2 The Standard Model of Electroweak and Strong Interactions

2.1 Particle Content and Gauge Group of the SM

The SM of particle physics is based on the gauge group[1]

$$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y. \tag{2.1}$$

It is a very successful model for the description of three out of the four fundamental interactions in nature. The electromagnetic, weak, and strong interactions between the known elementary particles at the currently available energy scales are predicted and described by the SM in terms of a number of parameters that can be determined in experiment or calculated from a more fundamental theory.

The gauge symmetry $\mathrm{SU}(3)_C$ is the symmetry of QCD, the theory of strong interactions. $\mathrm{SU}(2)_L$ is the gauge group of weak interactions, and the $\mathrm{U}(1)_Y$ force is felt by all particles with a nonzero hypercharge quantum number Y. Whereas $\mathrm{SU}(3)_C$ remains exact, $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ is spontaneously broken down to $\mathrm{U}(1)_{\mathrm{em}}$, the symmetry group of QED (see Section 2.3 for details). The quantum numbers of $\mathrm{SU}(3)_C$, $\mathrm{SU}(2)_L$, and $\mathrm{U}(1)_Y$ are called color, weak isospin, and hypercharge, respectively. While hypercharge can in principle be any real number, there are three colors (red, green, blue) and two weak isospin states ($\pm\frac{1}{2}$). The transformation properties of all SM fermion and gauge fields under the symmetry group $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ are given in Table 2.1, where we show the representations in the case of $\mathrm{SU}(3)_C$ and $\mathrm{SU}(2)_L$ and the hypercharge Y in the case of $\mathrm{U}(1)_Y$, which is normalized as follows. Note that in the literature Y is sometimes replaced by Y/2.

$$Q = T_3 + Y. \tag{2.2}$$

This is the well-known Gell-Mann–Nishima relation. Here Q is the electric charge and T_3 the third component of the weak isospin belonging to $\mathrm{SU}(2)_L$. The fermions of the SM are grouped in several ways: On the one hand we distinguish between quarks and leptons, where only quarks feel QCD and thus transform nontrivially under $\mathrm{SU}(3)_C$. Quarks are triplets **3** under $\mathrm{SU}(3)_C$ with quantum numbers red, green, and blue; antiquarks are antitriplets $\bar{\mathbf{3}}$ with quantum numbers antired, antigreen, and antiblue, whereas leptons are $\mathrm{SU}(3)_C$ singlets and thus color neutral. On the other hand, there are $\mathrm{SU}(2)_L$ doublets (left-handed fields) and $\mathrm{SU}(2)_L$ singlets (all right-handed fields). Therefore only the doublets

[1] This chapter has been written by Jennifer Girrbach-Noe.

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

Table 2.1 Particle content of the SM and transformation properties.

Fermions: three generations			$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
e_R	μ_R	τ_R	**1**	**1**	-1
$L_1 = (\nu_e, e_L)^\top$	$L_2 = \left(\nu_\mu, \mu_L\right)^\top$	$L_3 = (\nu_\tau, \tau_L)^\top$	**1**	**2**	$-\frac{1}{2}$
u_R	c_R	t_R	**3**	**1**	$\frac{2}{3}$
d_R	s_R	b_R	**3**	**1**	$-\frac{1}{3}$
$Q_1 = (u_L, d_L)^\top$	$Q_2 = (c_L, s_L)^\top$	$Q_3 = (t_L, b_L)^\top$	**3**	**2**	$\frac{1}{6}$
Gauge bosons					
	G^a_μ	$a = 1{-}8$	**8**	**1**	0
	W^a_μ	$a = 1, 2, 3$	**1**	**3**	0
	B_μ		**1**	**1**	0
Higgs					
	$\Phi = \left(\phi^+, \phi^0\right)^\top$		**1**	**2**	$\frac{1}{2}$

participate in the charged weak interactions mediated by $W^\pm$ bosons. The first (second) component of an $SU(2)_L$ doublet has weak isospin $T_3 = \frac{1}{2}$ ($T_3 = -\frac{1}{2}$). Because all representations of SU(2) are real (see Section 1.7.1), antiparticles also have isospin and not anti-isospin. Furthermore the fermions exist in three copies, called generations. The particles of the second and third generations have exactly the same quantum numbers as the corresponding particles of the first generation (as shown in Table 2.1) but they are different in mass. The requirement of local gauge invariance predicts the existence of massless gauge bosons for each gauge group that belong to adjoint representations of these groups. We have then eight gluons G^a_μ for $SU(3)_C$, three W^a_μ for $SU(2)_L$, and one B_μ for $U(1)_Y$. After electroweak symmetry breaking W^a_μ and B_μ mix into $W^\pm_\mu$, Z^0 and γ where only γ stays massless and belongs to the unbroken $U(1)_{\text{em}}$ symmetry, the symmetry of QED.

It is now time to encode all this information in an appropriate Lagrangian.

2.2 Short Overview: Lagrangian of the SM

The SM Lagrangian can be split into four parts given in (2.3), (2.8), (2.12), and (2.13). The kinetic terms of the SM fermions together with their couplings to gauge bosons follow from

$$\mathscr{L}^{\text{SM}}_{\text{fermion}} = \sum_{j=1,2,3} \bar{L}_j i\not{D} L_j + \bar{e}_{Rj} i\not{D} e_{Rj} + \bar{Q}_j i\not{D} Q_j + \bar{u}_{Rj} i\not{D} u_{Rj} + \bar{d}_{Rj} i\not{D} d_{Rj}, \tag{2.3}$$

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

where we used the shorthand notation $\not{D} = D_\mu \gamma^\mu$. The covariant derivatives are given as

$$D_\mu = \partial_\mu - ig_1 Y B_\mu - ig_2 \frac{\sigma^a}{2} W_\mu^a \qquad \text{for LH lepton doublets } L_j, \tag{2.4}$$

$$D_\mu = \partial_\mu - ig_1 Y B_\mu \qquad \text{for RH lepton singlets } e_{Rj}, \tag{2.5}$$

$$D_\mu = \partial_\mu - ig_1 Y B_\mu - ig_2 \frac{\sigma^a}{2} W_\mu^a - ig_s \frac{\lambda^a}{2} G_\mu^a \qquad \text{for LH quark doublets } Q_j, \tag{2.6}$$

$$D_\mu = \partial_\mu - ig_1 Y B_\mu - ig_s \frac{\lambda^a}{2} G_\mu^a \qquad \text{for RH quark singlets } u_{Rj}, d_{Rj}, \tag{2.7}$$

and are the same for all three generations $j = 1, 2, 3$. Here Y denotes the hypercharge quantum number, σ^a are the Pauli matrices defined in (A.17), λ^a are the Gell-Mann matrices defined in (A.19)–(A.21), and G_μ^a are the gluon fields. The couplings for $\mathrm{SU}(3)_C$, $\mathrm{SU}(2)_L$, and $\mathrm{U}(1)_Y$ are denoted by g_s, g_2, and g_1, respectively. In the more general considerations of Chapter 1 we used always g as a coupling constant.

The kinetic term of the gauge bosons is given by

$$\mathscr{L}_{\text{gauge}}^{\text{SM}} = -\frac{1}{4} G_{\mu\nu}^a G^{\mu\nu,a} - \frac{1}{4} W_{\mu\nu}^a W^{\mu\nu,a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \tag{2.8}$$

where

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_s f^{abc} G_\mu^b G_\nu^c, \tag{2.9}$$

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g_2 \varepsilon^{abc} W_\mu^b W_\nu^c, \tag{2.10}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{2.11}$$

From the first two terms in the Lagrangian (2.8) three gauge boson and four gauge boson self-couplings follow. The couplings of the gauge bosons to the fermions come from the covariant derivatives D_μ.

Explicit mass terms for both fermions and gauge bosons are not allowed in the SM because $-m\bar{\psi}\psi = -m(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$ couples left- and right-handed fields that transform differently under $\mathrm{SU}(2)_L$ and also $M^2 A_\mu^a A^{\mu,a}$ for gauge bosons breaks the gauge symmetry (see discussion in Sections 1.6 and 1.8). Thus, from (2.3) and (2.8) alone we do not get any mass terms. The Higgs mechanism, discussed in general terms in the previous chapter, provides a solution to this problem, and we will discuss it in some detail in Section 2.3. For completeness we give here only the relevant contribution of the Higgs doublet Φ to the Lagrangian of the SM:

$$\mathscr{L}_\Phi = \left(D_\mu \Phi\right)^\dagger \left(D^\mu \Phi\right) - V(\Phi). \tag{2.12}$$

It contains the kinetic term for the Higgs with its couplings to gauge bosons and the Higgs potential. The covariant derivative for the Higgs is the same as for left-handed lepton

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

doublets in (2.4) except for the value of the hypercharge that differs by sign as seen in Table 2.1.

The Yukawa sector of the SM contains the couplings of the Higgs to the fermions. It is very crucial for this book because it determines the flavor structure. The quantum numbers of the Higgs are such that it couples in a gauge invariant manner to an $SU(2)_L$ doublet and singlet so that the Yukawa part of the Lagrangian reads:

$$\mathscr{L}_Y = -\bar{Q}\Phi\Upsilon^D d_R - \bar{Q}\Phi^c\,\Upsilon^U u_R - \bar{L}\Phi\Upsilon^E e_R + h.c. \tag{2.13}$$

Here the fermion fields are three-component vectors such that all three generations are included, $\boldsymbol{\Upsilon}^{D,U,E}$ are 3×3 Yukawa-matrices, and Φ^c is the charged conjugated Higgs field. More details will be given in Section 2.5. There is no Yukawa interactions with neutrinos because of the absence of right-handed neutrinos in the SM.

2.3 Spontaneous Symmetry Breakdown in the SM

In the SM the Higgs field $\Phi = \left(\phi^+, \phi^0\right)^\top$ is responsible for the breakdown of the electroweak to the electromagnetic gauge symmetry

$$\mathrm{SU(2)}_L \times \mathrm{U(1)}_Y \to \mathrm{U(1)}_{\mathrm{em}}, \tag{2.14}$$

and for generating $W^\pm$, Z^0, and fermion masses. The Higgs field is a color neutral $SU(2)_L$ doublet with hypercharge $Y = \frac{1}{2}$ whose neutral component develops a vacuum expectation value (vev)

$$\langle 0|\Phi|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}, \qquad v = \frac{\mu}{\sqrt{\lambda}}, \tag{2.15}$$

which minimizes the Higgs potential of the SM[2]

$$V(\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda\left(\Phi^\dagger\Phi\right)^2, \quad \mu, \lambda > 0. \tag{2.16}$$

The Higgs potential introduces two new parameters: the "negative" mass term $-\mu^2$ and the Higgs self coupling λ. The charge operator,

$$\hat{Q} = \hat{T}_3 + \hat{Y} = \frac{1}{2}\sigma_3 + \frac{1}{2}\mathbb{1}_{2\times 2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \tag{2.17}$$

leaves $(0, v)^\top$ invariant, and thus $\mathrm{U(1)}_{\mathrm{em}}$ stays unbroken. As we will see in Section 3.1 the mass of the $W^\pm$ boson M_W can be related to the Fermi constant G_F, which is

[2] In Sections 1.10.2 and 1.10.3 the sign in front of μ in the Higgs potential is different. There we demonstrated that only for $\mu^2 < 0$ spontaneous symmetry breaking was possible. Here we make the replacement $\mu^2 \to -\mu^2$ such that now SSB arises for $\mu^2 > 0$. Then μ^2 is directly related to the squared Higgs mass m_h^2.

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

known very precisely from the muon decay (see (3.3)). This important relation is given as follows

$$\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8M_W^2}. \tag{2.18}$$

But as we will soon derive, the vev v is related to M_W in such a manner that eventually the value of G_F gives uniquely the numerical value of v. In this chapter we do not include any higher-order electroweak corrections so that the following expressions, although rather accurate, represent the SM at the so-called tree-level. We will improve on it in later chapters in cases in which it is necessary.

To derive the formulas for the masses M_W and M_Z we will now look closer at the Higgs part of the SM Lagrangian in (2.12). The covariant derivative is given explicitly as follows

$$D_\mu = \partial_\mu - ig_1 Y B_\mu - ig_2 \frac{\sigma^a}{2} W_\mu^a = \partial_\mu - i \begin{pmatrix} g_1 Y B_\mu + \frac{g_2}{2} W_\mu^3 & \frac{g_2}{\sqrt{2}} W_\mu^+ \\ \frac{g_2}{\sqrt{2}} W_\mu^- & g_1 Y B_\mu - \frac{g_2}{2} W_\mu^3 \end{pmatrix}, \tag{2.19}$$

where we replaced the real $W_\mu^{1,2}$ by $W_\mu^\pm$

$$W_\mu^- = \frac{1}{\sqrt{2}} \left(W_\mu^1 + i W_\mu^2 \right), \quad W_\mu^+ = \left[W_\mu^- \right]^\dagger = \frac{1}{\sqrt{2}} \left(W_\mu^1 - i W_\mu^2 \right), \tag{2.20}$$

which are eigenvectors to T_3 and also to Q due to $Y = 0$. As mentioned in Section 1.7.1 gauge bosons transform under the adjoint representations of the gauge group, which is given by the structure constants. Thus in SU(2) we have $(T_3)_{ab} = i\varepsilon_{a3b}$:

$$T_3 \begin{pmatrix} W_\mu^1 \\ W_\mu^2 \\ W_\mu^3 \end{pmatrix} = i \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} W_\mu^1 \\ W_\mu^2 \\ W_\mu^3 \end{pmatrix} = \begin{pmatrix} -iW_\mu^2 \\ iW_\mu^1 \\ 0 \end{pmatrix}. \tag{2.21}$$

Consequently

$$T_3 W_\mu^\pm = Q W_\mu^\pm = \pm W_\mu^\pm, \tag{2.22}$$

i.e., $W_\mu^\pm$ have electric charges ± 1, whereas W_μ^3 and B_μ are electrically neutral.

We next define

$$\phi^0(x) = \frac{1}{\sqrt{2}} \left(v + h(x) + iG^0(x) \right), \tag{2.23}$$

$$\phi^+(x) = G^+(x), \quad G^-(x) = \left[G^+(x) \right]^\dagger, \tag{2.24}$$

where $G^\pm$ and G^0 are the Goldstone bosons and h will become the physical Higgs boson field. Inserting Φ into $V(\Phi)$ and keeping only terms proportional to h^2, we find as the Higgs mass

$$m_h = \sqrt{2}\mu = \sqrt{2\lambda} v. \tag{2.25}$$

One complication occurs, namely mixing between B_μ and W_μ^3, so that they cannot be mass eigenstates. Because $\mathrm{U}(1)_{\mathrm{em}}$ is unbroken one of the mass eigenstates must be massless and identified with the photon A_μ. The second one is massive. It is Z_μ. As we have to

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

keep the normalization of the kinetic terms of the gauge fields in (2.8) only an orthogonal transformation from (B_μ, W_μ^3) to (A_μ, Z_μ) characterized by the Weinberg mixing angle ϑ_W is allowed:[3]

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\vartheta_W & \sin\vartheta_W \\ -\sin\vartheta_W & \cos\vartheta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}, \tag{2.26}$$

or its inverse

$$\begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} = \begin{pmatrix} \cos\vartheta_W & -\sin\vartheta_W \\ \sin\vartheta_W & \cos\vartheta_W \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}. \tag{2.27}$$

Inserting this back into (2.12) and (2.19) and requiring that A_μ couples only to electrically charged particles and not to ϕ^0, we get the following relations for the Weinberg angle and the couplings e, g_1 and g_2:

$$\tan\vartheta_W = \frac{g_1}{g_2}, \quad e = g_2 \sin\vartheta_W = g_1 \cos\vartheta_W. \tag{2.28}$$

Gauge invariant mass terms for the gauge bosons arise from "mixed" terms in the covariant derivative where one picks up a vev v from Φ and, for example, a W^+. Thus the gauge boson masses can be calculated from

$$\left[D_\mu \begin{pmatrix} 0 \\ v \end{pmatrix}\right]^\dagger D^\mu \begin{pmatrix} 0 \\ v \end{pmatrix} \stackrel{!}{=} M_W^2 W_\mu^- W^{\mu+} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu + \frac{1}{2} m_\gamma A_\mu A^\mu, \tag{2.29}$$

$$\Rightarrow M_W = \frac{g_2 v}{2}, \quad M_Z = \frac{g_2 v}{2\cos\vartheta_W}, \quad m_\gamma = 0. \tag{2.30}$$

In summary, then,

$$M_W = \frac{g_2 v}{2}, \quad M_Z = \frac{g_Z v}{2}, \quad g_Z = \frac{g_2}{\cos\vartheta_W} = \sqrt{g_1^2 + g_2^2}, \tag{2.31}$$

and consequently

$$\frac{M_W}{M_Z} = \cos\vartheta_W. \tag{2.32}$$

[3] Note that the sign in front of $\sin\vartheta_W$ is not unique, and different conventions appear in the literature. Appendix B should be useful in this respect.

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

Inserting the expression for M_W in (2.30) into (2.18), we find with our normalization in (2.15)

$$v = 2^{-\frac{1}{4}} G_F^{-\frac{1}{2}} = 246 \text{ GeV}. \tag{2.33}$$

The numerical value follows from (3.10) to which we will return in the next chapter. However, we should warn the reader that instead of (2.15) also $\langle \Phi \rangle = (0, v)^\top$ is common in the literature, such that in this case one has $v = 174$ GeV.

We observe that SSB implies relations between M_W, M_Z, e, $g_{1,2}$, which can be tested experimentally. Another important test is the manner in which the fermion masses are generated through the Higgs mechanism. While this topic will be discussed at later stages in this book, let us note that replacing Φ by its vev in (2.13) we get terms like $y_e \bar{L} \phi e_R \to \frac{1}{\sqrt{2}} y_e v e_L e_R$, where we set $y_e = Y^E_{11}$ such that we can identify the electron mass as $m_e = \frac{1}{\sqrt{2}} y_e v$. In the case of quarks the complication arises due to mixing between different quarks carrying the same electric charge. This implies that Yukawa matrices for up-quarks and down-quarks cannot be diagonalized simultaneously with very profound phenomenological consequences. This will be the subject of Section 2.5.

2.4 Gauge Boson Self-Interactions

The gauge bosons of the SM are the eight gluons G^a_μ of $\mathrm{SU}(3)_C$, $W^{1,2,3}_\mu$ of $\mathrm{SU}(2)_L$, and B_μ of $\mathrm{U}(1)_Y$. As seen in Section 2.3 after electroweak symmetry breaking, W^3_μ and B_μ mix into mass eigenstates Z^0 and A_μ. In this section we have a closer look at the kinetic terms of the gauge bosons as written in the Lagrangian in (2.8). We start with the first term for gluons because here the problem of mixing does not occur. It can be rewritten as (see also general discussion in Section 1.8):

$$\mathscr{L}^g_{\text{gauge}} = -\frac{1}{2} \left(\partial_\mu G^a_\nu\right) \left(\partial^\mu G^{\nu,a}\right) + \frac{1}{2} \left(\partial_\mu G^a_\nu\right) \left(\partial^\nu G^{\mu,a}\right) + \mathscr{L}_{\text{fix}} \tag{2.34}$$

$$- g_s f^{abc} \left(\partial_\mu G^{\nu,a}\right) G^{\mu,b} G^{\nu,c} \tag{2.35}$$

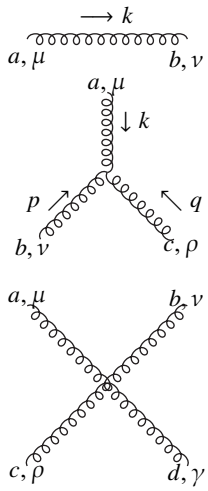
$$- \frac{1}{4} g_s^2 f^{abc} f^{ade} G^b_\mu G^c_\nu G^{\mu,d} G^{\nu,e}. \tag{2.36}$$

The first line determines the propagator, the second line describes triple gluon couplings, and the third line describes four-gluon couplings.

As discussed in any book on gauge field theories, to derive a propagator for a massless gauge boson we need an additional term $\mathscr{L}_{\text{fix}}$ that fixes the gauge. A_μ and G^a_μ, being massless, have two unphysical degrees of freedom. The spin-0 contribution is set to zero through the Lorenz condition $\partial_\mu A^\mu = 0$. However, we still have one degree of freedom left. The generalization of the Lorenz gauge are the so-called R_ξ gauges. Instead of fixing the gauge we add a Lagrange multiplier $1/(2\xi)$ to the Lagrangian

$$\mathscr{L}_{\text{fix}} = \frac{1}{2\xi} \left(\partial_\mu G^{\mu,a}\right)^2. \tag{2.37}$$

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006



$$\frac{-i\delta_{ab}}{k^2+i\delta}\left(g^{\mu\nu}-(1-\xi)\frac{k^\mu k^\nu}{k^2}\right)$$

$$g_s f^{abc}\left[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu\right]$$

$$\begin{aligned}-ig_s^2\Big[&f^{abe}f^{cde}\left(g^{\mu\rho}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\rho}\right)+\\ &f^{ace}f^{bde}\left(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\sigma}g^{\nu\rho}\right)+\\ &f^{ade}f^{bce}\left(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\rho}g^{\nu\sigma}\right)\Big]\end{aligned}$$

Figure 2.1 Propagator, triple vertex, and quartic vertex of the Gluon.

The Laundau gauge corresponds to $\xi \to 0$, the Feynman–'t Hooft gauge to $\xi = 1$. We get the Feynman rules as shown in Figure 2.1.

For $SU(2)_L$ in the basis of $W_\mu^{1,2,3}$ the Lagrangian and the triple and quartic coupling look exactly the same with the replacement $f^{abd} \to \varepsilon^{abd}$ and $g_s \to g_2$. Furthermore one simplification occurs because $f^{abe}f^{cde} = \varepsilon^{abe}\varepsilon^{cde} = \delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc}$ such that only the $W_\mu^1 W_\mu^2 W_\mu^3$ coupling is left over. However mixing makes it a little more complicated. For example, the triple W^+W^-A vertex has the coupling e in front and the $W^+W^-Z^0$ vertex $e\cot\vartheta_W$. Because it is an unitary rotation the kinetic terms for the propagator do not contain mixed terms:

$$\begin{aligned}&\frac{1}{4}\left(\partial_\mu W_\nu^a - \partial_\nu W_\mu^a\right)\left(\partial^\mu W^{\nu,a} - \partial^\nu W^{\mu,a}\right) + \frac{1}{4}\left(\partial_\mu B_\nu - \partial_\nu B_\mu\right)\left(\partial^\mu B^\nu - \partial^\nu B^\mu\right)\\ &\quad= \frac{1}{2}\left(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+\right)\left(\partial^\mu W^{\nu-} - \partial^\nu W^{\mu-}\right)\\ &\qquad+ \frac{1}{4}\left(\partial_\mu Z_\nu - \partial_\nu Z_\mu\right)\left(\partial^\mu Z^\nu - \partial^\nu Z^\mu\right) + \frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right).\end{aligned} \tag{2.38}$$

Because the electromagnetic force is based on a U(1) symmetry, there are no triple and quartic photon couplings, but only W^+W^-A, W^+W^-AZ, and W^+W^-AA vertices. We collect the Feynman rules in the SM in Appendix B.

2.5 Flavor Structure of the SM

2.5.1 Rotation from Flavor to Mass Eigenstates

The Yukawa sector of the SM contains most of its free parameters: six quark masses, three charged lepton masses, three mixing angles, and one phase. Neutrinos are massless in the

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

SM, although we know that they have masses. We will improve on it soon but this will imply new parameters.

The interaction Lagrangian of (2.3) is invariant under a $[\mathrm{U}(3)]^5$ flavor symmetry

$$Q_L \to V_L^u\, Q, \quad u_R \to V_R^u\, u_R, \quad d_R \to V_R^d\, d_R, \tag{2.39}$$

$$L \to V_L^e\, L, \quad e_R \to V_R^e\, e_R, \tag{2.40}$$

where $V_L^u, V_R^u, V_R^d, V_L^e$, and V_R^e are unitary 3×3 matrices. However, the Yukawa sector (2.13) breaks $[\mathrm{U}(3)]^5$ down to $[U(1)]^4$, which results in the conservation of baryon number B and of the three individual lepton flavor numbers $L_{e,\mu,\tau}$. These are, however, accidental symmetries of the SM, i.e., they are not imposed from the beginning and do not have to be an exact symmetry in models beyond the SM. We will discuss these symmetries in more detail in Section 15.1.2.

As a side remark, let us recall that the gauge group of the SM in (2.1) is a subgroup of the $[\mathrm{U}(3)]^5$ flavor symmetry and with the quantum numbers of all SM particles in Table 2.1 is free from the so-called triangle anomalies as required by the renormalizability of the SM. It is interesting that among the leftover U(1) global symmetries only $\mathrm{U}(1)_{B-L}$ is anomalyfree and as such could be gauged. Even $B + L$ is broken by anomalies. However, Majorana masses for neutrinos discussed later break L and consequently also $B - L$. The issue of triangle anomalies and their cancellations is a very important constraint on NP models, and we will return to it in Chapter 15 illustrating it on a few examples. General discussion can be found in many textbooks on field theory. See in particular [18].

The Yukawa matrices in (2.13) are a priori arbitrary 3×3 matrices. But we can use the $[\mathrm{U}(3)]^5$ flavor symmetry of the interaction Lagrangian in (2.3) to simplify them. The question then arises whether we could diagonalize all Yukawa matrices simultaneously by just using the rotations in (2.39).

It is easy to convince oneself that this is impossible. In fact, quite generally Yukawa matrices can be diagonalized through biunitary transformations:

$$(V_L^d)^\dagger \Upsilon^D V_R^d = \hat{\Upsilon}^D, \quad (V_L^u)^\dagger \Upsilon^U V_R^u = \hat{\Upsilon}^U, \quad (V_L^e)^\dagger \Upsilon^E V_R^e = \hat{\Upsilon}^E. \tag{2.41}$$

Because left-handed fields are embedded into doublets Q and L, we have to rotate the members of the doublets with the same matrix, i.e., u_L and d_L are rotated in the same way and also ν_L and e_L. Usually one rotates Q with V_L^u and L with V_L^e as written in (2.39) and (2.40). Once this is done these rotations are fixed.

Now from (2.41) we can see that four matrices in the quark sector are needed to diagonalize both $\Upsilon^{U,D}$, but we have only three of them: V_L^u, V_R^u, and V_R^d. The matrix V_L^d is missing in (2.39). Equivalently we could perform the first rotation in (2.39) with the help of V_L^d but then V_L^u would be missing. We thus conclude that the matrices Υ^D and Υ^U cannot be simultaneously diagonalized by rotations, which leaves the interaction Lagrangian invariant. On the other hand, without right-handed neutrinos that are not present in the original version of the SM, the lepton Yukawa matrix Υ^E can be diagonalized.

In summary after these rotations we get

$$\mathscr{L}_Y = -\bar{Q}\Phi(V_L^u)^\dagger V_L^d \hat{\Upsilon}^D d_R - \bar{Q}\Phi^c \hat{\Upsilon}^U u_R - \bar{L}\Phi\hat{\Upsilon}^E e_R + h.c. \tag{2.42}$$

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

with the first term being nondiagonal. To diagonalize this term and consequently get the mass eigenstate basis we must perform an additional rotation of the down-quarks

$$d_L \to d'_L = (V_L^u)^\dagger V_L^d \, d_L, \tag{2.43}$$

where d'_L are the original flavor eigenstates from where we started and d_L the mass eigenstates. But this additional rotation modifies the interaction Lagrangian in (2.3), which written in terms of d_L involves the matrix $(V_L^u)^\dagger V_L^d$.

This mismatch between mass and flavor eigenstates is responsible for flavor transitions in the SM, and the matrix $(V_L^u)^\dagger V_L^d$ is just the CKM matrix, named after Cabibbo, Kobayashi, and Maskawa [19, 20]:

$$V_{\text{CKM}} = (V_L^u)^\dagger V_L^d, \qquad d'_L = V_{\text{CKM}} \, d_L. \tag{2.44}$$

Using the common notation for the entries of the CKM matrix we have then a very important relation between flavor and mass eigenstates

$$\begin{pmatrix} d'_L \\ s'_L \\ b'_L \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}. \tag{2.45}$$

Note that we now resolved d_L into its three components $(d, s, b)_L$, and we will do the same with $u_L = (u, c, t)_L$ and corresponding right-handed fields. The elements of the CKM matrix will play a very profound role in this book. As we will see later V_{ud} determines the strength of $d \to u$ flavor transitions and similar for other elements. The basis where all Yukawa couplings are simultaneously diagonal is called mass eigenstate basis. If we now insert the vev of Φ into (2.42) we get diagonal 3×3 mass matrices:

$$\hat{M}^U = \text{diag}\,(m_u, m_c, m_t) = \frac{v}{\sqrt{2}} \hat{\Upsilon}^U = \frac{v}{\sqrt{2}} \cdot \text{diag}\,(y_u, y_c, y_t)\,, \tag{2.46}$$

$$\hat{M}^D = \text{diag}\,(m_d, m_s, m_b) = \frac{v}{\sqrt{2}} \hat{\Upsilon}^D = \frac{v}{\sqrt{2}} \cdot \text{diag}\,(y_d, y_s, y_b)\,, \tag{2.47}$$

$$\hat{M}^E = \text{diag}\left(m_e, m_\mu, m_\tau\right) = \frac{v}{\sqrt{2}} \hat{\Upsilon}^E = \frac{v}{\sqrt{2}} \cdot \text{diag}\left(y_e, y_\mu, y_\tau\right). \tag{2.48}$$

2.5.2 Including Right-Handed Neutrinos

From the experimental observation of neutrino oscillations we know that neutrinos must have a small mass. However, from the construction of the SM, as we have seen earlier, neutrinos stay exactly massless. This fact tells us already that the SM is not the whole story and that there must be some kind of new physics beyond the SM that allows to generate

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

neutrino masses. The literature on this topic is very rich. Here we describe first the simplest solutions to this problem. We will briefly discuss the role of neutrinos in weak decays in Section 17.5.

There are various possibilities to generate massive neutrinos. While it is possible to generate them through a coupling of an additional Higgs triplet to the usual left-handed neutrinos, let us have a look first at how neutrino masses can be generated with the help of the SM Higgs doublet. This requires the existence of right-handed neutrinos. Right-handed neutrinos, if they exist, are singlets under the SM gauge group and transform as

$$\nu_R \sim (\mathbf{1}, \mathbf{1}, 0)\,. \tag{2.49}$$

Thus, in principle they do not feel the SM gauge interactions at all. However they do have an effect on the $W^{\pm}$ couplings as soon as neutrinos are massive. In analogy to the quark sector we can add an additional neutrino Yukawa term in (2.13), which will generate a Dirac neutrino mass term after electroweak symmetry breaking in the same manner as in the quark sector. Thus we add a term

$$-\bar{L}\Phi^c\,\Upsilon^\nu \nu_R \tag{2.50}$$

to (2.13). The neutrino Yukawa matrix can also be diagonalized by a biunitary transformation

$$(V_L^\nu)^\dagger \Upsilon^\nu V_R^\nu = \hat{\Upsilon}^\nu, \tag{2.51}$$

and in addition to (2.40) we have an additional rotation for the right-handed neutrinos $\nu_R \to V_R^\nu \nu_R$. With this setup this works exactly as for quarks, and in (2.42) we have to add $-\bar{L}\Phi^c (V_L^e)^\dagger V_L^\nu \hat{\Upsilon}^\nu \nu_R$. As at this stage neutrinos are still in the flavor basis, additional rotation on neutrino fields has to be performed bringing us to the mass eigenstates. Evidently this rotation is made by the mixing matrix in the lepton sector

$$U_{\text{PMNS}} = (V_L^e)^\dagger V_L^\nu, \tag{2.52}$$

the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix [21, 22]. Rotation to the mass eigenstates in the lepton sector is then done through

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \tag{2.53}$$

where ν_ℓ with $\ell = e, \mu, \tau$ are flavor eigenstates and ν_i, with $i = 1, 2, 3$ mass eigenstates. Comparing with the CKM matrix in (2.45), we note that whereas the PMNS matrix relates neutrinos in the mass and interaction (flavor) bases, the CKM matrix is doing it for down-quarks. This is only the convention used in the literature.

This generation of neutrino masses assumes that neutrinos are Dirac particles and is a straightforward generalization of the one for quark masses. But the tiny masses of neutrinos, when compared even with electron mass, suggest that a different mechanism

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

for mass generation involving right-handed neutrinos could be at work here. Indeed as neutrinos carry no electric charge they could also be Majorana particles. As right-handed neutrinos are SM singlets in this case, it is also allowed to add a Majorana mass term to (2.13):

$$-\frac{1}{2}\nu_R^\top C\,\mathsf{M}_N\,\nu_R. \tag{2.54}$$

Here C is the charge conjugation operator and M_N is an arbitrary Majorana mass matrix. Because this mass term is not protected by any symmetry (the Dirac mass terms are protected by the chiral symmetry) M_N can be much larger than the electroweak scale. This Majorana mass term together with (2.50) leads to the seesaw mechanism that can explain the light neutrino masses observed in nature with a high Majorana mass scale. For the PMNS matrix one just has to exchange the matrix V_L^ν in (2.52) by another matrix U_χ that diagonalizes the effective neutrino mass matrix that one gets through the seesaw mechanism. We refer to an excellent book of Bilenky [23], in which this sector of particle physics is described.

While the latter mechanism for mass generation has no direct analog in the quark sector because quarks carry electric charges, there exist other means of generating quark masses then just coupling quarks to the Higgs. This brings us to one of the most important puzzles in the SM.

2.5.3 The Flavor Puzzle in the SM

We have seen earlier how quarks and leptons get their masses in the SM and from where the CKM and PMNS matrices arise. However, this does not explain the values of the quark and lepton masses and of the observed structure of the unitary CKM and PMNS matrices. With the discovery of the Higgs boson and the measurement of Yukawa couplings we know that this mechanism for mass generation seems to be correct. Yet the Yukawa couplings and the elements of CKM and PMNS matrices are still free parameters, and in particular we would like to understand the observed mass spectrum. Indeed, the masses of the SM particles span a very wide range: from a heavy top quark of roughly 170 GeV to a tiny electron mass in the MeV region and even much lighter neutrino masses in the eV region (see Table D.1).

Qualitatively the CKM and PMNS matrices look as follows[4]

$$V_{\text{CKM}} = \left(\begin{array}{ccc} \bullet & \cdot & \cdot \\ \cdot & \bullet & \cdot \\ \cdot & \cdot & \bullet \end{array}\right), \qquad U_{\text{PMNS}} = \left(\begin{array}{ccc} \bullet & \bullet & \cdot \\ \cdot & \bullet & \bullet \\ \cdot & \bullet & \bullet \end{array}\right). \tag{2.55}$$

The hierarchical structure of the CKM matrix is clearly visible, whereas the magnitudes of the entries of the PMNS matrix are of the same order. The explanation of the origin of these patterns and in particular why they differ from each other remains an important goal of the theorists.

[4] The radius of the circles illustrate the absolute values of the entries, i.e., $r \simeq \sqrt{|V_{ij}|}$.

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

The flavor puzzle of the SM can be summarized as follows: Why do the Yukawa couplings cover a large range of $O(10^{-6})$ to $O(1)$? Why are the mixing angles of the CKM matrix so small and those of the PMNS matrix rather large? Why are there exactly three particle generations?

There is a very rich literature on the generation of fermion masses and related CKM and PMNS matrices by some new dynamics, but we cannot describe these efforts here. Useful references can be found in the first paragraphs of [24]. Moreover, one should keep in mind that quark and lepton masses and the parameters of CKM and PMNS matrices could remain as given parameters of nature forever, and there is no dynamical explanation of their values. More important for the time being is the derivation of the Feynman rules for electroweak interactions of quarks and leptons. We begin with neutral currents coupled to the heavy gauge boson Z and the photon A. Subsequently we will discuss charged currents coupled to $W^{\pm}$.

2.5.4 Neutral Currents

To derive the couplings of quarks and leptons to the photon A_μ and to Z_μ we first insert the mixing in (2.26) back into the covariant derivatives (2.4)–(2.7) using the relations in (2.28). For doublets we can also use directly (2.19) with appropriate values of the hypercharge Y. For *lepton doublets* L_j we get, for example,

$$D_\mu = \partial_\mu - ie \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix}, \tag{2.56}$$

where

$$K_{11} = \left(Y + \frac{1}{2}\right) A_\mu - \frac{2Y \tan^2 \vartheta_W - 1}{2 \tan \vartheta_W} Z_\mu = \frac{1}{2 \sin \vartheta_W \cos \vartheta_W} Z_\mu, \tag{2.57}$$

$$K_{12} = \frac{1}{\sqrt{2} \sin \vartheta_W} W_\mu^+, \tag{2.58}$$

$$K_{21} = \frac{1}{\sqrt{2} \sin \vartheta_W} W_\mu^-, \tag{2.59}$$

$$K_{22} = \left(Y - \frac{1}{2}\right) A_\mu - \frac{2Y \tan^2 \vartheta_W + 1}{2 \tan \vartheta_W} Z_\mu = -A_\mu - \frac{-\sin^2 \vartheta_W + \frac{1}{2}}{\sin \vartheta_W \cos \vartheta_W} Z_\mu, \tag{2.60}$$

and in evaluating K_{11} and K_{22}, we set $Y = -1/2$ in accordance with Table 2.1.

For *lepton singlets* e_{Rj} we find

$$D_\mu = \partial_\mu - i \frac{e}{\cos \vartheta_W} Y \left(\cos \vartheta_W A_\mu - \sin \vartheta_W Z_\mu\right)\Big|_{Y=-1} = \partial_\mu + ieA_\mu - ie \tan \vartheta_W Z_\mu. \tag{2.61}$$

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

Inserting this back into (2.3) and only considering the couplings to neutral gauge bosons Z_μ and A_μ we get

$$\mathscr{L}_I^{\text{SM}} \supset \sum_{j=1,2,3} \bar{L}_j i\not{D} L_j + \bar{e}_{Rj} i\not{D} e_{Rj} \supset$$

$$\sum_{j=1,2,3} \Bigg[\frac{e}{2\sin\vartheta_W \cos\vartheta_W} \bar{\nu}_{Lj} \gamma^\mu Z_\mu \nu_{Lj} - e\bar{e}_{Lj}\gamma^\mu A_\mu e_{Lj} - e\bar{e}_{Rj}\gamma^\mu A_\mu e_{Rj} \tag{2.62}$$

$$+ \frac{e}{\sin\vartheta_W \cos\vartheta_W} \left(-\frac{1}{2} + \sin^2\vartheta_W \right) \bar{e}_{Lj}\gamma^\mu Z_\mu e_{Lj} + \frac{e}{\sin\vartheta_W \cos\vartheta_W} \sin^2\vartheta_W \bar{e}_{Rj}\gamma^\mu Z_\mu e_{Rj} \Bigg].$$

As it should be, the photon couples in the same manner to left- and right-handed particles. The same procedure can be used for quarks, and the result can be simplified and summarized as follows:

$$\mathscr{L}_I^{\text{SM}} \supset \frac{e}{\sin\vartheta_W \cos\vartheta_W} \left(T_3 - \sin^2\vartheta_W Q_f \right) \bar{f}\gamma^\mu Z_\mu f + eQ_f \bar{f}\gamma^\mu A_\mu f, \tag{2.63}$$

where f is RH or LH fermion (quark or lepton) with weak isospin T_3 ($T_3 = 0$ for RH fermions and $T_3 = \pm\frac{1}{2}$ for LH fermions) and electric charge Q_f in units of e, e.g., $Q_e = -1$ for the electron. If one multiplies (2.63) by i one gets directly the Feynman rules for the interaction of Z_μ and A_μ with SM fermions. We list these rules in Appendix B.

The expression in (2.63) is valid for all three generations. The CKM matrix that occurred in the Yukawa sector as explained in Section 2.5 does not occur in interactions with the Z boson and the photon. This is due to the unitarity of the CKM matrix. For $V_\mu = Z_\mu$, $V_\mu = G_\mu$, or $V_\mu = A_\mu$ neutral currents involving down-type quarks do not change after the rotation to mass eigenstates in (2.43):

$$\sum_{j=1,2,3} \bar{d}_{Lj}\gamma_\mu d_{Lj} V^\mu \to \sum_{j,k=1,2,3} \bar{d}_{Lj} \underbrace{\left(V_{\text{CKM}}^\dagger V_{\text{CKM}} \right)_{jk}}_{=\delta_{jk}} \gamma_\mu d_{Lk} V^\mu. \tag{2.64}$$

This is a very important result and is called tree-level GIM mechanism:

There are no flavor-changing neutral currents (FCNCs) at tree level in the SM. This consequence of the unitarity of the CKM matrix is called GIM mechanism, named after Glashow, Iliopoulos, and Maiani [6].

However the GIM mechanism is also active in many loop processes. FCNCs occur at loop level but not at tree level within the SM. In the so-called penguin diagrams, transitions like $b \to s$ and $s \to d$ are possible (see Figure 2.2). In the loop all three up-type quarks appear. The structure of such a penguin diagram is

$$P = \lambda_u f(m_u, M_W) + \lambda_c f(m_c, M_W) + \lambda_t f(m_t, M_W) \tag{2.65}$$

$$= \lambda_c \left(f(m_c, M_W) - f(m_u, M_W) \right) + \lambda_t \left(f(m_t, M_W) - f(m_u, M_W) \right), \qquad \lambda_i = V_{id} V_{is}^*,$$

with a loop function f that depends on M_W and the quark mass entering the loop. In the second line we used the unitarity of the CKM matrix

$$\lambda_u + \lambda_c + \lambda_t = 0. \tag{2.66}$$

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

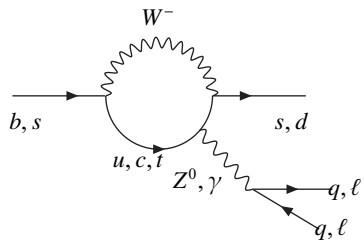


Figure 2.2 FCNC at loop level: penguin diagram for a $b \to s$ or $s \to d$ transition.

The important virtue of the GIM mechanism is that it does not only forbid tree-level FCNCs but also suppresses FCNCs at loop level for observables in which only light-quarks matter. It would also work if the masses of quarks with a given charge would be the same. Indeed for degenerate masses $m_u = m_c = m_t$ the penguin diagram in (2.65) would vanish, and this is a general property of FCNCs in the SM as we will see in other chapters of this book. For different masses a GIM suppression factor occurs when only the c and u quark contributions are relevant. Indeed, in this case

$$(f(m_c, M_W) - f(m_u, M_W)) \propto \frac{m_c^2 - m_u^2}{M_W^2} \approx 2 \cdot 10^{-4}, \tag{2.67}$$

which allows to suppress naturally certain observables in the K meson system to a desired level. However, due to $m_t > M_W \gg m_{u,c}$ there is no GIM suppression when the contributions of the top quark in the loops dominate a given observable. What is interesting is that the pattern of the violation of GIM mechanism through the top quark contributions appears until now to be confirmed within experimental and theoretical uncertainties by present data. We will witness this in the phenomenological parts of this book. But as we will stress there, beyond SM this structure could be violated, and FCNCs could take place already at tree level.

2.5.5 Charged Currents

The couplings of charged gauge bosons $W_\mu^\pm$ to fermions result from the covariant derivative in (2.3) and can be directly derived from (2.58) and (2.59). When rotating to the mass eigenstates the CKM and PMNS matrices occur:

$$\begin{aligned}
\mathscr{L}^{CC} &= \frac{g_2}{\sqrt{2}} \left(\bar{u}_j \gamma^\mu P_L d_j W_\mu^+ + \bar{\nu}_j \gamma^\mu P_L \ell_j W_\mu^+ \right) + h.c. \\
&\to \frac{g_2}{\sqrt{2}} \left(\bar{u}_j \gamma^\mu V_{jk} P_L d_k W_\mu^+ + \bar{\nu}_j \gamma^\mu U_{kj}^* P_L \ell_k W_\mu^+ \right) + h.c. \\
&= \frac{g_2}{\sqrt{2}} (\bar{u} \gamma^\mu V_{\text{CKM}} P_L d \, W_\mu^+ + \bar{\nu} \gamma^\mu U_{\text{PMNS}}^\dagger P_L \ell \, W_\mu^+) + h.c.
\end{aligned} \tag{2.68}$$

The complex conjugation in the PMNS matrix should be noticed, but this should be clear from the definition of this matrix in (2.52).

In the original formulation of the SM without neutrino oscillations one has to set $U_{\text{PMNS}} = \mathbb{1}$. The corresponding Feynman rules for the mass eigenstates are shown in

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

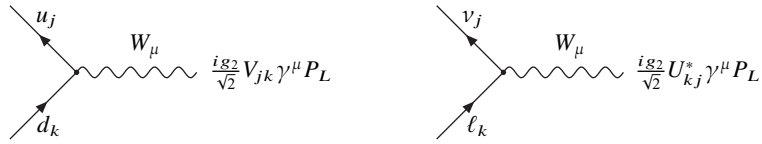


Figure 2.3 Feynman rules for charged W-fermion couplings. Here we already take into account neutrino mixing.

Figure 2.3. As one can see flavor violation or flavor transitions occur now in gauge interactions. However, one should emphasize that the gauge sector is in principle flavor blind: (2.3) doesn't distinguish between the different generations, and it is invariant under a rotation in flavor space as in (2.39) and (2.40). But the Yukawa sector is no longer flavor blind. So the origin of flavor violation comes from the Yukawa interactions. Only after diagonalizing all Yukawa matrices, i.e., in the mass eigenstate basis, flavor violation is transferred to the charged currents of the gauge sector.

2.5.6 CKM Matrix, CP Violation, and the Unitarity Triangle

The CKM matrix was already defined in (2.44). As a unitary 3×3 matrix, it can be parametrized in general by three angles and six phases. But not all of them are physical. The diagonal Yukawa couplings $\hat{\Upsilon}^U$, $\hat{\Upsilon}^D$ possess a $[U(1)]^6$ symmetry, i.e.,

$$u_j^{L,R} \to e^{i\varphi_j^u} u_j^{L,R}, \quad d_j^{L,R} \to e^{i\varphi_j^d} d_j^{L,R}, \tag{2.69}$$

leave $\bar{u}_L \hat{\Upsilon}^U u_R$ and $\bar{d}_L \hat{\Upsilon}^D d_R$ invariant but not the W couplings. With this we can rotate away five out of the six phases via

$$V_{ij}^{\text{CKM}} \to e^{i(\varphi_j^d - \varphi_i^u)} V_{ij}^{\text{CKM}}. \tag{2.70}$$

The only leftover phase in the SM is called Kobayashi–Maskawa phase [20]. It is the only origin of CP violation in the SM[5] and always connected with flavor changing transitions. Because there is only one single phase in the CKM matrix, CP violating processes are correlated with each other in the SM. The standard parametrization [25] of the CKM matrix with three angles and one phase is given as

$$V_{\text{CKM}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.71}$$

and consequently

$$\boxed{V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}}. \tag{2.72}$$

[5] We do not consider here the θ term of QCD, which is related to the strong CP violation that we will briefly discuss in Section 17.3.

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

The abbreviations s_{ij} and c_{ij} stand for $\sin\theta_{ij}$ and $\cos\theta_{ij}$. If there were only two generations, no physical phase would be left over but only one mixing angle, the Cabibbo angle θ_C.

From experiment we know that the CKM matrix is a hierarchical matrix. It is close to the unit matrix with small off-diagonal entries. In particular to an excellent approximation:

$$c_{12} = 1 - \frac{\lambda^2}{2}, \qquad c_{13} = 1, \qquad c_{23} = 1, \tag{2.73}$$

with λ being one of the Wolfenstein parameters, discussed later, that is equal to the $\sin\theta_C$ or equivalently to s_{12}. The virtue of the parametrization (2.72) supplemented by (2.73) is that by measuring $|V_{us}|$, $|V_{ub}|$, and $|V_{cb}|$ in tree-level decays one can determine s_{12}, s_{13}, and s_{23} simply through:

$$s_{12} = |V_{us}|, \qquad s_{13} = |V_{ub}|, \qquad s_{23} = |V_{cb}|. \tag{2.74}$$

This relation and (2.73) express the known fact that θ_{ij} can be chosen without losing the generality to be in the first quadrant. While then the phase δ could have in principle any value in the range $[0, 2\pi]$, we know already for some time that nature chooses it to be in the first quadrant as well. The values of all these parameters will play an important role in this book, and we will discuss their determination in the next chapters.

The hierarchical structure of the CKM matrix is nicely represented by its Wolfenstein parametrization [26]. It is an expansion in the parameter $\lambda = |V_{us}| = 0.225$. The other three parameters A, ρ, and η of this parametrization lie between zero and one. Thus

$$V_{\rm CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4). \tag{2.75}$$

In the flavor precision era one should at least include $O(\lambda^4)$ corrections and possibly higher-order terms. An efficient and systematic way of finding higher-order terms in λ has been proposed in [27]. One goes back to the standard parametrization (2.72) and *defines* the parameters $(\lambda, A, \varrho, \eta)$ through [27]

$$s_{12} = \lambda, \qquad s_{23} = A\lambda^2, \qquad s_{13}e^{-i\delta} = A\lambda^3(\varrho - i\eta), \tag{2.76}$$

to *all orders* in λ. It follows that

$$\varrho = \frac{s_{13}}{s_{12}s_{23}}\cos\delta, \qquad \eta = \frac{s_{13}}{s_{12}s_{23}}\sin\delta. \tag{2.77}$$

Equations (2.76) and (2.77) represent simply the change of variables from the standard ones to the ones of Wolfenstein. Making this change of variables in the standard parametrization (2.72) we find the CKM matrix as a function of $(\lambda, A, \varrho, \eta)$, which satisfies unitarity exactly! Expanding next each element in powers of λ, we recover the matrix in (2.75) and in addition find explicit corrections of $O(\lambda^4)$ and higher-order terms.[6]

[6] A similar parametrization has been proposed by Branco and Lavoura [28]. See [7] for a general discussion.

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

The result of this exercise can be found in [29]. One finds that by definition V_{ub} remains unchanged, and the corrections to V_{us} and V_{cb} appear only at $O(\lambda^7)$ and $O(\lambda^8)$, respectively. Consequently to a very good accuracy we have:

$$V_{us} = \lambda, \qquad V_{cb} = A\lambda^2, \tag{2.78}$$

$$V_{ub} = A\lambda^3(\varrho - i\eta), \qquad V_{td} = A\lambda^3(1 - \bar{\varrho} - i\bar{\eta}) \tag{2.79}$$

with

$$\boxed{\bar{\varrho} = \varrho\left(1 - \frac{\lambda^2}{2}\right), \qquad \bar{\eta} = \eta\left(1 - \frac{\lambda^2}{2}\right).} \tag{2.80}$$

Useful analytic expressions for $\lambda_i = V_{id}V_{is}^*$ with $i = c, t$ are the following ones

$$\mathrm{Im}\lambda_t = -\mathrm{Im}\lambda_c = \eta A^2\lambda^5 = |V_{ub}||V_{cb}|\sin\delta, \tag{2.81}$$

$$\mathrm{Re}\lambda_c = -\lambda\left(1 - \frac{\lambda^2}{2}\right), \tag{2.82}$$

$$\mathrm{Re}\lambda_t = -\left(1 - \frac{\lambda^2}{2}\right)A^2\lambda^5(1 - \bar{\varrho}), \tag{2.83}$$

and a more accurate formula for $\mathrm{Re}\lambda_t$ can be derived ($\gamma = \delta$)

$$\mathrm{Re}\lambda_t \simeq |V_{ub}||V_{cb}|\cos\gamma(1 - 2\lambda^2) + (|V_{ub}|^2 - |V_{cb}|^2)\lambda\left(1 - \frac{\lambda^2}{2}\right). \tag{2.84}$$

While these expressions are useful for the derivation of analytic formulas for various observables, in numerical calculations these days it is better to use exact standard parametrization as given in (2.72). To this end, as suggested to my knowledge by CKMfitters, $\bar{\varrho}$ and $\bar{\eta}$ can be defined in a phase convention-independent manner through

$$\boxed{\bar{\varrho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}.} \tag{2.85}$$

One can check that in the standard parametrization this definition agrees to an excellent accuracy with the original one in (2.80).

Before continuing our discussion of the CKM matrix, it should be emphasized that in contrast to strong and electromagnetic interactions, which are both invariant under parity P, charge conjugation C, and time reversal T, the weak interaction violates P and C maximally, and the combination CP is violated by the phase δ of the CKM matrix. In the standard phase convention we have under CP

$$\bar{u}_L\gamma_\mu d_L \xrightarrow{CP} -\bar{d}_L\gamma^\mu u_L, \quad W_\mu^\pm \xrightarrow{CP} -W^{\mp\mu}. \tag{2.86}$$

Thus, this means for charged currents:

$$\begin{aligned}\mathscr{L}_q^{CC} &= -\frac{g_2}{\sqrt{2}}\left(\bar{u}_j\gamma^\mu V_{jk}P_L d_k W_\mu^+ + \bar{d}_k\gamma^\mu V_{jk}^* P_L u_j W_\mu^-\right)\\ &\xrightarrow{CP} -\frac{g_2}{\sqrt{2}}\left(\bar{d}_k\gamma_\mu V_{jk}P_L u_j W^{\mu-} + \bar{u}_j\gamma_\mu V_{jk}^* P_L d_k W^{\mu+}\right).\end{aligned} \tag{2.87}$$

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

This Lagrangian is only invariant under CP if $V_{jk} = V_{jk}^*$ for all $j, k = 1, 2, 3$. Furthermore, to get CP violation all three generations must be involved in the process because with only two generations there exist no physical phases.

The unitarity of the CKM matrix leads first to the following set of equations:

$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1, \tag{2.88}$$

$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1, \tag{2.89}$$

$$|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1, \tag{2.90}$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1, \tag{2.91}$$

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1, \tag{2.92}$$

$$|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1, \tag{2.93}$$

that describe the normalization of the columns and rows of the CKM matrix, respectively. They are sometimes used to test the unitarity of this matrix.

The remaining six relations originate from the orthogonality of different columns and rows and read

$$V_{ik}V_{jk}^* = \delta_{ij}, \qquad V_{ik}V_{ij}^* = \delta_{kj}, \qquad i \neq j. \tag{2.94}$$

They can be represented as six "unitarity" triangles in the complex plane [30, 31]. The area of all triangles is the same and a measure for CP violation in the SM. It is described by the Jarlskog invariant J [32, 33]

$$J = \mathrm{Im}\left[V_{td}^* V_{tb} V_{ub}^* V_{ud}\right] = c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13}\sin\delta \simeq A^2\lambda^6\eta. \tag{2.95}$$

The best-known *unitarity triangle* (UT) is the one following from the relation

$$\boxed{V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0.} \tag{2.96}$$

Phenomenologically this relation played an important role in the last twenty-five years because it involves simultaneously the elements V_{ub}, V_{cb}, and V_{td}, which were and still are under extensive discussion.

The sides of *this* unitarity triangle are all of the order $O(\lambda^3)$, and it is constructed from the relation (2.96) divided by $V_{cd}V_{cb}^*$. The resulting triangle is shown in Figure 2.4.

It will be useful to have a collection of the most important properties of this triangle:

- Using simple trigonometry one can express $\sin(2\phi_i)$, $\phi_i = \alpha, \beta, \gamma$, in terms of $(\bar{\varrho}, \bar{\eta})$ as follows:

$$\sin(2\alpha) = \frac{2\bar{\eta}(\bar{\eta}^2 + \bar{\varrho}^2 - \bar{\varrho})}{(\bar{\varrho}^2 + \bar{\eta}^2)((1-\bar{\varrho})^2 + \bar{\eta}^2)}, \tag{2.97}$$

$$\sin(2\beta) = \frac{2\bar{\eta}(1-\bar{\varrho})}{(1-\bar{\varrho})^2 + \bar{\eta}^2}, \tag{2.98}$$

$$\sin(2\gamma) = \frac{2\bar{\varrho}\bar{\eta}}{\bar{\varrho}^2 + \bar{\eta}^2} = \frac{2\varrho\eta}{\varrho^2 + \eta^2}. \tag{2.99}$$

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

- The lengths CA and BA in the rescaled triangle to be denoted by R_b and R_t, respectively, are given by

$$\boxed{R_b \equiv \frac{|V_{ud}V_{ub}^*|}{|V_{cd}V_{cb}^*|} = \sqrt{\bar{\varrho}^2 + \bar{\eta}^2} = \left(1 - \frac{\lambda^2}{2}\right)\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|,} \tag{2.100}$$

$$\boxed{R_t \equiv \frac{|V_{td}V_{tb}^*|}{|V_{cd}V_{cb}^*|} = \sqrt{(1-\bar{\varrho})^2 + \bar{\eta}^2} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right|.} \tag{2.101}$$

- R_b and R_t can also be determined by measuring two of the angles of the UT:

$$R_b = \frac{\sin(\beta)}{\sin(\alpha)} = \frac{\sin(\alpha+\gamma)}{\sin(\alpha)} = \frac{\sin(\beta)}{\sin(\gamma+\beta)}, \tag{2.102}$$

$$R_t = \frac{\sin(\gamma)}{\sin(\alpha)} = \frac{\sin(\alpha+\beta)}{\sin(\alpha)} = \frac{\sin(\gamma)}{\sin(\gamma+\beta)}. \tag{2.103}$$

- The angles β and γ of the unitarity triangle are related directly to the complex phases of the CKM elements V_{td} and V_{ub}, respectively, through

$$V_{td} = |V_{td}|e^{-i\beta}, \quad V_{ub} = |V_{ub}|e^{-i\gamma}. \tag{2.104}$$

- The angle α can be obtained through the relation

$$\alpha + \beta + \gamma = 180^\circ \tag{2.105}$$

expressing the unitarity of the CKM matrix.

Now note that the unitarity relation (2.96) can also be rewritten as

$$R_b e^{i\gamma} + R_t e^{-i\beta} = 1\ . \tag{2.106}$$

This formula shows transparently that the knowledge of (R_t, β) allows to determine (R_b, γ) through

$$\boxed{R_b = \sqrt{1 + R_t^2 - 2R_t\cos\beta}, \qquad \cot\gamma = \frac{1 - R_t\cos\beta}{R_t\sin\beta}.} \tag{2.107}$$

Similarly, (R_t, β) can be expressed through (R_b, γ):

$$\boxed{R_t = \sqrt{1 + R_b^2 - 2R_b\cos\gamma}, \qquad \cot\beta = \frac{1 - R_b\cos\gamma}{R_b\sin\gamma}.} \tag{2.108}$$

These relations are remarkable. They imply that the knowledge of the coupling V_{td} between t and d quarks allows to deduce the strength of the corresponding coupling V_{ub} between u and b quark and vice versa.

There are other useful trygonometrical relations that can be used to construct the rescaled unitarity triangle by measuring e.g., the angle γ and β as given in (2.102) and (2.103). The complete list of such relations can be found in [34], and we will encounter some of them later in this book.

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

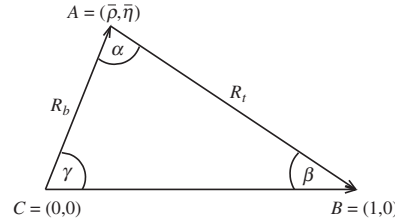


Figure 2.4 Unitarity Triangle. In Japan $\beta = \phi_1$, $\alpha = \phi_2$, and $\gamma = \phi_3$.

The triangle depicted in Figure 2.4 together with $|V_{us}|$ and $|V_{cb}|$ gives a full description of the CKM matrix. Looking at the expressions for R_b and R_t, we observe that within the SM the measurements of four CP *conserving* decays sensitive to $|V_{us}|$, $|V_{ub}|$, $|V_{cb}|$, and $|V_{td}|$ can tell us whether CP violation ($\eta \neq 0$) is predicted in the SM. This is a very remarkable property of the Kobayashi–Maskawa picture of CP violation: Quark mixing and CP violation are closely related to each other. Overconstraining the sides and angles of the unitarity triangle with experimental data is an important test of the CKM mechanism of the SM. Possible deviations are hints of contributions from NP. The processes used presently in the construction of this triangle will be discussed in Section 8.4 and its present status in Chapter 12. Yet already now we list the values of four CKM parameters obtained from present SM global fits. The CKMfitter collaboration finds

$$A = 0.8403^{+0.0056}_{-0.0201}, \quad \lambda = 0.224747^{+000254}_{-000059}, \quad \bar{\varrho} = 0.1577^{0.0096}_{-0.0074}, \quad \bar{\eta} = 0.3493^{0.0095}_{-0.0071}, \tag{2.109}$$

and the UTfitter collaboration

$$A = 0.826 \pm 0.012, \quad \lambda = 0.2255 \pm 0.0005, \quad \bar{\varrho} = 0.148 \pm 0.013, \quad \bar{\eta} = 0.348 \pm 0.010, \tag{2.110}$$

in good agreement with the CKMfitter determination. More details are given in Table 8.1.

2.6 Lepton Flavor Violation

The PMNS matrix defined in (2.52) and (2.53) that occurs in charged currents in (2.68) describes flavor transitions in the lepton sector in the same manner as the CKM matrix describes quark flavor transitions. The standard parametrization of the PMNS matrix is the same as for the CKM matrix and given in (2.72). But due to the possibility of Majorana neutrinos one has less phase rotation available such that two additional Majorana phases are left over. In this case one has to multiply (2.72) with the diagonal phase matrix

$$P = \text{diag}\left(e^{\frac{i\alpha_1}{2}}, e^{\frac{i\alpha_2}{2}}, 1\right). \tag{2.111}$$

Experiments observed a deficit in muon antineutrinos that are produced in cosmic rays in the atmosphere, which was known as *atmospheric neutrino problem*. Similarly less electron neutrinos as produced in the sun were finally detected on earth (*solar neutrino*

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

problem). Both problems can be explained by neutrino oscillations, which necessarily implies nonvanishing mass differences of the neutrinos $\Delta m^2_{ij} = m^2_{\nu_i} - m^2_{\nu_j}$ and a violation of the individual lepton number – both in contradiction to the SM. However the absolute mass scale for neutrino masses is still unknown, and only upper bounds are currently available. Furthermore we do not know if ν_3 is heavier (normal hierarchy) or lighter (inverted hierarchy) than ν_1 and ν_2 (ν_1 and ν_2 are defined to be those neutrinos that describe the solar mixing). Compared with the mixing angles of the CKM matrix, the PMNS matrix has two large mixing angles, the atmospheric $\theta_{23} = \theta_{\rm atm}$ and solar mixing angle $\theta_{12} = \theta_{\rm sol}$ and is thus not hierarchical at all. The third mixing angle, the reactor angle θ_{13}, was long thought to be around zero, but recent experiments measured $\theta_{13} \approx 8^\circ$, which is even larger than the corresponding mixing angle in the CKM matrix. The CP phase δ of the PMNS matrix hasn't been measured so far, but some information on it can be obtained from global fits [1335]. The latter paper contains also a very useful material for mixing angles and mass differences. The current experimental values are [35]

$$\begin{aligned} &\sin^2\theta_{12} = 0.310 \pm 0.013, && \Delta m^2_{21} = (7.53 \pm 0.18)\cdot 10^{-5}\ \text{eV}^2, \\ &\sin^2\theta_{23} = 0.582 \pm 0.019, && \Delta m^2_{32} = \begin{cases} -(2.56 \pm 0.04)\cdot 10^{-3}\ \text{eV}^2 & \text{inverted}, \\ (2.51 \pm 0.05)\cdot 10^{-3}\ \text{eV}^2 & \text{normal}, \end{cases} \\ &\sin^2\theta_{13} = 0.0220 \pm 0.0007. \end{aligned} \tag{2.112}$$

So far only lepton flavor violation in neutrino physics has been measured, namely in neutrino oscillations. However, no charged lepton-flavor violating decays have been observed to date. In the SM enriched by light neutrino masses lepton-flavor violating decays $\ell_j \to \ell_i\gamma$ occur at unobservable small rates because the transition amplitudes are suppressed by a factor of $(m^2_{\nu_j} - m^2_{\nu_i})/M^2_W$.

In Section 17.2 we will discuss charged lepton-flavor violating decays in some detail, demonstrating explicitly that their branching ratios in the SM are tiny and that any observation of such decays would be a clear signal of physics beyond the SM. On the other hand, we will basically not discuss neutrino oscillations that are already elaborated in many textbooks. An excellent account of this subject is given in [23]. However, we will list in Section 17.5 processes related to neutrinos that are important in the search for NP. Here neutrinoless double β decay plays a prominent role.

2.7 Quantumchromodynamics (QCD)

2.7.1 QCD Lagrangian

At the beginning of this chapter we looked briefly at QCD. Even if weak interactions are the driving force in weak decays, strong interactions play a very important role in this book simply because the decaying objects in the case of quark physics are mesons, bound states of quarks and antiquarks. Therefore we have to discuss in more detail this theory. Here we only collect a few basic facts about QCD that will be discussed in more detail in later chapters. We begin with the QCD Lagrangian that has the structure of a nonabelian gauge

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

theory based on the SU(3) symmetry group usually denoted by $SU(3)_C$. In the process of quantization ghost fields have to be introduced. We will not expose them here. Also the gauge has to be fixed. We will dominantly work in the Feynman gauge and consequently set the gauge parameter to $\xi = 1$. The QCD Lagrangian then reads

$$\begin{aligned}\mathscr{L}_{QCD} = &-\frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu}) - \frac{1}{2}(\partial^\mu A^a_\mu)^2 \\ &+ \bar{q}_\alpha (i \not{\partial} - m_q) q_\alpha + g_s \bar{q}_\alpha T^a_{\alpha\beta} \gamma^\mu q_\beta A^a_\mu \\ &- \frac{g_s}{2} f^{abc}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) A^{b\mu} A^{c\nu} - \frac{g_s^2}{4} f^{abe} f^{cde} A^a_\mu A^b_\nu A^{c\mu} A^{d\nu}. \end{aligned} \tag{2.113}$$

Here A^a_μ are the gluon fields with $a, b, c = 1, \ldots 8$ and $q = (q_1, q_2, q_3)$ is the color triplet of quark flavor q, $q = u, d, s, c, b, t$. We will denote the colors of quarks by Greek letters α, β, γ. The parameter g_s is the QCD coupling so that

$$\alpha_s = \frac{g_s^2}{4\pi}. \tag{2.114}$$

Finally, T^a and f^{abc} are the generators and structure constants of $SU(3)_C$, respectively. From this Lagrangian one can derive the Feynman rules for QCD. They are given in Appendix B.

2.7.2 Asymptotic Freedom

Probably the most important property of QCD is asymptotic freedom [36–39], which means that at large-energy scales or equivalently short-distance scales the QCD coupling α_s is small. This implies that calculations of strong interaction effects can be performed at short-distance scales within perturbation theory or more precisely in renormalization group improved perturbation theory. For the time being this statement is only a warning that we will have to go beyond usual perturbation theory to obtain useful results. The full technology will be exposed in Section 4.3, and subsequent sections related to renormalization group methods. On the other hand at large distances QCD is supposed to confine quarks and gluons in hadrons, and α_s must be large at low-energy scales. Thus α_s must be a scale-dependent quantity, and in fact QCD predicts this dependence.

We have already stated before that in an unbroken nonabelian gauge theory there is only one free parameter, the gauge coupling constant, which has to be determined from experiment. Now, the gauge coupling in a four-dimensional gauge theory is a dimensionless quantity, and its dependence on the energy scale Q implies that there must exist still another scale $\Lambda_{\rm QCD}$ so that α_s depends only on $Q/\Lambda_{\rm QCD}$ and remains dimensionless despite the dependence on the energy scale. Therefore instead of the gauge coupling also $\Lambda_{\rm QCD}$ can be regarded as the sole free parameter of QCD and as the fundamental scale of QCD. The fact that the description of QCD can be made either in terms a fundamental scale $\Lambda_{\rm QCD}$, which obviously is dimensionful, or a dimensionless quantity α_s is sometimes called *dimensional transmutation.* It is the property of renormalizable theories, which have an intrinsic scale, the renormalization scale.

In the late 1970s and in 1980s it was $\Lambda_{\rm QCD}$ or more precisely the so-called $\Lambda_{\overline{\rm MS}}$, to be introduced later, that played the role in the phenomenology as it could be extracted from

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

violation of Bjorken scaling in deep-inelastic electron-proton collisions. However, in the 1990s the strength of strong interactions could be determined more precisely at LEP in e^+e^- annihilation into hadrons and from lattice QCD calculations. It turned out then to be more useful to quote the value of α_s at the scale $\mu = M_Z$ instead of the value of $\Lambda_{\overline{\rm MS}}$. But once this is done, the theory predicts uniquely the value of α_s at any other scale at which perturbative calculations make sense. This typically means $\mu \geq 1\,\text{GeV}$. Of course, this information can also be translated into the corresponding value of $\Lambda_{\overline{\rm MS}}$.

The scale dependence of α_s is governed by a renormalization group equation, which we will derive and discuss in Section 4.3.

Quark masses enter the SM Lagrangian for electroweak interactions and also the QCD Lagrangian (2.113). We did not count them as parameters of QCD because they depend on flavor, and strong interactions are flavor blind. The SM cannot explain their values, but there are models trying to explain them. We will not discuss such models in this book as their number is very large. They can all be found with the help of INSPIRE or even Google.

Similar to α_s, quark masses in the Lagrangian depend on the energy scale and are renormalization scheme dependent. We will discuss this in Section 4.3.

2.7.3 Hadrons: Mesons and Baryons

The strong force can bound quarks into hadrons. Hadrons are strongly interacting particles and can be sorted into mesons and baryons. Mesons consist of a quark and an antiquark $q_i\bar{q}_j$, have integer spin, and are thus bosons. Two groups of lightest mesons containing (u, d, s) quarks are pions and kaons, which will play an important role in this book. However, since the middle of the 1970s also heavier mesons containing heavier quarks, $D^0(D_s^\pm)$ and $B^0_{d,s}, B^\pm$ mesons became even more important in testing the SM. Baryons consist of three quarks $q_iq_jq_k$ (or three antiquarks $\bar{q}_i\bar{q}_j\bar{q}_k$), have half-integer spin, and are thus fermions. The two lightest baryons are the proton and the neutron. Quarkonia are bound states as $c\bar{c}$ (charmonium) and $b\bar{b}$ (bottomium) and can be treated similar to the hydrogen atom (pe^-) or positronium (e^+e^-). Hadrons including top quarks do not exist because the top quark decays too fast before it could form a bound state. In this book we will dominantly deal with the decays of mesons. Neutron and proton will play a role in the context of electric dipole moments in Section 17.3. We will not discuss quarkonia in this book and refer to [40–42]. Looking at citations of these reviews one can easily find out more recent reviews.

We will mainly discuss mesons here. The decays of mesons, the bound states of quarks, and antiquarks, will play a very prominent role in this book, and it is important to recall their valence structure.

π mesons:

$$\boxed{\pi^+ = (u\bar{d}), \quad \pi^0 = \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}, \quad \pi^- = (\bar{u}d).} \tag{2.115}$$

K mesons:

$$\boxed{K^+ = (u\bar{s}), \quad K^- = (\bar{u}s), \quad K^0 = (d\bar{s}), \quad \bar{K}^0 = (\bar{d}s).} \tag{2.116}$$

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

***B* mesons:**

$$\boxed{B^+ = (u\bar{b}), \quad B^- = (\bar{u}b), \quad B_d^0 = (d\bar{b}), \quad \bar{B}_d^0 = (\bar{d}b)} \tag{2.117}$$

$$\boxed{B_s^0 = (s\bar{b}), \quad \bar{B}_s^0 = (\bar{s}b), \quad B_c^+ = (c\bar{b}), \quad B_c^- = (\bar{c}b).} \tag{2.118}$$

***D* mesons:**

$$\boxed{D^0 = (c\bar{u}), \quad \bar{D}^0 = (\bar{c}u), \quad D^+ = (c\bar{d}), \quad D^- = (\bar{c}d),} \tag{2.119}$$

$$\boxed{D_s^+ = (c\bar{s}), \quad D_s^- = (\bar{c}s).} \tag{2.120}$$

Note that K and B mesons contain a $\bar{s}$ and $\bar{b}$, respectively, and not an s and b quark. This has historical reasons. However, the D meson contains a c and not $\bar{c}$.

Mesons can be characterized by their J^{PC} quantum number, where J is the spin of the meson

$$\vec{J} = \vec{L} + \vec{S}, \qquad \vec{S} = \vec{s}_q + \vec{s}_{\bar{q}}, \tag{2.121}$$

composed of the angular momentum $\vec{L}$ and the total spin of the two quarks $\vec{S}$. For the lighter meson systems the quark and antiquark do not have a relative angular momentum and are in a state with $L = 0$. We will mostly consider such mesons. Parity and the charge conjugation quantum number are given, respectively, as

$$P = (-1)^{L+1}, \qquad C = (-1)^{L+S}. \tag{2.122}$$

The mesons built out of the three lightest quarks u, d, s can be grouped into multiplets of an approximate flavor SU(3) symmetry. If this symmetry would be exact, then there would be a singlet and an octet according to

$$\mathbf{3}_q \otimes \bar{\mathbf{3}}_{\bar{q}} = \mathbf{1} \oplus \mathbf{8}. \tag{2.123}$$

But because the flavor symmetry is broken, the singlet and the octet mix. For $L = 0$ we have pseudoscalar mesons, where the spins of the two quarks sum to zero: $J^{PC} = 0^{-+}$. Vector mesons have $S = 1$ such that $J^{PC} = 1^{--}$. In Figure 2.5 we arrange the pseudoscalar mesons and vector mesons in the (S, T_3^s) plane where S is the strangeness quantum number and T_3^s the third component of the *strong* isospin with

$$T_3^s(u) = -T_3^s(\bar{u}) = \frac{1}{2}, \quad T_3^s(d) = -T_3^s(\bar{d}) = -\frac{1}{2}, \qquad T_3^s(s) = T_3^s(\bar{s}) = 0. \tag{2.124}$$

The ρ mesons have the same quark content as the pions just as the spins of the two quarks point in the same direction. Similar for the K^* mesons. The pseudoscalars η and η' are given in the SU(3) symmetry limit ($m_u = m_d = m_s$) by:

$$\eta \simeq \eta_8 = \frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right) \qquad \text{member of SU(3) octet} \tag{2.125}$$

$$\eta' \simeq \eta_0 = \frac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right), \qquad \text{SU(3) singlet.} \tag{2.126}$$

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006

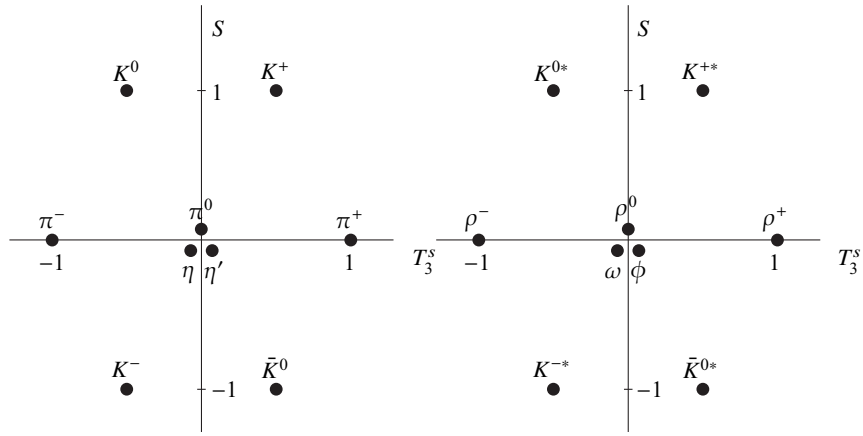


Figure 2.5 Meson multiplets: pseudoscalar mesons (left) and vector mesons (right).

Once the SU(3) is broken, a mixing angle of order -20° appears between η_8 and η_0 such that physical mass eigenstates η and η' are mixtures of η_8 and η_0. However, for the vector mesons we have a larger mixing between the singlet and the octet and the flavor content of vector mesons differs from the one of η and η'.

$$\omega = \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right), \qquad \phi = s\bar{s}. \tag{2.127}$$

Neutral mesons as K^0, B_d^0, B_s^0, and D^0 have an interesting property: Via the weak, interactions they can oscillate into their antiparticles. This will be discussed in detail in Chapters 6 and 8.

Our discussion of baryons will be very limited with the exception of the electric dipole moments (EDMs) in Section 17.3. The reason is that hadronic uncertainties in baryon decays are significantly larger than in the case of mesons, which is related to their more complicated structure. Consequently the tests of the SM in this sector are much more difficult, although the decays of Λ can also provide useful tests of the SM and its extensions. Rather exceptional are EDMs as here SM contributions are so small that any measurement of a nonvanishing EDM would be a clear signal of new physics.

2.8 Final Remarks

The information on the SM given in this chapter is sufficient for making the next steps and will be extended in the rest of the book. Yet, not all aspects of the SM can be discussed by us. There are numerous books on the dynamics of the SM where information on other topics, including baryons, can be found. See in particular [8]. Among recent ones, I can strongly recommend Langacker book [10]. Nice short introductions to flavor physics including most recent developments can be found in [43, 44]. More detailed aspects will be found in later chapters. Some of them can be found in a number of reviews and lectures, in particular in [7, 9, 29, 45] and very recently in [46]. Other lectures and reviews will be quoted as we proceed.

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 18 Nov 2026 at 06:30:58, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781139524100.006