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Turbulent spectra in the solar wind plasma

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Abstract. Observations of interstellar scintillations at radio wavelengths reveal a Kolmogorov-like scaling of the electron density spectrum with a spectral slope of −5/3 over six decades in wavenumber space. A similar turbulent density spectrum in the solar wind plasma has been reported. The energy transfer process in the magnetized solar wind plasma over such extended length scales remains an unresolved paradox of modern turbulence theories, raising the especially intriguing question of how a compressible magnetized solar wind exhibits a turbulent spectrum that is a characteristic of an incompressible hydrodynamic fluid. To address these questions, we have undertaken three-dimensional time-dependent numerical simulations of a compressible magnetohydrodynamic fluid describing super-Alfvénic, supersonic and strongly magnetized plasma. It is shown that the observed Kolmogorov-like (−5/3) spectrum can develop in the solar wind plasma by supersonic plasma motions that dissipate into highly subsonic motion that passively convect density fluctuations.

1. Introduction

It is a remarkable that observations of electron density fluctuations in both the interstellar medium (ISM) and solar wind (SW) plasma exhibit an omnidirectional Kolmogorov-like power spectrum $k^{-5/3}$ (Armstrong et al. 1981, 1990). There is a zoo of observations that suggest a $k^{-5/3}$ Kolmogorov-like power spectrum of turbulence in the SW plasma. For instance, Goldstein et al. (1995) reported 0.12-s average Mariner 10 magnetometer data that exhibit magnetic field fluctuation spectrum very close to −5/3 with a clearly defined dissipation range at higher frequencies. The power density spectra of magnetic field fluctuations observed by Helios 1, 2 and Ulysses between 0.3 and 1 AU indicate the dependence of the power spectra relative to the heliocentric distance (Bruno and Carbone 2005). The higher frequency component of the spectra is consistent with the Kolmogorov-like 5/3 spectrum. The spectral properties of the SW observations are described extensively in a comprehensive review paper by Bruno and Carbone (2005). Marsch & Tu (1990) and Tu et al. (1991) reported a Kolmogorov-like density, proton temperature and magnetic field fluctuations spectra in the SW plasma. Bellamy et al. (2005) reported an extensive spectral analyses of the plasma density based on the Voyager 2 spacecraft to investigate the spectral characteristics and fluctuation level of density turbulence from 1 to 60 AU, corresponding to the period 1977–1999. They found that the density spectra associated with high-frequency (above about

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10^{-4} Hz) SW turbulence in the outer heliosphere have spectral index that is close to a Kolmogorov-like $k^{-5/3}$ power law. While the low-frequency component of the spectrum follows a k^{-2} law, the high-frequency part of the spectrum shows a slight decrease in the spectral index. A nearly incompressible theory has been proposed to explain the observed SW density spectrum by Zank and Matthaeus (1993) and Matthaeus et al. (1991). The latter confirmed that density spectra near 3 AU have a spectral index very similar to the magnetic and velocity field spectral indices. In a most detailed analysis of density spectra interior to 1 AU, Marsch and Tu (1990) showed that spectra of density fluctuations vary as Kolmogorov-like $k^{-5/3}$ law. Leamon et al. (1998) have measured SW magnetic field spectrum near 1 AU, which shows a Kolmogorov-like $k^{-5/3}$ power law.

Perhaps, the most striking point about the above observations is that the SW is a fully compressible and a magnetized medium, yet it exhibits a Kolmogorov-like power spectrum $k^{-5/3}$ over an extended wavenumber space. Such a spectrum is characteristic of incompressible isotropic (wavenumbers are identical $|k| = |k_x| = |k_y| = |k_z|$) and homogeneous (does not vary with respect to the background flow) hydrodynamic turbulence. The observation yields two paradoxes: (1) why a compressible SW fluid behaves as though it were incompressible (Marsch and Tu 1990; Tu et al. 1991; Goldstein et al. 1995; Leamon et al. 1998; Bruno and Carbone 2005; Bellamy et al. 2005) and (2) why the density fluctuations, an apparently quintessential compressive characteristic of magnetized SW turbulence, yield a Kolmogorov power law spectrum characteristic of incompressible hydrodynamic turbulence. These questions have to be answered if we are to address the outstanding question regarding the origin of the SW density power law spectrum. In this paper we address these issues within the context of fully compressible magnetohydrodynamics (MHD) turbulence to understand how and why a supersonic, super Alfvénic and low-plasma-β (where β is the ratio of plasma pressure and magnetic pressure) SW fluid should exhibit a Kolmogorov-like wavenumber spectrum in density. Our results are valid for all length scales that constitute isotropic and homogeneous SW fluid.

In a compressible fluid, the characteristic motions often exceed or are comparable to the local sound speed. Accordingly, a local fluctuating Mach number defined as $M_{s_0} = U_0/C_{s_0}, C_{s_0}^2 = \gamma p_0/\rho_0$ (where U_0 and C_{s_0} are the characteristic and sound speeds of a turbulent fluid and p_0, ρ_0 and γ are the pressure, density and adiabatic index of magnetoplasma respectively) changes. It primarily expresses the degree of compressibility of the magnetofluid. A compressible magnetofluid contains fast and slow magnetoacoustic modes. By contrast, an incompressible magnetofluid corresponds to a fluid in which such fast-scale modes are absent and that contains purely vortical modes. The incompressible and compressible magnetoplasma fluids therefore differ considerably from each other. Sound waves do not exist in an incompressible fluid; hence the turbulent Mach number $M_{s_0} \to 0$. Turbulent Mach number in a compressible fluid can depend on local parameters; it is therefore useful to define a fluctuating Mach number that varies in space and time $M_s(\mathbf{r}, t)$ ($\mathbf{r} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$ is a three-dimensional space vector). The fluctuating Mach number $M_s(\mathbf{r}, t)$ is different from the fixed Mach number M_{s_0}. The latter can be viewed as a consequence of the normalizing quantities that represent large-scale flows. We use the fluctuating (small-scale) Mach number $M_s(\mathbf{r}, t)$ as a prime diagnostic to investigate the dynamics of *multiscale coupling* in a super-Alfvénic, supersonic and strongly magnetized compressible MHD plasma. All the small-scale fluctuating parameters are measured in terms of their respective normalized quantities.

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In Sec. 2, we describe our simulation model. Section 3 deals with the nonlinear simulation results. The mode coupling interaction leading to a Kolmogorov-like spectrum is described in Sec. 4, whereas Sec. 5 contains summary.

2. MHD model

The fluid model describing nonlinear turbulent processes in the magnetofluid plasma, in the presence of a background magnetic field, can be cast into plasma density (ρ_p), velocity ($\mathbf{U}_p$), magnetic field ($\mathbf{B}$) and pressure (P_p) components according to the conservative form

$$\frac{\partial \mathbf{F}_p}{\partial t} + \nabla \cdot \mathbf{Q}_p = \mathscr{Q}, \tag{2.1}$$

where

$$\mathbf{F}_p = \begin{bmatrix} \rho_p \\ \rho_p \mathbf{U}_p \\ \mathbf{B} \\ e_p \end{bmatrix}, \quad \mathbf{Q}_p = \begin{bmatrix} \rho_p \mathbf{U}_p \\ \rho_p \mathbf{U}_p \mathbf{U}_p + \frac{P_p}{\gamma - 1} + \frac{B^2}{8\pi} - \mathbf{B}\mathbf{B} \\ \mathbf{U}_p \mathbf{B} - \mathbf{B}\mathbf{U}_p \\ e_p \mathbf{U}_p - \mathbf{B}(\mathbf{U}_p \cdot \mathbf{B}) \end{bmatrix},$$

$$\mathscr{Q} = \begin{bmatrix} 0 \\ \mathbf{f}_M(\mathbf{r}, t) + \mu \nabla^2 \mathbf{U} + \eta \nabla(\nabla \cdot \mathbf{U}) \\ \eta \nabla^2 \mathbf{B} \\ 0 \end{bmatrix}$$

and

$$e_p = \frac{1}{2}\rho_p U_p^2 + \frac{P_p}{\gamma - 1} + \frac{B^2}{8\pi}.$$

Equations (2.1) are normalized by typical length scale ℓ_0 and timescale $t_0 = \ell_0 / V_A$ in our simulations such that $\bar{\nabla} = \ell_0 \nabla, \partial/\partial \bar{t} = t_0 \partial/\partial t, \bar{\mathbf{U}}_p = \mathbf{U}_p / V_A, \bar{\mathbf{B}} = \mathbf{B}/V_A (4\pi\rho_0)^{1/2}$, $\bar{P} = P/\rho_0 V_A^2, \bar{e}_p = e_p/\rho_0 V_A^2, \bar{\rho} = \rho/\rho_0$. The bars are removed from the normalized equations (2.1). The Alfvén speed is denoted by $V_A = B_0/(4\pi\rho_0)^{1/2}$. Compressible MHD model was employed by Cho and Lazarian (2003) to study magnetic field spectrum in magnetized plasma. In the context of ISM, Kissmann et al. (2008) found that density power spectra exhibit a departure from the classical Kolmogorov phenomenology.

The right-hand side in the momentum equation denotes a forcing function ($\mathbf{f}_M(\mathbf{r}, t)$) that essentially influences the plasma momentum at the larger length scale in our simulation model. With the help of this function, we drive energy in the large-scale eddies to sustain the magnetized turbulent interactions. In the absence of forcing, the turbulence continues to decay freely.

3. Simulation results

Nonlinear mode coupling interaction studies in three dimensions are performed to investigate the multiscale evolution of decaying/driven compressible MHD turbulence. The characteristic wavenumbers k and frequencies ω over which MHD is valid are defined by $k\rho_i < 1$, $\omega < \omega_{c_i} < \omega_{p_i}$, where ρ_i is ion gyro radius and ω_{ci} and ω_{p_i} are respectively the ion cyclotron and ion plasma frequencies. In the simulations, all

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the fluctuations are initialized isotropically with random phases and amplitudes in Fourier space and an initial shape close to k^{-2} (k is the Fourier mode, which is normalized to the characteristic turbulent length scale l_0). No mean magnetic or velocity field is assumed in the initial fluctuations, but it may be generated locally by nonlinear interactions. This ensures that the initial fluctuations are fairly isotropic and that no anisotropy is introduced by the initial data. However a background constant magnetic field is assumed along the z-direction in our simulations that is consistent with the SW background constant magnetic field. Since we are interested in a local region of the SW plasma, the computational domain employs a three-dimensional periodic box of volume π^3. Kinetic and magnetic energies are also equipartitioned between the initial velocity and the magnetic fields. The latter helps treat the transverse or shear Alfvén and fast/slow magnetosonic waves on an equal footing, at least during the early phase of the simulations. MHD turbulence evolves under the action of nonlinear interactions in which larger eddies transfer their energy to smaller ones through a forward cascade. During this process, MHD turbulent fluctuations are dissipated gradually due to the finite Reynolds number, thereby damping small-scale motion as well. This results in a net decay of turbulent sonic Mach number M_s which was demonstrated in Shaikh and Zank (2006). The turbulent sonic Mach number continues to decay from a supersonic ($M_s > 1$) to a subsonic ($M_s < 1$) regime. This indicates that dissipative effects predominantly cause supersonic MHD plasma fluctuations to damp strongly, leaving primarily subsonic fluctuations in the MHD fluid. Note that the dissipation is effective only at the small scales, whereas the large scales and the inertial range turbulent fluctuations remain unaffected by direct dissipation. Since there is no mechanism that drives turbulence at the larger scales, spectral transfer in driven turbulence, in general, follows a similar cascade process as in the decaying turbulence case, but it may be more complex depending upon what modes (irrotational or solenoidal) are being excited. In the energy-containing eddies, the large-scale energy simply migrates towards the smaller scales by virtue of nonlinear cascades in the inertial range and is dissipated at the smallest turbulent length scales. Spectral transfer in globally isotropic and homogeneous hydrodynamic and magnetohydrodynamic turbulence is the widely accepted paradigm (Kolmogorov 1941; Iroshnikov 1963; Kraichnan 1965) that leads to Kolmogorov-like energy spectra. The most striking effect, however, to emerge from the decay of the turbulent sonic Mach number is that the density fluctuations begin to scale quadratically with the subsonic turbulent Mach number as soon as the compressive plasma enters the subsonic regime, i.e. $\delta\rho \sim \mathcal{O}(M_s^2)$ when $M_s < 1$ (Shaikh and Zank 2006). This signifies an essentially *weak* compressibility in the magnetoplasma and can be referred to as a *nearly incompressible* state (Matthaeus et al. 1988; Zank and Mstthaeus 1990, 1993).

As a direct consequence of the magnetoplasma being near incompressible, the density fluctuations exhibit a weak compressibility in the gas and are convected predominantly passively in the background incompressible fluid flow field. This hypothesis can be verified straightforwardly by investigating the density spectrum which should be slave to the incompressible velocity spectrum. This is shown in Fig. 1 which illustrates that the density fluctuations follow the velocity fluctuations in the inertial regime after the long-time (several Alfvén transit time) evolution of MHD turbulence. The transition of compressible magnetoplasma from a(n) (initially) supersonic to a subsonic or nearly incompressible regime is gradual. This means that the magnetofluid contains supersonic and super-Alfvénic modes initially

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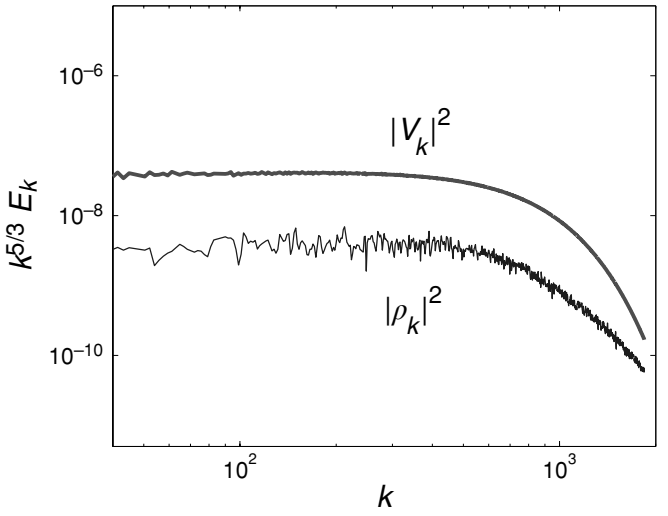


Figure 1. Velocity fluctuations are dominated by shear Alfvénic motion and thus exhibit a Kolmogorov-like $k^{-5/3}$ spectrum, where k is the Fourier mode (top curve). Density fluctuations are passively convected by the nearly incompressible shear Alfvénic motion and follow a similar spectrum in the inertial range (bottom curve). The numerical resolution in three dimensions is 512^3. The turbulent Reynolds numbers are $\tilde{R}_e = \tilde{R}_m \approx 200$.

in which highly compressible density fluctuations do not follow the velocity spectrum. It is the eventual decay of the turbulent Mach number to a subsonic regime that is responsible for the density fluctuations following the velocity fluctuations. In the subsonic regime, the compressibility weakens substantially so that the density fluctuations are advected only passively. A passively convected fluid exhibits a similar inertial range spectra as that of its background flow field (Macomb 1990). Likewise, subsonic density fluctuations in our simulations exhibit a Kolmogorov-like $k^{-5/3}$ spectra similar to the background velocity fluctuations in the inertial range. This, we believe, provides a plausible explanation for the Kolmogorov-like density spectrum observed in the SW plasma; i.e. they are convected passively in a field of nearly incompressible velocity fluctuations and acquire identical spectral features (as shown in Fig. 1). The passive scalar evolution of the density fluctuations is associated essentially with incompressibility and can be understood directly from the continuity equation as follows. Expressing the fluid continuity equation as $(\partial_t + \mathbf{U}\cdot\nabla)\ln\rho = -\nabla\cdot\mathbf{U}$, where the right-hand side represents compressibility of the velocity fluctuations, shows that the density field is advected passively when the velocity field of the fluid is nearly incompressible with $\nabla\cdot\mathbf{U} \simeq 0$. This is in part demonstrated in Shaikh and Zank (2007).

It is noted that the energy spectra depicted in Fig. 1 are globally isotropic. They can be locally anisotropic if there exists a self-consistently generated large-scale or mean magnetic field, i.e. $\lambda_\parallel \neq \lambda_\perp$, where $\lambda_\parallel$ and $\lambda_\perp$ are the length scales of turbulent eddies oriented relative to the mean magnetic field. Local anisotropy in SW turbulence can be mediated by nonlinear interactions that lead to the formation of large-scale or mean magnetic field. These large-scale local eddies act as guide fields for small-scale turbulent fluctuations and excite Alfvén waves locally. The Alfvén waves can potentially inhibit turbulent cascades along the direction of propagation.

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Consequently, energy transfer across and along the *local* mean magnetic fields occurs at different rates, and thus $\lambda_\parallel > \lambda_\perp$. In locally isotropic magnetofluid turbulence, $\lambda_\parallel \simeq \lambda_\perp$. Any disparity between the parallel and the perpendicular local turbulent fluctuations (or the scale-dependent anisotropy) is revealed quantitatively from the second-order structure function. The second-order structure function, corresponding to the energy associated with the Kolmogorov-like spectrum, describes turbulent eddy structure in the *local* real space, where $\lambda_\parallel$ and $\lambda_\perp$ are respectively the parallel and perpendicular length scales of the eddies relative to the local mean magnetic field. It is defined as $S_2(\ell) = \langle |\mathbf{U}(\mathbf{x}+\ell,t) - \mathbf{U}(\mathbf{x},t)|^2 \rangle$, where $\mathbf{U}$ denotes velocity or magnetic field fluctuations, ℓ is either the parallel or the perpendicular wavelength and $\langle \cdots \rangle$ represents an average over all $\mathbf{x}$. The local anisotropy in the turbulent cascade is demonstrated in Shaikh and Zank (2007). The local large-scale fluctuations in SW turbulence exhibits anisotropy, whereas the global energy spectrum (see Fig. 1) is representative of Kolmogorov-like isotropic turbulence. It has been argued that an anisotropic cascade in magnetofluid turbulence modifies the Kolmogorov-like $k^{-5/3}$ spectrum to the Iroshnikov–Kriachnan-like (IK-like) $k^{-3/2}$ spectrum (Kolmogorov 1941; Iroshnikov 1963; Kraichnan 1965). The physical arguments invoked to justify the IK-like spectrum are based on the self-interaction of colliding Alfvénic wave packets that tend to flatten out the MHD spectrum (Ng et al. 2003), yielding a $k^{-3/2}$ spectrum. This issue has been a subject of intense debate and is beyond the scope of this work. Our results described in Fig. 1 are further consistent with observational evidence (Marsch and Tu 1990; Tu et al. 1991; Goldstein et al. 1995; Leamon et al. 1998; Bellamy et al. 2005; Bruno and Carbone 2005).

4. Mode coupling interactions

It is important to investigate the mode coupling interactions in order to understand the turbulent cascade of energy that predominantly migrates, in our simulations, from the compressible modes to the incompressible modes. This is because the energy cascades are dominated by the processes that lead to the incompressible-like inertial range spectra in Fig. 1. Alternatively, we seek to understand how an initially non-solenoidal velocity field evolves towards a solenoidal field as it explains the transition of the compressible magnetoplasma from a supersonic to a subsonic or nearly incompressible state that yields a Kolmogorov-like $k^{-5/3}$ density spectrum. Since a non-solenoidal velocity field consists of compressible (fast/slow magnetosonic) modes, its eventual transformation into incompressible (Alfvén) MHD modes therefore requires a detailed understanding of nonlinear mode interactions and the subsequent energy cascade processes. To understand the nonlinear mode coupling between these MHD modes, we introduce diagnostics that distinguish energy cascades between Alfvénic and slow/fast magnetosonic fluctuations. Since Alfvénic fluctuations are transverse, the propagation wave vector is orthogonal to the oscillations, i.e. $\mathbf{k} \perp \mathbf{U}$, and the average spectral energy contained in these (shear Alfvénic modes $\langle k_{\mathrm{SAM}} \rangle$) fluctuations can be computed from the orthogonal fluctuations. On the other hand, slow/fast magnetosonic modes $\langle k_{\mathrm{SFM}} \rangle$ propagate longitudinally along the fluctuations, i.e. $\mathbf{k} \parallel \mathbf{U}$, and thus their contribution is determined from the longitudinal oscillations. The evolution of the modal energy is depicted in Fig. 2. Although the modal energies in k_{SAM} and k_{SFM} modes are identical initially, the disparity in the cascade rate causes the energy in

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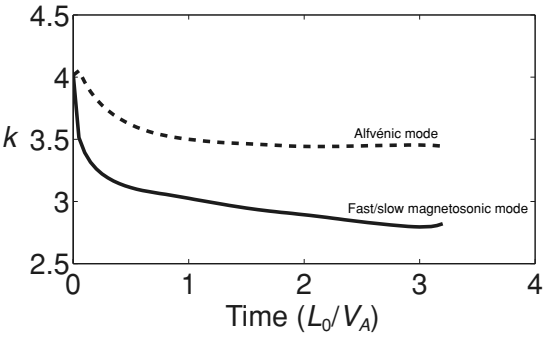


Figure 2. Spectral energy transfer amongst shear Alfvén modes ($\langle k_{\rm SAM}\rangle^2 \simeq \sum_{\mathbf{k}} |i\mathbf{k} \times \mathbf{U}_{\mathbf{k}}|^2 / \sum_{\mathbf{k}} |\mathbf{U}_{\mathbf{k}}|^2$) and slow/fast magnetosonic waves ($\langle k_{\rm SFM}\rangle^2 \simeq \sum_{\mathbf{k}} |i\mathbf{k} \cdot \mathbf{U}_{\mathbf{k}}|^2 / \sum_{\mathbf{k}} |\mathbf{U}_{\mathbf{k}}|^2$) during a three-dimensional dissipative compressible MHD simulation is shown. These modes have identical energy initially so that $\mathrm{k_{SAM}} \approx \mathrm{k_{SFM}}(t=0)$. As time progresses, turbulence decay and spectral transfer due to $\mathrm{k_{SFM}}$ is suppressed significantly.

longitudinal fluctuations to decay far more rapidly than the energy in the Alfvénic modes. The Alfvénic modes, after a modest initial decay, sustain the energy cascade processes by actively transferring spectral power amongst various Fourier modes in the stationary state. By contrast, the fast/slow magnetosonic modes progressively weaken and suppress the energy cascades. The $k_{\rm SFM}$ mode represents collectively a dynamical evolution of small-scale fast-plus-slow magnetosonic cascades and does not necessarily distinguish the individual constituents (i.e. the fast and slow modes) due to their wave vector alignment relative to the magnetic field. The physical implication, however, that emerges from Fig. 2 is that the fast/slow magnetosonic waves *do not* contribute efficiently to the energy cascade process and that the cascades are governed predominantly by non-dissipative Alfvénic modes that survive collisional damping in compressible MHD turbulence. This suggests that because of the decay of the fast/slow magnetosonic modes in compressible MHD plasmas, supersonic turbulent motions become dominated by subsonic motions and the nonlinear interactions are sustained primarily by Alfvénic modes thereafter, the latter being incompressible. The effect of inhibiting the fast/slow magnetosonic wave cascade is that the compressible magnetoplasma relaxes dynamically to a nearly incompressible state in the subsonic turbulent regime, and the solenoidal component of the fluid velocity makes a negligible contribution, i.e. $\nabla \cdot \mathbf{U} \ll 1$ but not 0. The nearly incompressible state, leading to a Kolmogorov-like $k^{-5/3}$ spectrum of density fluctuations, described in our simulations is further consistent with the theoretical predictions of Zank and Matthaeus (1990, 1993).

5. Summary

Within the paradigm of our model, we find that the nonlinear interaction time for Alfvén waves increases compared to that of the (magneto)acoustic waves. Consequently, the plasma motion becomes increasingly incompressible on Alfvénic timescales, and low-plasma-β fluctuations are eventually transformed into high-plasma-β fluctuations. During this gradual transformation to incompressibility, the compressible fast/slow magnetosonic modes do not couple well with the Alfvén modes. The cascades are therefore progressively dominated by shear Alfvén modes,

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while the compressible fast/slow magnetosonic waves suppress nonlinear cascades by dissipating the longitudinal fluctuations. This physical picture suggests that a nearly incompressible state develops naturally from a compressive SW magnetoplasma and that the density fluctuations, scaling quadratically with the subsonic turbulent Mach number, exhibit a characteristic Kolmogorov-like $k^{-5/3}$ spectrum that results from passive convection in a field of nearly incompressible velocity fluctuations.

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References

Armstrong, J. W., Cordes, J. M. and Rickett, B. J. 1981 Density power spectrum in the local interstellar medium. *Nature* **291**, 561–564.

Armstrong, J. W., Rickett, B. J. and Spangler, S. 1995 Electron density power spectrum in the local interstellar medium. *Astorphys. J.* **443**, 209–221.

Bellamy, B. R., Cairns, I. H. and Smith, C. W. 2005 Voyager spectra of density turbulence from 1 AU to the outer heliosphere. *J. Geophys. Res.* **110**, A10104. DOI:10.1029/2004JA010952.

Bhattacharjee, A., Ng, C. S. and Spangler, S. R. 1998 Weakly compressible magnetohydrodynamic turbulence in the solar wind and the interstellar medium. *Astrophys. J.* **494**, 409.

Bruno, R. and Carbone, V. 2005 The solar wind as a Turbulence Laboratory. *Living Rev. Solar Phys.* **2**, 4.

Cho and Lazarian 2003 *MNRAS* **345**, 325.

Goldstein, M. L., Roberts, D. A. and Matthaeus, W. H. 1995 *Ann. Rev. Astron. Astrophys.* **33**, 283–325.

Iroshnikov, P. S. 1963 Turbulence of a conducting fluid in a strong magnetic field. *Astron. Zh.* **40**, 742.

Kissmann, R., Kleimann, J., Fichtner, H. and Grauer, R. 2008 Local turbulence simulations for the multiphase ISM. *MNRAS* **391**, 1577.

Kolmogorov, A. N. 1941 On degeneration of isotropic turbulence in an incompressible viscous liquid. *Dokl. Akad. Nauk SSSR* **31**, 538–541.

Kraichnan, R. H. 1965 Inertial range spectrum in hydromagnetic turbulence. *Phys. Fluids* **8**, 1385–1387.

Leamon, R., Smith, C. W., Ness, N. F. and Matthaeus, W. H. 1998 Observational constraints on the dynamics of the interplanetary magnetic field dissipation range. *J. Geophys. Res.* **103**(A3), 4775–4787.

Matthaeus, W. H. and Brown, M. 1988 Nearly incompressible magnetohydrodynamics at low Mach number. *Phys. Fluids* **31**, 3634–3644.

Marsch, E. and Tu, C.-Y. 1990 On the radial evolution of MHD turbulence in the inner heliosphere. *J. Geophys. Res.* **95**, 8211–8229.

Marsch, E. and Tu, C.-Y. 1990 Spectral and spatial evolution of compressible turbulence in the inner SW. *J. Geophys. Res.* **95**, 11,945–11,956.

Matthaeus, W. H., Klein, L. W., Ghosh, S. and Brown, M. R. 1991 Nearly incompressible magnetohydrodynamics, pseudosound, and solar wind fluctuations. *J. Geophys. Res.* **96**, 5421–5435.

McComb, W. D. 1990 *The Physics of Fluid Turbulence*. Claredon: Oxford University Press.

https://doi.org/10.1017/S0022377809990237 Published online by Cambridge University Press

Montgomery, D. C., Brown, M. R. and Matthaeus, W. H. 1987 Density fluctuation spectra in magnetohydrodynamic turbulence. *J. Geophys. Res.* **92**, 282–284.

Ng, C. S., Bhattacharjee, A., Germaschewiski, K. and Galtier, S. 2003, Anisotropic fluid turbulence in the interstellar medium and SW. *Phys. Plasmas* **10**, 1954.

Shaikh, D. and Zank, G. P. 2006 *The Astrophysical Journal* **640**, L195–L198.

Shaikh, D. and Zank, G. P. 2007 Turbulence and Nonlinear Processes in Astrophysical Plasmas: Sixth Annual International Astrophysics Conference. *AIP Conf. Proc.* **932**, 111.

Shaikh, D. and Zank, G. P. 2007 *Astrophys. J.* **656**, L17–L20.

Tu, C.-Y., Marsch, E. and Rosenbauer, H., 1991 Temperature fluctuation spectra in the inner solar wind. *Ann. Geophys.* **9**, 748–753.

Zank, G. P. and Matthaeus, W. H. 1993 Nearly incompressible fluids. II: Magnetohydrodynamics, turbulence, and waves. *Phys. Fluids* **A5**, 257–273.

Zank, G. P. and Matthaeus, W. H. 1990 Nearly incompressible hydrodynamics and heat conduction. *Phys. Rev. Lett.* **64**, 1243–1246.

Zank, G. P. and Matthaeus, W. H. 1991 The equations of nearly incompressible fluids. I: Hydrodynamics, turbulence, and waves. *Phys. Fluids* A **3**, 69–82.

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