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A three-DOF translational manipulator with decoupled geometry

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SUMMARY
A new 3-DOF translational parallel manipulator is presented in this paper. This manipulator has decoupled motion in for x, y, and z axes, and employs only revolute joints. The structure and kinematics of the manipulator are studied. The mechanism singularity is examined and the calibration methods are presented.

KEYWORDS: Decoupled motion; Redundant constraints; Singularity; Calibration.

I. INTRODUCTION

Since Gough first introduced a 6-DOF parallel mechanism the variable-length-strut octahedral hexapod as a tire detecting device,[1] and afterwards Stewart used the same mechanism for a flight simulator,[2] parallel mechanisms have been used in many practical applications, such as heavy load platforms, good kinematic and dynamic performance machines, fine motion and precise positioning manipulators, etc.

In recent years, theoretical researches of parallel mechanisms have increased rapidly in a variety of areas, especially in mechanism design methodology. Hunt[3] studied the structural kinematics of parallel manipulators on the basis of screw theory and enumerated promising kinematic structures. Pernette and Henein etc.[4] presented the method for the design of a 3-DOF parallel translating manipulator with U-P-U joints kinematic chains. The planar 2-DOF parallel manipulators are 5-bar linkages with 5 revolute joints[5] or 3 revolute joints and 2 prismatic pairs. The planar 3-DOF parallel manipulators[6,7] are 8-bar linkages with 2 ternary links connected through three in-parallel legs, each leg consisting of two links. The Delta robot[8] is 3 translational degrees of freedom. More recently, several types of composite pairs and new kinds of sub-chains with specific degrees of freedom proposed by Gao and Li, etc.[9] and several new type of 2-, 3-, 4- and 5-DOF parallel robotic mechanisms are obtained.

However, most of these manipulators have coupled motion between the position and orientation of the end-effector. Recent research on 3-DOF parallel manipulators has been leaning toward the decoupling of the position and orientation of the end-effector and the elimination of complicated multi-DOF joints. Tsai presented the design of a 3-DOF translational platform that employs only revolute joints and planar parallelograms[10] and a spatial 3-*UPU* (universal-prismatic-universal) parallel manipulator.[11] Zhao and Huang studied the kinematics of an overconstrained 3-*RRC* (revolute-revolute-cylindrical) translational manipulator.[12] Baron and Bernier studied the design of a 3-DOF Star manipulator.[13] Carricato and Parenti-Castelli presented a family of 3-DOF parallel manipulators with pure translational motion.[14] Kong and Gosselin[15] studied the kinematics and singularity of a 3-CRR translational parallel manipulator, and recently, Kim and Tsai discussed the optimal design[16] and evaluation[17] about the similar manipulator named *Cartesian*.

In this paper, a new 3-DOF translational parallel manipulator that employs only revolute joints is presented. This manipulator has decoupled motion in x, y, and z axes. The kinematic structure of the manipulator is described. The kinematics and singularity of the manipulator is studied and two calibration methods are introduced.

II. KINEMATIC STRUCTURE

The kinematic structure and geometry of the 3-DOF translational parallel manipulator is shown in Fig. 1, where the end-effector is connected to the fixed frame by three limbs. Each limb includes a planar parallelogram and a link and other three joints. The structure size of the jth limb is represented by l_{j1}, l_{j2}. The joints A_j for $j = 1, 2, 3$ are connected to the fixed frame, while the joints B_j for $j = 1, 2, 3$ are connected to the end-effector. Furthermore, the three axes of the revolute joints B_j for $j = 1, 2, 3$ are parallel to the x, y, and z axes, respectively. The distance from A_j to M_{j3} is l_{j1}. The rotational actuator can be applied to M_{j1} or M'_{j1} for $j = 1, 2, 3$.

If the lengths of l_{j1} for $j = 1, 2, 3$ are equal, then the workspace of the manipulator is a cube. The position of the platform is defined by θ_j for $j = 1, 2, 3$. When $\theta_j = 0$, the end-effector is just at the center of the cube.

One advantage of this manipulator is that it is economic in production for only revolute joints are applied. Another advantage is that the end-effector can easily reach high speed, as a stroke of the end-effector only needs the actuator to rotate within 180°.

Generally, the planar parallelogram has only one translational DOF (degree of freedom). Hence each planar parallelogram can be considered as a 1-DOF translational pair

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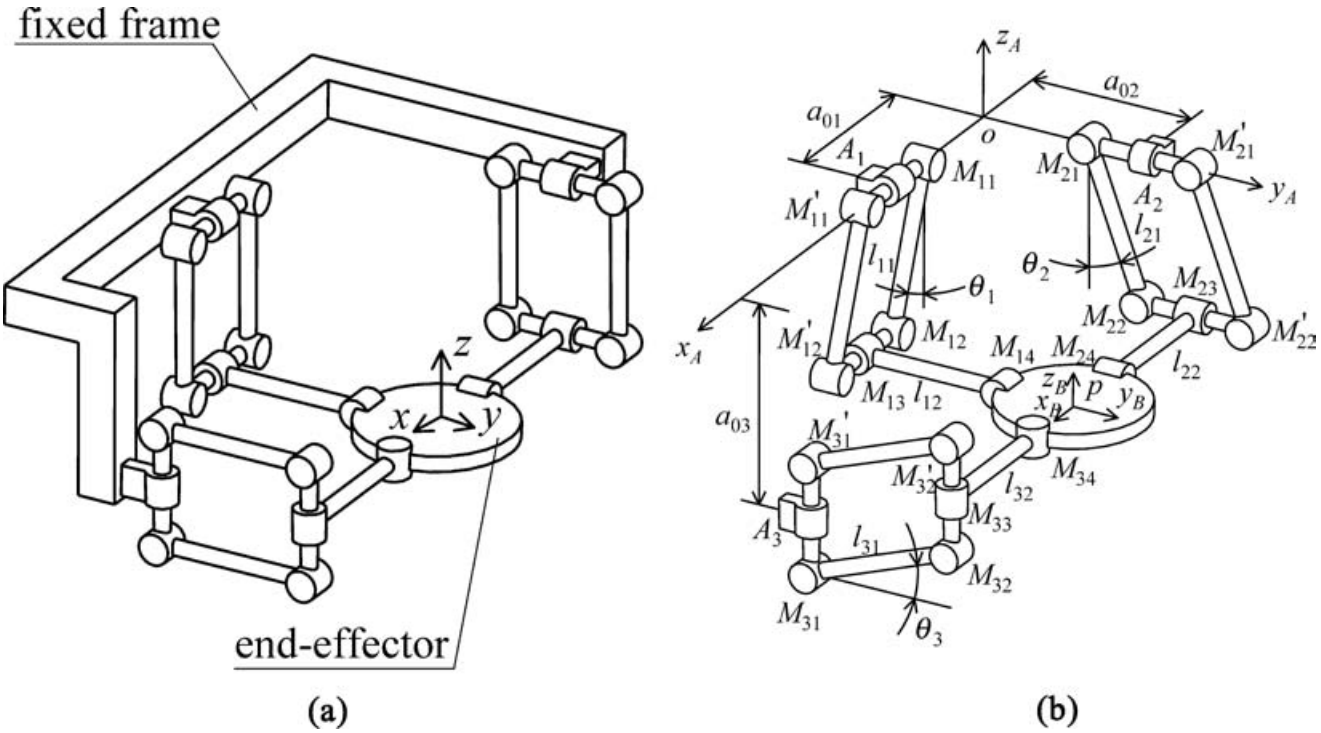


Fig. 1. 3-DOF translational parallel manipulator with 3-RRRRRRR joints (a) Structure (b) Geometry.

with two links connected to each side of it. Equivalently, each limb consists of three joints, one pair and three links, so the DOFs of the mechanism can be calculated as:[18]

$$F = \lambda(n - m - 1) + \sum_{i=1}^{m} f_i$$
$$= 6(11 - 12 - 1) + 3(1 + 1 + 1 + 1) = 0 \qquad (1)$$

where F denotes the DOFs of the mechanism, n and m represent the number of links and number of joints (or pairs), respectively, f_i denotes DOFs associated with the ith joint (or pair), and λ is a constant which value is 6 for spatial mechanism or 3 for planar (or spherical) mechanism.

From the result of equation (1), the mechanism has no freedom. However, this is not the real case, so we should examine the mechanism in another way.[19]

For convenience, we let \$ be the special Plücker coordinates for describing the displacement of the output link of a limb in a parallel mechanism, which is

$$\$_j = (v_{xj} \quad v_{yj} \quad v_{zj} \quad \omega_{\alpha j} \quad \omega_{\beta j} \quad \omega_{\gamma j}) \qquad (2)$$

where, $v_j(v_{xj} \quad v_{yj} \quad v_{zj})$ expresses the translation of the output link of the limb j in three directions, and $\omega_j(\omega_{\alpha j} \quad \omega_{\beta j} \quad \omega_{\gamma j})$ represents the rotation of the output link of the limb j with respect to three Eular's angles, α, β and γ. The special Plücker coordinates $v_{xj}, v_{yj}, v_{zj}, \omega_{\alpha j}, \omega_{\beta j}, \omega_{\gamma j}$ can be taken as 1 or 0. When taking 1, it means that the limb j has that DOF, when taking 0, it means that the limb j has no that DOF. Then the way to determine the DOFs of the parallel mechanism can be expressed as the following *Freedom Criterion*.

The special Plücker coordinates of the end-effector equal to the intersection of the special Plücker coordinates of all the limbs connecting the fixed frame to the end-effector.

This criterion can be described in the following *General Equation* (3), which is very useful not only for mechanism analysis but also for mechanism design.

$$\$ = \$_1 \cap \$_2 \cdots \cap \$_n \qquad (3)$$

In the present mechanism, $n = 3$, and as the planar parallelogram is considered as a 1-DOF planar translational pair, it follows that

$$\$_1 = (1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0) \qquad (4)$$
$$\$_2 = (1 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0) \qquad (5)$$
$$\$_3 = (1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 1) \qquad (6)$$

Then the DOFs of the end-effector will be determined from equation (3), that is

$$\$ = (1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0) \qquad (7)$$

It is pointed out that the mechanism has three translational DOFs.

If we examine equations (4), (5), and (6), we can find out that each limb provides two rotational constraints to the end-effector, or in another words, each rotational parameter of the end-effector is constrained by two limbs. As a result, there are three redundant constraints on rotation. This is why the result of equation (1) is zero.

III. KINEMATICS

The forward and inverse kinematic analyses are trivial since the decoupled characteristic exists. Referring to Fig. 1(b), each limb constrains point P to lie on a plane which passes through lines $\overline{M_{j3}B_j}$, for $j = 1, 2, 3$, and is perpendicular

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to the axis of x, y, and z, respectively. The position of jth plane is determined only by θ_j whenever the length l_{j1} is given. Consequently, the position of P is determined by the intersection of three planes, i.e. the intersection of θ_j for $j = 1, 2, 3$. The simple kinematic relation can be written as

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} a_{01} + l_{11}\sin\theta_1 \\ a_{02} + l_{21}\sin\theta_2 \\ -a_{03} + l_{31}\sin\theta_3 \end{bmatrix} \tag{8}$$

where $\mathbf{p} = [p_x,\ p_y,\ p_z]^T$ denotes the position vector of the end-effector.

Taking the time derivative of equation (8) and applying the principle of virtual work, we obtain

$$\begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot\theta_3 \end{bmatrix} = J^{-1} \begin{bmatrix} \dot p_x \\ \dot p_y \\ \dot p_z \end{bmatrix} \tag{9}$$

and

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = J^T \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} \tag{10}$$

where, $\mathbf{f} = [f_x\ f_y\ f_z]^T$ is the vector of the end-effector output force, T_j for $j = 1, 2$, and 3, denotes the jth rotational actuator torque, and J is a diagonal matrix that holds

$$J = \begin{bmatrix} l_{11}\cos\theta_1 & & \\ & l_{21}\cos\theta_2 & \\ & & l_{31}\cos\theta_3 \end{bmatrix} \tag{11}$$

IV. SINGULARITY DISCUSSION

From equation (9) we can find that the rotational actuator speed is nonlinear to the velocity of the end-effector. Moreover, if $\theta_j = \pm 90^\circ$, then $\det|J| = 0$, for any expected velocity of the end-effector, the rotational speed of the actuator will be infinite. When θ_j is not equal but close to $\pm 90^\circ$, then $\det|J| \to 0$, the required rotational speed of the actuator may be still too high to be reached. So the value of the θ_j must be designed in an appropriate range whenever the speed limit of the end-effector is given.

Suppose the desired velocity of the end-effector is v_e, and the permissible rotational speed of the actuator is n_e, then the absolute maximum value of the θ_j for $j = 1, 2, 3$ can be obtained from equation (9)

$$\cos|\theta_j| = \frac{v_e}{l_{j1}n_e} \tag{12}$$

Let

$$\theta_{jm} = \arccos\frac{v_e}{l_{j1}n_e} \tag{13}$$

Then θ_j should satisfy

$$-\theta_{jm} \le \theta_j \le \theta_{jm} \tag{14}$$

Whenever the mechanism design satisfies equation (14), no singularity will exist.

V. ORIGINAL POSITION CALIBRATION

From a manufacturing point of view, it is always important to develop a simple and efficient calibration method to determine the initial value of actuators when the end effector stands at its original position. For the present mechanism, the actuator's parameter to be determined is the initial angle of $\theta_j (j = 1, 2, 3)$.

For convenience, we suppose

(1) The input $\theta_j (j = 1, 2, 3)$ is within $[-\theta_{jm}, \theta_{jm}]$, where $\theta_{jm} > 0$, and $\theta_j = \theta_{jm}$ denotes the initial position of the jth limb.

(2) In the initial position (see Fig. 2), the angle between the link $l_{j1} (j = 1, 2, 3)$ and the axis $u (u = x, y, z)$ is $\theta_{0j} (j = 1, 2, 3)$, then the initial value θ_{jm} of θ_j can be determined as

$$\theta_{jm} = \frac{\pi}{2} - \theta_{0j} \tag{15}$$

So we can determine θ_{0j} first, then θ_{jm}.

The steps of the calibration can be as follows: From the initial position θ_{0j} of the arm in Fig. 2, rotate the driving arm twice in a specified angle θ_s, which satisfies

$$2\theta_s + \theta_{0j} \le \pi \tag{16}$$

During the process, record the two moving distances l_0 and l of the end-effector in the direction of axis $u (u = x, y, z)$, they satisfy

$$\begin{cases} l_{j1}\cos\theta_{0j} - l_{j1}\cos(\theta_{0j} + \theta_s) = l_0 \\ l_{j1}\cos\theta_{0j} - l_{j1}\cos(\theta_{0j} + 2\theta_s) = l_0 + l \end{cases} \tag{17}$$

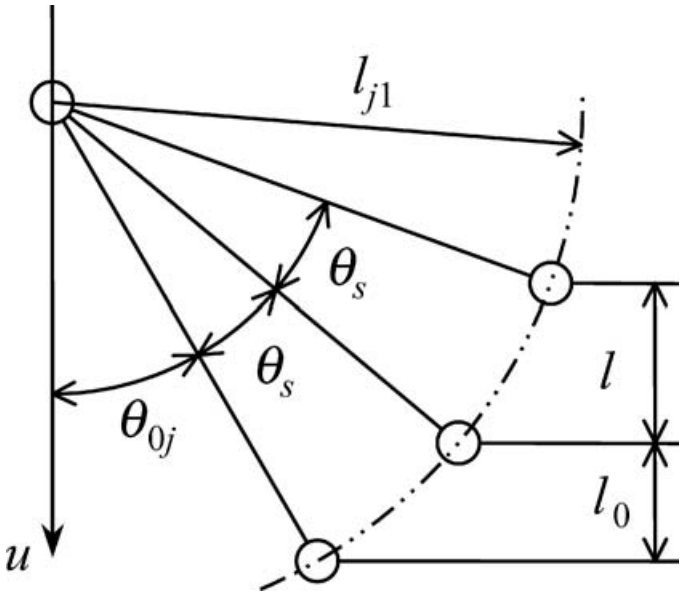


Fig. 2. The initial position θ_{0j}.

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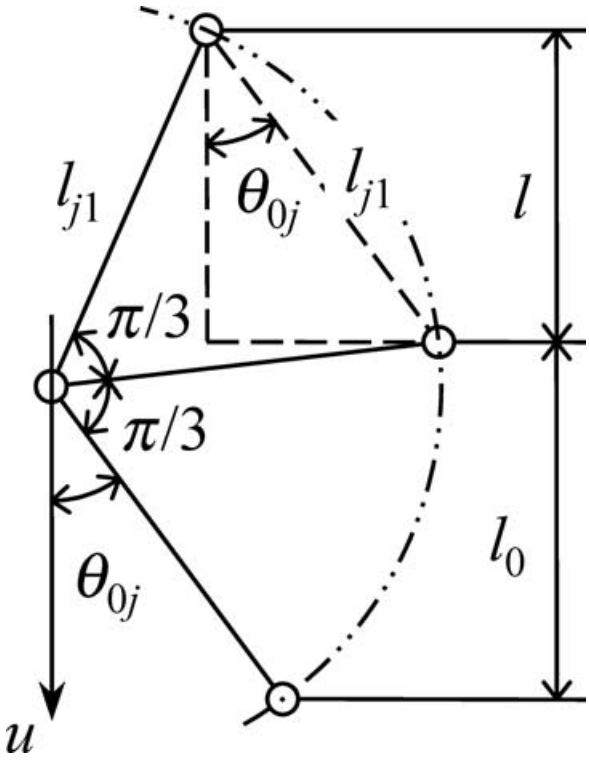


Fig. 3. The geometric significance of equation (19).

expand $\cos(\theta_{0j} + \theta_s)$ and $\cos(\theta_{0j} + 2\theta_s)$ in equation (17), and eliminate $\sin\theta_{0j}$, we obtain

$$\cos\theta_{0j} = \frac{l_0 + l - 2l_0\cos\theta_s}{2l_{j1}(1 - \cos\theta_s)} \tag{18}$$

If $\theta_0 \leq \pi/3$ and let $\theta = \pi/3$, then equation (18) yields

$$\cos\theta_{0j} = \frac{l}{l_{j1}} \tag{19}$$

The geometric significance of the equation (19) is shown in Fig. 3, which is very convenient for industrial applications. Then, we can obtain θ_{jm} from equation (15).

VI. CONCLUSIONS

A new 3-DOF translational parallel manipulator is considered. This manipulator employs only revolute joints and has three rotational redundant constraints. The forward and inverse kinematic analyses are carried through. Two simple and efficient calibration methods are put forward.

This manipulator has some outstanding advantages. Firstly, the x, y, and z motions of the manipulator are decoupled. Secondly, the manipulator is economical in production and simple in calibration. Thirdly, the end-effector of the manipulator can easily attach high speed. Finally, singularities exist only at the edge of the manipulator's workspace and can be easily avoided by proper design. The advantages show that this manipulator has an extensive application potential in industry.

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