product can be rewritten as $\frac{1}{2}\left(\sum_{n=0}^{\infty}\frac{1}{2^n}\right)\left(\sum_{n=0}^{\infty}\frac{1}{3^n}\right)\left(\sum_{n=0}^{\infty}\frac{1}{5^n}\right)\ldots$ which, by the unique factorization of integers into primes, expands to give $\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n}$.

From (2) we thus obtain

$$\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{5}\right)\left(1+\frac{1}{7}\right)\ldots > \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n} \tag{3}$$

Finally, from (1) and (3), we have

$$e^S = \left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{5}\right)\left(1+\frac{1}{7}\right)\ldots. > \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n},$$

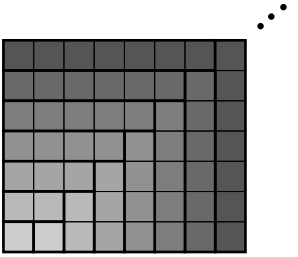
which contradicts the fact that the harmonic series diverges.

10.1017/mag.2025.18 © The Authors, 2025
Published by Cambridge University Press
on behalf of The Mathematical Association

ZHENGHAN ZANG
Tonbridge School,
Kent TN9 1JP

109.02 Sum of integers formula – another visual demonstration

This Note deals with a visual proof of the sum of positive integers formula: $2\sum_{i=1}^{n} i = n(n+1)$, where $n \in \mathbb{N}$.



Note that the diagram shows an array of shaped strings, where the length of the strings are 2, 4, 6, . . . , 2n respectively.

10.1017/mag.2025.19 © The Authors, 2025
Published by Cambridge University Press
on behalf of The Mathematical Association

BIKASH CHAKRABORTY
Department of Mathematics,
Ramakrishna Mission Vivekananda Centenary College,
Rahara, West Bengal 700 118, India
e-mail: *bikashchakraborty.math@yahoo.com,*
bikashchakrabortyy@gmail.com
SUSMITA GHOSH
Directorate of Open and Distance Learning,
University of Kalyani, West Bengal 741 245, India
e-mail: *sumghosh564@gmail.com*

https://doi.org/10.1017/mag.2025.19 Published online by Cambridge University Press