Matter for debate

Nature-nurture: finding-feeding

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Mathematicians know that the path from adolescent competence to adult professional life is full of unexpected twists and turns, and is fraught with potential pitfalls. Even when faced with very able adolescents, it is not at all clear which of them will flourish in the long run, or how one can best support their future development (when they eventually become young adults) in a way that takes their mathematical ability and inclinations seriously – whether in pure or applied mathematics, in computer science or software development, in engineering, mathematical biology, financial mathematics or linguistics, in management, or in some totally different profession.

Each student's 'basic ability' clearly affects their path from adolescence to adulthood. But other factors often appear to be more important in deciding

- whether someone goes on to study mathematics in the first place (and to what level);
- whether they begin in mathematics, but then pursue a career that leaves mathematics behind; or
- whether their mathematical bent ultimately expresses itself in other ways.

Each adolescent develops their own kind of inner robustness and psychology, which often determines whether they learn to 'enjoy' the fruitful frustration of repeated failure (with occasional success) which is so characteristic of mathematics: that is, whether they develop the kind of stubbornness that sees 'initial failure' as a challenge, and as an incentive to persevere. Even where this hurdle is overcome, and an initial foothold is gained in the world of mathematics research, the opportunities needed for survival (or their absence) involve a considerable degree of chance: whether one's early work is 'timely', who one knows or meets, whether someone puts in a kind word, or simply whether one happens to be in the right place at the right time.

The mathematical community can provide crucial opportunities, and can offer encouragement and support during adolescence. But we have rather little evidence as to what kinds of opportunities are most effective. Is it even possible to identify systematically potentially talented young mathematicians during adolescence, and then to devise and provide the kind of opportunities that effectively encourage them to cultivate whatever talent they might have? This short report concerns one example of such provision – and its outcomes.

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We know that some kinds of intervention during adolescence can be effective – at least in the short term. For example, there are schools and mathematics teachers who, over a given period, produce a steady stream of high quality 17-18 year olds. However, what follows after age 18 often fails to confirm the early promise. Such examples remind us that special provision is never 'definitively diagnostic', but gives rise to a long-term distribution of outcomes. Hence the question we should consider is that of

- assessing the effectiveness of different forms of provision
- by comparing the required inputs with the long-term distribution of outcomes to which they give rise.

Our example concerns an intervention, whose modest input (namely a highly focused, five-day summer school for around fifty students, aged 15 and 16) should not be expected to have any discernible long-term impact.

Most of these students had also engaged with the challenge of trying to write solutions to accessible, but serious, mathematical problems (in the form of the Junior and Intermediate 'Olympiads', targeted at around 600 UK students in each school year). Each invitation to attend the Summer School reflected the invitee's success in presenting solutions to such problems *in the year of participation*: however, such success was rarely a one-off – so many of the students had probably tackled the corresponding papers in the previous year or two.

The report is interesting partly because we managed to trace (almost) the *complete cohort* of participants, and partly because the summer school in question took place in the mid-1990s, so the participants are now all approaching the age of 40 – making it possible to form a sober assessment of the 'distribution of outcomes'.

We find the data striking, but encourage readers to draw their own conclusions. That is, we encourage those with experience of other forms of intervention during adolescence to identify the interesting questions which this example raises, and to assess (in the light of their own experience) whether other similarly documented enquiries have produced similar long-term outcome distributions (though examples seem to be scarce): (e.g. [1]), and hence whether or not it might be worth examining more closely the principles underlying the example described here.

What was this cohort like?

Any group of adolescents selected nationally on the basis of their performance in school mathematics will contain some students for whom mathematics is their sole strength. But the vast majority are likely to be outstanding in a number of subjects – and a significant fraction will be good at almost everything (as was the case with the group considered here). For such multi-talented students, the influences from school and family almost never prioritise mathematics. Hence most of those who excel in mathematics before the age of 16 are likely to be nudged away from

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studying both A level Maths and Further Maths as being 'too narrow' a programme; or they may take Further Maths as an indulgence (as a 4th, or 5th A level subject), rather than as a preparation for serious further study.

The National Mathematics Summer School was conceived as an explicitly mathematical intervention – not as a reward for 'generally able' students. This was one of the very early national summer schools, but the essential features remained consistent until relatively recently, those features being

- to select students who had already demonstrated both commitment and ability (e.g. by requiring a strong performance in the Intermediate Olympiad) – focusing on those with the potential to continue their mathematical studies; and then
- to provide a 'mathematically serious' diet of a broad kind (more focused than 'fun', and less focused than higher Olympiad preparation).

In addition to whatever was routinely available in the participants' own schools, their selection and participation was the culmination of several years of tackling annual problem solving papers with a serious mathematical purpose, designed for fairly large numbers – starting each year with a multiple choice paper (aimed at 25-30% of all secondary students), followed by a 2-hour paper for each year group, requiring written solutions to a small number of questions (involving around 600 students in each secondary year group).

- The overall focus (of both the preliminary events and the summer school) was unashamedly mathematical and serious – while trying not to assume much by way of prior knowledge.
- The summer school itself was intensive, varied, residential (five nights), and explicitly incorporated a social element with the same goals as the mathematical part.

In other words, those attending already had a shared prior mathematical experience which went beyond 'school maths', and which was intended to capture serious mathematics in adolescent form (without assuming 'higher' knowledge). The mathematical content of the 5 days developed this experience intensively, and in unexpected ways. For example, the social interactions between the students laid the foundation for continued friendships and many unseen contacts (before the era of mobile phones and e-mail), and this would seem to have contributed to their choices of university and subject.

Forty-six students aged 14-16 accepted the invitation to attend. They were organised into seven 'teams', with each team being led by one of the seven students preparing for that year's IMO (6 team members plus 1 reserve: these seven additional students are not included in the 46 students discussed here). The 'teams' were mainly social: they provided a 'tutorial' setting for the interactive lectures 9.00-10.30 each morning (with each team

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sitting together and being supported by their team leader), and they stayed together as a group for the afternoon workshops. The teams also formed the units for the varied individual and group problem-solving competitions that ran each day from 11.00-12.30. In particular, whatever long-term 'success' the event may have had owed much to the way the team leaders and the adult contributors worked with these groups.

In those early days we had already changed the mix of students in the IMO team – without ever lowering the bar. We had moved from 2 girls in total during the 24 years up to 1990, to 1-2 girls each year during the period 1991-1995. And we had reversed the 'private : state' school ratio for team members from 5 : 1 in 1990, to 1 : 5 in 1995. But we had not yet developed simple rules to balance each year's summer school cohort. So among the cohort discussed here, 28 of the 46 participants came from private schools (23 male, 5 female); 10 participants came from selective state schools (8 male, 2 female); and 8 participants (all male) came from comprehensive schools.

Where are they now?

I have managed to trace all 46 participants up to graduation, and the positions held by 45 of the 46 today.

- 7 of the 46 students went on to represent the UK at the International Mathematical Olympiad
- 2 of these (and 1 team leader) went on to be Senior Wranglers at Cambridge.

28 received places to study serious mathematics degrees (27 at Oxford or Cambridge, and 1 at Surrey); 4 studied Natural Sciences at Cambridge; 1 studied Physics at Oxford; 2 studied Engineering at Cambridge; 2 studied Medicine at Cambridge; 3 studied Computer Science – 2 at Cambridge, and 1 at UCL; 1 studied Marine Engineering in Strathclyde; 1 studied Accountancy in Aberdeen; 1 studied Architecture in Hong Kong; 1 studied Art at Bournemouth School of Art; 1 studied Biomedical Sciences at Sheffield Hallam University.

- 25 students began straight Mathematics courses: 1 of these changed to Engineering.
- In addition, 3 students applied to study 'Mathematics and Philosophy' at Oxford, one of whom switched to 'History and English' before matriculating. Both of the others graduated in Mathematics and Philosophy and went on to become partners in leading consultancy firms, one moving on to become managing director in a large national concern.

Of the 24 students who graduated in straight Mathematics, 8 moved directly into work. Of these 8:

- 1 was Prince Philip Scholar (a Hong Kong award) at St John's,

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Cambridge (but vanished after graduation).

- 1 is now an Anglican vicar – still enjoying mathematics and teaching on the side.
- 1 joined Legal & General and is now Head of Modelling in the Pensions Policy Institute.
- 1 went into insurance and is now a director of AXA.
- 1 spent 10 years in computer games (until the company folded), 4 years with Dreamworks on animated movie production, and is currently a software engineer at Google.
- 1 moved into software services for major international, London-based banks, and now works for a large private hedge fund.
- 1 became an actuary and is now with Wills, Towers and Watson.
- 1 was also a serious gymnast, and did Mathematics at Surrey in order to stay near his local club and his coach. He went on to become a Commonwealth Games gold medallist, and is now an accountant and a top-flight gymnastics judge (including having served at the 2012 Olympics).

Of the 24 students who graduated in straight Mathematics, 16 went on to complete either Part III in Cambridge, or a Master's degree in Oxford. Of these 16:

- 1 moved from Cambridge to do a Master's degree at Oxford, then joined the fast track Civil Service.
- 1 moved into software development, and is now senior developer for a radical new bank.
- 1 has been working on his PhD ever since 2002, and is still part of the local and national mathematics community.
- 13 completed PhDs in mathematics:
  - 1 had studied several different disciplines at undergraduate and at postgraduate level, with his PhD thesis winning that year's international prize for outstanding dissertations in the fields of logic, language and information; he has since collaborated with leaders in many fields, and is co-founder and head of R&D in his world-leading firm.
  - 1 did a PhD in Algebraic Geometry in Cambridge, and is now a professor in the Mathematical Institute in Oxford.
  - 1 did a PhD in Number Theory and is now a professor in Imperial.
  - 1 did a PhD in Harvard, and is now a reader in Durham.
  - 1 did a PhD in Algebraic Topology at MIT, before

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positions in Johns Hopkins and the University of Georgia, and is currently Associate Professor in Amherst College.

- 1 completed a PhD in Mathematical Biology at Cambridge, and then went to the US, where he is now a professor in the University of Georgia.
- 1 did a PhD and held a post-doc position in Cambridge, moved to Southampton and back to Cambridge, before joining a start-up company doing machine learning, which was then bought out by Apple – for whom he is now machine learning research manager.
- 1 completed a DPhil on 'entanglement and the quantum hall effect' before post-doc positions in Leeds and Birmingham, and now works on genome sequencing in Cambridge.
- 1 did a PhD in Applied Mathematics at Cambridge, with a series of post doc positions, and now works in GCHQ.
- 1 did a PhD in Combinatorics, then joined the Civil Service (Foreign Office), and now works in GCHQ.
- 1 did a PhD on Brownian Motion at a Russell Group university, where he now works in financial mathematics (having held positions elsewhere).
- 1 did a PhD in Dynamical Systems at Cambridge, and is currently an Associate Professor in Auckland.
- 1 did a PhD in Functional Analysis in Cambridge; he then spent 3 years working in the Prison Service before switching to work with very disturbed children – retraining as a group psychotherapist and child therapeutic counsellor.

- Of the 19 students who did not graduate in mathematics:
- 4 studied Natural Sciences at Cambridge. Two of these completed PhDs in Cambridge: one recently moved to be Reader in Psychology at a Russell Group university; the other is now a Vice President of Barclays Investment Bank. One graduated in Physiology, went on to complete a PhD in Nottingham, and now works for FERA (Food and Environment Research Agency). The fourth had been on the fringe of the UK Informatics Olympiad team when at school, and later moved into IT security.
- 1 studied Physics at Oxford and is now a partner in the London offices of an international financial strategy firm.
- 2 studied Medicine at Cambridge. One is now Assistant Professor in the London School of Hygiene and Tropical

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Medicine with a PhD in Epidemiology; the other joined the army as a medic and recently qualified as an orthopaedic surgeon.

- 2 studied Computer Science in Cambridge. One now runs his own digital mapping company; the other started a Cambridge PhD there before moving into auditory neuroscience, started another PhD in UC San Diego only to discover that this led only to academia, so quit (graduating with a Masters), worked for six years at Google, then DE Shaw, and is now independent.
- 1 studied Computer Science at UCL, going on to complete a PhD there, worked as a post-doc in Medical Imaging, and now has a permanent lecturer position at UCL.
- 3 studied Engineering in Cambridge. One of these had switched from Mathematics, and now runs his own software company; one worked first for Mars before moving into software for major banks in Australia; one now works as a senior planner for a major construction company.
- 1 studied Marine Engineering in Strathclyde before working for BAE Systems (first in Barrow and then in Fayetteville, North Carolina).
- 1 switched from Mathematics and Philosophy at Oxford to History and English, and is now Finance Director for a missionary society.
- 1 returned to Hong Kong where she studied Architecture and now works for the Hong Kong government on public buildings and infrastructure projects.
- 1 studied Accountancy in Aberdeen (following in family footsteps) and is now an Ernst & Young Director.
- 1 studied Biomedical Sciences at Sheffield Hallam University and is now Project Director (Oncology) for a large clinical research organisation.
- Though we did not know at the time, one student was already terminally ill at the time of the summer school. He started A level studies the next term, but had to withdraw before Christmas, and died soon after.

Conclusion

One possible response to this 'distribution of outcomes' might be to suggest that these were evidently the kind of students who would have pursued such careers even without the combined experience of working on Olympiad questions and attending summer school. If this were the case, then one would expect to see a similar 'distribution of outcomes' in many similar groups – and the evidence shows that this is not so (it may be enough to consider the world-lines of the much more select group represented on the UK IMO Register [2]). Moreover, having observed the progress of a dozen

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or so of these summer-school students over a number of years, and having communicated with most of the others (or their parents) in the course of this retrospective study, I am struck by their own testimony of how the experience of attending summer school contributed to their subsequent choices – especially in encouraging them to aim high, whether by persevering with mathematics, or by following a slightly different path. So I doubt whether the 'distribution of outcomes' can be so easily explained.

Another possible response might be to infer that the 30% or so of these students who went in highly non-mathematical directions were clever students, whose world-lines confirm that mathematics in general, and the summer school in particular, left very little impression. From the recent testimony of the students and their parents, contacted out of the blue after 22 years, this would seem to be far too negative a view – even in those cases where the invitation bore no evident fruit. For example, the family of the student who was terminally ill at the time of the summer school were deeply appreciative (even after 22 years), and went out of their way to link this experience with the fact that his younger brother later went to Trinity College to read Mathematics and is now a mathematics teacher at a good international school in Switzerland. One of the last to be traced, who left both the UK and mathematics behind at age 18, commented: 'Although I did not go into mathematics, the Summer School was one of the highlights of my life'. Another, whose world-line might appear to contradict the declared goals of the summer school wrote: 'my CV would give a false impression, and would probably make you think that in my case at least, there was no discernible impact …. However, there was plenty.'

It would be hard to imagine a collection of subsequent world-lines that better illustrate the intended impact of the summer school. In this I would even include the one student who was present almost by accident. This student's teacher knew that he had been the top scholar (in one of the UK's best schools) at age 13; he therefore noticed the student's remarkable score of zero on the popular multiple-choice paper (aimed at 25-30% of the national age group) – and, on closer inspection, observed that the score had been achieved by consistently choosing the option after the correct answer! He therefore entered the student as an exceptional candidate for the ensuing Olympiad, where he showed what he was capable of. But his real love was art and design, which he continues to pursue in a most original way.

The whole thrust of the summer school, and of what went before, was to convey what it is that makes mathematics 'special'. The central goal was

- to select a group of potentially committed students;
- to provide a mathematical experience which might capture their imagination; and then
- to encourage them to persevere with the subject, to be willing to question any peers or adults who might see it as peripheral or narrow, and to consider seriously developing whatever mathematical talent they might possess.

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The summer school programme also highlighted values that were far broader than just mathematics – including teamwork, drama, music and philosophy. For example, we built in a talent evening where everyone was encouraged to put on a small show (whether musical, or juggling, or reciting poetry, or acting a skit).

My own contact with participants in the years following the summer school was largely limited to those who subsequently took part in the IMO. Hence any influence on the subsequent choices made would seem to derive from behind-the-scenes interactions between participants, and from the silent fermentation of ideas.

Many well-meaning, and well-documented interventions in mathematics have no discernible long-term positive impact. The above evidence shows that: with appropriate prior experience, sensitive selection, and a suitable programme and follow-up, it is possible effectively to encourage and to support the long-term mathematical development of promising adolescents. But the nature, and the timing, of the intervention matter. It would be interesting to hear of other brief interventions during adolescence that have led to a comparable (mathematically-skewed) distribution of outcomes.

References

1. *Program in Mathematics for Young Scientists*, Alumn1 Community, accessed April 2018 at https://www.promys.org/alum-community/about
2. *UK IMO Register*, accessed April 2018 at https://www.imo-register.org.uk

10.1017/mag.2018.115

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The Mathematical Gazette Information Sheets

During 2018, as an experiment, we are producing an information sheet for each issue. Such sheets are slightly enhanced lists of contents.

The enhancements are to two aspects. First, books which are reviewed are listed (along with the author and reviewer). Secondly, the DOIs are included and, as the sheet is in pdf format, these are included as hyperlinks.

The three sheets for 2018 are available from

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