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Archytas and Optics

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Argument

This paper is detective work. I aim to show that the brilliant Pythagorean mathematician Archytas of Tarentum was the founder of ancient Greek mathematical optics. The evidence is indirect. (1) A fragment of Aristotle preserved in Iamblichus is one of two doxographical notices to mention Pythagorean work in optics. (2) Apuleius credits Archytas with a theory of visual rays which saves the principle that the angle of reflection is equal to the angle of incidence. I argue that the source from which Apuleius got this information was the *Catoptrics* of Archimedes, the genuineness of which I defend against Knorr's hypothesis that it is the Euclidean *Catoptrics*, which had been misattributed to Archimedes. (3) The omission of optics from the mathematical curriculum in Plato's *Republic*, and the *Timaeus*' wholly physical account of mirror-images, can be explained as polemical, for it is well attested that optics was practiced in the Academy. The reason Plato does not mention optics is that he objected to Archytas using mathematics to understand the physical world rather than to transcend it.

The starting point for this paper is an absence. My conclusion is that the absence is a significant chapter in the early history of mathematical optics in ancient Greece.

1. The mathematical curriculum outlined in Plato's *Republic* comprises arithmetic, plane and solid geometry, astronomy, harmonics. There is not a word about optics. This absence would be of no significance if mathematical optics did not exist at the time Plato wrote *Republic* VII. It certainly did exist soon afterwards, when Aristotle wrote the *Posterior Analytics* and classified optics as a science subordinate to geometry in the same way as harmonics is subordinate to arithmetic, mechanics to stereometry (I 7, 75b 14–7; 13, 78b 35–9). Mathematical harmonics is an important item in Plato's curriculum, singled out for special mention as useful for the understanding of Beauty and the Good (531c). But it is not only harmonics that has this function. The educative purpose of the entire curriculum is to prepare future rulers for the understanding of ultimate values on which ethics and politics in the ideal city depend. If Plato knew about optics when he wrote *Republic* VII and enthused about harmonics, he must have decided that optics would not advance his educational goals. Optics might even be positively harmful. It is not difficult to see why.

The mathematical program is designed to lead the mind upwards and away from sensible things. It starts with arithmetic and plane geometry, to get the student used to abstraction, to focusing on numbers and figures which can only be grasped in thought,

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not by the senses. It proceeds via solid geometry to a surprisingly abstract treatment of astronomy and harmonics, in which visual and acoustic observation are dismissed in favor of purely mathematical "problems" (530b, 531c). Optics would bring the student back to the sensible realm, which throughout the *Republic* is typified by the visible. Τὰ ὁρώμενα, visible things, are the metonym for all that the philosophers must use mathematics to struggle free from. The study of optics would undo the good work by focusing the mind on ὁρώμενα, or rather – and worse – on the *appearances* of visible things.

Thus far my thesis is conditional: *If* mathematical optics existed at the time Plato wrote the *Republic*, its absence from the curriculum is likely to have the significance just sketched. My next task is to find grounds for affirming the antecedent of this conditional.

I begin with Aristotle. He is certainly familiar with mathematical optics, which he mentions in a number of places. He wrote an ὀπτικόν of his own (DL V 26), and was praised for his presentation of the subject by no less an authority than Hero of Alexandria (*Cat*. 318. 6–7 Nix and Schmidt). Most tellingly, when debating philosophy of mathematics with the Platonists in *Metaphysics* MN, he takes it for granted that one of the mathematical objects whose ontological status is in dispute is the ὄψις, the line of sight which is the central concept of Greek optics. The clear implication is that optics was practiced, or at least discussed, in the Academy where the Platonists held sway. This is confirmed by the Suda's recording that Philip of Opus wrote two books on optics, two on catoptrics,[1] and by Philodemus' testimony (*Ind. Herc.* col. Y, 15–17) to advances in optics and mechanics made in the Academy during Plato's lifetime.

To discover what sort of optics Aristotle is familiar with, we may look to his account of how its subject matter differs from that of ordinary geometry:

> While geometry investigates physical lines but not *qua* physical, optics investigates mathematical lines, but *qua* physical, not *qua* mathematical. (*Ph*. II 2, 94a 9–12; tr. Hardie and Gaye)

Geometry starts from the contours and shapes of physical bodies and treats them *as if* they existed independently of the bodies whose contours and shapes they are. These abstract*ed* (rather than abstract) lines and shapes have all and only the properties which are contained in or demonstrable from their definitions. That is what it means to say that geometry investigates physical lines but not *qua* physical. Optics then adds to these geometrical items certain physical properties relevant to the understanding of visual appearances, first and foremost motion. Many of the optician's lines are rays traveling outwards from the eye. Some opticians add more such properties than others. Ptolemy does, while Euclid does not, ascribe strength and weakness to the rays according to their distance from the axis of the visual cone. Euclid does not, while other opticians

[1] *Sv*. Φιλόσοφος; the entry can only be about Philip, whose name has dropped out.

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do, use ray-length as a distance cue. But regardless of these additional features, the lines do not cease to be mathematical lines, abstracted items which have no more properties (mathematical or physical) than are contained in or demonstrable from their initial specification. You cannot legitimately infer that if the rays move, or have degrees of strength, they are corporeal[2] and therefore have standard properties of physical bodies such as weight and thickness. That is what it means to say that optics investigates mathematical lines, but *qua* physical, not *qua* mathematical.[3]

Thus understood, Aristotle's account seems to me a most helpful guide to the surviving corpus of ancient mathematical optics. I infer that optics in his day looked much like the optics we know from the Hellenistic period and later. The subject must have already reached some degree of sophistication to call for Aristotle's carefully layered analysis.

Granted, this evidence does not reach all the way back to the time when Plato was writing the *Republic*; it is consistent with optics and mechanics having come to the Academy's attention a bit later. However, it is clear from the text of the *Republic* itself that *some* mathematical disciplines of the time are deliberately excluded from the curriculum. One exclusion is calendaric astronomy as practiced by Meton and Euctemon in the late fifth century, which attempts to find in the observed phenomena whole-number ratios (συμμετρίαι) that will account for the relation of night to day, of these to the month, and of the month to the year (530ab). Another example is Pythagorean harmonics, which is described as a mathematical analysis of the scales used in actual music:

> They seek the numbers in these heard concords (συμφωνίαι) and do not ascend to problems to consider which numbers are concordant, which are not, and why each are so. (531c)

This is not the occasion to ponder what Plato can have had in view when making Socrates propose an astronomy of the invisible and a harmonics of the inaudible.[4] What matters is that Socrates deliberately excludes existing mathematical disciplines that would focus the mind on sensible things. And in the course of doing so he quotes a remark from, as he puts it, "the Pythagoreans" to the effect that astronomy and harmonics are "sister sciences" (530d: ἀδελφαὶ ἐπιστήμαι). We can identify the author of that saying. It was a Pythagorean closer in age to Plato than to Socrates: the philosopher, statesman, general, and mathematician of genius, Archytas of Tarentum, whose frag. 1 claims that geometry and arithmetic, astronomy and harmonics are all sisters.[5]

[2] As Alexander infers to start a polemic against all forms of ray theory at *In Sens*. 28.16 ff.
[3] On Aristotle's *qua* operator, see the now classic elucidation in Lear 1982.
[4] Suggestions in Burnyeat 2000, 47–63.
[5] For defense of the authenticity of fragment 1 against Burkert's suspicion (1972, 379–80 n. 46) that it was manufactured to fit Plato's quotation, see Huffman 1985.

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Here is the passage where Socrates quotes from Archytas:

> "It is probable," I said, "that as the eyes are framed for astronomy, so the ears are framed for harmonic motion, and that these two sciences are sisters of one another, as the Pythagoreans say – and we agree, Glaucon, do we not?"
>
> "We do," he said. (530d)

Socrates has already taken astronomy up to the same abstract level as geometry. He will now preserve the "sisterhood" of astronomy and harmonics by redirecting the latter to the same abstract level as arithmetic. In other words, he agrees with Archytas' coupling of astronomy and harmonics alongside geometry and arithmetic, but condemns his empirical approach in harmonics, which seeks numbers in the observed phenomena instead of purely mathematical "problems." Socrates continues:

> "Then," I said, "since the task is so great, shall we not inquire of them [the Pythagoreans] how they speak of these [sciences] and whether they have any other [science] to add?[6] And in all this we will be on the watch for what concerns us."
>
> "What is that?"
>
> "To prevent our fosterlings trying to learn anything incomplete (ἀτελές), anything that does not come out at the destination which, as we were saying just now about astronomy, ought to be the goal of it all." (530e)

As I read this, Socrates hints at other branches of Pythagorean mathematics he would wish to exclude. He does not name any, but it is worth recalling that Archytas was the founder of a discipline in which Archimedes was later to excel, mathematical mechanics.[7] Socrates would surely not want that on his curriculum. If, as I hope to show, Archytas was also interested in mathematical optics, this too will join the black-list of mathematical studies excluded from the *Republic's* curriculum.

2. My journey back to Archytas starts from a passage of Iamblichus (*Comm. Math.* 25, p. 78.8–18) which Walter Burkert has identified as part of a hitherto unrecognized fragment of Aristotle. In Burkert's presentation, the reference to optics is square-bracketed as Iamblichus' addition, thus:

> οἱ δὲ Πυθαγόρειοι διατρίψαντες ἐν τοῖς μαθήμασι καὶ τό τε ἀκριβὲς τῶν λόγων ἀγαπήσαντες, ὅτι μόνα εἶχεν ἀποδείξεις ὧν μεταχειρίζοντο ἄνθρωποι, καὶ ὁμολογούμενα ὁρῶντες [ἔνιοσν] τὰ περὶ τὴν ἁρμονίαν, ὅτι δι' ἀριθμῶν, [καὶ τὰ περὶ τὴν ὄψιν μαθήματα διὰ <διὰ>γραμμάτων], ὅλως αἴτια τῶν ὄντων ταῦτα ᾠήθησαν εἶναι καὶ τὰς τούτων ἀρχάς, ὥστε τῷ βουλομένῳ θεωρεῖν τὰ ὄντα πῶς ἔχει, εἰς ταῦτα βλεπτέον εἶναι, τοῖς ἀριθμοὺς [καὶ τὰ γεωμετρούμενα εἴδη τῶν ὄντων] καὶ λόγους, διὰ τὸ δηλοῦσθαι πάντα διὰ τούτων.

[6] Both περὶ αὐτῶν and πρὸς τουτοῖς (idiomatically switching to neuter) refer to the two sister sciences.

[7] DL VIII 83 with Kühn's widely (but not universally) accepted emendation μαθηματικαῖς for μηχανικαῖς; cf. also DK47A10a. My belief in Diogenes' plain and credible testimony, backed by Vitruvius VII *Praef.* 14 (= DK47B7), has not been shaken by Huffman's doubts (Huffman forthcoming).

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> The Pythagoreans, having devoted themselves to mathematics, and admiring the accuracy of its reasonings, because it alone among human activities knows of proofs, and seeing [equally] the facts about harmony, that they happen on account of numbers, generally admitted [and (seeing) the mathematics of optics depending on (dia)grams], they deemed these (facts of mathematics) and their principles to be, generally, causative of existing things, so that whoever wishes to comprehend the true nature of existing things should turn their attention to these, that is to numbers [and the geometrical entities among existing things] and proportions, because it is by them that everything is made clear. (Burkert 1972, 50 n. 112, and 447–8)

To vindicate the antecedent of my conditional, I shall need to remove all Burkert's square brackets and rewrite his translation.[8]

But first, the marks of Aristotle's pen. Burkert notes (a) the parallels of language with Aristotle's account of Pythagoreanism in *Metaphysics* A5, and (b) the sandwiching of the passage between two other extracts from Aristotle. I would emphasize (c) that the passage follows the same historiographic pattern as A5: the Pythagoreans occupied themselves with mathematics and, as a result, they took the principles of mathematics to be the principles of everything.

At the same time there are striking differences from A5. According to A5, it was the likenesses (ὁμοιώματα) between numbers and things (985b 27–8) that inspired the generalization from the principles of mathematics to the principles of all things. The fragment, by contrast, locates the inspiration in the precision of mathematical *proof*. In A5 proof is never mentioned; on the contrary, Aristotle disparages Pythagorean thought as a mish-mash of numerological fantasies (esp. 986a 6–8). And whereas A5 makes number the key to the cosmos – Pythagoreans explain everything by numbers – the fragment invokes the forms studied by geometry and ratios (τὰ γεωμετρούμενα εἴδη τῶν ὄντων καὶ λόγους) as well as numbers. Numbers may be primary, but geometry has cosmic significance too.

What are we to make of this combination of similarity and difference between our passage and *Metaphysics* A5? The contrast between Iamblichus' proofs and A5's likenesses would be striking enough on its own, even if one accepted the brackets by which Burkert athetizes the reference to geometry as well as the clause about optics. If A5 is Aristotle's canonical picture of the Pythagoreans, there seem to be two views one could take of the Iamblichus excerpt. *Either* it is a non-canonical picture of those same Pythagoreans – most likely, of Philolaus, who since Burkert has been widely (though not universally) accepted as the main source for A5 – *or* it is a picture of some non-canonical Pythagoreans.

I propose to argue for the second alternative. I am persuaded by Burkert that our passage is indeed from Aristotle, but my argument will not depend on the

[8] Strictly, his translator's translation: in the original German edition only the Greek is given. But the translation is published with Burkert's approval.

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attribution. What matters is that Iamblichus presents (from some source or other) a picture of a Pythagorean philosophy which emphasizes proof and is different from the Pythagoreanism of A5, where likeness is the key because Aristotle is looking for the antecedents of Plato's theory that sensible things are to be explained as likenesses of the Forms and, ultimately, Form numbers (*Met.* A6).

3. The first step in the argument is to defend the references to optics and geometry against Burkert's excisions. In Festa's Teubner text of Iamblichus the only excision is the word ὅτι before δι' ἀριθμῶν in line 4. But Burkert's concern is to restore the text of Aristotle. To this end he retains ὅτι and excises both the mention of optics in line 4 and the phrase about geometrical objects in line 7. What remains is all about number, harmony, proportion, so that the fragment's picture of Pythagoreanism is assimilated to the familiar account in A5.[9] Burkert says nothing about the excerpt's emphasis on mathematical proof. The main reason for his drastic surgery is precisely that geometry plays no role in the canonical picture given by Aristotle and confirmed by Aristoxenus (frag. 23 Wehrli). He suggests that Iamblichus added the references to optics and geometry "to emphasize the many-sidedness of Pythagorean μαθήματα."

My first puzzle is why, if that was Iamblichus' motivation, he should start by adducing a minor branch of mathematics such as optics, instead of geometry itself. Second, Burkert's translation of the remark about harmonics in lines 3–4, which he admits as Aristotelian, barely makes sense. Consequently, if ὅτι means "that," I would prefer to follow Festa in deleting it. But of course, ὅτι need not mean "that." It can also mean "because" (as at line 2), and so translated it yields much better sense, as follows: "seeing that the facts about harmony are generally admitted *because* they are through numbers," scilicet "because they are established or proved through numbers."

Just this is what the fragment is reporting. The Pythagoreans were impressed by mathematics because only in mathematics does one find proofs (ἀποδείξεις, line 2). It is proofs – in the case of harmonics, proofs by means of number theory – that bring it about that discoveries about harmony win widespread acceptance.[10] So likewise, I suggest the sentence continues, mathematical theories about vision win widespread acceptance because they are established through διαγράμματα. (Understand ὅτι from ὅτι δι' ἀριθμῶν or suppose that a second ὅτι διά dropped out in line 4 because the scribe's eye jumped ahead to διαγράμματα).[11] And διαγράμματα here naturally does not mean diagrams as such, but rather proofs conducted with a diagram in the manner we can

[9] Burkert's excisions and interpretation are followed by Huffman 1993, 70 n. 18, 114.

[10] I would compare ὁμολογούμενα with Plato, *Rep.* 510d ὁμολογουμένως, 533c ὁμολογία. Huffman (forthcoming) compares Archytas frag. 3 where proportion brings about political concord.

[11] In which case Vitelli's angled brackets really belong around the first διά. The alternative of reading simply διὰ γραμμῶν, which in later mathematical authors may contrast with δι' ἀριθμῶν (Heath [1921] 1981, II 257–8; Netz 1999, 36), finds no parallel in extant Aristotle.

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see exemplified on any page of Euclid, including the *Optics*.[12] To translate once more in the light of the interpretation now on offer:

> The Pythagoreans devoted themselves to mathematics. They both admired the accuracy of its reasonings, because it alone among human activities contains proofs, and they saw that general agreement is given in equal measure to theorems concerning attunement, because they are <established> through numbers, and to the mathematical studies that deal with vision, because they are <established> through diagrams. This led them to think that these things[13] and their principles are quite generally the causes of existing things. Consequently, these are what anyone who wishes to comprehend the true nature of existing things should turn their attention to – the numbers and geometrical forms of existing things and ratios – because it is by them that everything is made clear.

If this is a reasonable translation, we can see why the passage should mention optics and not geometry in general. It is *applied* number theory, as in harmonics, and *applied* geometry, as in optics, from which this Pythagorean train of thought begins. Because the power of mathematical proof can be brought to bear on the objects of hearing and vision, achieving in these areas results that are ὁμολογούμενα, generally admitted, for this reason – and here comes the characteristically Pythagorean generalization signaled by ὅλως in line 5 – it is mathematics that will give us an understanding of the world in general.

I submit that the sense and sequence of thought now won from the first 5 lines of our passage is so far superior to that in Burkert's translation that we have sufficient reason to strike out his first two sets of square brackets. Anyone worried by the word ἔνισον (a ἅπαξ λεγόμενον) can easily emend to ἐπ' ἴσον (Vitelli – a good Aristotelian phrase) or ἐξ ἴσου (Burkert) or ἐν ἴσῳ (a nice parallel at Thuc. IV 65), according to taste.[14] I shall move on to the third segment Burkert would excise, the mention of the forms studied by geometry: καὶ τὰ γεωμετρούμενα εἴδη τῶν ὄντων (line 7).

Burkert says that τὰ γεωμετρούμενα is a school term from late antiquity, citing Plutarch and others. I reply that the verb γεωμετρεῖν had been available since the fifth century B.C. as a transitive verb meaning "to measure, to survey land." In two quite evocative contexts (Aristophanes, *Birds* 995; Plato, *Theaetetus* 173e) it carries a strong suggestion of applied geometry, not just pacing out the length of a field. Now a transitive verb

[12] In Plato's *Hippias Minor* 366c–367e διαγράμματα stand to geometry as calculations (λογίσμοί) to λογιστική; they are the whole of what the geometer busies himself with. The same word at *Cratylus* 436c 8-d 8 only makes sense if translated "proofs."

[13] It is unclear whether ταῦτα are the numbers and diagrams or the results obtained through them.

[14] There remains Burkert's complaint about the absence of an article before λόγους. Easy as it would be to supply <τοὺς>, Huffman (forthcoming) has a better answer. ἀριθμούς and εἴδη both go with the dependent genitive τῶν ὄντων: they are the numbers and the forms *of* existing things. To talk of the ratios or proportions *of existing things* would make little sense, since ratios and proportions are relations between numbers or between geometrical magnitudes. Hence καὶ τοὺς λόγους is sensibly placed, without an article, after τῶν ὄντων, whose numbers and forms enter into ratios and proportions with one another.

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can be put into the passive by any native speaker. It is true that in extant Aristotle γεωμετρεῖν occurs only in its intransitive use "to practice geometry." But that is no reason to deny that he has the transitive use available in his lexicon. In which case, the passive is there as well.[15]

That leaves εἴδη. Even extant Aristotle uses this word in connection with geometry and mathematics generally: τὰ μαθήματα περὶ εἴδη ἐστίν (*APo.*I 13, 79a 7–8). It is a difficult passage, on which the commentators are not much help, but I am quite sure that Aristotle is not here claiming that mathematics is about Platonic Forms. He is making some sort of contrast between substance and attribute. The εἴδη studied by geometry and other branches of mathematics are attributes or aspects of substantial beings, although they are treated in abstraction from the subjects they belong to. Alternatively, εἴδη might refer to the diagrams used by the mathematicians – compare κέχρηται τοῖς εἴδεσιν (79a 7) with τοῖς ὁρωμένοις εἴδεσιν προσχρῶνται at Plato *Rep.* 510d 5 – the point being that these diagrams represent attributes abstracted from substantial beings. At the worst, the passage shows we can defend the word, even if we do not fully understand it.

This completes my restoration of a picture of a non-canonical Pythagorean philosophy which emphasizes mathematical proof. If the picture was not drawn by Aristotle, as it probably was, it still conforms to the same historiographic pattern as his canonical picture in *Metaphysics* A5, which is probably based on Philolaus. The next step is to propose Archytas as the original of the non-canonical portrait reproduced by Iamblichus.

4. Aristotle wrote a 3-volume work *On the Philosophy of Archytas*, and we hear also of a book of extracts he made "from the *Timaeus* and the works of Archytas" (DL V 25). Compared with his two books dealing with the Pythagoreans in general, this is a sign of serious interest and, perhaps, respect. Aristotle would not share Plato's dismissive attitude to applied mathematics, so if Archytas' achievement in optics was as notable as we know it was in harmonics, he would touch his cap to a Pythagorean so different from those he scorns in *Metaphysics* A5.

This brings me to Archytas frag. 4, which emphasizes proof:

> With respect to wisdom, arithmetic (λογιστικά) seems to be much superior to the other disciplines and to deal with what it wishes more clearly even than geometry.[16] And again, in cases where geometry is deficient, arithmetic completes proofs (ἀποδείξας), and

[15] A good example to shake up preconceptions about the passive is the astonishing ἀστρονομεῖται at Plato *Rep.* 530c. Giuseppe Cambiano kindly pointed me to DL VIII 88, where Hermippus (third century B.C.) is cited for the information that Eudoxus wrote both ἀστρολογούμενα and γεωμετρούμενα. This is quite likely to reflect Hermippus' own wording, long before Plutarch.

[16] I follow Huffman (forthcoming) in the view that there was no need for DK to mark a lacuna between this sentence and the next.

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> likewise, if, on the one hand, the subject is forms (εἰδέων), [it completes] the treatment of forms . . . [17]

Burkert impugns the fragment for Platonic language and mathematical nonsense (Burkert 1972, 200–221 n. 14). The first objection – to "the telltale εἰδέων πραγματεία" – is easily met. In Plato's *Republic* geometers are said to talk about visible forms (510d: ὁρωμένοις εἴδεσι) in order to think about the invisible forms they resemble. Even the invisible forms cannot be Platonic Forms, because the Platonic Form of, e.g., Square is necessarily unique, while Pythagoras' theorem states a relation between three distinct squares.[18] Archytas does not make the characteristically Platonic distinction between the visible and the invisible, but the fragment can hardly be faulted for saying, like Aristotle in the passage just cited, that mathematics, or some mathematics, deals with forms. It is common sense that geometry, for example, is about the shapes of things. Granted, Archytas seems here to contrast geometry with the treatment of forms. But good as it would be to know what he might mean by εἴδη,[19] to defend the fragment it is enough to insist that the word need not stink of Platonism. The more serious issue is whether the assertion that arithmetic helps where geometry fails is "nonsense mathematically."

There is surely no need to agree with Burkert that frag. 4 advances the deeply incompetent idea that incommensurables can be determined arithmetically. One alternative, suggested by Wilbur Knorr, is that the very notion of incommensurability involves arithmetic, for it is the notion of two magnitudes which fail to have to one another the ratio of an integer to an integer (Knorr 1975, 310–11). (But the same might be said of many geometrical notions, e.g. doubling the cube.) Another suggestion, due to Ian Mueller, is that the fragment is only saying that arithmetic yields true results where geometry yields false ones, since "the Pythagoreans used the numerical relations of musical intervals to argue that, e.g., there was no such thing as a half tone (the tone being represented by 9:8 and there being no numbers *n, m, k,* such that *n*:*m* :: 9:8 and *n*:*k* :: *k*:*m*), whereas the representation of the intervals as straight lines would imply the existence of a half tone" (Mueller 1991, n. 12). Both these proposals are mathematically competent, but neither to my mind does justice to the fragment's emphasis on proof. Can we think of a case where arithmetic completes a proof that geometry cannot complete?

Well, the classic (so-called Pythagorean) proof of the incommensurability of the side and diagonal of a square proceeds by arguing that if the side and diagonal were

[17] The sense of lines 10–11 is doubtful, and translation a gamble, because the fragment breaks off in mid-sentence with no response to μέν.

[18] More on this point in Burnyeat 2000, 35–42.

[19] Cambiano offers an admittedly speculative but interesting hypothesis based on Archytas' report (DK45.2–3) of the commendably thorough Eurytus, pupil of Philolaus (Cambiano 1998, 316–322). Eurytus' pebbles were set out in an attempt to arrive at some sort of numerical specification of different εἴδη (= kinds, as in Philolaus frag. 5) of animal or plant.

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commensurable, the same number would be both odd and even, which is impossible. A principle of arithmetic is called in to complete the proof of a result in geometry. Now historians think that this proof is old, and possibly even Pythagorean, because it contains a special step for unity (μόνας). This is required if the proof is to satisfy someone who holds that one is both odd and even. A strange idea, no doubt, but it is found in Philolaus frag. 5 and reported for Archytas (DK 47A21). Thus for Archytas the incommensurability proof would not work unless it ruled out the single case where the same number not only can be, but is, both odd and even. Add that extra step, as we find it in Euclid *Elements* X, Appendix 27, and the proof becomes compelling. It is a proof that makes good use of antique number theory.

Objection: this does not establish incommensurability as a case where geometry cannot manage on its own, because another way of proving the same result is by the wholly geometrical method of mutual subtraction (ἀνθυφαίρεσις). *Elements* X 2 shows that, quite generally, if no common measure of two magnitudes will ever be found by mutual subtraction, the magnitudes are incommensurable. Reply: if that *general* proof was not yet available (because Theaetetus and Eudoxus had not yet laid the basis for the study of irrationals in *Elements* X), one might reasonably respond to a particular case of mutual subtraction by saying, "I can see that the process *will* never end, but I cannot see that it is *impossible* for it to end." Aristotle at *Metaphysics* Θ 4, 1047b 5–6, envisages an opponent saying, "It is not impossible for the diagonal to be measured – it just never will be measured." His reply is that something which is possible, but will never come about, can be supposed true without any impossibility resulting, whereas the hypothesis that the diagonal has been measured does result in an impossibility – and here he is bound to be thinking of the impossibility of the same number being both odd and even (cf. *APr.* I 23, 41a 25–30). If proof by mutual subtraction is what Aristotle means to contrast with the familiar *reductio*,[20] the supposition that Archytas frag. 4 reflects similar thinking would lend it a controversial edge.

You may think I am on the verge of suggesting that Archytas might himself be the author of the *reductio* proof and writes frag. 4 to boost its superiority to proof by mutual subtraction. No verifiable historical fact stands in the way, but I shall be more modest. Frag. 4 is genuine. Hence Archytas is the first Pythagorean we know of to emphasize the importance of mathematical proof. Indeed, he is the only Pythagorean we know of to speak about mathematical proof. He has every right to do so, since he is by far and away the most skilled Pythagorean practitioner of proof; his duplication of the cube (DK 47A14) is a stunning example of sophisticated Greek mathematics.[21] Archytas is eminently suited to be the original of Iamblichus' non-canonical portrait.

[20] Here I am indebted to David Sedley's presentation on Θ 4 at a Cambridge seminar in 1995 (cf. also Knorr 1975, 34).

[21] Cf. also the *reductio* proof preserved by Boethius in DK47A19, that a superparticular ratio cannot be divided into equal parts by insertion of a mean proportional.

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5. We are now ready for optics. What the word διαγράμματα in the Iamblichus passage leads us to expect is real mathematical proofs with diagrams, such as we find in Euclid's *Optics* and *Catoptrics*. If we can confirm this, Archytas will emerge as very probably the founder of the discipline of mathematical optics as well as mechanics. Let me admit at once that the central concept of the ὄψις, the straight line of sight that travels outwards from the eye even as far as the Sun, is first attested for the fifth-century mathematician Hippocrates of Chios. It is to be found in Aristotle's report of his theory that the tails of comets are to be explained as phenomena of reflection (*Meteor.* I 6, 342b 35–343a 20; DK 42.5–6). But Aristotle's report does not indicate a particularly mathematical treatment of the issue, still less a general mathematical study of reflection. We know that Archytas was indebted to Hippocrates for his proof that the problem of duplicating a cube could be reduced to that of finding two mean proportionals. It would be entirely in keeping with our evidence for relations between these two mathematicians for Archytas to have borrowed a concept from Hippocrates and put it to constructive further use. In frag. 1 Archytas himself looks back at his predecessors and remarks on their outstanding achievements in astronomy, geometry, arithmetic, and harmonics. No mention of stereometry, mechanics, or optics: a silence which may suggest that before Archytas these disciplines did not exist or did not achieve results of significance.[22]

So on to the evidence for Archytas' interest in optics, more particularly catoptrics. Our source is the *Apologia* of Apuleius, written in the middle of the second century A.D. In chap. 15 Apuleius enumerates four accounts of why we see ourselves in a mirror:[23]

> Quid, quod nec ob haec debet tantummodo philosophus speculum invisere? Nam saepe oportet non modo similitudinem suam, verum etiam ipsius similitudinis rationem considerare: num, ut ait Epicurus, (1) profectae a nobis imagines velut quaedam exuviae iugi fluore a corporibus manantes, cum leve aliquid et solidum offenderunt, illisae reflectantur et retro expressae contraversim respondeant an, ut alii philosophi disputant, (2) radii nostri seu (i) mediis oculis proliquati et lumini extrario mixti atque ita uniti, ut Plato arbitratur, seu (ii) tantum oculis profecti sine ullo foris amminiculo, ut Archytas putat, seu (iii) intentu aëris coacti, ut Stoici rentur, cum alicui corpori inciderunt spisso et splendido et levi, paribus angulis quibus inciderant resultent ad faciem suam reduces atque ita, quod extra tangant ac visant, id intra speculum imaginentur.
>
> What about the point that these are not the only reasons for a philosopher to look into a mirror? Often enough he is bound to contemplate, not only his own likeness, but also the explanation of that likeness.[24] Is Epicurus right when he asserts that (1) images

[22] Aristotle twice implies that a proper understanding of the phenomena of reflection is relatively recent (*Meteor.* II 9, 370a 16–21; *Sens.* 2, 438a 5–10).

[23] The excerpt in DK47A25 stops mid-sentence with Archytas, omitting the Stoics.

[24] My rendering of "oportet" emphasizes that a philosopher is just the person you can expect to go looking for an explanation. The τόπος "philosophy begins from wonder" is first found at Plato, *Theaetetus* 155d, where the example is another phenomenon of reflection: the rainbow.

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> proceed forth from us – as it were a kind of slough that continually streams from our bodies – which, when they strike anything smooth and solid, are reflected by the shock and reversed in such wise as to give back an image turned to face its original? Or should we accept the view maintained by other philosophers that (2) our own rays – either (i) rays filtered forth from the centre of the eyes to mix and thereby unite with the light outside, as Plato holds, or (ii) rays advancing only from our eyes without any external support (*sine ullo foris amminiculo*), as Archytas thinks, or (iii) rays constituted by the tension of the air, as the Stoics suppose – when they strike a dense and shiny and smooth body, rebound back to the face they came from at an angle equal to the angle of incidence, thereby making an image within the mirror of the thing they approach and touch without? (tr. after Butler)

Three of these theories are familiar from elsewhere, but nowhere in other doxographical sources is Archytas credited by name with an account of vision or reflection.[25] The four theories come in two groups, distinguished by the direction in which vision in general is supposed to work. The view of Epicurus, that something travels from the object seen to the perceiving subject, stands opposed to three versions of the idea that visual rays (*radii* = Greek ὄψεις or ἀκτῖνες) go out from the eye to the object perceived. But "going out" is importantly ambiguous. Spatial traveling is appropriate to the theories of Plato and Archytas, but not to the Stoics. Their theory involves directionality (from eye to object), but not travel. The rays are lines of tension that form a cone in the air between eye and object, and seeing is compared with feeling something at a distance through a stick.[26] It is Plato and Archytas who require the rays to travel from eye to object. What we need to understand is the difference between their rival theories of the outwardly traveling ray. Apuleius marks the difference by saying that Archytas' rays travel *sine ullo foris amminiculo*, without any external support. What does this mean?

Amminiculum denotes something to lean on for support. The word is used of the prop for a vine and other supporting devices. That made me think it should be the Latin equivalent of the Greek ἐπέρεισμα. Subsequently I found the related noun ἐπέρεισις used by Alexander of Aphrodisias (perhaps quoting Chrysippus) to indicate what light would have if (contrary to Aristotle's philosophy) it was a body (*SVF* II 432). It would have some bodily support. I conclude that Archytas' rays travel without requiring any external vehicle.

By contrast, Plato's *Timaeus* is emphatic that vision is a body. It is a body (45b: σῶμα) of fire, streaming out from within the eye, which coalesces with the fire of daylight outside to form a single body (45c: ἓν σῶμα) as it comes up against the object seen. The

[25] No mention of Archytas in a very similar report of Stoic, Epicurean, and Platonic theories of vision by Apuleius' contemporary Aulus Gellius V 16.

[26] The sources assembled in *SVF* II 863–871 are not unequivocally against travel, but Ingenkamp agrees that the stick comparison is decisive (Ingenkamp 1971). Hence in Apuleius we can and should read *facti* with the MSS and Butler [1909] 1970 and Hunnink 1997, not the emendation *coacti* adopted from Purser 1907, 366–7, by Butler and Owen [1914] 1983 and Vallette's Budé (1924), still less the more bizarre emendations mentioned in the Butler-Owen apparatus. The modest "perhaps" which Purser attached to his proposal has not been repeated.

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internal fire's need for external support is shown by what happens at night: the visual rays are extinguished when they emerge into the dark air. Apuleius reports Plato's view quite correctly in the words *mediis oculis proliquati et lumini extrario mixti atque ita uniti*. He thereby casts the external body of fiery light as the support (*amminiculum*) that Archytas' theory does without. Archytas' rays, traveling unsupported to the object seen, behave just like the ὄψεις in Euclid.[27]

In principle, this would be compatible with their being rays of fire, or some other bodily substance, coming out from within the eye, just so long as they do not require any *external* bodily support such as Plato insists upon.[28] But Apuleius' contrast between *mediis oculis proliquati* ("filtered forth from the centre of the eyes") and *tantum oculis profecti* ("advancing only from our eyes") suggests that Archytas' rays start *at* the eyes, not behind and inside them. Bodily substance would require the latter. The former fits the practice of Euclidean diagrams, which show the eye as a point from which the rays fan out. If Archytas was responsible for the optical διαγράμματα reported in the Iamblichus passage, he would most likely do the same. For all that matters for the mathematics is that a ray can be diagrammed as a geometrically straight line from eye to object.

The next paragraph of the *Timaeus* begins, "It is not difficult to see (κατιδεῖν οὐδὲν ἔτι χαλεπόν) how mirror images are formed" (46a). Note the ironical word-play: understanding reflection is as easy as looking into a mirror, which anyone can do. Just so, reflections in a mirror are easy to understand because they are produced by necessity (46b: ἐξ ἀνάγκης). Timaeus then explains how the mixture of inner and outer fire constituting the visual ray gives rise to left-right reversal and other phenomena of reflection standard in books on catoptrics. This account is entirely mechanical, without a trace of mathematics. There are those who infer that at the time Plato wrote the *Timaeus* mathematical optics did not yet exist; it started later with Philip of Opus (Lasserre 1987, 647–8; Simon 1988, 22, 46). But if we believe Apuleius, it would seem that catoptrics already existed. Archytas had some interest in it. He may even have formulated the principle that the angle of reflection is equal to the angle of incidence (*paribus angulis quibus inciderant resultent*), which Apuleius implies was common ground to all the ray-theorists mentioned. Hence, if we believe Apuleius, we should infer that Plato, when writing the *Timaeus*, deliberately declined to mention the possibility of a mathematical account of reflection. It is by polemical design that he chose the rather surprising example of reflection to lead into the dialogue's first statement of the key distinction between reason and necessity, αἴτια and συναίτια. The polemic is a polemic

[27] Compare Epicurus *Ep. Hdt.* 49, who distinguishes two versions of the outwardly traveling story he rejects: οὐδὲ διὰ ἀκτίνων ἢ ὁινδήποτε ῥευμάτων. The ῥεύματα presumably allude to the "stream of sight" at *Timaeus* 45c 3, while ἀκτῖνες is a standard term for the rays postulated in mathematical optics.

[28] Thus O'Brien: "Archytas is specifically distinguished from Plato as having held a theory of vision by outward-flowing fire alone" (O'Brien 1970, 157).

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against the idea of a mathematics of visual appearances. Hence the scornful word-play about how easy it is to understand mirror images.

This goes well beyond the *Republic*'s exclusion of various branches of mathematics from a curriculum designed to turn the minds of future rulers towards the intelligible realm. The *Republic* acknowledged that mathematics, when applied to the sensible world, has certain useful "by-products" (πάρεργα), as when a general uses arithmetic in marshalling his troops or geometry when laying out a camp site. These worldly benefits were faintly praised as "not small" (527c), but dismissed as irrelevant to knowledge of the Good.[29] In the *Timaeus* we learn that the teleological cause of our having eyes at all is to observe the heavens and thereby acquire the notion of number and time, thus to be launched into philosophy in its broadest sense, including physics (47ac). In effect, we are told that the proper use of mathematics is what human nature is for. The implication, if we believe Apuleius, is that a mathematics of mirror images is a misuse of one of our most important endowments.[30]

The point is rammed home by a quotation from the end of Euripides' *Phoenissae* (1762), which Timaeus applies to the lesser benefits of eyesight: let these be "mourned in vain" by a non-philosopher who has gone blind (47b 4–5). As usual in Plato, the original context of the quotation is pertinent to the context of re-use. The speaker is the self-blinded Oedipus, the man who sought knowledge it would have been better for him not to know. His next and final line is, "Being mortal, I must bear the necessities (ἀνάγκας) sent by the gods."

6. But is Apuleius to be believed? Where would the novelist famous for the tale of Lucius and the Golden Ass obtain recherché information on Archytas, information which is not found anywhere else? Apuleius is, of course, much more than a novelist. He is a Platonist philosopher, translator of the *Phaedo*, author of *De dogmate Platonis* and other philosophical works. He is learned in many fields, including mathematics: he wrote a work on astronomy (attested by John Lydus), a translation of Nicomachus' *Arithmetica*, and a treatise *De Musica* (both attested by Cassiodorus). The translation of Nicomachus served the Romans as their main source of information about Greek mathematics. As for the source of Apuleius' account of Archytas, read on to the next chapter of the *Apology*, where he lists a selection of the matters treated in a massive work (*volumine ingenti*) on catoptrics by Archimedes: the effect of plane, convex and concave mirrors on the apparent size of an image in comparison with its original; left-right reversal; apparent motion of the image in response to forward and backward motion of the original; the burning glass; rainbows, parhelia and other such phenomena. Archimedes, a contemporary of the Stoic Chrysippus, is the obvious source for Apuleius to have used – and Archimedes would have had special reason to study the writings of Archytas, the founder of mathematical mechanics.

[29] The several passages need to be read together, as a sequence (Burnyeat 2000, 9–13).

[30] Compare 68bd on the vanity, even impiety, of attempting to declare the proportions in which the different colors are mixed, *even if one knows what they are*.

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Admittedly, Archimedes' authorship of a *Catoptrics* has been impugned by Knorr, who argues that Apuleius was the unwitting victim of a mistake of identity (Knorr 1985). The treatise he is referring to is in fact the *Catoptrics* which has come down to us in the corpus of Euclid. The celebrated name of Archimedes had somehow become attached to the work, with the result that not only Apuleius, but also later authors such as Theon, Olympiodorus, and the scholiast on Euclid *Cat.* 1, cite it as the *Catoptrics* of Archimedes.[31] But the major extant writers on optics never mention Archimedes as one of their predecessors. There is no mention of him in (what remains of) Ptolemy's *Optics,* in Hero's *Catoptrics*, or, most tellingly (if we disregard the problems of transmission via the Arabic), in Diocles *On Burning Mirrors*, who expressly distinguishes four predecessors, among whom Archimedes is not included (Knorr 1985, n. 18).[32]

I am not persuaded. It looks to me as though Knorr makes quite a good case for rejecting the long-standard view that the *Catoptrics* ascribed to Euclid is a late compilation, by Theon or someone else, out of Ptolemy, Hero, and others. So let us date the Euclidean work early rather than late: B.C. rather than A.D., indeed earlier than Diocles, who appears expressly to reject a key part of the Euclidean treatment of burning mirrors (Knorr 1985, 80 and idem 1994, 19).[33] That makes it *possible* that the Euclidean work is the one cited by Theon, Olympiodorus, and the Euclid scholium as Archimedes' *Catoptrics.* But it does not yet make it *likely*.

To make the case for likelihood, Knorr emphasizes the overlap between the problems listed by Apuleius and the problems actually dealt with by Euclid. Our Euclidean *Catoptrics* does not in fact deal with the rainbow or other meteorological phenomena, so overlap is all it is, not a perfect match. Knorr has to postulate that the *Catoptrics* Apuleius consulted did have a meteorological section (prepared for by the otherwise unused Euclidean postulate 6, on refracted rays), but it was cut out by a later editor.

In fact, the problems are a fairly standard list which would be likely to turn up in any work on phenomena of reflection, physical or mathematical. The *Timaeus*' discussion of left-right reversal in differently shaped mirrors may be compared with Lucretius, for example, whose long account of visual illusions (*DRN* IV 269–323) covers both left-right reversal and the depth of mirror images. Knorr himself acknowledges overlap between Hero's *Catoptrics* and the Euclidean one. But my main objection is that the Euclidean *Catoptrics* occupies no more than 28 pages of the (rather small) Teubner text – considerably less than Knorr's 76-page article on the subject. Even with a section on

[31] The testimonia are collected in Heiberg's edition of Archimedes, Vol. II 549–551.

[32] But note that Diocles does not include Euclid either. On mathematicians' references to their predecessors, see now Mansfeld 1998.

[33] Knorr's case for attributing the work to the well-known Euclid himself, which becomes stronger in Knorr 1994, only makes it harder to imagine its being mistaken for Archimedes. Simon proposes that the work is from Euclid's time *or earlier* (Simon 1994).

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meteorological phenomena added back in, Apuleius could hardly have described that as a massive volume.[34]

Another of Knorr's arguments actually tells against the idea that Apuleius has been reading Euclid. Among the problems listed in chap. 16 is this: Why in plane mirrors does the image appear practically (*ferme*) equal to its original? That, says Knorr, "can hardly have appeared in a technical work by Archimedes" (Knorr 1985, n. 13). But nor is it what we read in the Euclidean *Catoptrics* 19, which does not qualify its proof that image and original appear equal. Certainly, the adverb has no place in a mathematical proof. But given the imperfectly even surface of ancient mirrors, Archimedes could well use it to make a more discursive point about applying the mathematics to physical reality (so Butler and Owen [1914] 1983). (Seneca *NQ* I 4.1 offers a suitable context: an explanation of the rainbow as a cloud's distorted mirroring of the sun.) Again, it was not in Euclid that Apuleius found the four theories of vision.

As for the question why Archimedes should mention Archytas alongside the more familiar ray theories of Plato and the Stoics, the most economical answer (besides any reason of South Italian patriotism) is signaled at the end of my excerpt from Apuleius: the differences between the three theories do not stop them agreeing on the fundamental mathematical principle that the angle of reflection equals the angle of incidence. For that is what explains why it is your own face you see when you look into a mirror. The principle may well have received its first formulation from Archytas.[35]

If this was Archimedes' attitude, it finds an interesting parallel in Geminus *ap.* Damianus 24.7–15. Schöne, who declares that mathematical optics does not concern itself with deciding between rival theories of how vision is effected (rays from the eye *vs.* images from the object seen or tension in the air between), just so long as the basic principles of optics are "saved" (σώζεται). Provided vision can be represented as a straight line and apparent size diagrammed by the angle at the eye, the mathematics goes ahead. Similarly, Archimedes would be saying that his catoptrical investigations will be compatible with any theory of vision that preserves the equality of the angles of incidence and reflection.[36]

To sum up, I propose we have good reason to believe Apuleius. Archimedes did write on catoptrics, citing Archytas as a mathematically-oriented predecessor to set against the heavily physical accounts of Plato and the Stoics. Consequently, it will be to Archytas that the following doxographical notice alludes:

[34] Although Knorr quotes the phrase (Knorr 1985, 32) and construes Theon as referring to two or more books of Archimedes' *Catoptrics* (ibid., 36, n. 27), nowhere does he face up to the problem of size. Knorr suggests that the lost meteorological part was the second of an edition in two papyrus rolls (Knorr 1994, 6), but by ancient standards that is not massive: Ptolemy's *Optics* (from the same century as Apuleius) was in 5 volumes. A further puzzle is the total absence of color from the Euclidean treatise: that would surely be needed for discussion of rainbows in the alleged second roll.

[35] Cf. Knorr 1994, 22, inferring from Apuleius that "Archytas . . . conceptualized optical phenomena via geometric rays and knew the equal angles property of reflected rays."

[36] Including the Epicurean theory: Lucr. IV 323.

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> The followers of Pythagoras and the mathematicians[37] [hold that mirror images are formed] by reflection of the visual ray (κατ' ἀνάκλασιν τῆς ὄψεως). The ray travels stretching out towards the mirror, but, on being struck by the dense smooth surface it encounters, it turns back upon itself like an arm that is stretched out and bent back towards the shoulder.[38] (*Dox. Gr.* 405)

Philip of Opus was no doubt inspired by Archytas' example. Now to do mathematical catoptrics, one needs to represent the visual ray (ὄψις) as a geometrically straight line. This concept had been available since Hippocrates of Chios in the fifth century and, for the reasons given, I believe Apuleius when he says that Archytas made use of it. It is this use of a geometrical concept that Plato did not like. That is why in the *Timaeus* he insists on the bodily nature of vision and reduces the explanation of reflection to the realm of Necessity. As in the *Republic*, so in the *Timaeus* his attitude to Archytas is polemical. Equally, to return to my original question, that is why optics has no place in the curriculum of the *Republic*. An optics of "problems," without reference to the visual realm, would be nothing but some rather trivial geometry.

Finally, a last glance at Iamblichus. I hope that by now this testimony seems worthy of credence as a portrait of a non-canonical Pythagorean who devised proofs to solve problems in catoptrics. But notice that the passage speaks of τὰ περὶ τὴν ὄψιν μαθήματα in the plural. If one of the studies that deal with vision is catoptrics, the other must be optics in the more exact sense of the theory of visual appearances such as we find in Euclid's *Optics*. Recall that Philip of Opus worked in optics as well as catoptrics. It so happens that South Italy during the fourth century B.C. was where the art of perspective painting for the theater underwent rapid development. Just the time and the place for someone to found a mathematical theory of perspective.[39] The most famous illustration of theatrical perspective is the Würzburg vase, painted in Tarentum around 360–350 B.C.[40] Archytas was living and writing in Tarentum, South Italy, during the first half of the fourth century B.C. I nominate him as that someone.

[37] Either (a) Pythagoreans and non-Pythagorean mathematicians like Hippocrates, or (b) Pythagoreans and, more specifically, those of them who are μαθηματικοί rather than ἀκουσματικοί.

[38] Cf. Archytas' use of his arm to dispute the idea of a finite universe (DK47A24).

[39] Many scholars have supposed, on the strength of Vitruvius VII *Praef.* 10–12, that the art of perspective was invented by the painter Agatharchus in the fifth century B.C., who wrote a book about it, which stimulated Anaxagoras and Democritus to write about perspective too. None of this is true, according to my analysis of the evidence in "'All the World's a Stage-Painting': Scenery, Optics, and Greek Epistemology" (unpublished). To summarize the conclusions relevant here: (a) What Vitruvius says Agatharchus was the first to do is write a book on a technical architectural theme, viz. making the stage building (σκήνη). (b) In this book he evidently said something about the use of recession to imply depth when the σκήνη is painted to represent a temple or other building. (c) Anaxagoras and Democritus responded to this in writing, but we should not assume they theorized about perspective as such. In fact, I believe they merely cited pictorial recession as an image or model for the general epistemological principle "Appearances are a sight of things unseen" (ὄψις τῶν ἀδήλων τὰ φαινόμενα). In any case, Vitruvius says nothing whatever about any of these writers taking a *mathematical* approach to their subject.

[40] Simon 1982, 23 and plate 10, is just one of many reproductions.

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