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THE CHANGING PLACE OF VISUAL REPRESENTATION IN ECONOMICS: PAUL SAMUELSON BETWEEN PRINCIPLE AND STRATEGY, 1941–1955

BY
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In this paper, we show that Paul Samuelson (1915–2009), renowned as one of the main advocates of the mathematization of economics, has also contributed to the change of the place of visual representation in the discipline. In his early works (e.g. Foundations of Economic Analysis published in 1947), he rejected diagrammatic analysis as a relevant tool of theorizing but used diagrams extensively, both as a pedagogic tool in his introductory textbook Economics (1948) and as a way of clarifying his theory of public expenditure (1954-5). We show that Samuelson's reluctance to use diagrams in his early works can be explained by his training at Chicago and Harvard and his rejecting Marshall's economics, whereas his adoption of visual language in Economics was a product of the peculiar context affecting American mass-education after WWII. A methodological debate which opposed him to Kenneth Boulding in 1948 led him to reconsider the place of visual representation in order to clarify conceptual controversies during subsequent debates on mathematical economics. Therefore, it can be said that the prominent place of visual language in the diffusion of economic ideas was stabilized in the mid-1950s, as mathematical language became the prevailing tool of economic theorizing. From this, we conclude that the idea that algebra simply upstaged geometry in the making of economic analysis must be qualified.

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I. INTRODUCTION

In the past twenty years or so, historians, sociologists, and methodologists of science have emphasized the significance of visual representation, testifying to what Bruno Latour describes as "the extraordinary obsession of scientists with papers, prints, diagrams, archives, abstracts and curves on graph paper" (Latour 1988, p. 39).[1] Their contributions allowed for a new focus on the history of science, presenting seemingly familiar episodes in a new fashion. Following this trend, some historians of economics have begun to interest themselves in visual representations, stressing their importance from the emergence of political economy in the eighteenth century to the making of nineteenth-century neoclassical economics (Charles 2003, 2004; Maas and Morgan 2002; Cook 2005). However, with the exception of Derobert and Theriot (2003) and De Marchi (2003), who studied the uses of a number of diagrams in postwar economics, little has been written on that highly significant period for the discipline.[2]

This is not to say that modern economics is free from visualization. It would require only a little imagination to realize how important visual representation has been in shaping the discipline's workaday tools, and how those tools are still firmly entrenched in the contemporary economist's activities. Yet it is hardly arguable that visual language is still seen as a powerful tool, one that could help push the boundaries of economic research. In the early 1960s, when Paul Samuelson (1915–2009) wrote a piece celebrating fellow economist Abba Lerner's sixtieth birthday, he made clear that diagrams were no longer the research engines they used to be. "At first," he observed, "Lerner was known as a diagram man. In those days [the early 1930s], graphs did represent the frontier of our science" (Samuelson 1964a, p. 170), referring to an evolution of the discipline to which he himself contributed. As the author of *Foundations of Economic Analysis*, published in 1947, Samuelson may be seen as one of the most ardent defenders of a type of mathematics, made of matrices and difference equations, which questioned the usefulness of the kind of diagrammatic analysis that was so central in the practice of economists less than ten years before, and this is one of the reasons which could explain why he has been pointed out as the founder of "modern economic theory."[3] On the other hand, the same Paul

[1]In methodology, see Larkin and Simon (1987). Recently, a special issue of *Foundations of Science* (2004) on "Model-Based Reasoning" gave a central place to visualization. In history of science, see Daston and Galison (1992), Lightman (2000), Hankins (1999, 2006), and the "Focus" section in *ISIS* 97 (1), March 2006, "Science and Visual Culture," pp. 75–132.

[2]Charles (2003) and (2004) both deal with the visual history of Quesnay's *Tableau Économique*. Maas and Morgan (2002) and Cook (2005) explore the visual aspect of neoclassical economic theory and it is noticeable that both were published in history of science journals. De Marchi (2003) studies the role of visualization in the representation of the gains from trade from Marshall to Samuelson, and Derobert and Theriot (2003) shows how the use of the Lorenz curve has changed throughout the twentieth century. All these contributions are revised versions of drafts that were presented at the 7th European Conference on the History of Economics held at the University of Quebec at Montreal in 2002, whose purpose was to explore "the role of visual imagination in the history of economics." Also of interest is Leonard (1999), which studies Otto Neurath's engagement with art in the representation of social order.

[3]See Brown and Solow (1983). Not without irony, Deirdre McCloskey (2002) suggests that modern economics, because it relies on qualitative theorems and significance tests, should be called "Samuelsonian" economics.

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Samuelson was also the author of *Economics: An Introductory Analysis* (first published in 1948), a textbook devoted to undergraduate students that gained him a reputation both within and outside of the economics' profession. *Economics* was also a much more visual book than *Foundations*. It has often been asserted that his diagrammatic presentation of the theory of income determination strongly contributed to the dissemination of Keynesianism in the United States.[4] In view of the increasing importance of visual representation both in economics textbooks and in the field of economic education over the past fifty years, it is incontestable that Samuelson also played a role in this significant feature of recent economics which has not yet been thoroughly examined by historians.[5]

The changing place of visual representation from a useful tool at the core of economic analysis to a pedagogical device may seem too obvious to be investigated. It can be viewed as a consequence of the mathematization of the discipline, or of the transition from partial to general equilibrium analysis, two subjects that have been examined in detail by historians of economics. Yet the relation between the use of visual representation and those important developments of the discipline is anything but self-evident. Mathematical analysis can make use of many visual artifacts, and therefore an increasing use of mathematics could have resulted in more rather than less visualization.[6] Moreover, the increasing use of visualization in economics textbooks could hardly be explained solely by the changes in the tools of theorization. Our perspective in this article is that the changing place of visual representation should be seen in a context of differentiation of the audience. This was a process in which the push for mathematical economics played a significant, but not exclusive, role. The increasing number of students taking economics courses, the birth of a market for mass-education, and the ongoing commitment of some economists to diagrammatic methods in economics may also explain the changes affecting the place of visual representation in the discipline. Because Samuelson played a central role in these developments, we propose in this paper a contextualization of his attitude toward visual representation. We show that his reluctance to use diagrams in his early works such as *Foundations* and his endorsement of diagrams in *Economics* and subsequent articles were dictated by different circumstances, corresponding to different audiences. Samuelson promoted the use of algebra when much of economic theory continued to rely on geometric analysis and he wrote a very visual textbook when others continued to make extensive use of prose. The subsequent unsympathetic reviews and harsh debates suggest that what appears today as an immutable feature of the discipline was then highly controversial. Our narrative shows that the changing place of visual representation in the discipline parallels the increasing gap between the creation and the diffusion of scientific knowledge.

[4]See Elzinga (1992), Colander and Landreth (1996), Skousen (1997), and Samuelson (1997).

[5]Cohn et al. (2004) observe that visual representations have gained a prominent place in contemporary economic textbooks and give the example of Parkin's *Economics* (2000) with its 417 diagrams (Cohn et al. 2004, p. 41). Concomitantly, graphical analysis has increasingly been discussed in economic education (see, for instance, Wilkins 1992; Kugler and Andrews 1996). Robert Solow (1997) relates the profusion of diagrams, schemas, and tables in current textbooks to the increasing importance of formalization in the discipline.

[6]See for example Emmer (2005), which is devoted to the various relations existing between mathematicians and visual artists.

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Section 2 confronts Samuelson's interest in visual representations as a student and young scholar with the lack of such representations in his early work from 1941 to 1947, culminating with the algebraic *Foundations of Economic Analysis*. It is shown that Samuelson's search for operationally meaningful theorems led him to abandon the diagrammatical analysis that was predominant in the economics of the 1930s. Section 3 studies the visual apparatus of *Economics* (1948). We show that the visual treatment of *Economics* was a response to the constraints associated with mass education in the immediate postwar period. Section 4 examines the various presentations of Samuelson's theory of public expenditure in the 1950s. It is shown that his use of diagrams was motivated by the willingness to increase the communicability of economic results in the profession. Section 5 offers concluding remarks.

II. BREAKING AWAY FROM DIAGRAMMATICAL ANALYSIS: SAMUELSON'S *FOUNDATIONS* (1941–1947)

Early reviewers of Samuelson's *Foundations* often focused on its mathematics for two main reasons. First, though the book could hardly be considered the first attempt to offer a mathematical economic analysis, it was certainly the first which made algebra visible with such ostentation; in its 439 pages, less than hundred were free from any mathematical symbols, and the title page was adorned with physicist J. Willard Gibbs's famous quotation: "Mathematics is a language." Second, for most economists at the time, the mathematics of *Foundations* made it a difficult book. For that reason, many early reviews were written by mathematical economists who attempted to broaden the audience for the book by comparing Samuelson's mathematics to previous contributions and by assessing their accuracy. The importance of the book was acknowledged by the fact that most reviews appeared as journal articles or extended notes.[7] Kenneth Boulding's review article of 1948 was among the dissenting voices. Its author, a moderately mathematically trained economist and soon-to-be recipient of the John Bates Clark medal, intended his contribution as a general statement on the place of mathematics in economics. Like many other reviewers, Boulding realized that Samuelson's book constituted a shift in the way mathematics was used as a tool of theorization in economics, but raised a rather unusual viewpoint. Asking "what kind of mathematics is most useful?" he drew the distinction between the "analytical (algebraic)" and "the geometric method." Boulding pleaded in favor of the latter, which was "a very convenient way of dealing with discontinuities, the description of which is very awkward in analytical terms" (Boulding 1948, p. 191). By contrast, the analytical method used by Samuelson seemed too complicated and lacked generalizability.

Still, Boulding's disagreement with Samuelson's "analytical method" concerned more than the simple matter of the significance of discontinuities. One can understand his critique through the lens of the British diagrammatic economics in which he was trained.[8] This tradition can be traced back to Alfred Marshall's early

[7]See for example Baumol (1949) and Allen (1949).

[8]A former chemistry student from Liverpool, Boulding chose economics as a field of study during a visit to Oxford where he met Lionel Robbins. The latter advised him to read Marshall's *Principles* as an introduction to the discipline. See Boulding (1989), p. 639.

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papers, such as those on the "Theory of Foreign Trade," in which he asserted that "diagrams present simultaneously to the eye the chief forces which are at work, laid out, as it were, in a map; and thereby suggest results to which attention has not been directed by the use of the methods of mathematical analysis" (Marshall 1930 [1879], p. 5).[9] Marshall's ideas on the use of diagrammatic analysis were rooted in the tradition of Cambridge philosophy of science, which considered that the method of curves should be seen as distinct from the rules of mathematics and geometry.[10] Marshall's papers circulated privately, much to their author's discontent, and his method of curves was widely disseminated by his disciple Henry Cunynghame. That may explain why, two decades after Marshall's death, the tradition of diagrammatical analysis was still prevalent among British economists, even among those who had dismissed Marshall's partial equilibrium analysis in favor of the Walrasian general equilibrium framework. John Hicks's *Value and Capital*, published in 1939, reasserted Marshall's preference for diagrams, and strived to find a new diagrammatic method for the treatment of more than two or three simultaneous commodities.[11] Those economists did not necessarily draw on Marshall's philosophical background but they all shared a strong belief in the analytical and communicational virtues of diagrams, and though they were aware of their limits, they seemed reluctant to use algebraic analysis, which they pushed into appendices and endnotes.

In 1931, when Samuelson entered the University of Chicago at the age of sixteen, diagrammatic analysis was not as firmly entrenched in the practice of American economists as it was in Britain. Though Frank Knight, Jacob Viner, and Henry Simons were close to the economic theory that dominated England, theirs was more literary, less mathematical than the English version.[12] Among the Chicagoans, Jacob Viner, the greatest neoclassical economist ever according to Samuelson, followed Marshall's tradition and used diagrams to some extent, especially in his lectures on international trade. Viner, however, could not be considered a visual thinker in the same sense as the British economists already mentioned were. In his *Studies in the Theory of International Trade* (1937), diagrams were not really used as theoretical devices but rather as historical arguments, which the author used to synthesize and appraise past theories. For example, one of Viner's most famous diagrams in this book, which showed the intersection between the production possibility frontier and

[9]This conception was not shared by all of Marshall's contemporaries. Edgeworth, for example, adopted an opposite point of view (see his 1894 article on the "Theory of International Values").

[10]See Klein (1995), Weintraub (2002), and Cook (2005). Klein notes: "By the 1870s, graphs were not substitutes of equations or tables pegged, with apology at the end of a work for the mathematically illiterate; they were tools for exploring and describing phenomena that could not easily be captured by algebra, calculus or words" (Klein 1995, p. 113).

[11]Hicks wrote: "One often hears, particularly from those who are engaged in the study of the most intricate questions, a wish for some method of dealing, at once, with more than two of three variables. Simple problems of two or three variables can be dealt with, quite efficiently, by geometrical diagrams; but when the problem becomes more complex, the familiar geometric method fails. What is to be done? The obvious answer is, Have recourse to algebra. But, quite apart from the fact that many economists are not very good at algebra, the sort of algebraic methods commonly employed, while they are of some use in setting out problems, are much less efficient as a means of argument than diagrams appear to be, when diagrams can be used" (Hicks 1939, p. 1).

[12]In addition, institutionalism was still highly influential in American economics until the late forties (see Yonay 1998).

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the collective indifference curve (see p. 521), was used by the author to introduce a critique of Gottfried Haberler's theory.[13] Viner's blackboard errors were as notorious as his renown as an impressive and frightening professor. Samuelson, who attended Viner's graduate course in the academic year of 1934–1935 though he was an undergraduate in his senior year, gained a reputation as a *wunderkind* by correcting them. Samuelson notes that "Viner had a custom of coming to class with complicated diagrams to be copied on the blackboard. Such transcriptions are notoriously subject to minor errors in which curves intersect on the wrong side of axes and so forth" (Samuelson 1972, p. 8). It is unclear whether those "petty aberrations" hurt Viner's reputation among students, but they probably gave Samuelson a rather jaded view of the rigor and accuracy of diagrams.[14]

On the other hand, when Samuelson entered Harvard University after his graduation in 1935, he was particularly influenced by the ideas and personality of Edwin B. Wilson who taught mathematical economics there.[15] Born in 1879, Wilson was a Yale-educated physicist and mathematician, interested in various social sciences and not only in economic theory. He was Josiah Willard Gibbs's only protégé. Though few students and scholars actually attended his lectures at Harvard, his advocacy for a careful application of the mathematical apparatus used in physics to the social sciences was of importance for the genesis of Samuelson's *Foundations*.[16] In 1928, while Wilson was participating in a roundtable on the future of quantitative economics, joining some neoclassical economists such as Viner and Frank Taussig to oppose the views of Wesley Mitchell, he pleaded for the use of qualitative mathematics in economics. "Many persons seem to believe that mathematics is essentially quantitative, but the first stages in analysis are generally qualitative and mathematics has a large branch which deals with qualitative matters" (in Mills 1928, p. 37). Interestingly, Viner's contribution to the debate was to argue that only the use of a more technical apparatus would lead to a better interaction between theoretical economics and empirical research. Answering the criticism that the foundations of neoclassical economics were too abstract to allow for a good recognition of facts, Viner claimed that, on the contrary, more abstract foundations would help build a more rigorous economic theory. These suggestions, expressed by both Viner and Wilson on the occasion of this roundtable, constituted one of the main orientations of modern neoclassical economics in the post-war period. *Foundations*,

[13]It is very different from the way Abba Lerner used the same diagram in his first published article (1932. Lerner used the diagram to represent various combinations of cost conditions without actually discussing its theoretical background.

[14]Notorious was also the fact that Viner did not draw his diagrams himself. Y. K. Wong did. Wong was a Chinese statistician who, in 1931, attempted unsuccessfully to correct Viner's long-run cost curve envelope, arguing—and rightly so—that it could not be drawn through the minimum points of the short-run average cost curves. This was one of the most famous errors in the history of economics. When Viner, after admitting before the classroom that Wong had been right about the envelope, said privately to the young Samuelson he nevertheless kept on thinking he could trace it, the latter replied: "With a good *thick* pencil, you can do it" (Samuelson 1972, p. 9).

[15]It is noticeable that Samuelson later explained he was drawn to Harvard because of his interest in Chamberlin's theory of monopolistic competition "by miscalculation" (Samuelson 1998, p. 1376).

[16]Samuelson (1998) notes that, apart from himself, the only students in Wilson's seminar were Abram Bergson, Sidney Alexander, and Joseph Schumpeter.

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an adaptation of Samuelson's PhD dissertation, "The Observational Significance of Economic Theory: A Study in the Foundations of Analytical Economics," written under Wilson's supervision, was undoubtedly one of the cornerstones of this larger movement.

More precisely, Samuelson's purpose in *Foundations* was to determine *operationally meaningful theorems*, *i.e*. theorems that would turn abstract economic concepts into a set of observable or measurable data.[17] Provided that these theorems were determined, economic theory could be tested and eventually accepted or refuted in the Popperian sense of the term. According to Samuelson, this process of operationalization would reveal "the unmistakable signs of decadence which were clearly present in economic theory prior to 1930" (Samuelson 1947, p. 4). Samuelson's dedicated tools to show the existence of such theorems were comparative statics and dynamics. Samuelson's only methodological justification for his extended use of algebra and matrices against any other method of inquiry was essentially strategic (*Ibid*., p. 6). Actually, his justification was not so much in the text itself as in the intellectual authorities outside of the economics' profession to which he appealed. In addition to Gibbs's sentence on the title page, Samuelson began his introduction by quoting mathematician Eliakim H. Moore's defense of generalization by abstraction. Turning Marshall's "Burn the Mathematics" upside down, he then dismissed "[t]he laborious literary working over essentially simple mathematical concepts such as is characteristic of much of modern economic theory" as "not only unrewarding from the standpoint of advancing the science" but "involv[ing] as well mental gymnastics of a peculiarly depraved type" (*Ibid*., p. 6).

His critique was not so much addressed to the non-mathematical mode of reasoning as to the process of translation from the mathematical to the literary mode of exposition. Given that for some economists diagrams were the appropriate tools to address the poorly mathematically-trained reader, it is a little surprising that *Foundations* had so few of them. While mathematical economics books, such as Irving Fisher's *Mathematical Investigation in the Theory of Value and Prices* or Hicks's *Value and Capital*, contained a number of innovative visual artifacts meant to ease the way for more complicated mathematics, Samuelson's book deliberately avoided them. The six diagrams included in *Foundations* were intended as mere illustrative devices. This was particularly obvious from the section devoted to index numbers. Earlier contributions on index numbers, such as Hans Staehle's and Abba Lerner's articles in the *Review of Economic Studies*, involved an extensive use of diagrams.[18] By contrast, Samuelson used only one diagram, which added little information to the words used to conduct the demonstration. Though Samuelson sporadically referred to geometry and curves throughout the book, he depicted those using words, not diagrams (see, for example, his treatment of cost curves, p. 79). Two

[17]Exemplary of this research program was also Samuelson's theory of consumer's behavior he had first published in 1938, though his attempt to eschew the theory of utility in this article was slightly undermined in several subsequent articles and in *Foundations* (see Hands and Mirowski 1998).

[18]On the other hand, when Samuelson criticized the "fondness" economists had developed for "certain dimensionless expressions called elasticity coefficients," noting that they were only "mental exercises for beginning students" (Samuelson 1947, p. 125), he was certainly referring to some of the works Lerner and his fellow LSE scholars had done at the beginning of the previous decade.

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of the diagrams were reinventions of Marshall's curves in his "Pure Theory of Foreign Trade," which Samuelson used only to show that his own mathematical analysis was a generalization of Marshall's (*Ibid*., p. 266).[19] In addition, Samuelson dismissed all economic concepts which could not benefit from his search for operationally meaningful theorems. Most of those, indeed, had been defined and explored visually by previous contributors, such as complementarity (*Ibid*., p. 183) and consumer's surplus (*Ibid*., p. 195).[20] In short, the quasi-absence of visual representation from *Foundations* reflected not so much the author's lack of interest in diagrams but rather a rough rejection of entire sections of economic theory which had previously been framed in diagrammatic analysis. Methodological justifications aside, the use of a different language of inquiry was required to enhance the book's novelty and reinforce the feeling that Samuelson was departing from previous economic theorizing. This strategy was especially unusual given that in theoretical books, unlike in scientific articles, greater attention was given to audience, so that fresh theories were usually presented in a rather polished and palatable fashion. In Hicks's *Value and Capital*, for example, the use of diagrammatic analysis testified to a bargain between accuracy and communication. For Samuelson, however, it seems that the greater accuracy allowed for by the use of algebra could not be bargained away in a book devoted to the advancement of economic theory.

Actually, as early reviewers unanimously observed, communication was not one of Samuelson's main concerns in *Foundations*. Metzler (1948) and Carter (1950) noted that the final mathematical appendix, which was intended as a brief textbook on difference equations, was hardly readable for the uninitiated. Carter added that *Foundations*' mathematics was "often obscure; and the obscurity often arises, not from the inherent difficulties of the subject, but from the use of a notation which is unexplained, or difficult to be remembered over several pages of argument, or excessively general" (Carter 1950, p. 51). Roy Allen found Samuelson's mathematics "not simplified enough for the economist and not rigorous enough to satisfy the mathematician" (Allen 1949, p. 114). George Stigler considered Samuelson's "failure to provide translations for the 'literary' economist a serious shortcoming of his work," pleading for a greater "responsibility to the canons of scholarship" (Stigler 1948, p. 605). William Baumol emphasized the more rhetorical side of *Foundations*' algebraic apparatus, suspecting that "one will be left with the feeling that it is occasionally overdone here" and assuming that Samuelson was "too unwilling to adapt himself to the needs and limitations of his potential readers" (Baumol 1949, p. 160).[21] Most reviewers argued that Samuelson's mathematics was unnecessarily complicated, but Boulding went further. "[T]here is among mathematical economists a certain feeling (from which Samuelson is by no means exempt)

[19]It is also interesting to note that all but one diagrams were included in two chapters that had been previously published in *Econometrica* (chapters IX and X). One might infer from this that Samuelson's original PhD dissertation was even less visual than the published book.

[20]Nicholas Georgescu-Roegen's "A diagrammatic analysis of complementarity" (1952) was a response to Samuelson's criticism.

[21]We are not insisting here on the number of typographical errors and other mistakes that the authors have raised in their reviews, for we know that careless proof correction was due to Samuelson's intricate relationship with Harvard University Press, as explained in Samuelson 1998.

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that there is something childish and elementary about geometrical treatments and that analytical treatment is the only ultimately satisfactory method'' (p. 191). Noting that Samuelson's algebra added ''much to the aesthetics of economics but surprisingly little to its substance,'' he remarked that in some cases algebra could even be inferior to diagrammatic analysis, the latter being ''much more suited to dealing with functions 'in the large' than the analytic [algebraic] method'' (*Ibid.*, p. 192). Samuelson succinctly answered Boulding's criticism in private correspondence: ''Any controversy between geometry and analysis,'' he noted, ''involves *strategy* not *principle*.''[22] His answer was not mere rhetoric. Samuelson did not dismiss diagrammatic analysis as an engine of discovery; ideally, geometry should be on an equal footing with algebra as an analytical device. His reluctance to use diagrams in *Foundations* reflected his opposition to careless and inaccurate practices by the uninitiated who had been misled by the apparent accessibility provided by the geometrical method of exposition. By adopting the algebraic mode of exposition, Samuelson thought he would erase the illusion of simplicity, and make the field of pure theory a domain for specialists. Communication, on the other hand, could only be worked out, as he wrote to Boulding, ''provided that this is not pushed so far as to slow down unduly the development at the frontier.''[23]

III. INTRODUCING ECONOMIC THEORY THROUGH VISUALIZATION: SAMUELSON'S *ECONOMICS* (1948)

Published in the wake of *Foundations*, *Economics: An Introductory Analysis* was a textbook intended for undergraduate students. Following Lorie Tarshis's *The Elements of Economics*, it was the second textbook to incorporate the Keynesian message. As noted by Kenneth Elzinga, its commercial success could be explained by two rather unusual diagrams: the circular flow diagram which had its roots in Frank Knight's wheel of wealth, and the Keynesian Cross, which ''became the standard format to teach the Keynesian system to principles students'' (Elzinga 1992, p. 863).[24]

Despite what Elzinga implies, the idea that diagrams helped students better understand economics was not widely accepted among the economics profession in the interwar period, and most economists were then reluctant to use visual representation for pedagogical purposes. Reviewing a number of textbooks and

[22]Samuelson to Boulding, July 13th 1948, *Kenneth Ewart Boulding Papers*, Bentley Historical Library, University of Michigan, Box 3. Emphasis by Samuelson. I am grateful to Philippe Fontaine for drawing my attention to this correspondence.

[23]Samuelson to Boulding, July 13th 1948, *Boulding Papers*, Box 3.

[24]According to Elzinga, and based on McGraw-Hill sales figures, 42,757 copies were sold for only the year of 1948 and the first edition sold 121,453. The highest sales encountered were for the sixth edition, which sold more than 400,000. For an account of the origin of Knight's wheel of wealth and its influence on Samuelson's circular flow diagram, see Patinkin (1973). On the influence of Samuelson's Keynesian Cross see also Blaug (1984): ''[Samuelson's] elementary textbook, *Economics*, containing his invention of the '45-degree Keynesian Cross' diagram to depict the determination of national income, was a major factor in the Keynesian conquest of economics departments in the years after World War II'' (Blaug 1984, p. 248).

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presenting his views on economic education, Charles E. Persons wrote in 1916: ''Mere outlines of lectures—notes made to save the students' ink—are valueless. Of similar sort are the elaborate endeavors to simplify the study of economics through the use of diagrams and illustrations. It is not thus that one thinks effectively of things economic. . . . Our function is not to painlessly and surreptitiously make lodgment of our doctrines in the student mind. It rather behooves us to stir him to high endeavor and give continual mental exercise to his mental muscle; to send him out finally well exercised in economic thinking and confident of his ability to perform well in that field'' (Persons 1916, p. 97). Such reluctance was discernable even among those economists who used diagrams extensively as research engines. In Marshall's *Principles*, most diagrams appeared in footnotes. His *Economics of Industry*, which was an abridged version of *Principles* intended for junior students, contained even fewer diagrams. Lerner's 1944 book, *Economics of Control*, had only a few diagrams, though it included some of the results its author had obtained graphically in a series of articles published between 1933 and 1936. When asked by his friend Mary Wise why he did not choose a graphical exposition, Lerner replied: ''I think you are right about the advisability of having put some diagrams in for the sake of making it easier, but I was much of the time thinking of getting others than university students of economics to read the book and these would have been frightened by diagrams and not helped by them much, if at all.''[25] Taussig's *Principles of Economics*, still one of the most influential textbooks in the early postwar period though its first edition had been published in 1911, was a very austere book with only a dozen diagrams. A notable exception, Boulding's textbook *Economic Analysis* (1941) had 158 diagrams, but it was conceived not only as a book for students but also as a theoretical contribution, which may explain why the complexity of its graphical apparatus was criticized by most early reviewers. Apart from this, there was the abiding feeling among the economics profession that diagrams were not for everyone, especially for the students taking economics courses as a minor subject.

The scarcity of visual representations in economics textbooks contrasted with the fact that American mass culture had become increasingly visual since the end of WWI. Created in 1912 by Paul Kellogg, the liberal magazine *Survey Graphic* promoted the use of visual representation as a way to diffuse the results of social scientific research to a larger audience, using statistical charts and various other visual artifacts. Under the influence of the photographic essays that were included in the periodical, the Farm Security Administration, created in 1937, hired a new generation of photographers to document poverty and to promote New Deal programs. Those photographs , which were also diffused in the newly created newsmagazines like *Look* and *US Camera*, contributed to a shift in the role of visual representation from a simple illustrative device to a tool of communication and persuasion intended for a larger audience.[26] The invention of a new technology to display images helped increase that movement. Before the war, most Americans used to go to movie theatres at least once a week. By the time *Economics* was published in 1948, American citizens were buying their first TV sets ''at increasingly dramatic

[25]Lerner to Wise, June 16th 1944, *Abba Ptachya Lerner Papers at the Library of Congress*, Manuscript Reading Room, Box 19 Folder 5.
[26]On the influence of the FSA photographs on US print culture, see Finnegan (2003).

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rates" (Baughman 2006, p. 30). In addition, the end of World War II fostered the introduction of visual aids into education in various fields. After Congress voted the GI Bill, which offered scholarships to those ex-servicemen who wanted to pursue higher education, the number of students increased significantly in US colleges and universities. Whereas less than 200,000 bachelor degrees were awarded annually by US universities before WWII, this figure rose to 271,000 in 1947–8 and 432,000 in 1949–50.[27] Many students took economics courses as a minor field of study as part of their undergraduate programs; they just wanted to gain basic knowledge in order to become engineers or businessmen. Visual aids could be helpful in this respect especially as they had proven their efficiency as pedagogical devices during wartime when time was of the essence.[28]

Thus, it is not surprising that in 1950, when the Committee on the Undergraduate Teaching of Economics and the Training of Economists published its report to the American Economic Association under the supervision of Horace Taylor, an entire section was devoted to the use of visual aids in the teaching of economics. The report highlighted the growing demand for visual materials—not only diagrammatic textbooks but also motion pictures and film strips—by those who taught economics at the introductory level.[29] Among the publishers who filled the demand was the firm McGraw-Hill which specialized in textbooks for engineers. Those books were full of schemas and patterns, and some of them were extensively used by the US army during WWII. Samuelson's *Economics* was McGraw-Hill's first foray into the field of economics textbooks. Its author was particularly well-placed to be aware of the changes affecting US education, as a young professor at MIT who mostly taught soon-to-be engineers, some of them coming from the US army contingent. As a researcher, Samuelson had worked since 1941 for the Radiation Laboratory, a research center which had been established at MIT by the National Defense Research Committee. His work there contributed to the improvement of the Navy MK-56 automatic gun aiming device.[30] Having completed his PhD dissertation, he temporarily left the field of economic theory and became entrenched in the culture of applied mathematicians working for the military.[31] In 1945, there were 8,000 students who had to take a one-year seminar in economics, a course that was quite unpopular at the time. Samuelson was asked to write an introductory textbook for those particular undergraduate students. In this context, it is likely that Samuelson's visual representations in *Economics* were influenced more by the visual culture of engineering than by the tradition of diagrammatic analysis he had somewhat dismissed in his *Foundations*.

[27]See Backhouse (2008), p. 523.

[28]Burlingame (1959) wrote: "During the war visual education was widely used in quick training programs. At that time when the requirement of speed was paramount, the people in command of training projects seized upon any device that could help quicken the instruction. Factory workers were taught by films how to use lathes or millers; men in the Army medical corps were shown films on first-aid procedures; soldiers were taught everything from machine-gun assembly to venereal prophylaxis by motion pictures" (Burlingame 1959, p. 407).

[29]See Taylor (1950), especially pp. 187–201.

[30]See Mirowski (2002), p. 225.

[31]In a 1950 article, Samuelson described himself as being "out of economics" during this period. See Samuelson (1950), p. 358.

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Economics' visual apparatus was different from what had been done in previous textbooks. It embraced the development of visualization as a mass medium. Actually, it was almost unrelated to the diagrammatic analysis of Boulding's textbook of 1941. On purely quantitative considerations, the first edition of *Economics* did not represent a step forward in visual economic education. It had only 75 diagrams, which did not constitute a major difference with Frederic Garver and Alvin Hansen's *Principles of Economics*, one of the economic textbooks students were assigned to read at Harvard. Yet the structure of *Economics*' visuals was completely different: Samuelson's book contained as many statistical charts as analytical diagrams, whereas analytical diagrams were predominant in Garver and Hansen. In the latter, the majority of diagrams were located in the microeconomics section, whereas Samuelson used more visual aids to introduce macroeconomics.

One of the most important visual elements in *Economics*, the circular flow diagram, inspired by Knight but drawn in the fashion of Fisher's hydraulic apparatus, was used to explain the Keynesian principle of the multiplier metaphorically (Samuelson 1948, p. 264 and Figure 1 below). Visually, it looked like one of the many engineering diagrams MIT undergraduate students were likely to encounter. It had two spigots, representing investment and saving. Of course, the equilibrium is reached when saving equals investment.

The circular flow diagram allowed for a ''restatement'' of the theory of income that Samuelson had introduced a few pages before in the form of the 45-degree diagram, a figure that he had previously used in a 1939 article in the *Journal of Political Economy*.[32] This diagram represented the intersection between the global demand and supply curves and the determination of national income (p. 260). The same level of national income could be alternately obtained by representing the intersection of the saving curve with the investment line (see Figure 2). This diagram, later known as the Keynesian Cross, was reproduced on the cover and the spine of the book.

In spite of its instant appeal for teaching purposes, the ''Keynesian Cross,'' like Marshallian supply and demand curves, did not allow for the internal mechanics to be seen. The curves helped determine the equilibrium but did not explain the process and the hypotheses underlying it. The circular flow schema helped complete the demonstration by describing the principle of the multiplier in a convenient way. The diagram was a relevant tool to display some results that were already known, but an analogy was necessary to depict the economic machinery. In other words, the schema brought in more explanatory power than the diagram.

Samuelson's figures in *Economics* were interesting not only because of their intrinsic properties but also because of the way they were displayed. Compared with those of Taussig or Garver and Hansen, Samuelson's visual representations were clear and simple, filled with captions, embellishments, symbols, stipples and stripes that eased the reader's comprehension. In addition, Samuelson offered many statistical charts taken from various sources and displayed them in a fashionable way. Parallelism was one of the numerous artifacts Samuelson used in order to gain both clarity and enhancement in his visual explanations. Samuelson's striving for visual

[32]Samuelson later explained (1959) that the article was a correction of one of Alvin Hansen's numerical examples in his 1938 book, *Full Recovery or Stagnation*. Hansen himself used the diagram in his visual exposition of Keynes' theory (1953).

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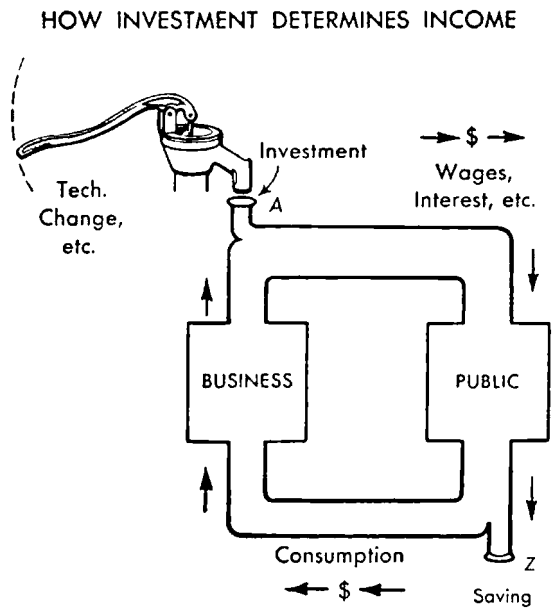


FIGURE 1. Samuelson's circular flow diagram (Samuelson 1948, p. 264)

simplicity made his textbook look totally different from Boulding's *Economic Analysis*. In the latter, diagrams were introduced to be carefully worked on by the students; they were anything but self-evident figures. On the other hand, the reader could read Samuelson's textbook without having any difficulty with its diagrams. They were not conceived as working tools but as means of arguments.

All in all, the use of visual elements in *Economics* was consistent with their scarcity in *Foundations*. Samuelson's textbook did not promote diagrammatic analysis as a convenient tool of theorizing, but as a satisfactory shorthand language that was entertaining enough to hold students' interest during their year of introduction to the principles of economics. Algebra was almost absent from *Economics* and diagrams were not intended as substitutes for other kinds of

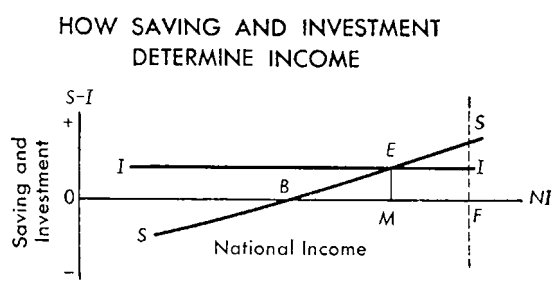


FIGURE 2. Samuelson's "Keynesian Cross" (from *Economics*: 259)

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mathematical symbols. In other words, diagrams were not used for their mathematical properties, but because students, especially those Samuelson taught at MIT, were assumed to be used to them. Given his opposition to the Marshallian method in *Foundations*, it is nonetheless consistent that Samuelson embraced diagrams as a tool of economic education while Marshall was reluctant to use diagrams as a pedagogical device in his *Principles*. Yet this is not to say that context did not matter. In Marshall's time, indeed, there was no distinct market for introductory textbooks as opposed to books devoted to advanced students and professional economists. By the time Samuelson wrote his textbook, there had emerged a gap between books aimed at novices and ones for experts, which Samuelson widened further by bringing into economics visual devices influenced by wartime army training manuals. With this textbook, the author took advantage of visualization as a device of persuasion and communication, whereas he considered diagrammatic analysis as a rather weak tool for scientific discovery. That may explain why Samuelson gave visual representation an increasing importance in subsequent debates on the use of mathematics in economics, as he appeared more concerned for the need of communication among the economics profession.

IV. CONFRONTING ECONOMISTS' VISUAL CULTURE: SAMUELSON'S ''THEORY OF PUBLIC EXPENDITURE'' (1952–1955)

The need for diagrams to communicate was not confined to the undergraduate audience of *Economics*. Samuelson reintroduced diagrams in his theory of public expenditure because they were also important for communication with his fellow economists. In 1954, he published a three-page article, ''The Pure Theory of Public Expenditure,'' which consisted of a mathematical investigation of the conditions of collective consumption goods pricing. As will be shown later, the inclusion of this note in this particular issue of the *Review of Economics and Statistics* was no accident: it was actually linked to a symposium on the place of mathematics in economics published in the same issue, involving eminent mathematical economists such as Tjalling Koopmans, Jan Tinbergen, Robert Solow, Jon Chipman, and Lawrence Klein. A year later, however, Samuelson published a diagrammatic exposition of his theory of public expenditure in reaction to a number of criticisms leveled at his mathematical note. This article was not his first contribution to diagrammatic analysis, but it was the first one to involve explicitly a process of translation of which he had firmly disapproved in the introduction of his *Foundations*. To understand this seemingly surprising turnaround, one needs to turn to the debates preceding it.[33]

In 1951, Samuelson participated in a session on ''Issues in Methodology'' at the annual meeting of the AEA. Among participants were Fritz Machlup and Boulding.

[33]Pickhardt (2006) shows that Samuelson's ''Pure Theory'' is less a contribution to public expenditure than a demonstration that mathematics is a useful shorthand language of economic theorizing. He relates the article to the debate around Novick's note, but also to a session in economic methodology at the AEA Annual Meeting in 1951 in which Samuelson participated.

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In his contribution, ''Economic Theory and Mathematics—An Appraisal,'' Samuelson answered Boulding's critical article on *Foundations*, though he did not make this target explicit. Samuelson's article addressed Boulding's critique point by point, beginning with the relevance of mathematics as a language. Then, Samuelson devoted one section to the debate between algebra and geometry.[34]

> Today, when an economic theorist deplores the use of mathematics, he usually speaks up for the virtues of geometrical diagrams as the alternatives. It was not always thus. Seventy years ago, when a man like Cairnes criticized the use of mathematics in economics, probably he meant by the term ''mathematics'' primarily geometrical diagrams. From the point of view of this lecture, the ancients were more nearly right than the modern critics. Geometry is a branch of mathematics, in exactly the same sense that mathematics is a branch of language. . . . [W]hat is not at all clear is . . . why any modern methodologist should find some virtue in two-dimensional graphs but should draw the line at third or higher dimensions. I suggest that the reason for such inconsistent methodological views must be found in the psychological and tactical problems which constitute the remaining part of my remarks (Samuelson 1952, pp. 59–60).

Samuelson did not really develop those remarks on psychology and strategy in his 1952 article, but in the debate of 1954 we mentioned above. At the origin of the controversy was Novick's two-page article in which he dismissed the idea of mathematics as a language, claiming that mathematics should be used only when quantification was involved.[35] All the other participants strongly disagreed with Novick—Solow was undoubtedly the roughest—, turning the whole symposium into a press tribute in favor of mathematical economics. Samuelson's contribution to this discussion shed some new light on his reluctance to provide a translation of his algebraic language, showing that his disdain for ''mental gymnastics of a peculiarly depraved type'' did not stand on methodological but rather sociological grounds:

> [O]ften poor mathematicians, such as Novick claims to be, do find it necessary to work through the logical conclusions of some branch of technical economics. This with work they can always do: for there is absolute truth in the saying: ''What one fool can learn so can another.'' Now what happens when finally they do grasp the contents of the theory? Quite usually, they genuinely say, ''Is that all there is to it? Why poor little me with the infinite ignorance can easily understand that—in fact I probably knew it all along. What was all the fuss about?'' . . . Only if you swallow Novick's deep-down view—that all [mathematical economists'] work has been

[34]Fritz Machlup, who was the chairman of the session, was also fond of diagrammatic analysis. Describing his experience as a scholar at Harvard University in 1935, Nicholas Georgescu-Roegen (1988, p. 22) wrote, ''Machlup usually attended only when the topic involved diagrammatical analysis in which he was a deft expert.'' This suggests that Samuelson's remarks were also directed at Machlup.

[35]Though Novick's critique could be regarded as a direct attack on Samuelson's mathematical economics, Mirowski (2002) showed that it was mainly a debate within RAND Corporation, between system analyst David Novick and members of the Cowles Commission. Samuelson was not part of any of these two groups, but he knew editor Seymour Harris very well and may have been invited to act as chairman of the symposium.

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> trivial—can you regard the problem of charlatanism as one of any importance. The real problem goes much deeper and will, I think, long persist (Samuelson et al. 1954, p. 382).

Samuelson's rhetoric rephrased the "strategy vs. principle" argument he had used in his letter to Boulding. As far as geometry and algebra are considered as methodologically equivalent, there is no need to recast the algebraic formulation, because it will not provide much understanding among the non-mathematically trained community but a feeling of deception about mathematical economics. Samuelson's viewpoint turned the debate on mathematics in economics into a matter of generation. This was reinforced by the younger participants, like Solow and Chipman, the latter arguing, "[E]conomics has gone a long way since the days when . . . it was considered a great accomplishment to be able to draw a marginal curve from an average one. . . . This is not to suggest that . . . geometrical economics did not have an important place in the history of economic thought. [It was], I believe [an] inevitable stage in the development of economics" (in Samuelson et al. 1954, p. 364).

Following this discussion, editor Seymour Harris added a note which slightly undermined the arguments of Samuelson and his allies. He quoted a study published in 1953, which showed that, among the PhD candidates, only 2 percent were judged good in mathematical practice by their professors, 41 percent were judged fair and 44 percent, poor (Samuelson et al. 1954, p. 382). Harris, who recognized himself having difficulties in understanding some of the articles he used to publish, pleaded for a better understanding between mathematical and literary economists. "Hence the mathematical economist has a responsibility to communicate his results to the non-mathematical economist, and also both in the choice of problems to be treated and in the manner of treatment not to overdo mathematical economics" (*Ibid.* pp. 385–6). Though Harris's note was supposed to qualify the other participants' attack on Novick's critique, it also undermined the methodological aspects of the debate, reducing it to a question of communication.

The inclusion of the "Pure Theory of Public Expenditure" in the same issue of the *Review of Economics and Statistics* made it part of the debate, as Samuelson explicitly introduced it as an alternative to "[his] own views on the deeper substantive methodological questions." He noted, not without provocation, that the paper contained "most of the imperfections of communication so often charged against mathematical economics" (in Samuelson et al. 1954, p. 380). Thus the article was only three pages long—"about the space of Novick's note." The mathematics of the article consisted of the determination of Pareto-optimal conditions of production of n "private consumption goods" and m "collective consumption goods," with standard assumptions concerning the production possibility functions F and individual's utility function U. Samuelson's main contribution in the paper was the second of three optimality conditions which showed that the sum of the individual marginal rates of substitution between the public good and the private good must be equal to the marginal rate of transformation between the two goods.

In subsequent issues of the *Review of Economics and Statistics*, Samuelson's "Pure Theory" was strongly criticized on various grounds by Stephen Enke and Julius Margolis, both collaborators with Novick at RAND Corporation. Standing up for Novick, Enke refused to draw a distinction between substantive and communication

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issues, considering Samuelson's inability to address non-mathematical economists a serious defect of his paper:

> First, it is unnecessarily unintelligible to most people. Many economists, interested in public finance and welfare, will want to understand what anyone of Samuelson's reputation has to contribute. Frustration will be their lot. Moreover, this refusal to communicate to more than a few is willful. . . . Mathematical shorthand may permit a three-page article, but a few more words would have added many readers, some of them capable of subsequent contributions to the theory (Enke 1955, p. 132).

On the other hand, Margolis's critique of the Pure Theory was more substantial as it questioned Samuelson's definition of "collective consumption goods." Samuelson's "Diagrammatic Exposition" of 1955 was his answer to both critical notes, with its clear and simple diagrams. Samuelson mentioned explicitly Margolis's article but not Enke's note, and yet the fact that Samuelson decided to write a diagrammatic exposition could not be unrelated to previous criticisms. Samuelson's reaction to Enke's remarks on unintelligibility resulted in a diagrammatic exposition rather than in a pure literary article. The fifth and last diagram in the article was a graphical translation of the second equation in the 1954 paper, showing the optimal condition as the equivalence of the public good's marginal cost and the sum of the individual's marginal rate of substitution between the two goods (see Figure 3 below).

By employing the graphical method, Samuelson highlighted the connection between his contribution and earlier works by Eric Lindahl and Howard Bowen, a point he had just suggested in his article of 1954. His own mathematical formulation, he argued, allowed for the generalization of an infinite number of Bowen and Lindahl diagrams. The fact that visualization had already been used in the public goods literature constituted however a more favorable ground for the use of diagrammatic analysis. Moreover, the frequent use of the imperative mode throughout the article was evidence that Samuelson was actually extending his pedagogical skills from students to the economics' profession. Samuelson's diagrams in his 1955 article were not comparable to those of Marshall, or to those of the diagrammatic analysis which was predominant in the 1930s. His visualization was much simpler than the complicated geometrical constructs of Lerner or Boulding; they were tools of diffusion of some previous research and not any longer the tools on which research itself relied. Though the article of 1955 can be considered as a concession by Samuelson, showing that the advancement of science itself could not be totally dissociated from a good understanding among the profession of the results that had already been expressed algebraically, it did not represent a turnaround in Samuelson's use of visual representation. On the contrary, it was evidence that visuals were successfully migrating from the core of economic research to its diffusion. Actually, it is rather striking that Samuelson's "diagrammatic representation of a theory of public expenditure" was published in a period which constituted both a peak in the history of diagrammatic analysis in economics and the beginning of its decline. According to Roger Backhouse's 1998 survey of US post-war economics through the study of three leading journals (*AER*, *QJE,* and *JPE*), the proportion of mathematical articles containing diagrams began to decrease in the mid-1950s, after two decades of steady rise. During the same period, the proportion of articles relying on algebra began to rise at a higher rate, as did the number of diagrams in economic textbooks.

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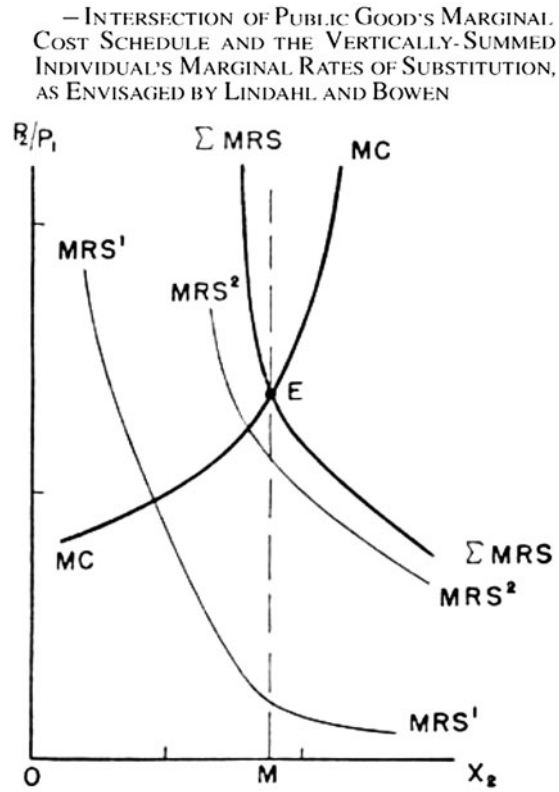


FIGURE 3. Intersection between public good's marginal cost curve and the sum of the individual's rates of substitution between private and public goods (Samuelson 1955, p. 354)

The sixth edition of *Economics* in 1964 had twice as many diagrams as the first edition of 1948, and still it would seem arid by today's standards.

V. CONCLUDING REMARKS

Over the course of less than two decades, the place of visual language in economics has dramatically changed. Visual representation became an expository device aimed at the mathematically unskilled rather than an engine of discovery at the frontier of the discipline. Samuelson played a leading role in this process by publishing the book which would set the standard for scientific writing at the research level, giving diagrams a subsidiary role and simultaneously setting a much higher visual standard for textbook writing. On one hand, his reluctance to use visual representation in his early scientific writings was prompted by his rejection of all the economic concepts which did not fit into his theoretical framework, most of them related to

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diagrammatic analysis. On the other hand, his endorsement of visual language as a pedagogical device can be explained by the peculiar audience he was trying to reach at MIT, in a context marked by the invention of new tools to provide mass education. This duality may explain why certain diagrams achieved almost mythical status among students even though teachers often despise them as a vulgar form of knowledge.

Norton Wise has argued, in the context of the history of science, that accounts of visualization should break free from the traditional division between algebra and geometry because "[g]eometric intuition never gets far without analytic abstraction and vice versa" (Wise 2006, p. 81). In contrast, the episode discussed here suggests that this distinction has to be taken into account because it reflects the way participants actually reasoned: the relevance of geometry versus algebra was one of the points at issue. Boulding, for example, explicitly introduced this distinction because he found it useful to discuss the future of mathematical economics. On the other hand, Samuelson's statement that "geometry is a branch of mathematics" seems to suggest that he did not find this distinction very useful for discussion. Perhaps a better mathematician than Boulding, Samuelson was more conscious of the controversies between Lagrange and Newton, which showed that both geometry and algebra contributed to the advancement of physics. It is customary to associate mathematics (algebra) with general equilibrium and diagrams (geometry) with partial equilibrium but that does not get us to the heart of the issue. Hicks, for example, tried to reconcile the use of diagrams with the general equilibrium framework, whereas Samuelson used algebra in two-variable models.

The division between Samuelson and Boulding, however, was not merely about dimensions. Though their debate does not only revolve around a distinction between the creation of scientific knowledge and its dissemination, that was an important issue that involved different ideals of scientific knowledge; it was not simply a dispute about communication. For Boulding, there should not be a gap between the creation and diffusion of knowledge, even for the sake of scientific rigor. For Samuelson, rigor was the only valid criterion of any serious economic research, beside which any other appreciation was subsidiary. From this, it follows that Samuelson's and Boulding's positions were actually incommensurable. Samuelson eschewed diagrams in *Foundations* mainly because he was using Marshall as a straw man while his real target there was the economics of the early 1930s. On the other hand, though his thought was not fully shaped at the time, Boulding was in the process of building a theoretical framework which he thought would help integrate the social sciences with the help of visual representation. Much later, he would write: "My own inclination is for graphical analysis of statistical data, simply because the core of our imaging process is spatial and temporal. I have argued that numbers are something of a figment of the human imagination, that the real world consists of shapes, sizes, structures, patterns, fittings, and so on, and very often, though not necessarily, unless we can visualize them in some form, numbers do not convey very much" (1988, pp. 113-4). Samuelson's point of view is the one that eventually prevailed in the second half of the twentieth century, resulting in an increasing gap between scientific creation, driven by the use of algebra, and its diffusion, driven by the use of visual representation.

Our study of Samuelson's use of visual representation between 1941 and 1955 has allowed us to deconstruct historically some of the characteristics of diagrams that

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contemporary economists may find both immutable and unquestionable, such as their obvious convenience for pedagogical purpose. Though we believe this deconstruction to be useful, we are aware that this is only a modest step toward the construction of a satisfying history of visual representation in modern economics. The latter would require much more attention to the place of visual culture both within and outside of the economics profession, as well as a deeper understanding of how economics has been diffused to a larger audience during the studied period. The fact that proper consideration to those elements is part of the very recent developments in the history of economics as a discipline explains why we should expect many more contributions on the subject in the future.

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