\exp(-0.3t^3))\ \text{rad}.$$

The controller presented above was designed to achieve position tracking as well as to facilitate a clear demonstration of the involvement of the velocity generating filter in the controller development and analysis. Use of this particular from of the controller in a tracking experiment suggested that since the two links are physically very different that the performance could be enhanced by refinements to the controller. To this end the control structure was modified to provide separate control gains for each link in the torque input and to increase from unity the weighting of the e and e_f terms in the velocity generating filter. First, the filter equation (5) was redefined using the scalar gains k_α and k_β as

$$\dot{p} = -(k_\alpha k + k_\alpha)p + (k_\alpha k^2 + kk_\alpha + k_\alpha + kk_\beta)e.$$

Secondly, separate gain terms were added for each of the two rows in the torque input, i.e., separate gain terms for each of the two links, through addition of the diagonal gain matrix $k_s \in \Re^{2\times 2}$ as

$$\tau = Y_d\hat{\Theta} + k_s(-ke_f + e).$$

The tracking experiment was performed with these enhancements to the controller. The control gains were adjusted until the best tracking performance was achieved. The gains used to achieve the best performance are

$$k_s = \begin{bmatrix}3.8 & 0\\ 0 & 1\end{bmatrix}, \quad k = 58, \quad k_\alpha = 23, \quad \text{and} \quad k_\beta = 1.2.$$

The adaptation matrix was set to

$$\Gamma = \text{diag}\{5 \quad 5 \quad 30 \quad 80 \quad 10\}$$

where the notation diag $\{\cdot\}$ is used to denote the elements of a diagonal matrix. For the implementation, the parameter estimates were all initialized to 50% of their true values. The integrations required in the update law of (11) and the filter in (5) were performed using a standard trapezoidal algorithm.

The results of the experiment are shown in Figure 3 through Figure 5. In Figures 3 and 4, the difference between the measured positions and the desired positions are plotted. The parameter estimates are shown in Figure 5.

Remark 7. Note that the above addition of k_s, k_α, k_β are supported by the theoretical developments presented

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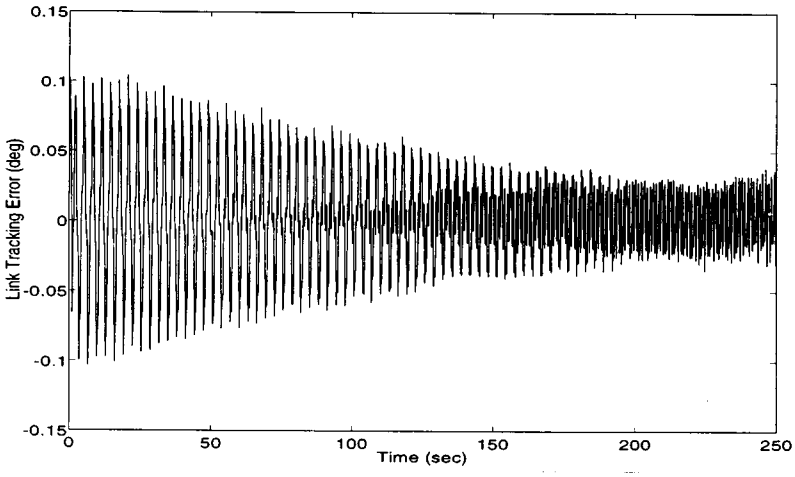


Fig. 3. Link Position Tracking Error – Link 1.

here with only minor modifications. The filtered tracking error must be redefined (see Remark 6.2) as $\eta = \dot{e} + k_\alpha e + k_\beta e_f$. The Lyapunov-like function in (43) is now modified as

$$V = \tfrac{1}{2} e^T k_s e + \tfrac{1}{2} e_f^T k_s e_f + \tfrac{1}{2} \eta^T M(q) \eta + \tfrac{1}{2} \tilde{\Theta}^T \Gamma^{-1} \tilde{\Theta}$$

from which Theorem 1 can be derived for the modified controller.

Remark 8. The control gains selected in the above experiments were "tuned" to achieve the best tracking performance. That is, by repeated trials and readjustment of the controller gains the tracking performance was improved. Such a tuning process relies on the insight and observations of the experimenter to achieve good tracking. There is no guarantee that optimal tracking has been achieved nor will the selected gains necessarily satisfy the sufficient conditions set forth in the type of analysis used to prove Theorem 1. In fact we have found that the conditions on the gains specified by the Lyapunov-type analysis provide little more than a guide to gain selection since these conditions are spawned by such a conservative analysis. It is worth noting that the steady state link tracking error in this experiment is roughly of the same order as the steady state link tracking errors found in the comparative study by Yao and Tomizuka[38] where a similar robot was used.

6 EXTENSIONS

As an alternative to the adaptation law of (11) for the estimation of the system parameters, it is possible to use fixed parameter estimates in the feedforward component of the torque input control. The choice of fixed parameter estimate will dictate the type of link tracking result that can be achieved. The following two corollaries describe three cases for the selection of the fixed parameter estimate: i) the parameter estimate is a "best guess" constant estimate of the actual parameters, ii) the parameter estimate is set to zero, and iii) the constant parameter estimate exactly matches the system values.

Corollary 1. Best Guess Parameter Estimate

Given the link position tracking controller in (7), (5), and (6) which incorporates the constant, "best guess" estimate of the system parameters, $\hat{\Theta}$, and the gain k of (8) redefined as

$$k = \frac{1}{m_1} \left(\alpha_o^2 k_N + \alpha_1^2 k_N + \frac{\zeta_4}{\varepsilon} + 1 \right) \tag{20}$$

where k_N, ε are positive nonlinear damping gains, α_o, α_1

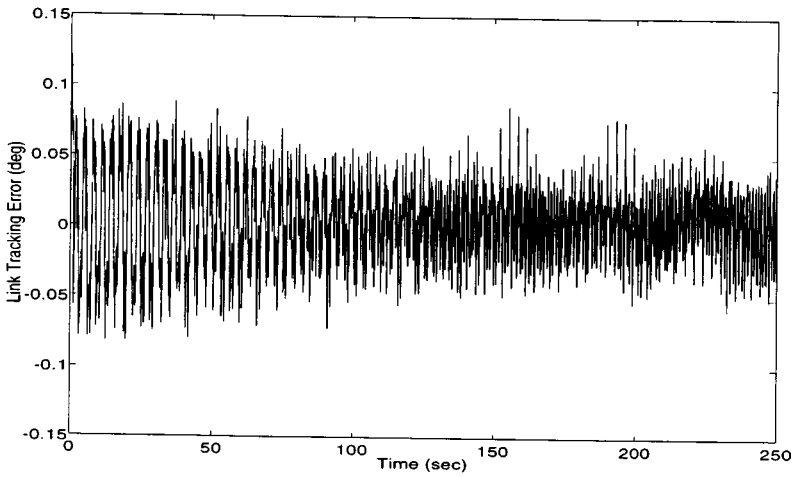


Fig. 4. Link Position Tracking Error – Link 2.

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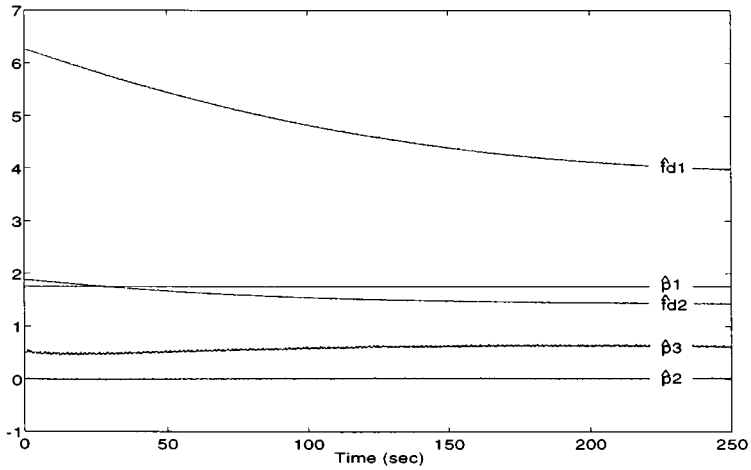


Fig. 5. Parameter Estimates.

were defined in (9), and (10), respectively, and the positive constant ζ_4 only depends on the bounds on the desired trajectory and the magnitude of the difference between the constant best guess estimate and the actual system parameters, if the control gain k_N of (20) is chosen such that

$$k_N > \frac{\lambda_2}{\lambda_1}\|z(0)\|^2 + 1 + \frac{\varepsilon}{\lambda_1 \beta}, \tag{21}$$

where β is some positive constant less than *one*,

$$z = [e^T \quad e_f^T \quad \eta^T]^T \in \Re^{3n}, \tag{22}$$

$$\lambda_1 = \tfrac{1}{2}\min\{1, m_1\}, \quad \text{and} \quad \lambda_2 = \tfrac{1}{2}\max\{1, m_2\} \tag{23}$$

then

$$\|z(t)\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}\|z(0)\|^2 \exp(-\beta t) + \frac{\varepsilon}{\beta\lambda_1}(1 - \exp(-\beta t))}\,. \tag{24}$$

Proof of this corollary requires only minor modifications to the analysis provided for Theorem 1.

Remark 9. The semiglobal uniform ultimately bounded tracking result given by (24) can be illuminated by writing the steady state bound on $\|z(t)\|$ as

$$\lim_{t\to\infty}\|z(t)\| = \sqrt{\frac{\varepsilon}{\lambda_1\beta}}. \tag{25}$$

equation (25) demonstrates that $\|z(t)\|$, and hence the steady state link position tracking error $\|e(t)\|$, can be made arbitrarily small by decreasing the value of ε (i.e., by increasing the control gain term $\frac{1}{\varepsilon}$ in (20)). It should be noted that the controller of (7) with $\hat{\Theta}$ set to the best guess, constant estimate is directly related to the approach given by Yuan and Stepanenko in reference 27.

Remark 10. The rate of convergence of $\|z(t)\|$ in (24) to the steady state bound given in (25) is limited by the maximum value of β, which will be smaller than *one*. This rate of convergence can be increased if the filtered tracking error of (14) is redefined to include the weighted sum of e_f and e in the manner

$$\eta = \dot{e} + k_\alpha e + k_\beta e_f$$

in which k_α and k_β are positive, constant scalars. This change in η will require minor changes to the velocity error generating filter in (5), (6) and the selection of the control gains k and k_N (see Vedagarbha *et al.*[39] for details).

Remark 11. Corollary 1 includes the case where $\hat{\Theta} = 0$. That is, even if there is no feedforward compensation in the control law, the semiglobal uniform ultimately bounded tracking result of (24) remains valid. This controller can be seen as a simple linear controller which is directly related to the controllers proposed in references [29] and [25]. Specifically, the simple linear controllers proposed in references 29 and 25 used second-order filters to generate link velocity information while the approach presented here uses a first-order filter.

Corollary 2 Known Parameter Case

Given the link position tracking controller in (7), (5), and (6) which incorporates the exact estimate of the system parameters, $\dot{\Theta} = \Theta$, and the gain k selected according to (8), (9), and (10), iff the control gain k_N of (8) is chosen such that

$$k_N > \frac{\lambda 2}{\lambda_1}\|z(0)\|^2 + 1, \tag{26}$$

where z, λ_1, and λ_2 were defined in (22) and (23), then

$$\|z(t)\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}}\|z(0)\|\exp\left(-\frac{\beta t}{2}\right) \tag{27}$$

in which β is some positive constant less than *one*. Proof of this corollary requires only minor modifications to the analysis provided for Theorem 1.

Remark 12. We can see from (27) that the use of the

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exact parameter values in the system leads to semiglobal exponential link position tracking. In addition, the rate of convergence can be increased if the controller is modified in accordance with Remark 6.2.

7 CONCLUSION

In this paper, an adaptive link position tracking controller has been formulated for the mechanical dynamics describing an n-link revolute robot. Through the use of a Lyapunov-like technique, we show that the controller achieves semi-global asymptotic link position-velocity tracking. Experimental results on a 2-link manipulator are used to validate the performance of the proposed controller. The primary contribution of this work is that we have formulated an adaptive link position tracking controller that does not require measurement of the link velocity. This reduction in the number of measurement sensors required was achieved through use of a dynamic filter applied to the link position tracking error.

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A ERROR SYSTEM DEVELOPMENT

In this appendix, the dynamics of the three error terms e, η, and e_f are formulated. These error terms contain the dynamic behavior of all the system states; therefore, analysis of these error dynamics will be used to make conclusions regarding the performance of the system under the influence of the proposed controller (in Appendix B).

The dynamics associated with e_f can be found by first differentiating the velocity error generating filter in (6) with respect to time in order to produce

$$\dot{e}_f = -k\dot{e} + \dot{p}. \tag{28}$$

Substituting into (28) for $\dot{p}$ from the first equation of the velocity error generating filter in (5) and then substituting $p = e_f + ke$, derived from (6), into the result yields

$$\dot{e}_f = -k(\dot{e} + e + e_f) - e_f + e. \tag{29}$$

The terms in equation (29) are grouped in order that the definition of η in (14) can be invoked to write the final form of the filtered velocity tracking error dynamics as

$$\dot{e}_f = -k\eta - e_f + e. \tag{30}$$

The dynamics for the link position tracking error are obtained by simply rewriting equation (14). That is, the link position tracking error dynamics are given by

$$\dot{e} = \eta - e_f - e. \tag{31}$$

The final error term to be described is η. The derivation begins with the time differentiation of the definition in (14) to produce

$$\dot{\eta} = \ddot{e} + \dot{e} + \dot{e}_f. \tag{32}$$

Both sides of the equality in (32) are multiplied by $M(q)$ and the term $\ddot{e}$ is expanded using $\ddot{e} = \ddot{q}_d - \ddot{q}$ in order to facilitate substitution from the robot equation. The substitution for $M(q)\ddot{q}$ from the robot equation (1) is made and yields

$$M(q)\dot{\eta} = M(q)\ddot{q}_d + V_m(q, \dot{q})\dot{q} + F_d\dot{q} + G(q) - \tau + M(q)\dot{e} + M(q)\dot{e}_f \tag{33}$$

into which further substitutions are made using $\dot{e}$ from (31) and $\dot{e}_f$ from (30) to produce

$$M(q)\dot{\eta} = M(q)\ddot{q}_d + V_m(q, \dot{q})\dot{q} + F_d\dot{q} + G(q) - \tau + M(q)\eta - 2M(q)e_f - kM(q)\eta. \tag{34}$$

At this point the term $Y_d\Theta$ of Property 4 is injected by adding and subtracting it to the right-hand side of (34) and the relationship $\dot{q} = \dot{q}_d - \dot{e}$ is used where needed, to yield

$$M(q)\dot{\eta} = M(q)\ddot{q}_d + V_m(q, \dot{q})\dot{q}_d + F_d\dot{q} + G(q) - \tau + M(q)\eta - 2M(q)e_f - kM(q)\eta + Y_d\Theta - Y_d\Theta - V_m(q, \dot{q})\dot{e}. \tag{35}$$

The variable $\tilde{Y}$ is now introduced with the following definition

$$\tilde{Y} = M(q)\ddot{q}_d + V_m(q, \dot{q})\dot{q}_d + G(q) + F_d\dot{q} - Y_d\Theta. \tag{36}$$

applying (36) to (35) yields

$$M(q)\dot{\eta} = \tilde{Y} - \tau + M(q)\eta - 2M(q)e_f - kM(q)\eta + Y_d\Theta - V_m(q, \dot{q})\dot{e}. \tag{37}$$

The last term containing the velocity, $\dot{e}$, in (37) is again rewritten by substituting from (31) to produce

$$M(q)\dot{\eta} = \tilde{Y} - \tau + M(q)\eta - 2M(q)e_f - kM(q)\eta + Y_d\Theta - V_m(q, \dot{q})\eta + V_m(q, \dot{q})e_f + V_m(q, \dot{q})e. \tag{38}$$

To simplify the notation, the auxiliary variable χ is defined as

$$\chi = V_m(q, \dot{q})e_f + V_m(q, \dot{q})e + \tilde{Y} + M(q)\eta - 2M(q)e_f, \tag{39}$$

which can be used to rewrite (38) as

$$M(q)\dot{\eta} = \chi - \tau + Y_d\Theta - kM(q)\eta - V_m(q, \dot{q})\eta. \tag{40}$$

A final step is to form the closed loop dynamics for η by substituting the link position tracking controller given in (7) into (40) to yield

$$M(q)\dot{\eta} = \chi - Y_d\hat{\Theta} + ke_f - e + Y_d\Theta - kM(q)\eta - V_m(q, \dot{q})\eta \tag{41}$$

which can be further refined using the parameter estimation error definition from (4) to yield

$$M(q)\dot{\eta} = \chi + Y_d\tilde{\Theta} + ke_f - e - kM(q)\eta - V_m(q, \dot{q})\eta. \tag{42}$$

B ANALYSIS OF TRACKING AND ADAPTATION PERFORMANCE

A Lyapunov-type analysis technique is used to make claims regarding the performance of the proposed controller in the context of link position tracking. Included in this performance analysis is a statement on

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the appropriateness of applying the proposed control (i.e., that all states and signals remain bounded). The analysis begins with introduction of the following non-negative function

$$V = \tfrac{1}{2}e^T e + \tfrac{1}{2}e_f^T e_f + \tfrac{1}{2}\eta^T M(q)\eta + \tfrac{1}{2}\tilde{\Theta}^T\Gamma^{-1}\tilde{\Theta}. \quad (43)$$

From (43), V is upper and lower bounded as

$$\lambda_1 \|x\|^2 \le V \le \lambda_2 \|x\|^2 \quad (44)$$

where the positive constants λ_1 and λ_2 are defined in (15), and x is defined in (13).

Equation (43) can be differentiated with respect to time to produce

$$\dot{V} = e^T\dot{e} + e_f^T\dot{e}_f + \tfrac{1}{2}\eta^T\dot{M}(q)\eta + \eta^T M(q)\dot{\eta} + \tilde{\Theta}^T\Gamma^{-1}\dot{\tilde{\Theta}}. \quad (45)$$

Substitution from the three error systems defined in Appendix A, equations (30), (31) and (42), produces

$$\dot{V} = -e^T e - e_f^T e_f + \tilde{\Theta}^T\Gamma^{-1}\dot{\tilde{\Theta}} - k\eta^T M(q)\eta + \eta^T\chi + \eta^T Y_d\tilde{\Theta} + \tfrac{1}{2}\eta^T\dot{M}(q)\eta - \eta^T V_m(q,\dot{q})\eta. \quad (46)$$

Property 2, the skewed symmetry property, can be used to reduce (46) to

$$\dot{V} = -e^T e - e_f^T e_f + \tilde{\Theta}^T\Gamma^{-1}\dot{\tilde{\Theta}} - k\eta^T M(q)\eta + \eta^T\chi + \eta^T Y_d\tilde{\Theta}. \quad (47)$$

The term containing $\tilde{\Theta}$ can be grouped and from (4) it can be seen that since Θ is constant, the substitution $\dot{\tilde{\Theta}} = -\dot{\hat{\Theta}}$ can be made in (47) to produce

$$\dot{V} = -e^T e - e_f^T e_f - k\eta^T M(q)\eta + \eta^T\chi + \tilde{\Theta}^T(Y_d^T\eta - \Gamma^{-1}\dot{\hat{\Theta}}). \quad (48)$$

The update law in (11) can be differentiated with respect to time, arranged in the form

$$\dot{\hat{\Theta}} = \Gamma Y_d^T\eta, \quad (49)$$

and substituted into (48) to produce

$$\dot{V} = -e^T e - e_f^T e_f - k\eta^T M(q)\eta + \eta^T\chi. \quad (50)$$

Property 1, boundedness of the inertia matrix, is used to place a larger upper bound on (50) as

$$\dot{V} \le -\|e\|^2 - \|e_f\|^2 - km_1\|\eta\|^2 + \|\eta\|\,\|\chi\| \quad (51)$$

where the quadratic terms have been rewritten in equivalent 2-norm notation.

The truncated version of the vector x, z defined in (17), allows a new bound for the auxiliary variable χ to be written as

$$\|\chi\| \le \alpha_o\|z\| + \alpha_1\|z\|^2, \quad (52)$$

where α_o and α_1 are defined in (9) and (10), respectively. Substituting (52) into (51) yields the new upper bound on $\dot{V}$ as

$$\dot{V} \le -\|e\|^2 - \|e_f\|^2 - km_1\|\eta\|^2 + \alpha_o\|\eta\|\,\|z\| + \alpha_1\|\eta\|\,\|z\|^2. \quad (53)$$

The definition of the gain k in (8) is used to rewrite (53) as

$$\dot{V} \le -\|e\|^2 - \|e_f\|^2 - \|\eta\|^2 + \alpha_o\|\eta\|\,\|z\| - \alpha_o^2 k_N\|\eta\|^2 + \alpha_1\|\eta\|\,\|z\|^2 - \alpha_1^2 k_N\|\eta\|^2 \quad (54)$$

where nonlinear damping [36] can now be applied to yield the following upper bound on $\dot{V}$

$$\dot{V} \le -\left(1 - \frac{1}{k_N}(1 + \|z\|^2)\right)\|z\|^2. \quad (55)$$

Equation (44) is used to write the final form for the upper bound on $\dot{V}$ as

$$\dot{V} \le -\left(1 - \frac{1}{k_N}\left(1 + \frac{V}{\lambda_1}\right)\right)\|z\|^2. \quad (56)$$

The sign of the upper bound on $\dot{V}$ is determined by the parenthetical term of (56), which must be positive in order to conclude the negative semidefiniteness of $\dot{V}$. That is, we must have

$$1 - \frac{1}{k_N}\left(1 + \frac{V}{\lambda_1}\right) > 0 \quad (57)$$

for $\dot{V}$ to be negative semidefinite. From (56) and (57), the analysis to this point can now be summarized as

$$\dot{V} \le -\beta\|z\|^2 \quad \text{for} \quad V < \lambda_1(k_N - 1) \quad (58)$$

where β is some positive constant. Now for $V < \lambda_1(k_N - 1)$, we know from (58) that $\dot{V}$ is negative semidefinite, i.e.,

$$\dot{V}(x,t) \le -\beta\|z\|^2\ \forall t \ge 0 \quad (59)$$

and hence

$$V(x,t) \le V(x(0),0)\ \forall t \ge 0. \quad (60)$$

Therefore a sufficient condition for (58) is given by

$$\dot{V} \le -\beta\|z\|^2 \quad \text{for} \quad V(x(0),0) < \lambda_1(k_N - 1) \quad (61)$$

or by (44) we have the sufficient condition for (61) as

$$\dot{V} \le -b\|z\|^2 \quad \text{for} \quad \|x(0)\| < \sqrt{\frac{\lambda_1}{\lambda_2}(k_N - 1)}. \quad (62)$$

The direct implication of (43) and (62) is that e, e_f, η, and $\tilde{\Theta}$ are bounded. The bound on e along with the required bound on q_d implies that q is bounded. Equation (31) implies that since e, e_f, and η are bounded then $\dot{e}$ is bounded, which can then be used with the bound on $\dot{q}_d$ to show that $\dot{q}$ is bounded. Since $\tilde{\Theta}$ is bounded, using the definition in (4) and the fact that Θ is a constant vector, then $\hat{\Theta}$ is bounded. In equation (30) it can be seen the bounds on e, e_f, and η imply that $\dot{e}_f$ is bounded. Since Y_d is a bounded function of bounded inputs we can conclude the link position tracking controller given in (7) is bounded. Finally, from (6), p is bounded and from (5) $\dot{p}$ is also bounded. Now from equation (34) and the fact that $M(q)^{-1}$ exists and is bounded then $\dot{\eta}$ is bounded. From (32) it can be seen that since $\dot{\eta}$, $\dot{e}$, and $\dot{e}_f$ are bounded then $\ddot{e}$ is bounded, which along with the bound on $\ddot{q}_d$ implies $\ddot{q}$ is bounded. It can be seen from (62) that $z \in \mathscr{R}_2^{3n}$ [reference 40]. Since $z \in \mathscr{R}_2^{3n}$, $\mathscr{R}_\infty^{3n}$ and $\dot{z} \in \mathscr{R}_\infty^{3n}$ then we can use Barbalat's Lemma [reference 40] to yield (16).

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