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RESEARCH ARTICLE

An optimisation based framework for robust underwater acoustic source localisation using Time-Difference of Arrival

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Abstract
Underwater acoustic source localisation is essential for marine monitoring, navigation of autonomous underwater vehicles and underwater surveillance. Time Difference of Arrival (TDOA) localisation is attractive because it avoids absolute time synchronisation; however, its accuracy degrades in realistic underwater channels due to multipath, measurement noise and environmental variability (e.g. sound-speed mismatch) as well as sensor geometry limitations. This paper proposes an optimisation-based TDOA localisation framework that integrates: (i) Kalman filtering (KF) for dynamic tracking; (ii) extended Kalman filtering (EKF) to handle nonlinear measurement models; and (iii) nonlinear least-squares (NLS) minimisation to refine the source position. A parametric analysis is also presented by varying key system parameters – primarily noise level and measurement uncertainty – to quantify performance trade-offs in terms of localisation error and convergence behaviour. Simulation results (static and moving source cases) show that LS provides high accuracy for low-noise/static cases, while KF/EKF are more robust for dynamic and high-noise scenarios; EKF achieves the fastest error decay due to explicit nonlinear modelling. These results demonstrate the proposed framework's effectiveness for robust underwater acoustic source localisation.

1. Introduction

Accurately locating underwater acoustic sources is essential for a variety of marine-related applications such as environmental monitoring, marine biology and underwater security operations. These applications range from tracking marine animals to aiding the navigation of autonomous underwater vehicles (AUVs), as demonstrated by Modalavalasa (2015). Among the several localisation techniques, Time Difference of Arrival (TDOA) is widely adopted due to its simplicity and effectiveness in source localisation (Xu, 2012). However, deploying TDOA in underwater environments poses several practical challenges. The underwater medium is highly unpredictable and complex, which makes precise localisation difficult (Jensen, 2011). For instance, environmental noise, signal reflection and multipath propagation severely affect the measurement accuracy of time delays (Shen and Ding, 2014). Multipath effects occur when signals reflect off the sea surface, the seabed or other underwater structures, leading to overlapping signal paths and distorted arrival times (Touze and Tourneret, 2008). Furthermore, environmental variables such as water temperature, pressure and salinity significantly influence the speed of sound underwater, thereby altering signal propagation characteristics (Jensen, 2011). Misaligned sensors or calibration drift can further degrade performance, especially in long-term deployments (Chaitanya, 2016). Hyperbolic multilateration techniques have also been evaluated for

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their positioning errors in underwater environments (Chaitanya, 2016). In addition to environmental factors, synchronisation issues between sensors complicate TDOA-based methods. Xu (2012) proposed a method to address localisation without requiring time synchronisation, highlighting the potential for improved flexibility. Several studies have incorporated Kalman filters to improve TDOA estimation, such as the work by Laveti on GPS position refinement (Laveti, 2016a) and comparative studies of least squares, SVD and QRD methods for positioning (Munoz, 2008). A significant body of research focuses on improving the robustness of localisation systems in the presence of such uncertainties. For example, Shen and Ding (2014) explored multiple source localisation using Time of Arrival (TOA) measurements, Touze and Tourneret (2008) studied localisation with a single hydrophone, highlighting the value of robust modelling in sparse sensor environments. GPS positioning in multipath environments has also been well documented (Soubielle, 2002), showing parallels to underwater acoustic challenges. Recent approaches focus on combining optimisation-based frameworks with traditional signal processing to mitigate these problems. By integrating Optimal Recursive Estimation and least-squares optimisation, these frameworks aim to reduce error in noisy conditions (Laveti, 2016b; Munoz, 2008). Kalman filtering strategies have proven particularly useful in improving GPS-based localisation performance (Kumar and Dutt, 2019), especially in estimating satellite clock and orbital errors (Bidikar, 2014).To accurately model sound propagation in water, works such as those by Jensen (2011) and Schmidt and Finn (2012) have made significant contributions through computational ocean acoustics, providing the basis for accurate environmental modelling in localisation. Advanced Kalman filter variants – such as the cubature Kalman filter (CKF) (Liu et al., 2012) and comparative analyses of EKF versus particle filters (Ko and Kim, 2012) – have further improved underwater tracking performance. Ribeiro (2004) and Elmezain et al. (2025) also provide comprehensive derivations of Kalman filtering principles and their extensions for nonlinear systems. Linearised approaches such as least-squares correction techniques have also shown promise in practical systems. Huang (2001) proposed a linear-correction least-squares approach for passive localisation, while Ampeliotis and Berberidis (2008) applied energy-based least squares in acoustic environments. The foundational mathematics of nonlinear and optimal least squares have been elaborated by Teunissen (1990) and Cohen and Migliorati (2017). With optimisation playing a growing role in localisation algorithms, modern gradient descent techniques are being explored for their efficiency. Ruder (2016) provided a foundational overview of gradient descent methods, which was extended in recent works focusing on large step sizes and non-monotonicity benefits (Wu, 2024). Complexity analysis of such optimisation problems has been explored since the 1980s, notably by Krentel (1986). Recent innovations also apply Kalman filtering in various domains, such as human tracking (Jung, 2024), aerospace target estimation (Wei, 2024), object detection (Zeng et al., 2025), bee tracking (Kongsilp et al., 2024) and robotic precision tracking (Liu and Li, 2024). Hybrid filtering strategies have emerged, such as radar-based deep learning fused with unscented Kalman filters (Patrick, 2024) and beam tracking using hybrid Kalman approaches (Zhang, 2024). Iterated unscented Kalman filters also offer better passive tracking performance under certain conditions (Zhan and Wan, 2007). In the field of visual and geometric tracking, Stenger et al. (2001) applied unscented Kalman filtering for hand modelling, while Rydell (2025) developed a complexity-reduction framework for triangulation with approximation bounds. Triangulation techniques continue to evolve, not only in engineering but also in multidisciplinary research including tourism and management (Heale and Forbes, 2013; Cornelissen, 2025; Manthiou et al., 2025). Finally, with the growing importance of underwater communication, recent works by Yue (2025) provide Monte Carlo-based modelling for non-line-of-sight underwater optical communication, further emphasising the need for reliable localisation frameworks in complex underwater environments. In light of these insights, this paper proposes a robust, adaptive TDOA-based localisation **framework** that integrates filtering, environmental modelling and optimisation to deliver consistent and accurate source localisation in challenging underwater conditions. Future directions may include the integration of machine learning-based estimators or real-time adaptive filters to further enhance the system's performance in dynamic and uncertain environments.

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A. Hybrid localisation techniques (TOA + AOA and TDOA + AOA)

In addition to pure TDOA multilateration, hybrid schemes combine time-based measurements with Angle of Arrival (AOA) to improve observability and reduce ambiguity in multipath/noise-limited conditions. Two commonly used hybrids are: (i) TOA + AOA and (ii) TDOA + AOA. These are briefly summarised here to address alternative approaches and enable a fair comparison with the proposed method.

- TOA + AOA. Each receiver i measures absolute Time-of-Arrival t_i (requires synchronisation) and bearing θ_i (AOA) of the arriving wavefront. TOA constrains the source to a sphere/circle around each receiver, while AOA constrains it to a ray/line. The joint estimate can be obtained by weighted least squares (WLS) over both range and bearing residuals.
- TDOA + AOA. Receivers measure TDOA values Δt_{i1} relative to a reference receiver 1 (no global synchronisation) together with AOA bearings θ_i. The combined cost function minimises (a) TDOA residuals of the form $(\|s-r_i\|-\|s-r_1\|)/c - \Delta t_{i1}$ and (b) AOA residuals of the form $\text{atan}^2(y-y_i, x-x_i) - \theta_i$.

Practical note: hybrid methods typically require directional hydrophones or array processing to estimate AOA, which increases hardware cost and calibration effort. In contrast, the proposed framework improves robustness using signal processing and optimisation on conventional TDOA measurements.

B. Optimal recursive estimation

The Kalman filter is a mathematical algorithm used for optimal recursive estimation. It efficiently estimates the state of a dynamic system from noisy observations over time (Liu et al., 2012). This method is particularly useful to estimate the state of a system that is constantly changing and is influenced by uncertainty. It is widely applied in various fields, including robotics, navigation systems, signal processing and economics. The Kalman filter operates recursively, which means that at each time step, it uses the previous state estimate and the new incoming measurements to update the current estimate (Ko and Kim, 2012). It combines predictions and corrections in a way that minimises the estimation error (Shen and Ding, 2014).

Step 1. State vector. The state of a system is typically represented as a vector, denoted by X_k, where k is the discrete time step, this state vector contains all the variables that describe the system at a given time.

Step 2. Prediction and correction.

2.1. Prediction. At each step, the Kalman filter predicts the next state based on a model of the system's dynamics.

State Prediction:

$$X_k = F_k X_{k-1} + B_k U_k$$

Covariance prediction:

$$P_k = F_k P_{k-1} F_k^T Q_k$$

where F_k is the state transition matrix, B_k is the control input matrix, U_k is the control input, P_k is the predicted covariance and Q_k is the Covariance.

2.2. Correction: The prediction is then corrected using new measurements. The correction considers the uncertainty in both the system's model and the measurement process.

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Kalman gain:

$$K_k = P_k H_k^T (H_k P_k H_k + R_k)^{-1}$$

State update:

$$X_k = X + (Z_k - H_k X_k)$$

Covariance update:

$$P_k = (I - K_k H_k) P_k$$

where K_k is the Kalman gain, I is the identity matrix, H_k is the measurement matrix and K_k is the measurement model.

Step 3. Process and measurement models.

3.1. Process model. This model describes how the system evolves over time. It is typically linear and can be written as

$$X_k = F_k X_{k-1} + B_k U_k + w_k$$

where w_k is the process noise, which is assumed to be Gaussian with zero mean and covariance Q_k.

3.2. Measurement model. This model relates the observed measurements to the true state of the system. It can be expressed as

$$Z_k = H_k X_k + V_k$$

where V_k is the measurement noise, assumed to be Gaussian with zero mean and covariance R_k.

Step 4. Covariance.

The Kalman filter maintains a covariance matrix P_k which represents the uncertainty in the system's state estimate. The covariance evolves over time and is used to weigh the trust in the prediction versus the measurement update. The Kalman filter is optimal in the sense that it minimises the mean squared error of the state estimate, assuming the process noise w_k and measurement noise V_k are Gaussian and have known covariances, the system is linear, and the initial state and covariance are known or can be approximated. In the case where the system or measurements are nonlinear, the Kalman filter can be extended to the extended Kalman filter (EKF) (Huang, 2001), which linearises the system at each step.

C. Least-squares optimisation

Least-squares optimisation is a powerful mathematical technique used to find the best-fit solution to an overdetermined system, where the number of equations exceeds the number of unknowns. The primary goal of least squares is to minimise the sum of the squared differences between the observed values and the predicted values from a model (Elmezain et al., 2025). For a simple linear regression, the objective is to minimise the sum of the squared differences between the observed values y_i and the model predictions z_i,

$$f(x) = \sum_{i=1}^{n} (y_i - z_i)^2$$

where n = number of data points.

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1. Linear least squares.

   Linear models are commonly used to establish a relationship between observed data and unknown parameters. A linear system can be represented in the following way:

$$Y = Ax + \varepsilon$$

where Y is a vector of observed values, A is a matrix representing the design matrix, x is the vector of unknown parameters and ε is the error vector.

The least squares problem then becomes

$$min_x||Ax - Y||_2^2$$

where $||.||_2$ represents the Euclidean norm. The solution to the linear least-squares problem can be found using the normal equation

$$X = (A^T A)^{-1} A^T Y$$

where X is the optimal parameter that minimises the squared error.

2. Nonlinear least squares.

   In many real-world scenarios, relationships between variables can be nonlinear. In such cases, the least-squares method can still be applied, but it becomes necessary to address a nonlinear optimisation problem (Ampeliotis and Berberidis, 2008). The objective remains to minimise the sum of squared residuals, although the model now includes nonlinear functions of the parameters. The general form of the nonlinear least-squares problem is

$$min_\theta \sum_{i=1}^{n} (y_i - f(x_i, \theta))^2$$

where $f(x_i, \theta)$ is the nonlinear function that predicts the value z_i given the parameters θ. With advanced methods for solving both linear and nonlinear problems, least-squares optimisation remains a core technique in applied mathematics and many engineering disciplines (Teunissen, 1990).

2. Methodology

2.1. Time difference of arrival

The fundamental principle of TDOA is to estimate the time delays between signal arrivals at multiple hydrophones (receivers) and use these differences to determine the source position. The process begins with an acoustic source emitting a signal that propagates through the medium at a known speed C. The signal is detected at multiple hydrophones placed at known positions $H_i. = (x_i, y_i, z_i)$, and the time at which each hydrophone receives the signal is recorded. The first step in TDOA localisation is to compute the Time of Arrival (TOA) at each hydrophone. The TOA is given by the equation:

$$\text{TOA}_i = \frac{d_i}{C}$$

where d_i is the Euclidean distance between the source and the i^{th} hydrophone. Since the actual position of the source is usually unknown, the absolute TOA values cannot be directly used for localisation. Thus, the exact distance between the source and each receiver is unknown, and the TDOA values must be extracted directly from the received signals using cross-correlation techniques. To determine the TDOA, it was computed by cross-correlation between the received signals. Given two signals $S_1(t)$ and $S_2(t)$ recorded at two hydrophones, the cross-correlation function is defined as

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$$R_{12}(t) = \int S_1(t)S_2(t+\tau)dt$$

where τ is the time lag between the signals.

2.2. Improving TDOA-based source localisation using Kalman filtering

The integration of Kalman filtering with TDOA-based localisation effectively improves the accuracy of source tracking in underwater environments. The approach compensates for measurement noise and environmental uncertainties by dynamically updating the estimated position and velocity, leading to robust and reliable localisation. The Kalman Filter models the source motion using a dynamic state-space representation. The state vector is defined as

$$S_t = \left[x_t, y_t, z_t, v_{xt}, v_{yt}, v_{zt}\right]^T$$

where x_t, y_t, z_t are the source coordinates and v_{xt}, v_{yt}, v_{zt} are the velocities along the respective axes.

The system follows a linear motion model:

$$S_{t+1} = FS_t + w_t$$

where F is the state transition matrix:

$$\mathrm{F} = \begin{bmatrix} I_{3x3} & dt \cdot I_{3x3} \\ 0_{3x3} & I_{3x3} \end{bmatrix}$$

and w_t represents process noise, modelled as a Gaussian distribution with covariance matrix Q.

2.2.1. Measurement model and Kalman Filter implementation

The measured TDOA values are related to the source position using

$$z_t = Hs_t + V_t$$

where H is the measurement matrix and V_t represents the measurement noise. The predicted state and covariance are estimated as

$$S_{t|t-1} = FS_{t-1|t-1}$$

$$P_{t|t-1} = FP_{t-1|t-1}F^T + Q$$

Using the newly obtained TDOA measurements, the Kalman gain is computed as

$$K_t = P_{t|t-1}H^T\left(HP_{t|t-1}H^T + R\right)^{-1}$$

The estimated state is then updated:

$$S_{t|t} = S_{t|t-1} + K_t\left(z_t - HS_{t|t-1}\right)$$

The updated covariance matrix is given by

$$P_{t|t} = (I - K_tH)P_{t|t-1}$$

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2.3. Improving TDOA based source localisation using extended Kalman filter

While the traditional Kalman filter provides optimal state estimation for linear systems, TDOA localisation is inherently nonlinear, especially due to the relationship between signal travel times and Euclidean distances. To address this, the extended Kalman filter (EKF) is introduced as a nonlinear extension of the Kalman filter. The EKF linearises the nonlinear system at each time step using a first-order Taylor expansion, allowing recursive estimation in systems where the process and/or measurement models are nonlinear.

2.3.1. Nonlinear process and measurement models

The state-space representation for EKF consists of

Nonlinear state transition:

$$x_k = f(x_{k-1}, u_k) + w_{k-1}$$

Nonlinear measurement equation:

$$z_k = h(x_k) + v_k$$

where x_k is the state vector (e.g. position and velocity), z_k is the measurement vector (TDOA values), $f(\cdot)$ is the nonlinear state transition function, $h(\cdot)$ is the nonlinear measurement function (based on range differences) and w_k is the process and measurement noise, assumed Gaussian.

2.3.2. Linearisation using Jacobians

To accommodate nonlinearities, the EKF computes the following Jacobians.

State transition Jacobian:

$$F_k = \frac{\delta f}{\delta x_{x=x_{k-1}}}$$

Measurement Jacobian:

$$H_k = \frac{\delta h}{\delta x_{x=x_{k-1}}}$$

These Jacobians are used in the prediction and update steps to propagate and correct the estimates specified in Kalman filtering.

2.4. Improving TDOA-based source localisation using least squares

Enhancements to TDOA-based underwater source localisation using least squares (LS) involve improving TDOA estimation, optimising the LS algorithm and refining the system geometry. LS estimates the source's position by minimising the difference between the true and predicted TDOA values. TDOA localisation is nonlinear because its fundamental equations involve square root distance relationships. Unlike linear least squares (LLS), which have a closed-form solution, nonlinear least squares (NLS) require iterative numerical methods to find the best-fit solution. The NLS problem is formulated as

$$min_{(x,y,z)} \sum_{i=1}^{n} \left(\frac{\sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2}}{C} - \Delta t_i \right)^2$$

where Δt_i is the actual TDOA, C is the speed of the signal and $d_i = \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2}$,

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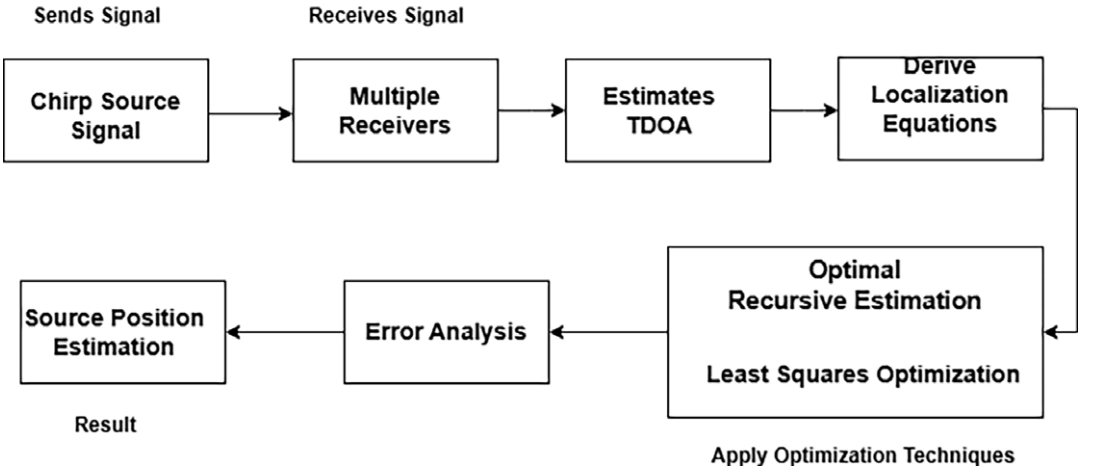


Figure 1. *Source localisation using TDOA with optimisation techniques.*

where $\frac{d_i}{C}$ is the predicted TDOA. Since the equation is nonlinear, analytical solutions (like LLS normal work. Instead, numerical optimisation techniques such as Gradient Descent, Levenberg–Marquardt Algorithm, and Nelder–Mead (fminsearch in MATLAB) are used to iteratively update the estimate until the error is minimised.

Figure 1 illustrates the methodology for underwater source localisation using TDOA in combination with optimisation techniques. This approach enhances localisation accuracy by compensating for environmental noise and measurement uncertainties. Additionally, the accuracy can be further improved through the use of Kalman filtering and least-squares methods.

3. Proposed algorithm, mathematical formulation and comparative discussion

Notation. Let the unknown source position be $s = [x, y, z]^{\mathrm{T}}$ and receiver i position be $r_{\mathrm{i}} = [x_{\mathrm{i}}, y_{\mathrm{i}}, z_{\mathrm{i}}]^{\mathrm{T}}$. The propagation speed is c. The geometric range is $d_{\mathrm{i}}(s) = \|s - r_{\mathrm{i}}\|$. For a reference receiver i, the model-based TDOA is $\tau_{\mathrm{i1}}(s) = (d_{\mathrm{i}}(s) - d_1(s))/c$. Measured TDOA values are denoted by $\hat{\tau}_{\mathrm{i1}}$.

Algorithm overview (integration of KF, EKF, and LS);

Step 1. Acquire hydrophone signals and estimate pairwise TDOA via GCC/cross-correlation.
Step 2. Initialise source estimate with nonlinear least squares (NLS) using the TDOA measurement model.
Step 3. For dynamic scenarios, run KF/EKF prediction using a constant-velocity motion model.
Step 4. Update the filter using the TDOA measurement function (EKF uses the Jacobian of $\tau_{\mathrm{i1}}(s)$).
Step 5. Optionally refine each filtered estimate with a short NLS refinement (1–3 iterations) to reduce residual bias.
Step 6. Output position estimate and error statistics.

Flow diagram (textual).

Signals → TDOA extraction → (Init: NLS) → Prediction (KF/EKF) → Update (TDOA residuals) → Optional NLS refinement → Final estimate.

Mathematical analysis. The proposed solution is formulated as a constrained nonlinear least-squares problem. Given M receivers, define the residual vector

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Table 1. *Comparison summary.*

Method	Extra sensing requirements	Strengths	Limitations
TDOA + LS/ KF/EKF (proposed)	Conventional hydrophones only	| Good robustness via filtering; EKF handles nonlinearity; LS accurate at low noise	Performance depends on geometry and TDOA quality; no directional constraint
TOA + AOA	Clock synchronisation+ AOA estimation	Strong observability; can reduce ambiguity with fewer nodes	Synchronisation burden; AOA calibration; multipath biases TOA/ AOA
TDOA + AOA	AOA estimation	Avoids global sync; improves geometry and disambiguation	Requires directional arrays; higher computation and calibration

$$r(s) = [\tau_{21}(s) - \hat{\tau}_{21}, ..., \tau_{M1}(s) - \hat{\tau}_{M1}]^T.$$

The NLS estimator solves

$$s* = \text{argmin}_s \; r(s)^T \; W \; r(s),$$

where W is a weighting matrix (often $W = R^{-1}$, R = covariance of the TDOA estimates). This formulation is maximum-likelihood under Gaussian TDOA errors and directly justifies the optimisation component.

For EKF updates, the measurement function is $h(s) = [\tau_{21}(s), \ldots, \tau_{M1}(s)]^{\mathrm{T}}$. The Jacobian $H = \partial h/\partial x$ evaluated at the predicted state $\hat{s}^-$ provides a first-order linearisation. For receiver i, the gradient is

$$\partial\tau_{i1}/\partial s = (1/c) * \left((s - r_i)/\|s - r_i\| - (s - r_1)/\|s - r_1\|\right).$$

Using this Jacobian in the EKF update reduces linearisation error compared with a linear KF measurement model and explains the faster error decay observed in the parametric noise study.

Comparative discussion with hybrid baselines. TOA + AOA and TDOA + AOA reduce geometric dilution by adding directional information, but require either synchronised clocks (TOA) or calibrated array processing (AOA). The proposed framework targets scenarios where only conventional hydrophones are available; it improves robustness via filtering + optimisation without changing the sensing hardware. Section 5 includes a qualitative comparison table; if AOA measurements are available, the same NLS/EKF formulation can be extended by augmenting the residual with AOA terms. Comparison summary is given in Table 1.

4. Simulation results

TDOA-based source localisation using Kalman filter and least-squares optimisation is evaluated with the following initial conditions: receiver positions are fixed and known; sound speed $c = 1500$ m/s; noise level = 80 dB; signal frequency = 1000 Hz; sampling rate = 10 kHz. First, a static source is considered, followed by a moving source with constant velocity. Three hydrophones are placed at known locations. The estimated source position is illustrated in Figure 2 (LS-based static localisation) and Figure 3 (KF-based static localisation).

In static source localisation using TDOA with least squares and Kalman filter optimisation, the least squares successfully estimated the source position with the lowest possible error when compared with the Kalman filter. Figures 2 and 3 visualise that the least squares is the best for static source localisation.

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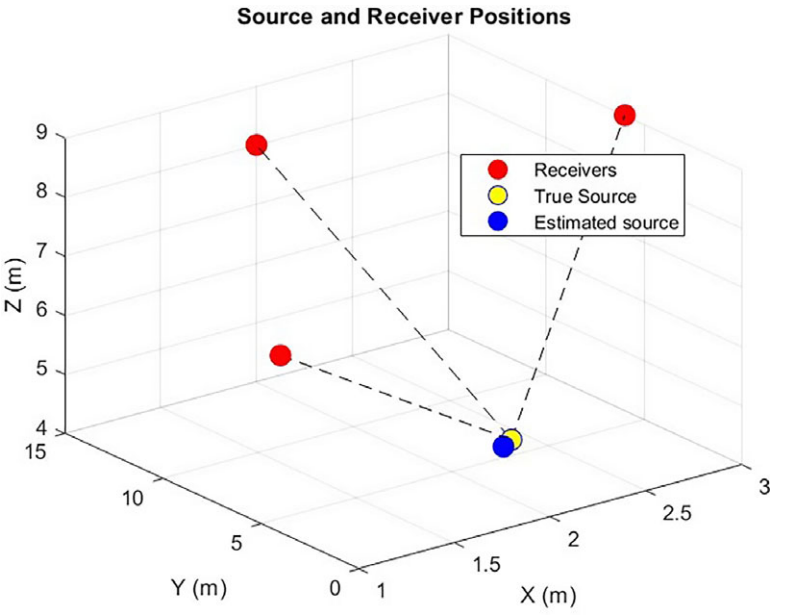


Figure 2. *Static source localisation using least squares.*

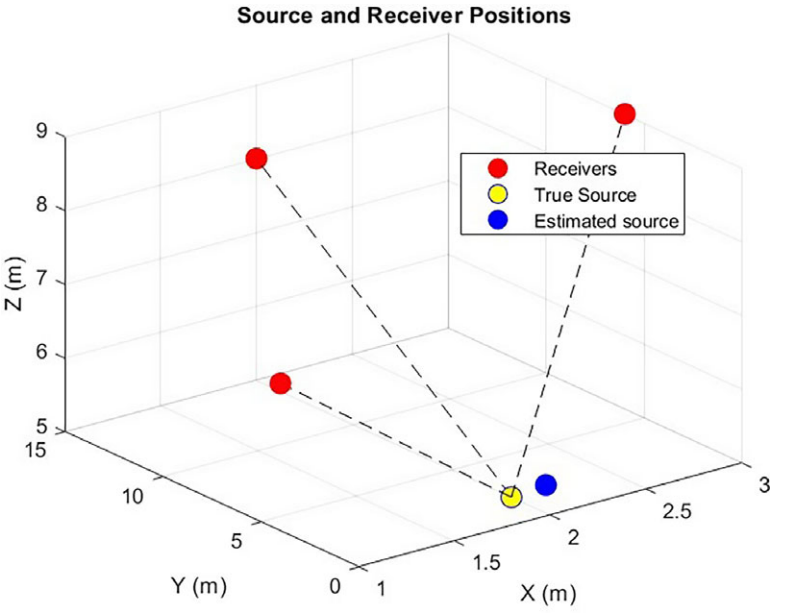


Figure 3. *Static source localisation using Kalman filter.*

In dynamic source localisation, it was considered that the source moves with a constant velocity and the source location is estimated by considering the noise that is naturally present in the underwater. The results obtained using the Kalman filter will give an idea that in dynamic source localisation by making use of the TDOA, the trajectory of the true source will vary when compared with the estimated source

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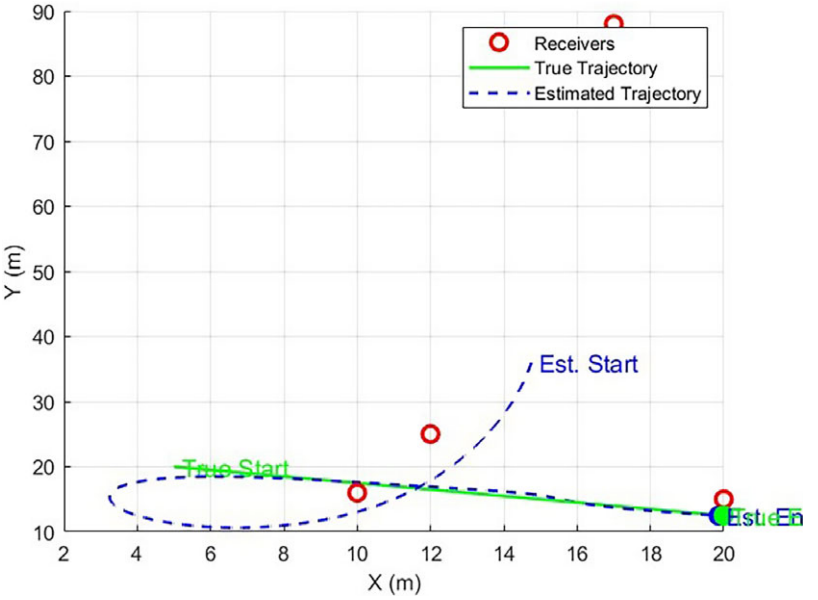


Figure 4. *Trajectory of the source using Kalman filter.*

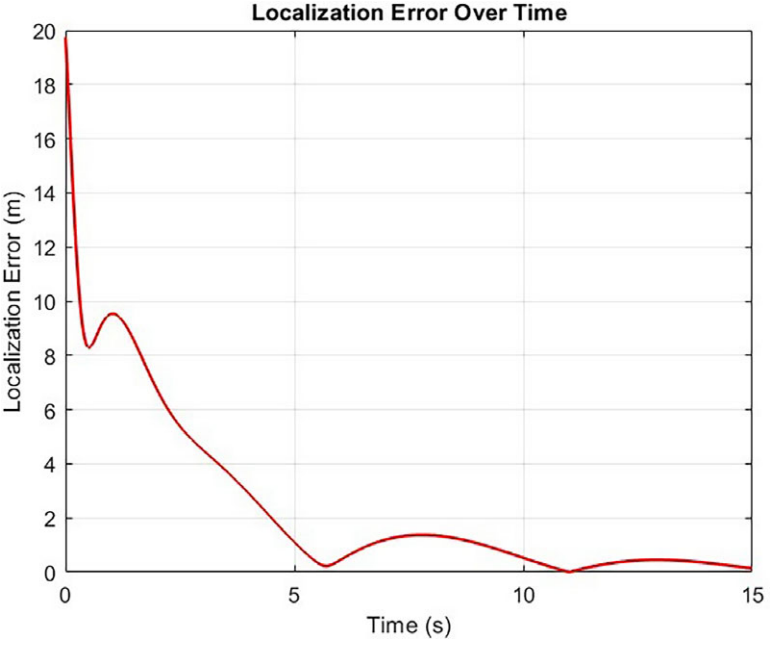


Figure 5. *Localisation error over time using Kalman filter.*

trajectory at the beginning, but will coincide with the true source trajectory as time progresses, as shown in Figure 4 (KF-based trajectory tracking).

The localisation error will decrease over time as the estimated source aligns with the true source as shown in Figure 5. Consequently, after the specified duration of 15 seconds, the error will be reduced (Figure 6). With the help of a Kalman filter, the localisation is performed over different noise levels and

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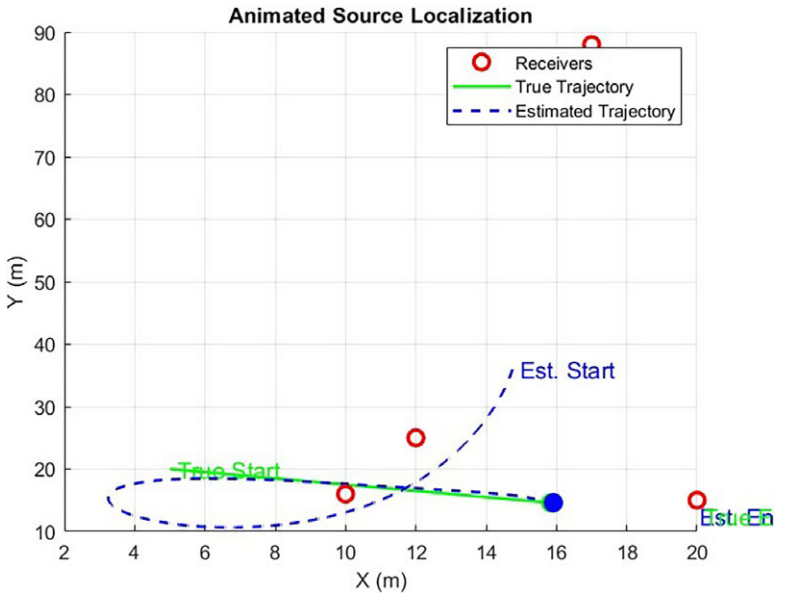


Figure 6. *Trajectory of the source using extended Kalman filter.*

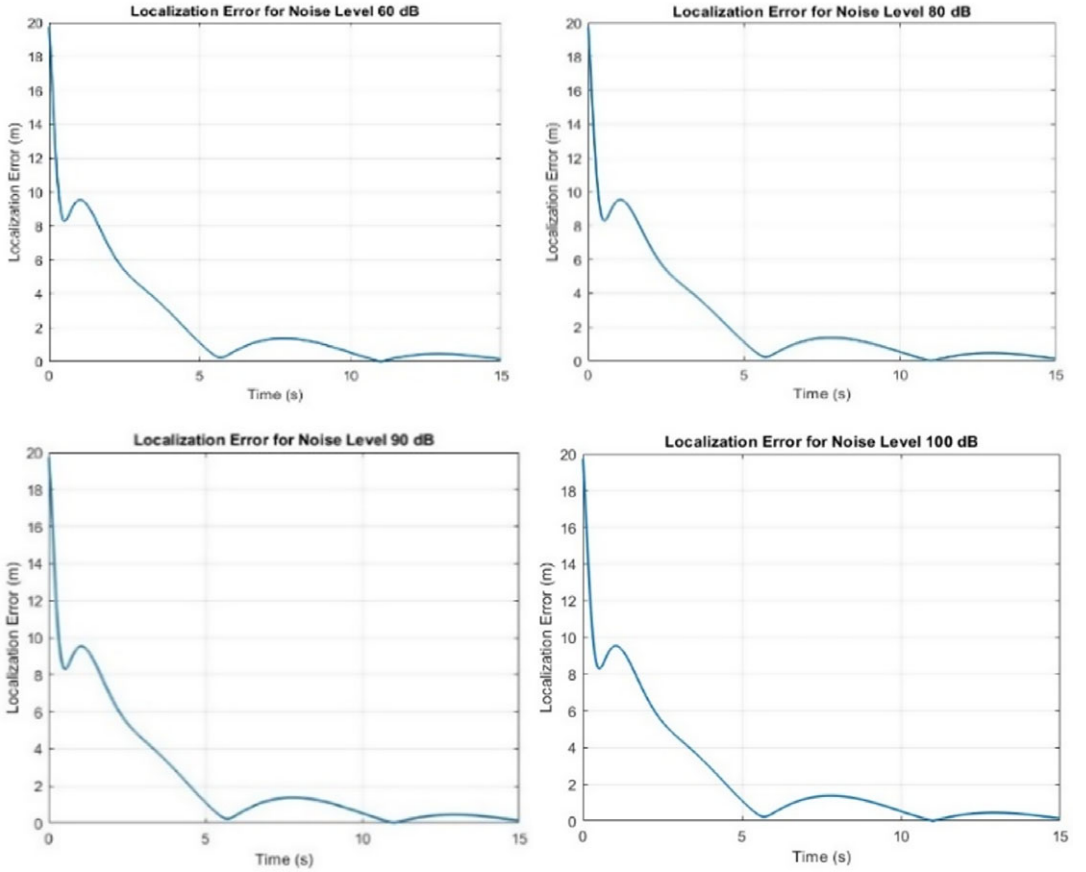


Figure 7. *Localisation error across different noise levels using Kalman filter.*

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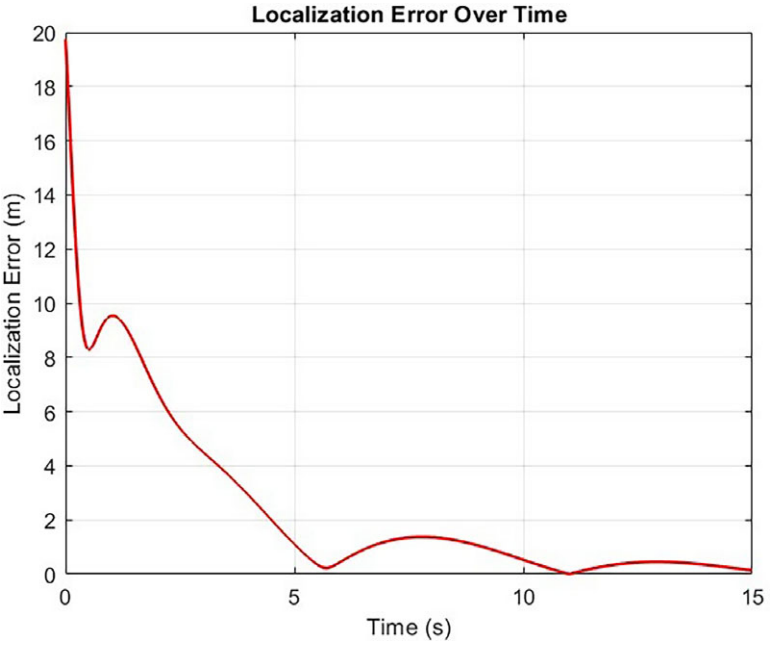


Figure 8. *Localisation error over time using extended Kalman filter.*

the results demonstrate that, regardless of the noise, the localisation error decreases over time as the estimated source aligns more closely with the actual source. Figure 7 illustrates the localisation error across different noise levels when using a Kalman filter.

It was observed that by using an Extended Kalman Filter (EKF), the accuracy can be achieved more significantly than with Kalman filter. As it considers nonlinearity in the existing localisation, as shown in Figure 6 (EKF error convergence). In the case of the extended Kalman filter, the noise over time decreases more rapidly than with the Kalman filter, as shown in Figure 8, which is also exhibits at different noise levels as shown in Figure 9, which exhibits localisation at different noise levels.

Source localisation using least-squares optimisation estimates the position of a moving source in which the estimated trajectory initially deviates, but later aligns closely with the true path, indicating the effectiveness of the localisation technique used, which is shown in Figure 10. The error increases over time, indicating that noise affects accuracy progressively. The nonlinear trend suggests growing inaccuracies in the localisation algorithm.

The localisation error will be increased concerning the noise in the underwater as in the case of least-squares optimisation. Figure 11 visualises the localisation error over different noise levels. Initially, the error will be minimum, but over time for any noise level, the error increases representing the divergence of the estimated source from the true source. The localisation error will be increased over time, as shown in Figure 12. This shows that least-squares optimisation is sensitive to the noise where noisy environments will have a drastic impact on the localisation accuracy.

Localisation using the Kalman filter, extended Kalman filter and least squares at a maximum noise level is shown in Tables 2, 3 and 4, respectively. At lower noise levels, the least squares will outperform the Kalman filter optimisation, but as the noise increases, the Kalman filter becomes more accurate than the least squares. The localisation error obtained using the Kalman filter will be reduced over time, but in the case of least squares, the error will remain the same (not much difference over time) As the estimated source position will coincide with the true source position, the error will be reduced. It is to be noted that by using the extended Kalman filtering, the localisation is further reduced as the estimated source position matches with the true source position. Further, the extended Kalman filter is the best option in the case of nonlinear environments as shown in this case.

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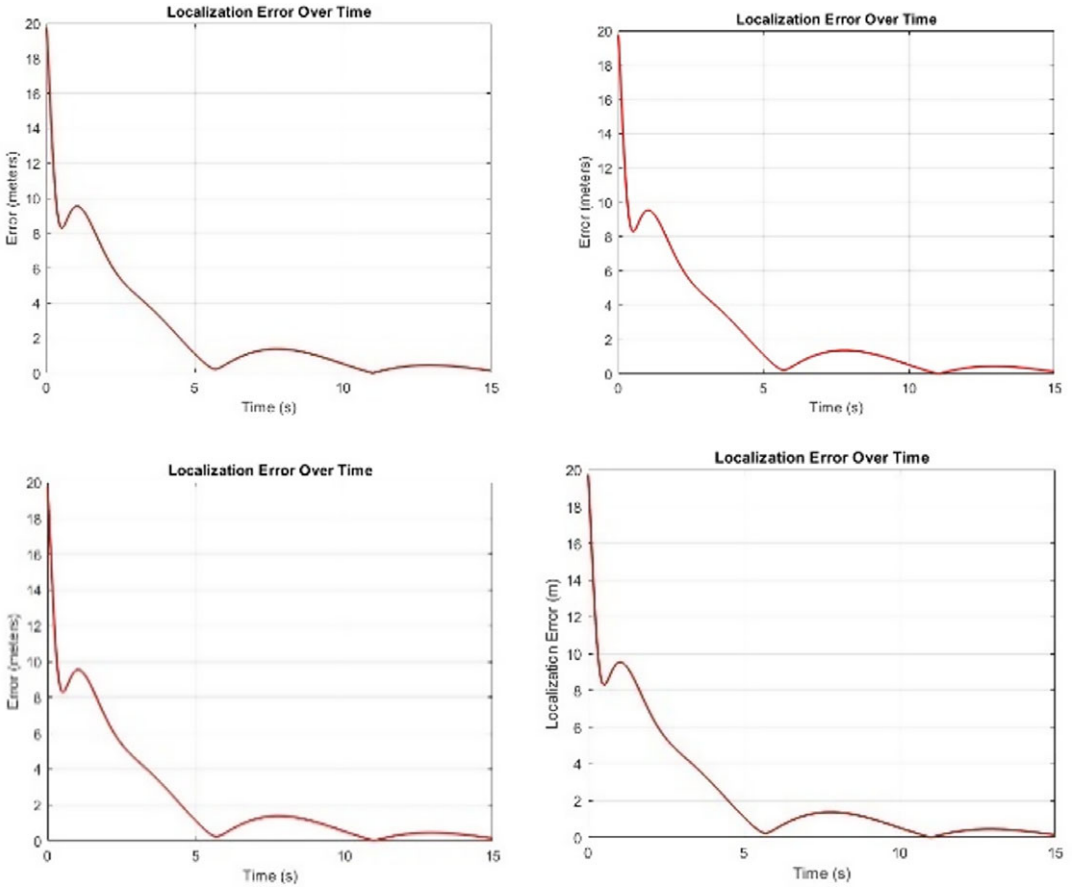


Figure 9. *Localisation error across different noise levels using extended Kalman filter .*

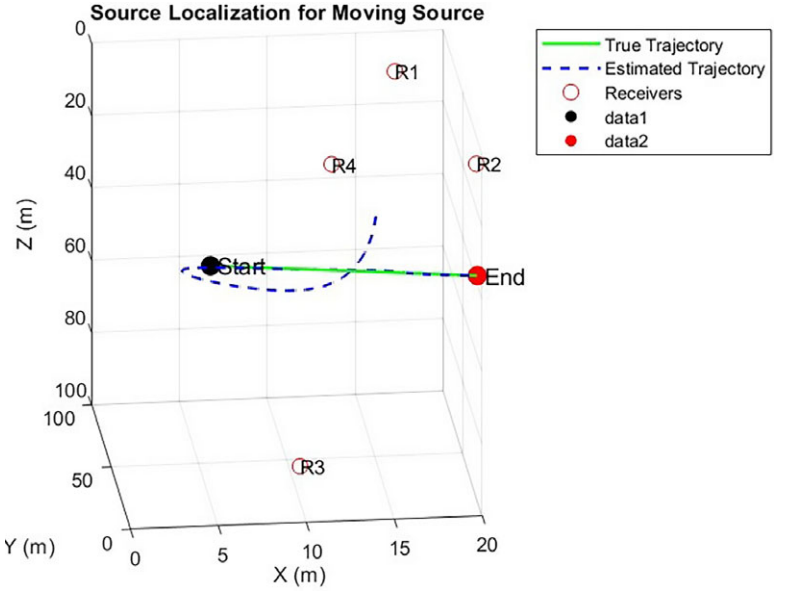


Figure 10. *Trajectory of the source using least squares.*

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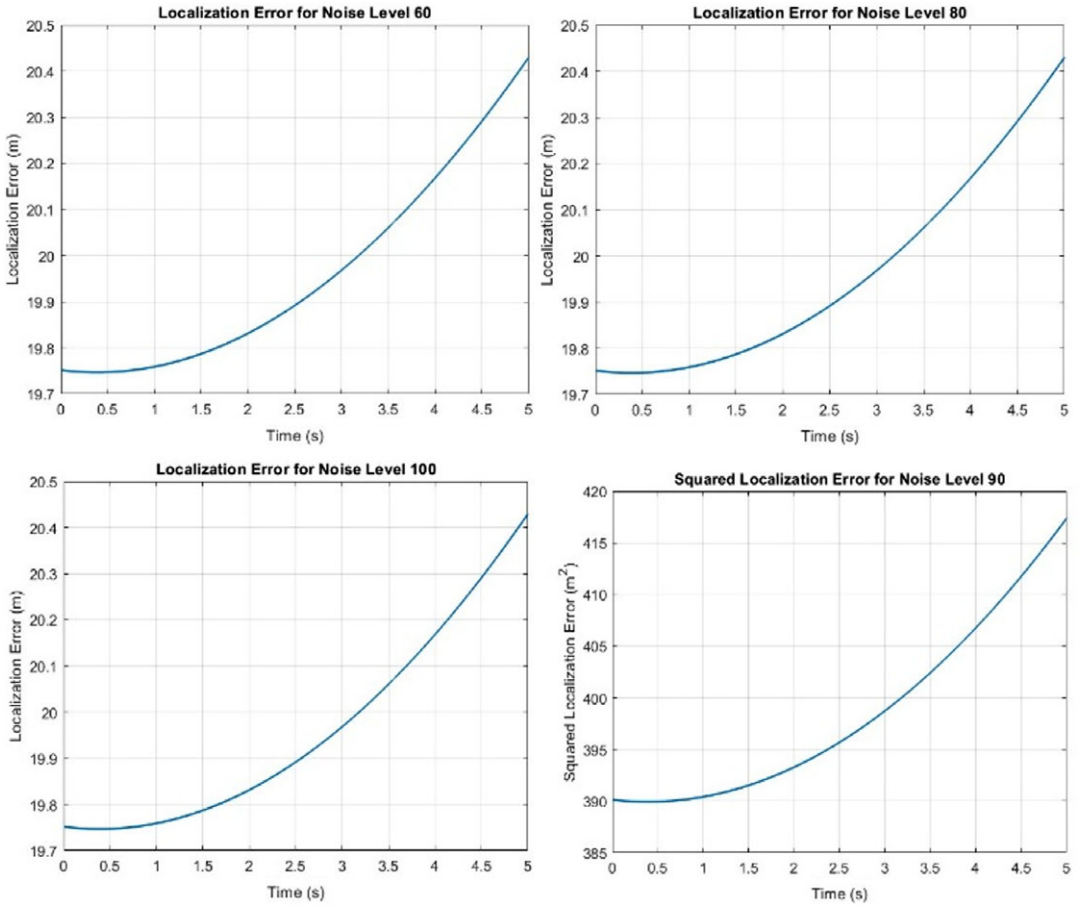


Figure 11. *Localisation error across different noise levels using least squares.*

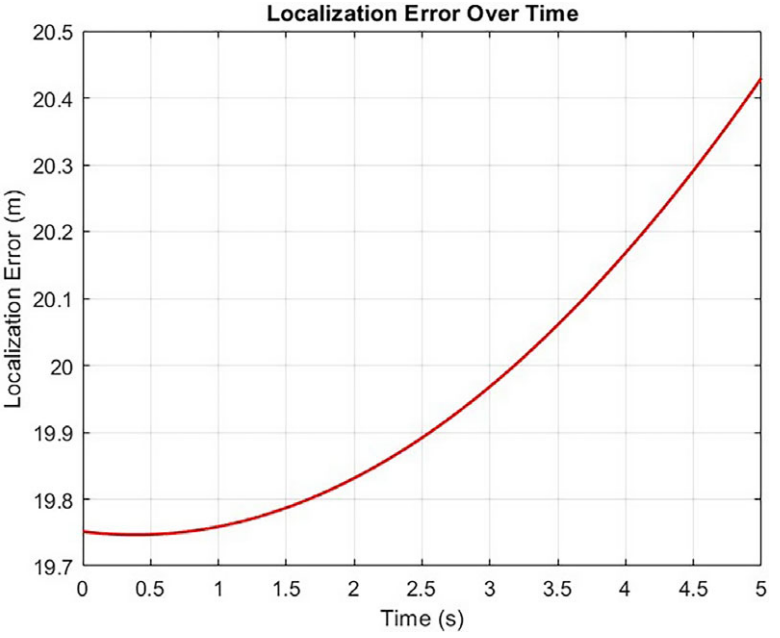


Figure 12. *Localisation error over time using least squares.*

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Table 2. *Localisation using Kalman filter at noise level 100 db.*

Time (sec)	Estimated source position	True source position	Mean square error
0	[10.78, 14.54, 40.45]	[5, 20, 35]	10.69
1	[4.46, 11.83, 35.98]	[6.73, 19.13, 35.35]	10.69
2	[3.69, 13.14, 35.18]	[7.00, 19.00, 35.40]	9.54
3	[5.42, 18.43, 35.09]	[8.73, 18.14, 35.75]	7.81
4	[6.17, 18.49, 35.31]	[9.00, 18.00, 35.80]	7.07
5	[10.87, 17.32, 36.23]	[10.75, 17.12, 36.15]	5.13
6	[11.41, 17.13, 36.31]	[11.00, 17.00, 36.20]	5.13
7	[14.12, 16.13, 36.68]	[12.75, 16.13, 36.55]	4.12
8	[14.36, 16.01, 36.72]	[13.00, 16.00, 36.60]	4.12
9	[15.29, 15.31, 36.95]	[14.75, 15.13, 36.95]	2.21
10	[15.53, 15.04, 37.04]	[15.00, 15.00, 37.00]	3.52
11	[16.46, 14.11, 37.33]	[16.74, 14.13, 37.35]	2.98
12	[16.65, 13.97, 37.38]	[17.00, 14.00, 37.40]	2.94
13	[18.37, 13.08, 37.74]	[18.75, 13.12, 37.75]	2.75
14	[19.21, 12.73, 37.88]	[19.00, 13.00, 37.80]	2.52
15	[19.86, 12.47, 38.00]	[20.00, 12.50, 38.00]	2.22

Table 3. *Localisation using extended Kalman filter at noise level at 100 db.*

Time (sec)	True source position	Estimated source position	Mean square error
0	[10.78, 14.54, 40.45]	[5, 20, 35]	10.6
2	[3.69, 13.14, 35.18]	[7.00, 19.00, 35.40]	9.42
4	[11.41, 17.13, 36.31]	[11.00, 17.00, 36.20]	5.13
6	[11.48, 16.76, 36.30]	[11.00, 17.00, 36.20]	3.99
8	[13.48, 15.76, 36.70]	[13.00, 16.00, 36.60]	3.89
10	[15.47, 14.76, 37.09]	[15.00, 15.00, 37.00]	3.53
12	[17.48, 13.76, 37.50]	[17.00, 14.00, 37.40]	2.95
14	[19.46, 12.77, 37.89]	[19.00, 13.00, 37.80]	2.42
15	[19.86, 12.47, 38.00]	[20.00, 12.50, 38.00]	2.22

Table 4. *Localisation using least squares at noise level 100 db.*

Time (sec)	Estimated source position	True source position	Mean square error
0	[5.45, 19.77, 35.09]	[5, 20, 35]	5.69
1	[6.00, 19.50, 35.20]	[6.73, 19.13, 35.35]	4.33
2	[7.46, 18.77, 35.49]	[7.00, 19.00, 35.40]	4.22
3	[8.00, 18.50, 35.60]	[8.73, 18.14, 35.75]	4.12
4	[9.47, 17.76, 35.89]	[9.00, 18.00, 35.80]	4.08
5	[10.00, 17.50, 36.00]	[10.75, 17.12, 36.15]	4.01
6	[11.48, 16.76, 36.30]	[11.00, 17.00, 36.20]	3.99
7	[12.00, 16.50, 36.40]	[12.75, 16.13, 36.55]	3.91
8	[13.48, 15.76, 36.70]	[13.00, 16.00, 36.60]	3.89
9	[14.00, 15.50, 36.80]	[14.75, 15.13, 36.95]	3.78
10	[15.47, 14.76, 37.09]	[15.00, 15.00, 37.00]	3.53
11	[16.00, 14.50, 37.20]	[16.74, 14.13, 37.35]	3.53
12	[17.48, 13.76, 37.50]	[17.00, 14.00, 37.40]	3.25
13	[18.00, 13.50, 37.60]	[18.75, 13.12, 37.75]	3.25
14	[19.46, 12.77, 37.89]	[19.00, 13.00, 37.80]	3.25
15	[20.00, 12.50, 38.00]	[20.00, 12.50, 38.00]	3.25

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5. Conclusion

This paper presented an optimisation-based framework for robust underwater acoustic source localisation using TDOA measurements. The framework combines nonlinear least-squares (NLS) positioning with recursive Bayesian tracking (KF) and its nonlinear extension (EKF). The key benefit is that improved robustness is achieved without requiring additional sensing modalities (e.g. AOA arrays) or global time synchronisation.

A parametric analysis was carried out by varying the noise level of the received signals and examining localisation error versus time and versus noise. The results indicate a clear trade-off: (i) LS yields highly accurate estimates for static/low-noise conditions, but becomes sensitive as noise increases; (ii) KF improves robustness for moving sources by exploiting temporal consistency; and (iii) EKF further reduces error and converges faster because it explicitly models the nonlinear TDOA measurement function through Jacobian linearisation.

Compared with hybrid baselines (TOA + AOA, TDOA + AOA), the proposed approach is most suitable when only conventional hydrophones are available, and when system complexity, calibration and cost must be minimised. When AOA measurements are available, the proposed NLS/EKF formulation can be extended by augmenting the measurement vector with bearing residuals to obtain further accuracy gains. Future work will include implementing and benchmarking these hybrid extensions experimentally and studying robustness under sound-speed uncertainty and multipath-dominant channels.

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