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Theory for plasma and dust voids in a complex plasma

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Abstract. Theories for the formation of plasma and dust voids (large-amplitude ion and dust holes) in a complex plasma with static and mobile negatively charged dust are presented. It is shown that a plasma (dust) void in a complex plasma with negatively charged static (mobile) dust can be formed due to the presence of negative (positive) plasma potential, which is built up owing to a non-isothermal ion (dust) distribution arising from ion (dust) trapping. The plasma voids shrink when the dust grains are added into the plasma, and they tend to disappear if the dust number density is sufficiently high. On the other hand, the dust voids shrink with an increase in the dust charge density, but enlarges with an increase in the dust temperature.

1. Introduction

The physics of charged dust particles, which are ubiquitous in most space and laboratory plasmas (Goertz 1989; Mendis and Rosenberg 1994; Verheest 2000; Shukla and Mamun 2002), and which make a plasma system very complex, have become an outstanding and challenging research topic not only because it has introduced a great variety of new phenomena associated with waves and instabilities (Rao et al. 1990; Shukla and Silin 1992; Melandsø 1996; Verheest 2000; Shukla 2001; Shukla and Mamun 2002) but also because it provides a number of new experimental events/facts, namely dust crystals (Thomas et al. 1994; Chu and I 1994; Hayashi and Tachibana 1994), Mach cones (Samsonov et al. 1999), voids (Dorier et al. 1995; Samsonov and Goree 1999; Morfill et al. 1999), etc.

Plasma or dust voids (plasma or dust free regions) inside the plasma or dust cloud have been observed in silane (Dorier et al. 1995) and other dust-laden plasmas (Morfill et al. 1999; Samsonov and Goree 1999). Morfill et al. (1999) observed centimeter-sized stable dust voids, which formed without any initial turbulent phase in their microgravity experiment (where dust particles are already sufficiently large). Samsonov and Goree (1999) observed that as dust particles in a sputtering plasma grew in diameter, the void was developed by a sudden onset of a filamentary mode in which both the ionization rate and the dust number density were modulated.

A number of theoretical models have been proposed to explain the physics of the void formation. Morfill et al. (1999) predicted that the thermophoretic force

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(a neutral temperature gradient force), which is more dominant than all other forces on a dust particle in the absence of gravity, is responsible for the formation of voids. Goree et al. (1999) explained the dust void formation by the balance of the electrostatic and ion drag forces on a dust particle. The balance of the electrostatic and ion drag forces involves electron depletion (reduction of the electron number density in a dust cloud due to the absorption on dust particles) and electron-impact ionization. The electron-impact ionization rate is, therefore, decreased in a dust cloud. However, in a void the comparatively higher ionization rate leads to an electric field that is directed outward from the void center. This gives rise to an outward ion flow that exerts an outward ion drag force on the dust particles. Therefore, in equilibrium there is a balance of forces on a dust particle: an inward electrostatic force and an outward ion drag force. Using the models based on ion drag force (Shukla and Mamun 2002; Goree et al. 1999; Nitter 1996), thermophoretic force (Shukla and Mamun 2002; Morfill et al. 1999; Nitter 1996), and some other forces (Shukla and Mamun 2002; Nitter 1996) linear (Ivlev et al. 1999; Ivlev and Morfill 2000; Wang et al. 2001; Avinash 2001) analyses have been made to predict the formation of dust voids in a dusty plasma.

However, computer simulation studies (Akdim and Goedheer 2001) show that the ion–neutral collisions increase the gas temperature by a maximum of 1 K, and that the thermophoretic force or the ion drag force cannot explain the appearance of the void. They also imply that the ion drag force model of a dust void only works if one accounts for the enhancement of the ion drag force (Khrapak et al. 2003). The observed voids are of large amplitudes. The force balance models are, therefore, inappropriate for their formation. Accordingly, nonlinear models (Mamun et al. 2001; Avinash et al. 2002; Jovanović and Shukla 2003; Mamun and Shukla 2004) have been developed to explain the formation of a plasma void in a dusty plasma. Specifically, Mamun et al. (2001) and Mamun and Shukla (2004) suggested that a one-dimensional plasma/dust void is formed due to the presence of ion/dust holes associated with a non-isothermal ion/dust distribution. Jovanović and Shukla (2003) have shown that cylindrical symmetric dust voids are caused by a large potential drop associated with the ion holes and double layers in a dusty plasma. Avinash et al. (2002) presented a nonlinear theory for the void formation in a colloidal plasma. Their model is based on the nonlinear evolution of a zero-frequency linear instability which grows in the nonlinear regime to form a void. Our present paper is aimed at presenting theoretical models for the formation of plasma voids (ion holes) in a plasma with negatively charged static dust, and the formation of dust voids (dust holes) in a dusty plasma containing non-isothermally distributed negatively charged dust.

This paper is organized as follows. We first consider a dusty plasma with negatively charged static dust, and study the formation of plasma voids in a dusty plasma with non-isothermally distributed ions and Boltzmann electrons (Sec. 2). We then consider the dust-phase-space vortex model of Schamel et al. (2001), and show the existence of a dust void in a dusty plasma with Boltzmann electron and ion density distributions (Sec. 3). We finally provide a brief discussion in Sec. 4.

2. Static dust: plasma voids (ion holes)

We consider a one-dimensional, collisionless, unmagnetized dusty plasma consisting of electrons following a Boltzmann distribution, ions obeying a trapped/vortex-like

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distribution, and negatively charged dust particles providing the stationary background. The electrostatic potential associated with the charge density perturbation is described by Poisson's equation

$$\frac{\partial^2\phi}{\partial\xi^2} = \exp(\sigma\phi) - \alpha n_{\rm i} + \alpha - 1, \tag{2.1}$$

where $n_{\rm i}$ is the ion number density normalized by its equilibrium value $n_{\rm i0}$, ϕ is the electrostatic plasma potential normalized by $k_{\rm B}T_{\rm i}/e$ ($T_{\rm i}$ is the ion temperature, $k_{\rm B}$ is the Boltzmann constant, and e is the magnitude of the electron charge), ξ is the space variable normalized by $\lambda_{\rm Dm} = (k_{\rm B}T_{\rm i}/4\pi n_{\rm e0}e^2)^{1/2}$, $\sigma = T_{\rm i}/T_{\rm e}$ and $\alpha = n_{\rm i0}/n_{\rm e0}$. We note that $\alpha = 1$ represents no dust particle present in the plasma, but $\alpha > 1$ represents the presence of negatively charged dust particles in the background. Thus, (2.1) is valid in a plasma with a uniform distribution of dust particles.

To model the ion distribution in the presence of trapped particles, we employ a vortex-like ion distribution of Schamel (1972) which solves the ion Vlasov equation (Schamel 1972; Schamel 2000). Thus, we have $f_{\rm i} = f_{\rm if} + f_{\rm it}$, where

$$f_{\rm if} = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left[\pm(v^2+2\phi)^{1/2} + u_0\right]^2\right) \quad \text{for } |v| > \sqrt{-2\phi}, \tag{2.2}$$

$$f_{\rm it} = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left[\beta(v^2+2\phi) + u_0^2\right]\right) \quad \text{for } |v| \leqslant \sqrt{-2\phi}. \tag{2.3}$$

Here v (u_0) is the ion (the frame) speed normalized by the ion thermal speed $v_{\rm Ti} = (k_{\rm B}T_{\rm i}/m_{\rm i})^{1/2}$, and $|\beta|$ ($= T_{\rm h}/T_{\rm it}$), which is the ratio of the free ion temperature $T_{\rm i}$ to the trapped ion temperature $T_{\rm it}$, is a parameter determining the number of trapped ions. We note that the ion distribution function, as prescribed above, is continuous in velocity space and satisfies the regularity requirements for an admissible Bernstein–Greene–Kruskal (BGK) solution (Bernstein et al. 1957).

It is obvious from (2.3) and (2.4) that $\beta = 1$ ($\beta = 0$) represents a Maxwellian (flat-topped) distribution, whereas $\beta < 0$ represents a vortex-like excavated trapped ion distribution, corresponding to an under-population of trapped ions. The situation $\beta < 0$ is of our present interest. Thus, integrating the ion distributions over the velocity space, the ion number density $n_{\rm i}$ for $\beta < 0$ can be expressed as (Schamel 1972; Schamel 2000)

$$n_{\rm i} = \exp\left(-\frac{u_0^2}{2}\right)\left[I(-\phi) + K\left(\frac{u_0^2}{2}, -\phi\right) + \frac{2}{\sqrt{\pi|\beta|}}W_{\rm D}(\sqrt{\beta\phi})\right], \tag{2.4}$$

where

$$I(x) = [1 - \mathrm{erf}(\sqrt{x})]\exp(x), \tag{2.5}$$

$$W_{\rm D}(x) = \exp(-x^2)\int_0^x \exp(y^2)\,dy, \tag{2.6}$$

$$K(x,y) = \frac{2}{\sqrt{\pi}}\int_0^{\pi/2} d\theta\sqrt{x}\cos\theta\exp(-y\tan^2\theta + x\cos^2\theta)\mathrm{erf}(\sqrt{x}\cos\theta). \tag{2.7}$$

We have now numerically solved (2.1) with (2.4) to examine the plasma potential profiles and associated electron and ion density depletions. The results are displayed in Fig. 1, which shows that the self-consistent solutions of Poisson's

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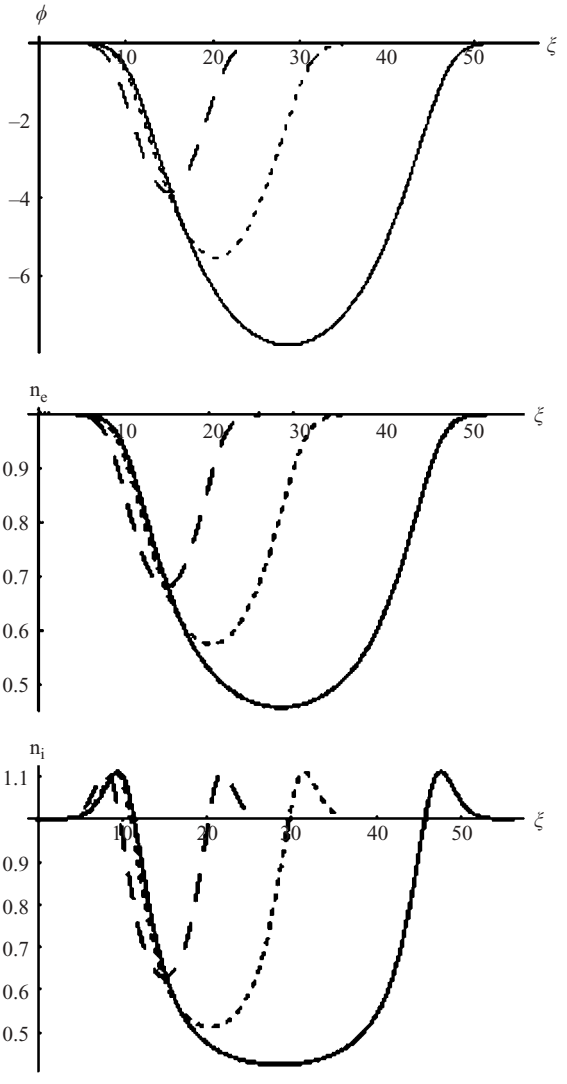


Figure 1. The upper, middle and lower plots show the variation of ϕ, n_e, and n_i with ξ, respectively, for $\beta = -0.9$, $u_0 = 0$, $\sigma = 0.1$, $\alpha = 1$ (full curve), $\alpha = 1.1$ (dotted curve), and $\alpha = 1.5$ (dashed curve).

equation give rise to the plasma density holes (plasma voids). Figure 1 also shows that as we increase the dust particle number density, the depth as well as the width of the voids/holes (localized potential structure, electron and ion density depletions) decrease. We note that ion density depletions can also be obtained by considering nonthermal electron distributions of Cairns et al. (1995, 1996).

3. Mobile dust: dust voids/holes

We consider a one-dimensional, collisionless, unmagnetized dusty plasma composed of electrons and ions following Boltzmann distributions, negatively charged dust particles obeying a trapped/vortex-like distribution. The electrostatic potential

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distribution associated with the charge density perturbation is obtained by Poisson's equation

$$\frac{d^2\phi}{d\xi^2} = \alpha_{\rm d} Z_{\rm d} n_{\rm d} + (1 - \alpha_{\rm d} Z_{\rm d}) \exp(\sigma_{\rm e}\phi) - \exp(-\phi), \tag{3.1}$$

where $n_{\rm e}$, $n_{\rm i}$, and $n_{\rm d}$ are the electron, ion and dust number densities, normalized by their equilibrium values $n_{\rm e0}$, $n_{\rm i0}$, and $n_{\rm d0}$, respectively, ϕ is the electrostatic plasma potential normalized by $k_{\rm B}T_{\rm i}/e$ ($T_{\rm i}$ is the ion temperature, $k_{\rm B}$ is the Boltzmann constant, and e is the magnitude of the electron charge), $Z_{\rm d}$ is the number of electrons residing onto the dust grain surface, ξ is the space variable normalized by $\lambda_{\rm Di} = (k_{\rm B}T_{\rm i}/4\pi n_{\rm i0}e^2)^{1/2}$, $\alpha_{\rm d} = n_{\rm d0}/n_{\rm i0}$ and $\sigma_{\rm e} = T_{\rm i}/T_{\rm e}$.

To model the dust distribution in the presence of trapped particles, we employ the Schamel distribution (Schamel 1972; Schamel 2000) which solves the dust Vlasov equation (Schamel et al. 2001). Thus, we have $f_{\rm d} = f_{\rm df} + f_{\rm dt}$, where

$$f_{\rm df} = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}(v^2 - 2Z_{\rm d}\sigma_{\rm d}\phi)\right], \tag{3.2}$$

for $|v| > \sqrt{-2Z_{\rm d}\sigma_{\rm d}\phi}$, and

$$f_{\rm dt} = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\beta_{\rm d}(v^2 - 2Z_{\rm d}\sigma_{\rm d}\phi)\right], \tag{3.3}$$

for $|v| \leqslant \sqrt{-2Z_{\rm d}\sigma_{\rm d}\phi}$. Here v is the dust particle speed normalized by the dust thermal speed $v_{\rm Td} = (k_{\rm B}T_{\rm d}/m_{\rm d})^{1/2}$, $\sigma_{\rm d} = T_{\rm i}/T_{\rm d}$, and $|\beta_{\rm d}|$ ($= T_{\rm d}/T_{\rm dt}$), which is the ratio of the free dust temperature $T_{\rm d}$ to the trapped dust temperature $T_{\rm dt}$, is a parameter determining the number of trapped dust particles.

It is obvious from (3.2) and (3.3) that $\beta_{\rm d} = 1$ ($\beta_{\rm d} = 0$) represents a Maxwellian (flat-topped) distribution, whereas $\beta_{\rm d} < 0$ represents a vortex-like excavated trapped dust distribution corresponding to an under-population of trapped dust particles. The situation $\beta_{\rm d} < 0$ is of our present interest. Thus, integrating the dust distributions over the velocity space, the dust number density $n_{\rm d}$ for $\beta_{\rm d} < 0$ can be expressed as

$$n_{\rm d} = I(Z_{\rm d}\sigma_{\rm d}\phi) + \frac{2}{\sqrt{\pi|\beta_{\rm d}|}} W_{\rm D}(\sqrt{-\beta_{\rm d}Z_{\rm d}\sigma_{\rm d}\phi}). \tag{3.4}$$

We are now interested in examining the possibility for the formation of a standing dust void in a dusty plasma. We have found self-consistent solutions of (3.1) with $n_{\rm d}$ given by (3.4), and examine the roles of dusty plasma parameters (particularly, $Z_{\rm d}$ and $\alpha_{\rm d}$) on these numerical solutions. The results are displayed in Figs. 2 and 3. Figures 2 and 3 show that the self-consistent solutions of Poisson's equation give rise to the dust density holes (dust voids). Figure 2 implies that the dust voids shrink when the dust grain charge is increased, and Fig. 3 shows that the dust voids shrink when the dust particle number density is increased.

4. Discussion

We have first considered a one-dimensional, collisionless, unmagnetized plasma composed of electrons, ions, and negatively charged dust particles. We have obtained self-consistent solutions of Poisson's equation, and investigated the role of negatively charged dust particles on the plasma potential structures and associated

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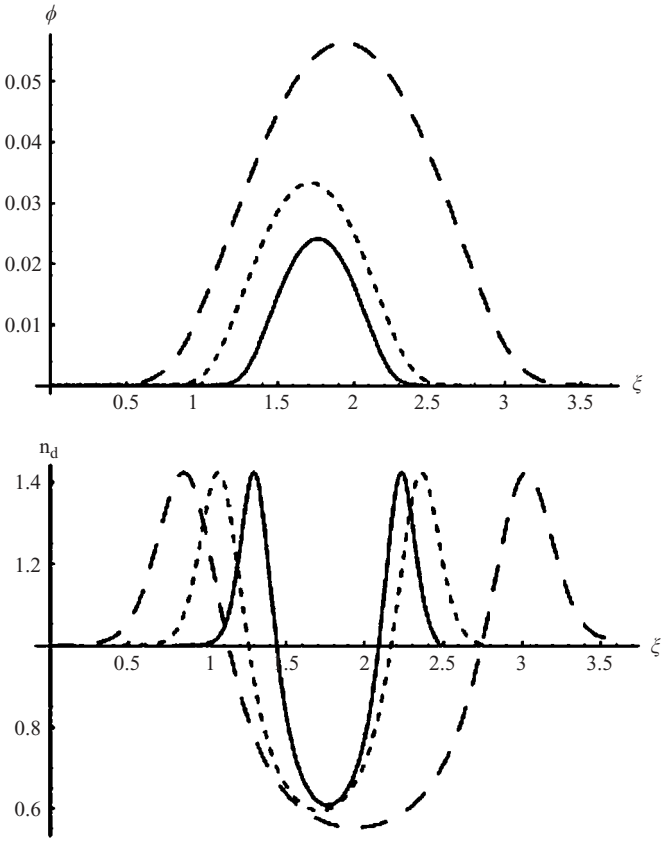


Figure 2. The direct solutions of Poisson's equation (3.1) for the potential profiles (upper plot) and the corresponding dust number density profiles (lower plot) for $\sigma_e = 0.1$, $\beta_d = -0.3$, $\sigma_d = 10$, $\alpha_d = 0.01$, $Z_d = 80$ (solid curves), $Z_d = 60$ (dotted curves), and $Z_d = 40$ (dashed curves).

electron and ion density depletions. We have shown that the plasma voids are self-consistent solutions of Poisson's equation in which the electron density response is Boltzmannean and the ion distribution is non-Maxwellian due to the ion trapping in large-amplitude plasma potentials. We have also found that as the dust particle number density is increased, the depth as well as the width of the voids (holes) decrease, and after a certain value of the dust particle number density, the voids (holes) disappear.

We have considered a one-dimensional, collisionless, unmagnetized plasma composed of Maxwellian electrons and ions, and non-Maxwellian negatively charged dust particles. The non-Maxwellian distribution of dust particles has been obtained due to their trapping in large-amplitude plasma potentials. We have obtained self-consistent solutions of Poisson's equation, and investigated the role of negatively charged dust particles on the plasma potential structures and associated dust density depletions (dust voids). It has been shown that the dust void shrinks (enlarges) when the dust charge density (dust temperature) is increased. The present part of our analysis of a dust void uses a Boltzmann ion distribution and a non-isothermal

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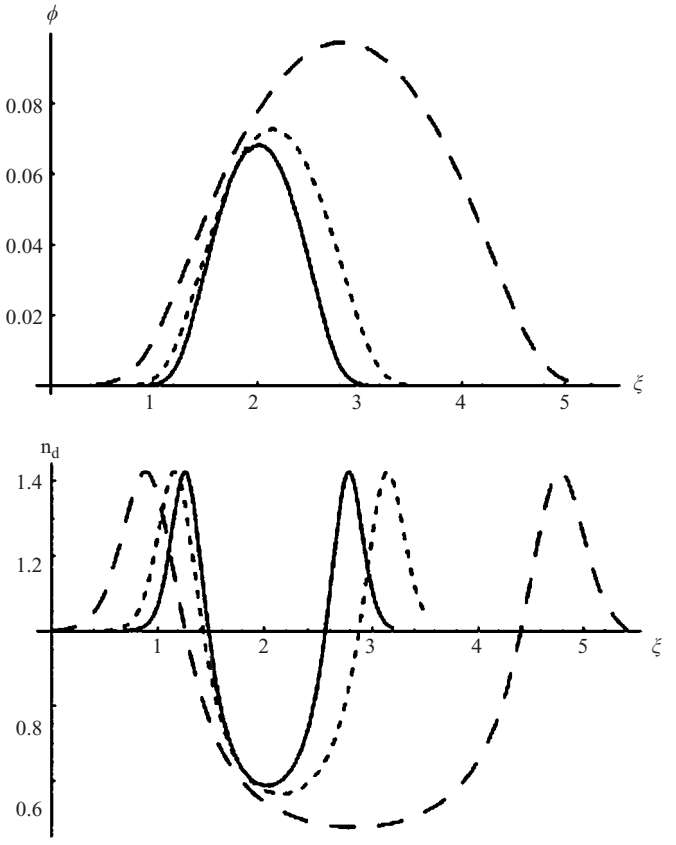


Figure 3. The direct solutions of Poisson's equation (3.1) for the potential profiles (upper plot) and the corresponding dust number density profiles (lower plot) for $\sigma_e = 0.1$, $\beta_d = -0.3$, $\sigma_d = 10$, $Z_d = 30$, $\alpha_d = 0.03$ (solid curves), $\alpha_d = 0.02$ (dotted curves), and $\alpha_d = 0.01$ (dashed curves).

dust particle distribution, in contrast to our previous work that employed non-isothermal ions and an ensemble of cold stationary dust grains.

To conclude, our results should be applicable to understand experimental observations of a dust void in plasmas if there are indications of dust grain heating and associated modification of ion or dust particle distribution function due to the quasi-linear effect. We also propose to conduct new laboratory experiments in which dust grains are of low charge state and are sufficiently heated due to a variety of processes (Sorasio et al. 2002), such as stochastic dust charge fluctuations, the variations of the plasma parameters, radiative heating, etc.

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It is our great pleasure to dedicate this paper to Professor Alan Cairns on the occasion of his 60th birthday.

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