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Modified Zakharov–Kuznetsov equation for a non-uniform electron–positron–ion magnetoplasma with kappa-distributed electrons

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We investigate the low-frequency (by comparison with the ion Larmor frequency) electrostatic solitary structures in a spatially non-uniform electron–positron–ion (e–p–i) magnetoplasma with non-Maxwellian electrons. A linear dispersion relation for the obliquely propagating ion acoustic drift wave is derived and it is shown that the non-Maxwellian electron population modifies the dispersion characteristics of the wave under consideration. We also carry out a nonlinear analysis and derive the modified Zakharov–Kuznetsov (MZK) equation for the coupled drift acoustic wave in a non-uniform magnetized plasma. We highlight the differences between the MZK equation and its homogeneous counterpart. We also find the solution of the MZK equation using the tangent hyperbolic method. It is observed that the electron spectral index κ, positron concentration, and propagation angle α alter the structure of the ion acoustic drift solitary waves. The results obtained in this paper may be beneficial to understanding the propagation characteristics of electrostatic drift solitary structures in the interstellar medium and in laboratory experiments where electron–positron plasmas have recently been created by impinging ultra-intense laser pulses on a solid density target at the Lawrence Livermore National Laboratory (LLNL).

1. Introduction

It has been observed that the linear and nonlinear propagation of waves in multicomponent plasmas is different to that in ordinary electron–ion plasmas (Rizzato 1988). The electron–positron–ion (e–p–i) plasma has an important role in the understanding of the plasmas in the early universe (Weinberg 1972; Misner, Thorne & Wheeler 1973; Rees 1983), in active galactic nuclei (Miller & Witta 1987), in pulsar magnetospheres (Michel 1982), and in the solar atmosphere (Goldreich & Julian 1969; Tandberg-Hansen & Emslie 1988). During the last decade, laser–plasma interaction has become a field of growing interest in various fundamental research and technological applications (Modena *et al.* 1995; Pukhov 2003; Esarey, Schroeder & Leemans 2009). Interesting nonlinear phenomena such as self-focusing, harmonic generation, parametric instabilities, soliton formation, and wave collapse have been observed. Ion acoustic two-soliton systems in unmagnetized quantum e–p–i plasmas

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were studied in Mandal *et al.* (2015). The nonlinear dynamics of intense laser pulses in e–p/e–p–i plasma has received a great deal of attention (Berezhiani & Mahajan 1994; Shukla, Marklund & Eliasson 2004; Asenjo *et al.* 2012; Cheng *et al.* 2013). Abundant production of e–p pairs in the collision of a multi-petawatt laser beam and a solid target is predicted, and the result shows that the positron density can be up to 10^{26} m^{-3} (Liang, Wilks & Tabak 1998; Ridgers *et al.* 2012). The positrons can be used to probe particle transport in tokamaks and, since they have sufficient lifetime, the two-component electron–ion plasma becomes a three component e–p–i one (Surko *et al.* 1986; Surko & Murphy 1990). The Advanced Satellite for Cosmology and Astrophysics (ASCA) has confirmed the presence of ions in electron–positron plasmas of astrophysical interest occurring in particular environments.

The ubiquitous presence of energetic superthermal particles in plasmas is confirmed by the long-tailed distributions in many space and laboratory plasma observations (Scudder & Olbert 1979; Masood *et al.* 2006; Masood & Schwartz 2008; Pierrard & Lazar 2010). Numerous models have been proposed to describe this effect on wave dynamics via phenomenological modification to the electron distribution function. One approach to non-Maxwellian plasma modelling is provided by the kappa distribution (Vasyliunas 1968; Hellberg *et al.* 2009; Kourakis, Sultana & Hellberg 2012) which was introduced by Vasyliunas to fit phenomenologically the power-law-like dependence of electron distribution functions observed in space. The spectral index kappa (κ), for which the kappa distribution is named, acts to modify the effective thermal speed in the distribution function (DF). At low values of κ, distributions exhibit strong superthermality which implies an enhancement in the superthermal component of the distribution by comparison with that of a Maxwellian. At very large values of κ, the distribution function approaches a Maxwellian distribution. It is pertinent to mention here that the physical origin of the distribution is still not clear (Yoon 2012; Yoon *et al.* 2012; Livadiotis & McComas 2013). Ion acoustic solitary waves in e–i and e–p–i plasmas have also been investigated in the presence of non-thermal populations of plasma species (Saha & Chatterjee 2014). It is reported that the presence of non-thermal species significantly alters the linear and nonlinear propagation of electrostatic perturbations (Samanta, Saha & Chatterjee 2013; Saha, Pal & Chatterjee 2014).

It is a well-established idea that the linear theory is valid so long as the wave amplitude is small and one may ignore the nonlinearity. However, there are numerous mechanisms by which unstable modes can saturate and attain large amplitudes. When the amplitude of a wave becomes sufficiently large, the nonlinear processes become important and lead to the localization of waves that brings forth different types of interesting nonlinear coherent structures such as solitons, shocks, vortices, etc. It is fairly well known from linear plasma theory that so-called ion drift instabilities exist in low-pressure inhomogeneous plasmas contained in a homogeneous magnetic field (Mikhailovskii 1974). The interest in these instabilities is connected with the widely held view that drift fluctuations can be the cause of the anomalous particle transport in a magnetically confined fusion plasma. The standard approach of the linear theory of drift instabilities is based on the representation of the drift waves as monochromatic ones with some fixed values of the wavenumbers and frequencies.

In their pioneering work, Kadomtsev & Petviashvilli (1970) explored the propagation characteristics of a soliton in a two-dimensional system. The work was later extended to model the soliton in a three-dimensional system by Zakharov and Kuznetsov (ZK) (Zakharov & Kuznetsov 1974). For a non-relativistic magnetized plasma, they obtained a three-dimensional differential equation, which is known as the ZK equation.

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However, the ZK equation may also be used for a two-dimensional magnetized system (Infeld & Rowlands 1990). Many investigations have been made of the propagation of two-dimensional solitary waves in plasmas by deriving the ZK equation (Iwasaki, Toh & Kawahara 1990; Mace & Hellberg 2001; Mushtaq & Shah 2005; Sahu & Roychoudhury 2012). Drift solitons were first studied in an electron–ion plasma by Gell (1977) in the intermediate region of parallel and perpendicular wavenumbers so that the coupling between ion acoustic and drift waves could be studied, and a ZK equation was derived in this regard. Mushtaq (2008) investigated the ion acoustic and drift solitary waves with intermediate parametric range of wavenumbers and arrived at a ZK-type equation. The stationary soliton solution for oblique drift solitary waves was presented and the effect of positron concentration and temperature were also discussed analytically and graphically.

In this paper, we consider a three-component e–p–i plasma in which ions are assumed to be mobile whereas electrons and positrons are considered inertialess since we are studying the waves on the ion time scale. Furthermore, electrons are assumed to follow the isotropic kappa distribution while positrons are assumed to follow the Boltzmann distribution. We study here the linear and weakly nonlinear properties of obliquely propagating ion acoustic and drift waves. We show that in the presence of superthermal electrons the dispersion properties of the waves are modified.

The manuscript is organized as follows. Basic governing equations are given in § 2. The linear dispersion relation of low-frequency electrostatic coupled ion acoustic and drift waves is presented in § 3. The modified Zakharov–Kuznetsov (MZK) equation and its solitary wave solution for a spatially inhomogeneous plasma are given in § 4. The results are presented and discussed in § 5, and finally a summary of the work is given in § 6.

2. Theoretical model

We consider a collisionless, magnetized, inhomogeneous plasma consisting of cold fluid ions, superthermal electrons, and Boltzmann positrons. The constant external magnetic field $B_0\hat{z}$ is considered to be along the z-axis, where $\hat{z}$ is the unit vector in the z-direction. The inhomogeneity in density is assumed to be along the negative x-axis, given by $\nabla n_0 = -\hat{\boldsymbol{x}}\, \mathrm{d}n_0/\mathrm{d}x$. The quasi-neutrality condition at equilibrium reads

$$n_{i0} + n_{p0} = n_{e0} \tag{2.1}$$

where n_{i0}, n_{p0}, and n_{e0} are the equilibrium number densities of ions, positrons, and electrons, respectively. The continuity equation is

$$\frac{\partial n_i}{\partial t} + \nabla_\perp \cdot (n_i \boldsymbol{v}_{i\perp}) + \frac{\partial (n_i v_{iz})}{\partial z} = 0, \tag{2.2}$$

whereas the ion dynamics is given by the following momentum equation:

$$\frac{\partial \boldsymbol{v}_i}{\partial t} + (\boldsymbol{v}_i \cdot \nabla)\boldsymbol{v}_i = \frac{e}{m_i}\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{v}_i \times \boldsymbol{B}_0\right) \tag{2.3}$$

where $\boldsymbol{E} = -\nabla\phi$ is the electric field (ϕ is the electrostatic potential), $\boldsymbol{v}_i$ is the ion fluid velocity, m_i is the ion mass, n_i is the ion number density, e is the magnitude of the electric charge, and c is the speed of light in vacuum. The standard three-dimensional

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isotropic kappa velocity distribution function $f_\kappa(v)$ can be expressed as (Summers & Thorne 1991; Baluku & Hellberg 2008; Hellberg *et al.* 2009)

$$f_\kappa(v) = \frac{n_0}{\theta^3}\left(\frac{1}{\pi\kappa}\right)^{3/2}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}\left(1+\frac{v^2-2e\phi/m_e}{\kappa\theta^2}\right)^{-(\kappa+1)}, \tag{2.4}$$

where $\theta = [((2\kappa-3)k_BT)/\kappa m_e]^{1/2}$ and k_B is the Boltzmann constant. From the definition of the kappa distribution function, it is required that $\kappa > 3/2$ (Pierrard & Lazar 2010). Integrating the kappa distribution over velocity space, one obtains the following expression for the total electron number density:

$$n_e = n_{e0}\left[1-\frac{e\phi}{\left(\kappa-\frac{3}{2}\right)T_e}\right]^{-(\kappa-1/2)}, \tag{2.5}$$

while the number density for the Boltzmann distributed positrons can be expressed as

$$n_p = n_{p0}\exp\left(-\sigma\frac{e\phi}{T_e}\right) \tag{2.6}$$

where T_j is the temperature of species j ($j = e$ for electrons and p for positrons) and $\sigma = T_e/T_p$. The plasma species are coupled through the Poisson equation

$$\nabla^2\phi = -4\pi e(n_i + n_p - n_e). \tag{2.7}$$

In the limit of small electrostatic perturbations, $e\phi/T_j \ll 1$, using (2.5) and (2.6) in (2.7), we obtain

$$(\beta - \lambda_{De}^2\nabla^2)\phi = 4\pi e\lambda_{De}^2 n_i \tag{2.8}$$

where $\lambda_{De} = \sqrt{T_e/4\pi n_{e0}e^2}$, $\beta = \alpha + (n_{p0}/n_{e0})\sigma$ and $\alpha = (\kappa-1/2)/(\kappa-3/2)$.

3. Linear analysis

Linearizing (2.3) under the drift approximation $|\partial/\partial t| \ll \omega_{ci}$ ($\omega_{ci} = eB_0/m_ic$ is the ion gyrofrequency), we obtain

$$v_{ix} \simeq -\frac{e}{m_i\omega_{ci}}\frac{\partial\phi}{\partial y}, \quad v_{iy} \simeq \frac{\mathrm{i}\omega e}{m_i\omega_{ci}^2}\frac{\partial\phi}{\partial y}, \quad v_{iz} \simeq -\frac{\mathrm{i}e}{m_i\omega}\frac{\partial\phi}{\partial z}. \tag{3.1a–c}$$

Using (3.1) in the linearized ion continuity equation (2.2), we find

$$n_i = \frac{en_{i0}}{m_i\omega^2}\left[\frac{\omega^2}{\omega_{ci}^2}\frac{\partial^2\phi}{\partial y^2} - \frac{\partial^2\phi}{\partial z^2} - \frac{\mathrm{i}\omega}{\omega_{ci}}\kappa_{ni}\frac{\partial\phi}{\partial y}\right] \tag{3.2}$$

where $\kappa_{ni} = |(1/n_{i0})\,\mathrm{d}n_{i0}/\mathrm{d}x|$. Using this equation, we can write (2.8) as follows:

$$(\beta - \lambda_{De}^2\nabla^2)\phi = \frac{c_s^2}{\omega^2}\frac{n_{i0}}{n_{e0}}\left[\frac{\omega^2}{\omega_{ci}^2}\frac{\partial^2\phi}{\partial y^2} - \frac{\partial^2\phi}{\partial z^2} - \frac{\mathrm{i}\omega}{\omega_{ci}}\kappa_{ni}\frac{\partial\phi}{\partial y}\right] \tag{3.3}$$

where $c_s = \sqrt{T_e/m_i}$ is the ion acoustic speed. Assuming the perturbation proportional to $\exp[\mathrm{i}(k_yy + k_zz - \omega t)]$ in (3.3), we obtain the following dispersion relation for

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the coupled ion acoustic drift waves in the presence of superthermal electrons in an inhomogeneous e–p–i plasma:

$$\left(\beta + \lambda_{De}^2 k^2 + \frac{n_{i0}}{n_{e0}}\rho_s^2 k_y^2\right)\omega^2 - \frac{n_{i0}}{n_{e0}}\omega_* \omega - \frac{n_{i0}}{n_{e0}}c_s^2 k_z^2 = 0 \tag{3.4}$$

where $\rho_s = c_s/\omega_{ci}$, $\omega_* = v_d^* k_y$ is the drift wave frequency, with $v_d^* = (cT_e/eB_0)\kappa_{ni}$. For $\alpha = 1$, i.e. for Boltzmann electrons and using the long-wavelength approximation ($\lambda_{De}^2 k^2 \ll 1$) this equation reduces to equation (10) of Haque & Saleem (2003). It is obvious that for a homogeneous density plasma, $\nabla n_0 = \mathbf{0}$, (3.4) gives the dispersion relation for modified ion acoustic waves in a magnetized plasma, with the presence of superthermal electrons, as

$$\omega^2 = \frac{c_s^2 k_z^2 (1-p)}{\beta + \lambda_{De}^2 k^2 + (1-p)\rho_s^2 k_y^2} \tag{3.5}$$

where $p = n_{p0}/n_{e0}$. For the long-wavelength approximation (in comparison with λ_{De} and ρ_s), (3.5) leads to a modified ion acoustic phase speed, given by

$$\frac{\omega^2}{k_z^2} = c_s^2 \left(\frac{1-p}{\alpha + p\sigma}\right). \tag{3.6}$$

Equation (3.6) shows that, due to the positron concentration (or population) in usual e–i plasmas, the phase speed of ion acoustic waves is reduced (Popel, Vladimirov & Shukla 1995). It also shows that as the velocity distribution function deviates from a Maxwellian one, spectral index κ decreases (smaller κ values represent relatively more superthermal particles), and hence the phase speed decreases further. On the other hand, for $\omega \gg c_s k_z$, (3.4) leads to the dispersion relation for the modified drift waves in a magnetized e–p–i plasma in the presence of superthermal electrons, given by

$$\omega = \frac{\omega_*(1-p)}{\beta + \lambda_{De}^2 k^2 + (1-p)\rho_s^2 k_y^2}. \tag{3.7}$$

Equation (3.7) shows that the drift wave frequency and phase speed decrease with decreasing κ.

4. Nonlinear analysis

In the limit of $\lambda_{De}^2 \nabla^2 \ll \beta$, $(\omega k_y/\omega_{ci} k_z)^2 \ll 1$, $|\omega \kappa_{ni} k_y/\omega_{ci} k_z^2| \ll 1$, (3.3) gives

$$\omega^2 \phi = -\frac{c_s^2}{\beta}(1-p)\frac{\partial^2 \phi}{\partial z^2}. \tag{4.1}$$

Equation (3.3) with the help of relation (4.1) becomes

$$(\beta - \lambda_{De}^2 \nabla^2)\phi = -\frac{c_s^2}{\omega^2}(1-p)\left[(1-p)\frac{\rho_s^2}{\beta}\frac{\partial^4 \phi}{\partial y^2 \partial z^2} + \frac{\partial^2 \phi}{\partial z^2} + \frac{\kappa_{ni}}{\omega_{ci}}\frac{c_s}{\sqrt{\beta}}\sqrt{(1-p)}\frac{\partial^2 \phi}{\partial y \partial z}\right]. \tag{4.2}$$

Assuming that higher-order terms are negligibly small, it yields

$$\omega \simeq \frac{c_s k_z}{\sqrt{\beta}}\sqrt{(1-p)}\left[1 - \frac{1}{2\beta}(\lambda_{De}^2 + (1-p)\rho_s^2)k_y^2 - \frac{1}{2\beta}\lambda_{De}^2 k_z^2\right] + \frac{1}{2\beta}\frac{n_{i0}}{n_{e0}}v_d^* k_y. \tag{4.3}$$

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Using (4.3) in the parallel component of (3.1), we get

$$\frac{e}{m_i}\frac{\partial\phi}{\partial z}=\frac{c_s}{\sqrt{\beta}}\sqrt{\frac{n_{i0}}{n_{e0}}}\left[\frac{\partial v_{iz}}{\partial z}+\frac{1}{2\beta}\left(\lambda_{De}^2+\frac{n_{i0}}{n_{e0}}\rho_s^2\right)\frac{\partial^3 v_{iz}}{\partial z\partial y^2}+\frac{1}{2\beta}\lambda_{De}^2\frac{\partial^3 v_{iz}}{\partial z^3}\right]+\frac{1}{2\beta}\frac{n_{i0}}{n_{e0}}v_d^*\frac{\partial v_{iz}}{\partial z}. \tag{4.4}$$

Using (4.4) in (2.3), we obtain the nonlinear equation for the low-frequency coupled ion acoustic and drift waves as

$$\frac{\partial v_{iz}}{\partial t}+v_{iz}\frac{\partial v_{iz}}{\partial z}+\frac{c_s}{\sqrt{\beta}}\sqrt{\frac{n_{i0}}{n_{e0}}}\left[\frac{\partial v_{iz}}{\partial z}+\frac{1}{2\beta}\left(\lambda_{De}^2+\frac{n_{i0}}{n_{e0}}\rho_s^2\right)\frac{\partial^3 v_{iz}}{\partial z\partial y^2}+\frac{1}{2\beta}\lambda_{De}^2\frac{\partial^3 v_{iz}}{\partial z^3}\right]+\frac{1}{2\beta}\frac{n_{i0}}{n_{e0}}v_d^*\frac{\partial v_{iz}}{\partial z}=0. \tag{4.5}$$

Equation (4.5) is the modified Zakharov–Kuznetsov (MZK) equation for describing the nonlinear solitary waves in inhomogeneous magnetized e–p–i plasmas with the presence of superthermal electrons. To find the solitary wave solution of (4.5), we rewrite it in a normalized form as

$$\frac{\partial v_{iz}}{\partial t}+v_{iz}\frac{\partial v_{iz}}{\partial z}+A\frac{\partial v_{iz}}{\partial z}+B\frac{\partial^3 v_{iz}}{\partial z\partial y^2}+C\frac{\partial^3 v_{iz}}{\partial z^3}+D\frac{\partial v_{iz}}{\partial y}=0 \tag{4.6}$$

where we have normalized the ion fluid velocity by the ion acoustic speed c_s, the space coordinates by the ion Larmor radius ρ_s, and time by the ion gyrofrequency ω_{ci}. The coefficients A, B, C, and D are given by

$$A=\frac{1}{\sqrt{\beta}}\sqrt{\frac{n_{i0}}{n_{e0}}},\quad B=\frac{A}{2\beta}\left(\frac{n_{i0}}{n_{e0}}+\frac{\lambda_{De}^2}{\rho_s^2}\right),\quad C=\frac{A}{2\beta}\frac{\lambda_{De}^2}{\rho_s^2},\quad D=\frac{n_{i0}v_d^*}{2\beta n_{e0}c_s}. \tag{4.7a–d}$$

It is worth mentioning here that the terms containing coefficients A and D are absent from the usual ZK equation. Using the tangent hyperbolic method (Malfliet 1992, 2004), the solitary wave solution of (4.6) can be put in the form

$$v_{iz}=12(B\alpha^2+C)\ \mathrm{sech}^2(z+\alpha y-ut) \tag{4.8}$$

where $u=(A+\alpha D)+4\alpha^2 B+4C$, and $\alpha<1$ is the angle of propagation for ion acoustic drift solitary waves. Note that the coefficients A and D alter the velocity of the nonlinear structure and hence play a role in the phase shift of the nonlinear solitary structure. By dropping the last term in (4.6), we obtain the limiting case of a spatially homogeneous plasma and obtain a solitary wave solution for ion acoustic waves which looks similar to (4.8) but gives a different phase shift for the nonlinear structure owing to the absence of diamagnetic drift velocity.

5. Results and discussion

In this section, we shall numerically investigate the dependence of the propagation characteristics of the coupled ion acoustic and drift solitary waves on the plasma parameters, such as electron spectral index κ, positron concentration, and obliqueness angle α. For illustration, we have chosen the parameters which are typically found in the interstellar medium (Klein & Kerp 2008): $n_{i0}\sim 10^{-3}$–10^{-2} cm^{-3}, $n_{p0}\sim 0.15\times10^{-3}$–$0.15\times10^{-2}$ cm^{-3}, $T_e=10^7$ K, $B_0=10^{-5}$ G. It should be noted that recent experiments performed at the Lawrence Livermore National Laboratory (LLNL)

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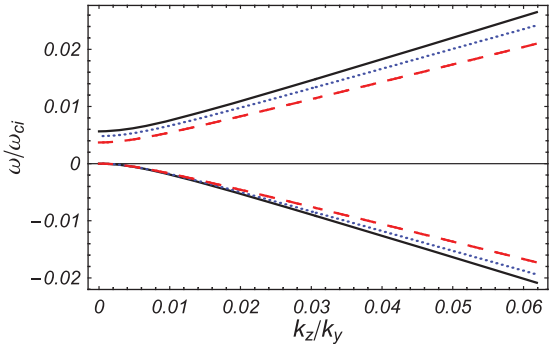


FIGURE 1. Linear dispersion relation for coupled drift and ion acoustic waves in a magnetized e–p–i plasma. Variation of frequency for different values of the electron spectral index κ is illustrated. Solid curve is for a Maxwellian distribution, dotted for $\kappa = 6$, and dashed for $\kappa = 3$.

have given rise to the exciting possibility of generation of hard bremsstrahlung, photonuclear reactions, which is preliminary evidence for positron–electron pair production (Cowan *et al.* 1999). It is well known that laser-induced plasmas are inhomogeneous and hence the present study becomes all the more important. The next generation ultra-intense laser experiments might open up new challenges for theoretical physicists. Moreover, a study of radiative blast waves in atomic cluster media using intense laser pulses has also been reported (Cowan *et al.* 1999; Masood *et al.* 2014). Careful application of these equations and similarities allow experiments to be scaled to astrophysical phenomena that have spatial and temporal scales that are greater by as much as 15–20 orders of magnitude. In this way, radiative blast waves in the laboratory have been scaled to those experienced in supernova remnants and the physics governing their dynamics investigated under controlled conditions (Palastro *et al.* 2008; Masood *et al.* 2014). It is pertinent to mention here that when the laser hits a solid target, fast electrons are produced and such a non-thermal population plays an important role in inertial confinement fusion by fast ignition (Tabak *et al.* 1994; Bell & Kingham 2003). Another application of the non-Maxwellian electron distribution function is the RF discharge. Accordingly, the generalized Lorentzian (κ) distribution function is used to simulate the electron distribution function. By this choice both cases are covered, since the limit of infinite κ represents the Maxwellian distribution function (Godyak, Piejak & Alexandrovich 1993).

Figure 1 explores the effect of electron spectral index κ on the frequency of the coupled ion acoustic and drift wave in a magnetized spatially inhomogeneous e–p–i plasma. It is found that on increasing the superthermal population of electrons (i.e. decreasing κ), the frequency of the wave under consideration decreases. This fact can lead to the enhanced Landau damping of the electrostatic waves as the gap between the ion thermal velocity and the phase velocity of the wave shrinks (Lee 2007). This also implies that increasing the non-thermal population of electrons (meaning decreasing κ) mitigates the restoring force and hence the observed decrement in frequency.

Figure 2 investigates the effect of electron spectral index κ on the propagation characteristics of the coupled ion acoustic and drift solitary wave. It is observed that enhancing the electron spectral index κ decreases the amplitude as well as the width of the solitary structure. A similar trend was observed by El-Shamy (2014) for an ion

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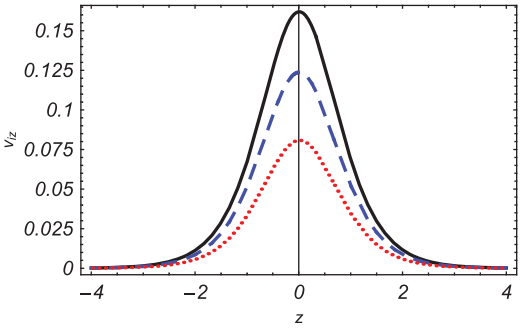


FIGURE 2. Variation of the amplitude of drift and ion acoustic solitons for different values of the electron spectral index κ. Solid curve is Maxwellian, dashed for $\kappa = 6$, and dotted for $\kappa = 3$. The other parameters are $p = 0.2$ and $\alpha = 0.2$.

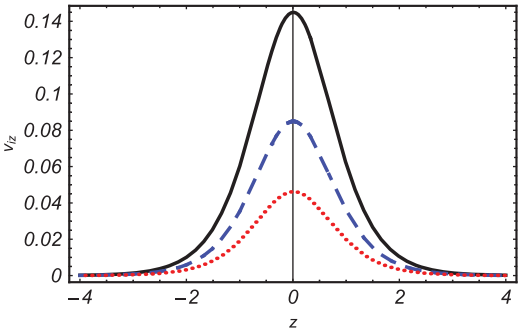


FIGURE 3. Variation of the amplitude of drift ion acoustic solitons for different values of positron concentration $p = n_{p0}/n_{e0}$. Solid curve is for $p = 0$, dashed for $p = 0.2$, and dotted for $p = 0.4$. The other parameters are $\kappa = 4$ and $\alpha = 0.2$.

acoustic wave in a homogeneous magnetized plasma.The physical reasoning given to explain figure 1 holds here as well.

Figure 3 shows the effect of positron concentration on the structure of the ion acoustic drift solitary wave. It is found that increasing positron concentration reduces the amplitude and also decreases the width of the solitary wave, which agrees well with the results presented by Popel *et al.* (1995) and El-Shamy (2014). This happens because of the quasi-neutrality condition, which dictates that an increase in positrons should be accompanied by a decrease in ion population. Since we are considering the ion dynamics, therefore, a decrease in ion population brings about a decrease both in the linear and nonlinear acoustic structures. It is pertinent to mention here that the results presented by Mushtaq (2008) showed the opposite trend. We doubt his results since increasing the positron concentration means a decrease in the ions in the system and should therefore lead to a decrease in the amplitude of the drift ion acoustic solitary wave.

Figure 4 explores the effect of increasing propagation angle on the ion acoustic drift solitary wave. Physically, an increase in angle of propagation means the inclusion of cyclotron as well as drift effects, as the particles experience neither of them along the direction of the magnetic field owing to the absence of Lorentz force. It is observed that increasing the propagation angle enhances both the amplitude and width of the

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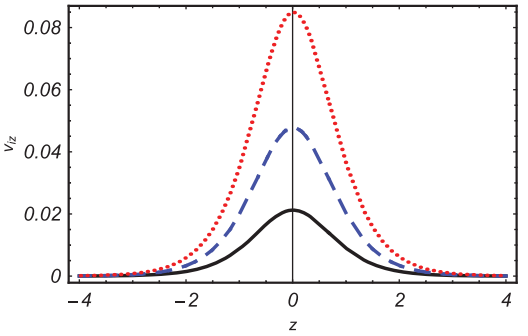


FIGURE 4. Variation of the amplitude of drift ion acoustic solitons for different values of the propagation angle α. Solid curve is for $\alpha = 0.1$, dashed for $\alpha = 0.15$, and dotted for $\alpha = 0.2$. The other parameters are $\kappa = 4$ and $p = 0.2$.

solitary wave, which agrees well with the results presented by Masood *et al.* (2013). Note that the results are valid only for small angles of propagation, as larger angles pose problems of stability for the solitary wave (Kadomtsev & Petviashvilli 1970; Zakharov & Kuznetsov 1974). It is pertinent to mention here that the acoustic drift solitary structures experience a greater shift by comparison with their counterparts in homogeneous plasma as evident from the solution given by (4.8). Finally, we mention that the spatial coordinates are normalized by the ion Larmor radius which is set to be 2000 miles. Therefore, the spatial extent of the ion acoustic drift solitary structures is estimated to be ∼7000 miles.

6. Conclusion

In this paper, we have studied the linear and nonlinear propagation of two-dimensional ion acoustic drift solitary structures in an inhomogeneous e–p–i magnetoplasma in the presence of superthermal electrons. It is found that the dispersion characteristics of the wave under consideration are modified by the non-Mawellian electrons. We have also presented a nonlinear analysis and derived the MZK equation for the coupled ion acoustic drift wave in a spatially inhomogeneous plasma. The solution of the MZK equation is obtained by employing the tangent hyperbolic method. Only compressive solitary structures exist in this frame. We have also discussed the limiting case of homogeneous plasma in both the linear and nonlinear regimes. It is found that the electron spectral index κ, positron concentration, and propagation angle α modulate the structure of ion acoustic drift solitary waves. For illustration, using the parameters typically found in the interstellar medium, the spatial extent of the solitary structure is estimated to be ∼7000 miles. The work presented here on obliquely propagating ion acoustic drift waves in magnetized e–p–i plasmas can improve our understanding of nonlinear excitations in astrophysical and laser-induced plasmas.

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