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A UNIFIED FRAMEWORK FOR OPTIMAL TAXATION WITH UNDIVERSIFIABLE RISK

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This paper considers a model of linear capital taxation for an economy where capital and labor income are subject to idiosyncratic uninsurable risk. To keep the model tractable, we assume that investment decisions are made before uncertainty is realized, so that the realization of the capital and labor income shocks only affects current consumption. In this setting, we are able to jointly analyze capital and labor income risk and derive analytical results regarding the optimal taxation of capital. We find that the optimal capital tax is positive in the long run if there is only capital income risk. The reason for this is that the capital tax provides insurance against capital income risk. Furthermore, for high levels of risk, increasing the capital tax may actually induce capital accumulation. On the other hand, if there is only labor income risk, the optimal capital tax is zero. The sign of the optimal tax can only be negative if the two types of risk are negatively correlated and labor income risk is large enough.

Keywords: Fiscal Policy, Optimal Taxation, Risk

1. INTRODUCTION

This model aims to study individual idiosyncratic risk in a simple framework with both labor income risk and capital income risk. The two types of risk have been studied separately in different models. However, there is no obvious way to bring them together in a tractable framework. This paper will be able to do so by making the simplifying assumption that investment decisions are made before uncertainty is realized. This means that in every period all the ex-ante identical agents will make the same investment decision, and their individual shocks will

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only affect the value of consumption in the current period. We are thus able to keep the representative agent nature of the model, although within each period the agents will have different consumption levels.

In representative agent Ramsey models, Chamley (1986) shows that capital taxes are usually zero in the long run, but during the transition they may be positive or negative depending on the elasticities of preferences (Frankel (1998)). Even in the long run, capital may be taxed or subsidized if there is incomplete factor taxation, such as in Correia (1996) or capital cannot be separated from entrepreneurial effort, like in Reis (2011). DeBonis and Spataro (2005) also show that if the planner discounts the future differently from households, then the optimal long-run capital tax may not be zero.

Optimal taxes with idiosyncratic labor income risk have been studied by Aiyagari (1994, 1995), who finds that it is optimal to tax capital income in the long run because idiosyncratic risk tends to increase capital accumulation due to precautionary savings. On the other hand, Panousi and Reis (2016) use a model based on Angeletos (2007) and Angeletos and Panousi (2009) to derive the optimal capital income tax when there is capital income risk and find that it may be optimal to tax or subsidize capital, depending on whether aggregate capital lies above or below its first best level. However, the two models are not quite comparable since Aiyagari (1994, 1995) needs exogenous borrowing constraints which would make the model in Angeletos (2007) and Angeletos and Panousi (2009) intractable. Furthermore, dynamic models with idiosyncratic uncertainty are in general very hard to solve analytically.

We propose an alternative modeling strategy for decisions which makes these kinds of models surprisingly simple and allows us to incorporate both kinds of idiosyncratic shocks in one single model. The assumption is that agents make their investment decisions before the realization of uncertainty for the current period. Although this is a simplifying assumption, the opposite assumption that is usually made, that agents optimize their choice of capital after they have all the information relative to the current shock, is also extreme. Reality probably lies somewhere in between. Our assumption is different from Gervais and Mennuni (2015), who assume that capital becomes immediately productive, and thus responds to the current capital income tax. In our model, capital invested today is only used in production tomorrow, but needs to be chosen before the shocks to today's income are realized.

Under our timing assumption, the decision for the state variable is made before uncertainty is realized, and as a consequence the state variable will be the same for all agents. This means we need to keep track of a single state variable, even though agents will get hit by a different shock in each period, which makes individual consumption random. Investment will still be affected by the existence of uncertainty, although it does not depend on its realization. Our timing assumption does not affect the deterministic version of our model since when the variance of the idiosyncratic shock goes to zero this model converges

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exactly to the traditional Ramsey model, where zero taxes on capital income are optimal.

One concern with our simplified framework would be that the dynamics in capital might not be able to replicate what happens under incomplete markets, since the investment decision is made before the realization of uncertainty. However, we find that this is not the case in qualitative terms, although the effects will of course be different quantitatively. In particular, in our model capital investment decreases when its riskiness increases, as happens in Panousi and Reis (2016). More interestingly, we are able to generate non-monotonicity of capital with respect to the tax rate, as Panousi (2015). In addition to that, the new framework allows us to explore the effect of the covariance between labor and capital income risk. For example, if labor income risk is negatively correlated with capital income risk, then increasing capital riskiness may increase the level of capital.

In our model if there are only labor income shocks, the optimal capital income tax is zero. This is different from the Aiyagari (1995) paper, where the optimal capital income tax is positive if there is labor income risk. This is not surprising, however, since the Aiyagari model has borrowing constraints that generate precautionary savings which are absent in our model. Since there is no over-saving for precautionary reasons, there is no need to use capital taxation.

On the other hand, if there is only capital income risk, the optimal tax is always positive in our model. Furthermore, the optimal tax seems to be increasing in the level of risk. These results are similar to Panousi and Reis (2016) for high levels of capital income risk, where the tax was positive and increasing with risk, but in Panousi and Reis it was also possible to get optimal capital subsidies for low-risk levels in order to induce higher risk-taking. In our model, the only way we are able to generate optimal capital subsidies is if the labor and capital shocks are negatively correlated and the labor shock is sufficiently strong. This happens because the main reason for taxation in this model is insurance, since taxation reduces the randomness in income. In addition to that, for high levels of risk, increasing the capital tax may actually increase the level of capital since it reduces its riskiness.

This paper is also related to Krueger and Ludwig (2018), who do optimal taxation in an overlapping generations model where agents face uninsurable labor income risk and are able to obtain an analytical solution with logarithmic preferences. Their approach complements our results since in their model the optimal tax is driven by precautionary savings and life cycle effects which are absent from our model.

The paper proceeds as follows. Section 2 sets up the model and derives the competitive equilibrium for the economy. Section 3 derives the optimal policy plan for the benevolent government. Section 4 concludes. The appendix shows the specification of a numerical example that is used to highlight the results of the paper.

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2. MODEL SETUP

This section sets up a model of linear capital income taxation. Agents are subject to an idiosyncratic capital income shock. They supply labor inelastically and lump-sum taxes are allowed. In this model, there is no need to use capital taxation in order to collect revenue, since lump-sum taxes are allowed, so any positive (or negative) taxes that arise are purely so for efficiency reasons.

2.1. Households

There is a continuum of measure one of ex-ante identical households who discount the future at rate β. Households derive utility from consumption c_t and supply one unit of labor inelastically. Each household's lifetime utility is thus given by

$$\sum_{t=0}^{\infty} \beta^t E\left[u(c_t)\right],$$

where u is increasing, concave, and meets the usual Inada conditions.

Each period, households receive income w_t from their labor plus a labor income shock ϵ_t, and the after tax return on their capital investment, where r_t denotes the deterministic return on capital, η_t denotes each household's individual shock to this return, and δ is the depreciation rate. Households can spend their income in consumption or invest in capital, and they have to pay lump-sum taxes. The shocks η_t and ϵ_t may be correlated but they follow a joint distribution $F(\eta, \epsilon)$ that is iid across time, has mean zero, and is bounded from below, so that they do not lead to negative consumption in the optimal solution. The households' budget constraint for each period is given by

$$c_t + k_{t+1} + T_t = w_t + \epsilon_t + (1 - \tau_t)(r_t + \eta_t)k_t + (1 - \delta)k_t. \quad (\mathbf{1})$$

The crucial assumption in this paper is that households choose k_{t+1} before the uncertainty in η_t and ϵ_t is realized. In addition to that, all households start period zero with the same initial level of capital k_0.

Given that shocks are iid over time, this implies that all households will make the same investment decisions in each period, which in fact allows us to maintain the representative agent of the model by limiting the effect of the idiosyncratic shock realization to current consumption. This result is proven in the following lemma.

LEMMA 1. *The optimal path of capital investment will be the same for all agents, regardless of the sequence of shocks.*

Proof. We will prove this lemma by showing that if two households (with potentially different histories of shocks) have the same level of capital in period t, then it is optimal for them to choose the same level of capital for period $t + 1$. We will do a proof by contradiction.

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Assume households with the same k_t and different histories of shocks did not choose the same allocations (which may also be a function of future shocks) starting in period t. Then they might have different values of expected utility starting in period t. But this means that it would be optimal for all households to choose the allocations associated with the highest of these expected utilities. Notice that this is feasible since it would not violate any past or future constraints. Finally, if the allocations are the same starting in period t, this necessarily means that k_{t+1} is the same for all agents with the same k_t since k_{t+1} cannot depend on the realization of future shocks, since by assumption it must be chosen before the realization of η_t and ϵ_t.

Given that all households start off with the same level of capital k_0, and agents with the same k_t have the same optimal choice of k_{t+1}, it follows that the optimal choice of capital is the same for all agents and is independent of the sequence of shocks. ■

Since the level of capital does not depend of the sequence of shocks, the households' budget constraint (1) implies that consumption will only depend on the current shock to η_t and ϵ_t.

Households maximize expected utility subject to their budget constraints. Writing the Lagrangian and taking first-order conditions, we reach the following Euler equation:

$$E\left[u'\left(c_t(\eta_t,\epsilon_t)\right)\right]=\beta E\left[u'\left(c_{t+1}(\eta_{t+1},\epsilon_{t+1})\right)\left(1+(1-\tau_{t+1})(r_{t+1}+\eta_{t+1})-\delta\right)\right]. \tag{2}$$

Together with the budget constraints, this equation determines the optimal solution to the households' problem.

2.2. Firms

The production function $F(k_t, l_t)$ has constant returns to scale and is concave. Furthermore, assume that the usual Inada conditions are met. The output of each firm is given by the production function plus the additive idiosyncratic return on the factors of production that it is using. So, the firm it acquires labor from household i and capital from household j, its output will be $F(k_t^j, l_t^i) + \eta_t^j k_t^j + \epsilon_t^i l_t^i$. Since firms behave competitively, they will pay $r_t + \eta_t^j$ for each unit of capital acquired from household j and $w_t + \epsilon_t^i$ for each unit of labor acquired from household i, where w and r are given by

$$r_t = F_k(k_t, l_t) \quad w_t = F_l(k_t, l_t). \tag{3}$$

The total output produced in the economy is still given by the production function $F(k_t, l_t)$ and is deterministic since $E[\eta_t] = E[\epsilon_t] = 0$.

Although we have assumed that there is idiosyncratic capital risk, there is still a perfectly competitive capital market. This way, each household receives the same return from the market, plus their individual productivity shock. The same is true for the labor income shock.

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2.3. Government and Market Clearing

Each period, the government uses lump-sum taxes (or transfers) to balance its budget, which is given by

$$T_t = E\left[-\tau_t(r_t + \eta_t)k_t\right] = -\tau_t r_t k_t. \tag{4}$$

The resource constraint for the economy in period t is given by

$$E\left[c_t(\eta_t, \epsilon_t)\right] = F(k_t, l_t) + (1-\delta)k_t - k_{t+1}. \tag{5}$$

Furthermore, total labor demand must equal supply, which is inelastic and equal to one, so that

$$l_t = 1. \tag{6}$$

2.4. Competitive Equilibrium

DEFINITION 1. *A competitive equilibrium is a sequence of allocations* $\{c_t(\eta_t, \epsilon_t), k_{t+1}, l_t\}_{t=0}^{\infty}$, *prices* $\{r_t, w_t\}_{t=0}^{\infty}$, *and taxes* $\{\tau_t, T_t\}_{t=0}^{\infty}$ *such that* (*i*) *given a sequence of prices and taxes, and given initial capital* k_0, *households maximize their expected utility subject to (1),* (*ii*) *given a sequence of prices, firms maximize profits,* (*iii*) *the government's budget constraints (4) are met, and* (*iv*) *markets clear, so that conditions (5) and (6) hold.*

In many representative agent taxation models, it is common to summarize the conditions for the competitive equilibrium in an implementability condition plus resource constraint. In this problem we are not able to do that due to the existence of lump-sum taxes and idiosyncratic risk, but we are still able to summarize the competitive equilibrium in two single equations.

LEMMA 2. *If a sequence of consumption, capital, and capital taxes* $\{c_t(\eta_t, \epsilon_t), k_{t+1}, \tau_t\}_{t=0}^{\infty}$ *meets the conditions below for a given* k_0, *then it can be implemented as a competitive equilibrium. If a sequence of consumption, capital, and capital income taxes meets the conditions below, then it can be implemented as a competitive equilibrium.*

$$c_t(\eta_t, \epsilon_t) = F(k_t, 1) + \epsilon_t + (1-\delta)k_t + (1-\tau_t)\eta_t k_t - k_{t+1} \qquad \forall \{\eta_t, \epsilon_t\}_{t=0}^{\infty} \tag{7}$$

$$E\left[u'\left(c_t(\eta_t, \epsilon_t)\right)\right] = \beta E\begin{bmatrix} u'\left(c_{t+1}(\eta_{t+1}, \epsilon_{t+1})\right) \\ (1 + (1-\tau_{t+1})(F_k(k_{t+1}, 1) + \eta_{t+1}) - \delta) \end{bmatrix} \quad t = 0, ..., \infty. \tag{8}$$

Proof. Given our definition of competitive equilibrium, we find conditions (1)–(6) are necessary and sufficient for a competitive equilibrium. By Walras' Law, we can disregard the resource constraint (5). The problem can be further

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simplified by replacing market prices lump-sum taxes, and labor from equations (3), (4), and (6) into the remaining equations, which leads to the conditions above. ■

The first equation that characterizes our competitive equilibrium is an "Individual Resource Constraint" that highlights the fact that households will have different levels of consumption because they are subject to different individual shocks. If we aggregate this condition, we will reach our usual aggregate resource constraint. The second equation is one single Euler equation for all households because the investment decision is made before the shocks are realized.

We can see from the equations above that the capital income tax plays two roles: it reduces the impact of capital income risk on consumption, which provides insurance, but it also decreases the return on capital for households.

Notice that in this model the only asset available to consumers is capital. If we had allowed government debt, then households would be able to hold different levels of debt depending on their individual shock, which would make all choices dependent on the past history of shocks. The only way to introduce government debt in the model and maintain our results is if households must choose how much debt to hold before the realization of shocks, similar to what happens with capital investment. Under this new assumption, Ricardian equivalence holds due to the existence of lump-sum taxes, so there is no welfare loss due to the fact that we impose budget balance, contrary to what happens in most Ramsey models (for a discussion on the welfare loss that may arise due to balanced-budget rules, see Stockman (2001)).

2.5. Steady-State Effects

In this section, we analyze the effect of capital taxes on the steady-state level of capital. In steady state, the level of capital is determined by the stationary solution to equations (7) and (8).

If there is no capital income risk ($\eta = 0$), then in steady-state households will choose capital such that $\beta\,(1 + (1 - \tau)F_k(k, 1) - \delta) = 1$, so the level of capital is equal to the one in an economy with no idiosyncratic risk. This is true even when there is individual uncertainty in the labor income, so this model does not exhibit precautionary savings motive, which means that all the effects that traditionally arise from that channel will not be present. In this case, an increase in the capital tax always discourages capital investment since it reduces the return on capital.

However, if there is capital income risk, a higher capital tax also reduces the covariance between consumption and the capital income shock, which increases steady-state capital. Whether increasing the capital tax reduces or increases the level of capital depends on the trade-off between these two effects.

Figure 1 illustrates the effect of different levels of capital taxes on the steady-state level of capital by comparing economies with different levels of capital income risk. The economies on the left have no labor income risk, whereas the

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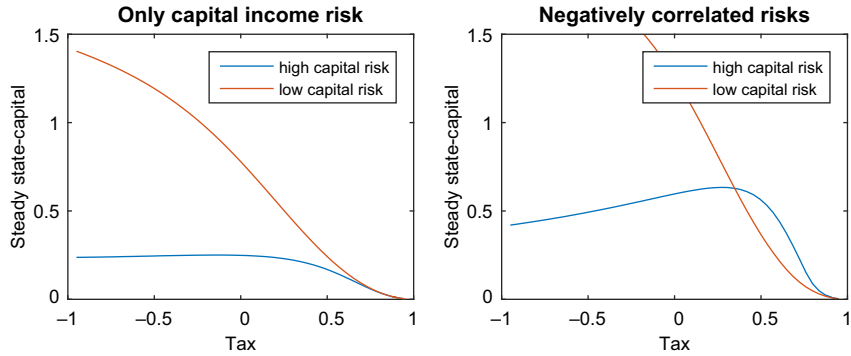


FIGURE 1. Monotonicity of capital with respect to the capital income tax for different levels of risk.

economies on the right have a labor income shock which is negatively correlated with the capital income shock. For this example, steady-state capital would be one if there were complete markets (no idiosyncratic risk) and no taxes. Details are provided in the appendix.

With low capital income risk, the economy essentially behaves as under complete markets, with capital decreasing monotonically with the tax. However, for high capital income risk, we can observe a non-monotonicity of the capital level with respect to the tax. This effect can be even stronger if we add a labor income shock, as we can see in the right pane.

This non-monotonicity of capital with respect to the capital income tax is also present in Panousi (2015), and despite our simplifying assumption we are able to fully replicate this feature of the model with idiosyncratic capital income risk even with our different timing assumption.

3. OPTIMAL CAPITAL TAXES

In this section, we determine the ex-ante optimal tax profile chosen by a benevolent planner who chooses the competitive equilibrium that maximizes the utility of a representative household. Given initial capital k_0, the planner chooses the sequence $\{c_t(\eta_t, \epsilon_t), k_{t+1}, \tau_t\}_{t=0}^{\infty}$ that maximizes expected utility subject to the conditions of Lemma 2

$$\max_{\{k_{t+1},\tau_t,c_t(\eta_t,\epsilon_t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t E\left[u\left(c_t(\eta_t, \epsilon_t)\right)\right]$$

$$st \quad c_t(\eta_t, \epsilon_t) + k_{t+1} = F(k_t, 1) + \epsilon_t + (1-\delta)k_t + (1-\tau_t)\eta_t k_t \qquad \forall \{\eta_t, \epsilon_t\}_{t=0}^{\infty}$$

$$E\left[u'\left(c_t(\eta_t, \epsilon_t)\right)\right] = \beta E\begin{bmatrix} u'\left(c_{t+1}(\eta_{t+1}, \epsilon_{t+1})\right) \\ (1+(1-\tau_{t+1})(F_k(k_{t+1}, 1) + \eta_{t+1}) - \delta) \end{bmatrix} \quad t = 0, ..., \infty.$$

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Associating condition (7) with the multiplier $\beta^t \rho_t(\eta_t, \epsilon_t) f(\eta_t, \epsilon_t)$, and condition (8) with the multiplier $\beta^t \mu_t$, we have the Lagrangian

$$\begin{aligned} L = &\sum_{t=0}^{\infty} \beta^t E\left[u\left(c_t(\eta_t, \epsilon_t)\right)\right] - \sum_{t=0}^{\infty} \beta^t \mu_t E\left[u'\left(c_t(\eta_t, \epsilon_t)\right)\right] \\ &- \sum_{t=0}^{\infty} \beta^t E\left\{\rho_t(\eta_t, \epsilon_t)\left[c_t(\eta_t, \epsilon_t) + k_{t+1} - F(k_t, 1) - \epsilon_t - (1-\delta)k_t - (1-\tau_t)\eta_t k_t\right]\right\} \\ &+ \sum_{t=0}^{\infty} \beta^{t+1} \mu_t E\left[u'\left(c_{t+1}(\eta_{t+1}, \epsilon_{t+1})\right)\left(1 + (1-\tau_{t+1})(F_k(k_{t+1}, 1) + \eta_{t+1}) - \delta\right)\right]. \end{aligned}$$

The first-order conditions for consumption, capital investment, and capital taxes are

$$\begin{aligned} u'\left(c_t(\eta_t, \epsilon_t)\right) = &\rho_t(\eta_t, \epsilon_t) + \mu_t u''\left(c_t(\eta_t, \epsilon_t)\right) \\ &- \mu_{t-1} u''\left(c_t(\eta_t, \epsilon_t)\right)\left(1 + (1-\tau_t)(F_k(k_t, 1) + \eta_t) - \delta\right), \\ E\left[\rho_t(\eta_t, \epsilon_t)\right] = &\beta E\left[\rho_{t+1}(\eta_{t+1}, \epsilon_{t+1})\left(1 + F_k(k_{t+1}, 1) + (1-\tau_{t+1})\eta_{t+1} - \delta\right)\right] \\ &+ \beta \mu_t E\left[u'\left(c_{t+1}(\eta_{t+1}, \epsilon_{t+1})\right)(1-\tau_{t+1}) F_{kk}(k_{t+1}, 1)\right] \end{aligned}$$

$$-E\left[\rho_t(\eta_t, \epsilon_t)\eta_t k_t\right] = \mu_{t-1} E\left[u'\left(c_t(\eta_t, \epsilon_t)\right)\left(F_k(k_t, 1) + \eta_t\right)\right].$$

The optimal tax for the initial period is given by $E\left[\rho_0(\eta_0, \epsilon_0)\eta_0\right] k_0 = 0$. Since η has zero expected value, this implies that the covariance between the shock and the multiplier on condition (7) must also be zero. When there is only capital income risk this is achieved with $\tau_0 = 1$, which makes consumption constant in all states of nature in the initial period. This is similar to many taxation models, since taxing initial capital does not distort the incentives to save since the initial level of capital is predetermined. However, here the reason to tax capital is not to collect revenue, since lump-sum taxation is allowed, but to provide insurance, since a capital tax of 100% eliminates the variation in income due to different rates of return on capital.

Let us now turn to analyzing the steady-state features of the optimal tax. To do this, we will first consider each type of risk separately, and then we turn to analyzing the joint effects of the two.

3.1. Only Capital Income Risk

In this section, we will assume that there is only capital income risk, so $\epsilon = 0$ always. In this setting, we will show that the optimal capital income tax is strictly positive in steady state as long as there is capital income risk, and that the optimal taxes are never in the region where capital is increasing with the tax rate, so that the optimal tax is higher than the tax that maximizes steady-state capital.

PROPOSITION 1. *In steady state, the optimal capital tax is strictly positive if there is only capital income risk.*

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Proof. We will show that it is not optimal to have zero capital taxes. Furthermore, when the capital tax is equal to zero, utility is increasing in the capital tax, which implies that utility achieves its maximum for a positive tax rate.

If capital taxes are zero, condition (8) does not bind in steady state, which implies that $\mu = 0$. With $\mu = 0$, the derivative of the Lagrangian with respect to the capital tax is given by

$$\left.\frac{\partial L}{\partial \tau}\right|_{\tau=0} = -E\left[\rho(\eta,\epsilon)\eta\right]k.$$

Furthermore, consumption varies positively with the shock η and from the first-order condition for consumption, we know that $u'\left(c_t(\eta,\epsilon)\right) = \rho_t(\eta,\epsilon)$. Since $\rho_t(\eta,\epsilon)$ is negatively correlated with η, this derivative is strictly positive, and utility can be increased by choosing a positive capital tax. ■

Thus, the existence of capital income risk is a reason that may justify positive capital income taxes.

PROPOSITION 2. *Optimal capital taxes are never in a region where capital is increasing with the tax rate.*

Proof. Suppose the capital tax τ_{t+1} is in a region where capital is increasing with the tax rate. Then there is a higher capital income tax that leads to the same capital choice (so it meets the Euler equation (8)) but leads to lower volatility in consumption in equation (7), and thus to higher utility, which means the initial value of τ_{t+1} cannot be optimal. ■

To see how the level of taxes varies with the degree of riskiness, we now turn to a numerical example. Figure 2 illustrates the optimal taxes in the presence of capital income risk. Details are provided in the appendix. The optimal tax increases monotonically with the magnitude of the shock. Although steady-state capital is almost always reduced by the optimal taxes, it does not significantly differ from the level of capital with no taxes. This is because for higher levels of risk, where the optimal tax is higher, the relationship between tax and capital becomes nonmonotone, as we saw in Figure 1. In this example, what lowers steady-state capital is the existence of idiosyncratic risk, and not the optimal tax per se. In the bottom panes, we can see that consumption becomes less volatile when the optimal taxes are used.

For this very stylized example, we are able to generate capital income taxes of up to 20%. However, our model will probably exacerbate the need for insurance since all the shock is absorbed by consumption in a single period. Even so, it taxes a very high level of capital income risk (100% of invested capital) to reach this number. This seems to suggest that even though the optimal capital income tax is positive, it is probably of a small magnitude.

In this model, capital income taxes are distortionary since they tax both the market and the idiosyncratic return on capital. If the government was allowed to

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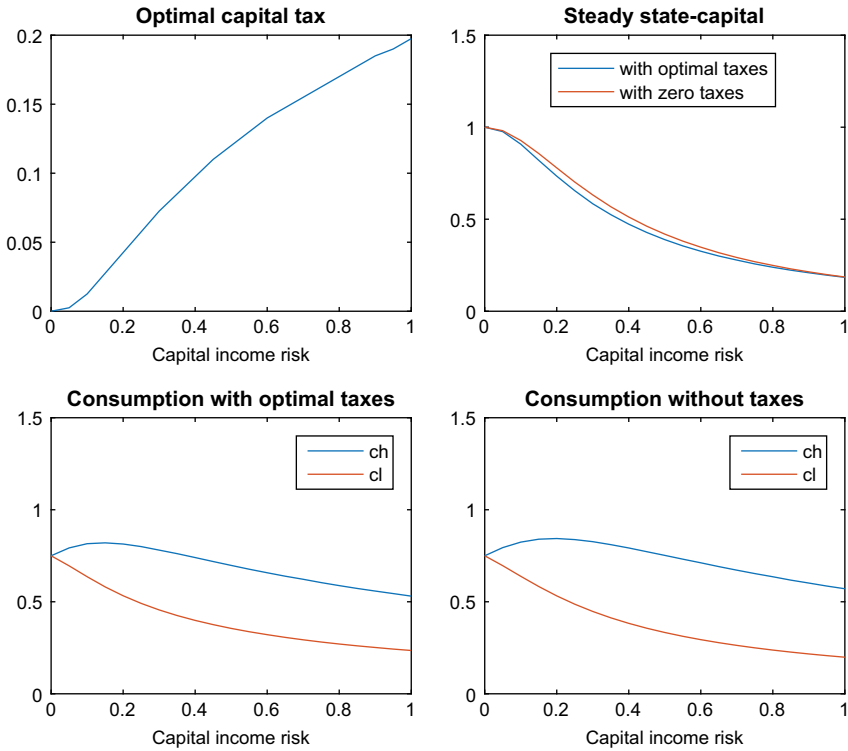


FIGURE 2. Optimal taxes with only capital income risk.

tax these two components at different rates, then it would be possible to achieve first best by using a zero tax on the market return r_t, together with a confiscatory tax on the excess return (this would actually be a subsidy whenever η is negative). In reality, this may not be very easy to achieve, specially since the market return may itself vary due to aggregate risk.

3.2. Only Labor Income Risk

In this section, we will assume that there is only labor income risk, so $\eta = 0$ always.

PROPOSITION 3. *In steady state, the optimal tax is zero if there is only labor income risk.*

Proof. If $\eta = 0$, the first-order condition for the optimal tax becomes $0 = \mu F_k(k, 1) E\left[u'\left(c(\eta, \epsilon)\right)\right]$, which implies that $\mu = 0$. If this is the case, the first-order condition for capital becomes $1 = \beta\left(1 + F_k(k_{t+1}, 1) - \delta\right)$, which implies that the capital income tax is zero. ■

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With only labor income risk, the optimal capital tax is zero in this model. This result differs from Aiyagari (1995), where the optimal tax on capital is positive, but it is not a surprising result since in Aiyagari there was a strong precautionary savings motive due to the borrowing constraints that were imposed.

We have assumed throughout that labor income taxes are not allowed. If we allowed labor income taxes in our setup, then with only labor income risk it would be optimal to fully tax labor income, in order to eliminate the uncertainty in labor income. This happens because labor is exogenous, so labor income taxes do not distort the labor supply choice. If instead labor was endogenous, then the optimal labor income tax would be determined by the trade-off between providing insurance and distorting labor supply.

3.3. Both Types of Risk

We now assume both types of risk are present, and are possibly correlated.

PROPOSITION 4. *The capital income tax can be either positive or negative when both types of risk are present.*

Proof. We will start by showing that at zero capital taxes, the first-order condition for the optimal tax can be either positive or negative, depending on the correlation between the two shocks.

If capital taxes are zero, condition (8) does not bind in steady state, which implies that $\mu = 0$. With $\mu = 0$, the derivative of the Lagrangian with respect to the capital tax is given by

$$\left.\frac{\partial L}{\partial \tau}\right|_{\tau=0} = -E\left[\rho(\eta,\epsilon)\eta\right]k.$$

From the first-order condition for consumption, we know that $u'\left(c(\eta,\epsilon)\right) = \rho_t(\eta,\epsilon)$. The level of consumption depends on the two shocks in the following way:

$$c(\eta,\epsilon) = F(k,1) - \delta k + \epsilon + \eta k.$$

If ϵ and η are positively correlated, then $E\left[\rho(\eta,\epsilon)\eta\right] < 0$, and the first-order condition is positive at zero taxes, which means that the optimal tax is positive.

If ϵ and η are negatively related, then the optimal tax can be either positive or negative. If $\epsilon + \eta k$ and η are positively correlated, then $E\left[\rho(\eta,\epsilon)\eta\right] < 0$, and the optimal tax is still positive. However, if $\epsilon + \eta k$ and η are negatively correlated, then $E\left[\rho(\eta,\epsilon)\eta\right] > 0$, and the optimal tax becomes a subsidy. ■

If there is capital and labor income risk, the optimal capital tax can be either positive or negative. If the two shocks are positively correlated, the optimal capital tax is positive. A more interesting case occurs when the two shocks are negatively correlated, since in this case the optimal capital tax can be positive or negative.

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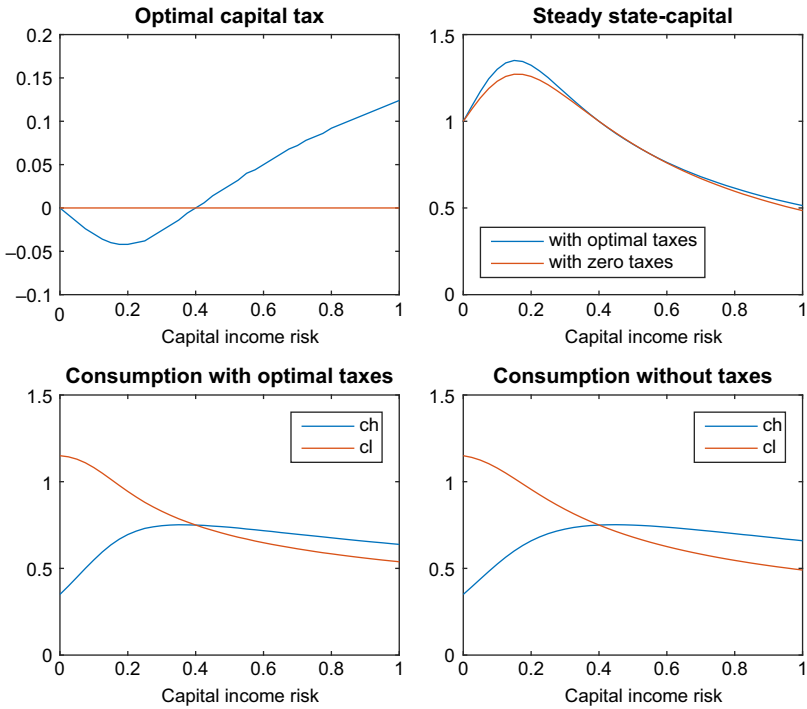


FIGURE 3. Optimal taxes with negatively correlated capital and labor income risk.

To gain a better understanding of the steady-state effects, we fix the variance of the labor income shock, but we allow the magnitude of the capital income shock to vary.

Figure 3 illustrates the optimal taxes in the presence of negatively correlated labor and capital income risk. Details are provided in the appendix. In this case, the optimal tax is negative for low capital income risk, and positive for high risk levels. This is reminiscent of Panousi and Reis (2016), where the optimal tax was positive for high levels of risk but negative for low levels of risk (although for different reasons). Now steady-state capital tends to be higher when optimal taxes are used. This happens for two reasons. First, for low levels of risk the optimal tax is a subsidy. Second, for high levels of risk the presence of the labor income shock exacerbates the non-monotonicity of capital with respect to the tax rate. For low levels of risk, capital is actually higher than under complete markets, since it provides insurance against the labor income shock. Just like in Figure 2, we can see that consumption becomes less volatile when the optimal taxes are used. This is true even when the optimal tax is a subsidy because in this case the capital income shock (which is relatively small) is actually insuring against the labor income shock.

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4. CONCLUDING REMARKS

If households are subject to idiosyncratic capital and labor income risk, it is optimal for taxes to provide partial insurance against the income shocks. In general, this is achieved through a positive capital income tax. However, if the labor income shock is large and negatively correlated with the capital income shock, the optimal policy may be a capital subsidy. Thus, the conventional wisdom that capital taxes should be zero in the long run no longer applies in this setting.

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APPENDIX A: PARAMETERS FOR NUMERICAL EXAMPLE

We assume individual uncertainty follows a discrete uniform distribution where the capital income shock η can take values θ and $-\theta$ with 50% probability each. By varying the value of θ, we can change the degree of uncertainty in the model. We consider two possibilities for the labor income shock ϵ. Either there is no labor income shock and $\epsilon = 0$, or there is a labor income shock which is perfectly negatively correlated with the capital income

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shock such that $\epsilon = 0.4$ when $\eta = -\theta$ and $\epsilon = -0.4$ when $\eta = \theta$. By varying θ, we vary the magnitude of the capital income shock keeping the labor income shock constant. Utility is isoelastic and the production function is Cobb–Douglas, with functions given by $u(c) = c^{1.5}$ and $F(k, 1) = k^{0.5}$. Furthermore, the discount factor is $\beta = 0.8$ and depreciation is $\delta = 0.25$.

For Figure 1, we consider a high capital risk of $\theta = 0.8$ and a low capital risk of $\theta = 0.2$.

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