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Singularity robust balancing of parallel manipulators following inconsistent trajectories

Mustafa Özdemir*

Department of Mechanical Engineering, Middle East Technical University, 06800 Ankara, Turkey

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SUMMARY

When compared to serial manipulators, parallel manipulators have small workspaces mainly due to their closed-loop structure. As opposed to type I singularities (or inverse kinematic singularities) that are generally encountered at the workspace boundaries, type II singularities characteristically arise within the workspace, and around them, the inverse dynamic solution becomes unbounded. Hence, a desired trajectory passing through a type II singular position cannot be achieved by the manipulator, and its useful workspace becomes further and substantially reduced. It has been previously shown in the literature that if the trajectory is replanned in such a way that the dynamic equations of motion of the manipulator are consistent at a type II singularity, i.e. if the trajectory is consistent, then the manipulator passes through this singular configuration in a controllable manner, while the inverse dynamic solution remains finite. An inconsistent trajectory, on the other hand, is stated in the literature to be unrealizable. However, although seems a promising technique, trajectory replanning itself is also a deviation from the originally desired trajectory, and there might be cases in applications where, due to some task-specific reasons, the desired trajectory, although inconsistent, is not allowed to be replanned to satisfy the consistency conditions. In this paper, a method of singularity robust balancing is proposed for parallel manipulators passing through type II singular configurations while following inconsistent trajectories. By this means, an originally unrealizable inconsistent trajectory passing through a type II singularity can be followed without any deviation, while the required actuator forces remain bounded after the manipulator is balanced according to the design methodology presented in this study. The effectiveness of the introduced method is shown through numerical simulations considering a planar 3-DOF 2-PRR parallel manipulator under different balancing scenarios.

KEYWORDS: Parallel manipulators; Inverse dynamics; Singularities; Type II singularities; Balancing.

1. Introduction

Parallel manipulators are receiving a growing attention because of their distinctive characteristics, such as high stiffness, high precision, high load-carrying capacity and high acceleration ability.[1,2] One of the main directions of the studies in this field is still, and increasingly, related to their singularities. Actually, their inverse kinematic singularities or type I singularities are generally encountered at the boundaries of the workspace,[3] and for this reason, do not cause a serious restriction in the applications. However, due to their closed-loop structure, another type of singularities is also encountered during their inverse dynamic solution. This type of singularities is called with different names in the literature, such as direct kinematic singularities or type II singularities,[3] force singularities,[4–6] actuation-related singularities,[7] drive or actuation singularities.[8,9] The studies on this topic started with determination of their loci.[10–12] However, type II singularities characteristically arise within the workspace,[3] and in their neighbourhood, the manipulator becomes uncontrollable since the inverse dynamic solution takes unboundedly large values.[4,5,7–9] As a result, the already small workspace of the manipulator is further and dramatically reduced by being divided into some subregions such that passing from one

* Corresponding author. E-mail: mozdemir@metu.edu.tr

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of these to another in a controllable manner is unfeasible, and therefore, path planning to avoid such singularities is not a promising solution to this problem.[5,8,9]

A feasible attempt to be able to fully use the whole workspace is the redundant actuation of the manipulator,[6,7,13] which also leads to the opportunity of optimizing the load distribution among the actuators and different task performance indices and backlash avoidance.[14–16] However, due to the additional actuators and sensors to be used, it might bring additional cost and complexity to the system. As another alternative remedy, some more recent studies showed that it is possible for a non-redundant parallel manipulator to pass through its type II singularities without any sacrifice from its controllability if the trajectory is appropriately planned.[5,8,9,17] This is achieved by properly constraining the values of the velocities and accelerations that the end-effector can be given at the singularity. By this means, any possible path passing through a type II singular position can be tracked by the manipulator, but each and every possible trajectory is not yet realizable. Therefore, in cases where the said constraints that the trajectory should meet in order to be realizable are too demanding and/or they are in a conflict with some other constraints arising from the essential specifications of the required task, the problem will still continue to occur.

In this article, a singularity robust balancing method is proposed for parallel manipulators in order to enable them to follow any desired trajectory along a desired path in a controllable manner while passing through type II singular positions. With this aim, first the consistency conditions of the dynamic equations of the manipulator are recalled, as derived in ref. [8]. Then, if the desired trajectory is inconsistent, i.e. unattainable as stated in the related literature, these consistency conditions are satisfied by appropriately using balancing masses and/or springs and/or dampers. As a numerical case study, a planar 3-DOF 2-PRR parallel manipulator is balanced for robust singularity passing while following an inconsistent trajectory by using different alternatives and the effectiveness of the suggested technique is demonstrated. Some possible future implications of the introduced method are also discussed.

2. Background Information

For an n DOF non-redundant parallel manipulator, let the number of joint variables be m and let $\mathbf{q}$ denote the m dimensional vector of joint variables. The $m - n$ loop closure constraint equations can be written as

$$\boldsymbol{\phi}(\mathbf{q}) = \mathbf{0}, \tag{1}$$

where $\boldsymbol{\phi}$ is an $m - n$ dimensional vector. Velocity loop closure equations can then be written as

$$\boldsymbol{\Phi}\dot{\mathbf{q}} = \mathbf{0}, \tag{2}$$

where $\boldsymbol{\Phi} = \boldsymbol{\Phi}(\mathbf{q}) = \frac{\partial \boldsymbol{\phi}}{\partial \mathbf{q}}$.

Choosing the vector of generalized coordinates to be $\mathbf{q}$, the equations of motion of the unconstrained system can be written as given below through the use of Lagrange's equations:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j \quad \text{for} \quad j = 1, \cdots, m. \tag{3}$$

Here, L is the Lagrangian and Q_j's are the generalized forces. Let the only external non-conservative forces acting on the unconstrained system be the actuator forces. Then, one can easily see that each of Q_j's which is associated with an actuated joint variable is simply equal to the corresponding actuator force of that joint, whereas the ones which are associated with the unactuated joint variables are simply zero.

Eqs. (3) can be rewritten in the matrix form as

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{N} = \mathbf{Q}, \tag{4}$$

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where $\mathbf{M} = \mathbf{M}(\mathbf{q})$ is the $m \times m$ generalized inertia matrix, $\mathbf{N} = \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}})$ is the m dimensional vector that collects the generalized Coriolis, centrifugal and gravity terms, and $\mathbf{Q}$ is the m dimensional vector of generalized forces.

Then, by imposing the loop closure constraint equations on Eq. (4), the dynamic behaviour of the parallel manipulator can be expressed in the joint space as

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{N} = \mathbf{Q} + \mathbf{\Phi}^T \boldsymbol{\lambda}, \tag{5}$$

where $\boldsymbol{\lambda}$ is the $m - n$ dimensional vector of Lagrange multipliers.

Once the joint positions, velocities and accelerations are known for the desired motion of the manipulator, Eq. (5) can be solved for the actuator forces that are required to achieve this motion unless the manipulator is at or near a type II singular configuration where the solution tends to infinity, as mentioned previously.

Assuming that the desired motion of the manipulator is free of type I singular configurations, Ider[8] derived the $m - r$ consistency conditions of the dynamic equations that the desired trajectory should meet at a type II singularity in order to be achievable, which can be recalled as follows[§]:

$$M_{s_k j}\ddot{q}_j + N_{s_k} = \alpha_{kp}\left(M_{pj}\ddot{q}_j + N_p\right) \quad \text{for} \quad p = 1, \cdots, m \ \ (p \neq s_k), \ k = 1, \cdots, m - r. \tag{6}$$

The derivation details and the definitions of s_k, α_{kp} and r can be found in ref. [8].

3. Singularity Robust Balancing Method

When a desired trajectory is inconsistent, it can be replanned to ensure the consistency of the manipulator dynamics at the singularity, as suggested in refs. [8, 9, 17]. However, there might be some operational cases where not only the desired path but also the desired trajectory along that path is indispensable, and therefore, such a reshaped trajectory might be unacceptable. In such cases, instead of modifying the trajectory, it will be sensible to use balancing elements, i.e. balancing masses and/or springs and/or dampers, such that the consistency conditions are met. Basically, the idea is to modify the dynamics of the manipulator as required by the consistency conditions via an appropriate application of the said balancing elements. Moreover, when compared to using redundant actuators, this might be expected to be a less expensive and less complex alternative.

After balanced, the left-hand side of Eq. (5) changes as follows:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{N} + \boldsymbol{\psi} = \mathbf{Q} + \mathbf{\Phi}^T \boldsymbol{\lambda}, \tag{7}$$

where $\boldsymbol{\psi} = \boldsymbol{\psi}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$ is an m dimensional vector that collects the terms arising in the dynamic equations of motion of the manipulator due to the added balancing masses and/or springs and/or dampers. A more explicit definition of $\boldsymbol{\psi}$ can be made on the basis of the changes in the Lagrangian and Rayleigh dissipation function due to the applied balancing. Let them be denoted by ΔL and ΔR, respectively. Then, one can write the elements of the $\boldsymbol{\psi}$ vector as follows:

$$\psi_j = \frac{d}{dt}\left(\frac{\partial \Delta L}{\partial \dot{q}_j}\right) - \frac{\partial \Delta L}{\partial q_j} + \frac{\partial \Delta R}{\partial \dot{q}_j}. \tag{8}$$

It should be noted that the numerical values of the parameters of the balancing elements that are in plan to be applied, like the masses, mass moments of inertia and mass centre locations of the balancing masses, and/or the spring stiffness coefficients and free lengths or unstretched angles of the balancing springs, and/or the damping coefficients of the balancing dampers, are unknown at this stage and all these parameters appear symbolically in Eq. (7).

[§] Modified from Mechanism and Machine Theory, Volume 40, Issue 1, S. Kemal Ider, Inverse dynamics of parallel manipulators in the presence of drive singularities, Pages 33–44, Copyright ©2004 Elsevier Ltd., with permission from Elsevier

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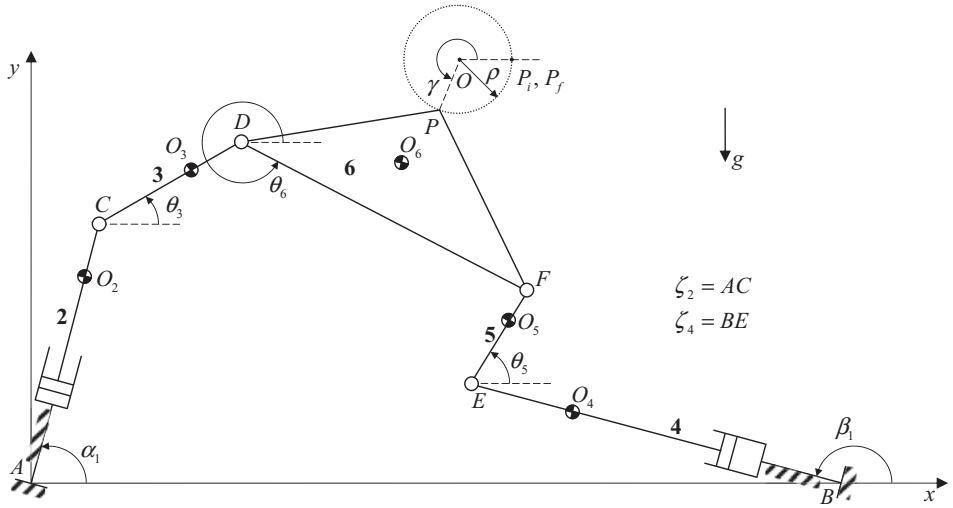


Fig. 1. A planar 3-DOF 2-PRR parallel manipulator.

The new consistency conditions to be satisfied at the singularity can then be written by modifying Eqs. (6) as follows:

$$M_{s_k j}\ddot{q}_j + N_{s_k} + \psi_{s_k} = \alpha_{kp}\left(M_{pj}\ddot{q}_j + N_p + \psi_p\right) \quad \text{for} \quad k = 1, \cdots, m - r. \tag{9}$$

Hence, the numerical values of the parameters of the balancing elements to be used should be obtained by solving the following rearranged forms of Eqs. (9) at the singular position:

$$\alpha_{kp}\psi_p - \psi_{s_k} = M_{s_k j}\ddot{q}_j + N_{s_k} - \alpha_{kp}\left(M_{pj}\ddot{q}_j + N_p\right) \quad \text{for} \quad k = 1, \cdots, m - r. \tag{10}$$

Eqs. (10) give the $m - r$ singularity robust balance design equations that have to be solved at the singularity for the numerical values of the parameters of the balancing elements to be used. Once the manipulator is balanced according to Eqs. (10), the originally inconsistent trajectory can now be achieved since it is then consistent for the balanced manipulator. Below are the five guidelines of the proposed method.

Guideline 1: If the desired trajectory is inconsistent, the numerical values regarding the balancing elements to be applied should be determined by solving Eqs. (10) at the singular position of the manipulator.

Guideline 2: The balancing elements should be applied such that the terms they bring into the Lagrangian and/or Rayleigh dissipation function contain at least one or more of the unactuated joint variables and/or their first time derivatives so that it is possible for the inconsistent rows of the matrix Eq. (5) to become consistent.

Guideline 3: Any balancing element that brings terms not containing any of the unactuated joint variables or their first time derivatives into the Lagrangian or Rayleigh dissipation function should be avoided for preventing useless and unnecessary modifications in the already consistent rows of the matrix Eq. (5).

Guideline 4: The actuator forces that are required to produce the desired motion should be determined by solving Eq. (7) once the manipulator is balanced for robust singularity passing.

Guideline 5: If more than one singular configuration exists for the desired motion, the manipulator should be balanced such that Eqs. (10) are fulfilled at all of these singular points.

4. Illustrative Examples

In this section, similar to the planar 3-DOF 2-RPR parallel manipulator constructed in ref. [9], a planar 3-DOF 2-PRR parallel manipulator (Fig. 1) is taken into consideration. First, its inverse dynamics, singular positions and consistency condition are studied using the methodology described in refs. [8, 9], and then, the proposed technique of this paper is applied to it.

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4.1. Modelling of the manipulator

Referring to Fig. 1, the vector of joint variables is $\mathbf{q} = \left[\, \zeta_2 \;\theta_3 \;\zeta_4 \;\theta_5 \;\theta_6 \,\right]^T$. Let $a_1 = AB$, $a_3 = CD$, $a_5 = EF$, $a_6 = DF$, $b_6 = DP$, α_6 be the angle measured in the counter-clockwise sense from the line DF to the line DP, and x_P and y_P the position coordinates of the end-effector point P. The $\mathbf{\Phi}$ matrix appearing in Eq. (2) can be written as follows:

$$\mathbf{\Phi} = \begin{bmatrix} \cos\alpha_1 & -a_3\sin\theta_3 & -\cos\beta_1 & a_5\sin\theta_5 & -a_6\sin\theta_6 \\ \sin\alpha_1 & a_3\cos\theta_3 & -\sin\beta_1 & -a_5\cos\theta_5 & a_6\cos\theta_6 \end{bmatrix}. \tag{11}$$

Furthermore, let ζ_2, θ_3 and ζ_4 be the actuated joint variables, T denote the motor torque corresponding to θ_3, and F_1 and F_2 the actuator forces corresponding to ζ_2 and ζ_4, respectively. Once the vector of generalized coordinates is selected as $\mathbf{q}$, the $\mathbf{M}$ matrix and the $\mathbf{N}$, $\mathbf{Q}$ and $\boldsymbol{\lambda}$ vectors appearing in Eq. (5) can be written as

$$\mathbf{M} = \begin{bmatrix} M_{11} & M_{12} & 0 & 0 & M_{15} \\ M_{21} & M_{22} & 0 & 0 & M_{25} \\ 0 & 0 & M_{33} & M_{34} & 0 \\ 0 & 0 & M_{43} & M_{44} & 0 \\ M_{51} & M_{52} & 0 & 0 & M_{55} \end{bmatrix}, \tag{12}$$

$$\mathbf{N} = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \end{bmatrix}, \tag{13}$$

$$\mathbf{Q} = \begin{bmatrix} F_1 \\ T \\ F_2 \\ 0 \\ 0 \end{bmatrix}, \tag{14}$$

$$\boldsymbol{\lambda} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}, \tag{15}$$

where

$$M_{11} = m_2 + m_3 + m_6, \tag{16}$$

$$M_{12} = M_{21} = (m_3 r_3 + m_6 a_3)\sin(\alpha_1 - \theta_3), \tag{17}$$

$$M_{15} = m_6 r_6 \sin(\alpha_1 - \beta_6 - \theta_6), \tag{18}$$

$$M_{22} = m_3 r_3^2 + I_3 + m_6 a_3^2, \tag{19}$$

$$M_{25} = M_{52} = m_6 a_3 r_6 \cos(\theta_6 + \beta_6 - \theta_3), \tag{20}$$

$$M_{33} = m_4 + m_5, \tag{21}$$

$$M_{34} = M_{43} = m_5 r_5 \sin(\beta_1 - \theta_5), \tag{22}$$

$$M_{44} = m_5 r_5^2 + I_5, \tag{23}$$

$$M_{51} = m_6 r_6 \sin(\alpha_1 - \theta_6 - \beta_6), \tag{24}$$

$$M_{55} = m_6 r_6^2 + I_6, \tag{25}$$

$$N_1 = (m_2 + m_3 + m_6)\, g \sin\alpha_1 - (m_3 r_3 + m_6 a_3)\,\dot{\theta}_3^2 \cos(\theta_3 - \alpha_1) - m_6 r_6 \dot{\theta}_6^2 \cos(\theta_6 + \beta_6 - \alpha_1), \tag{26}$$

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$$N_2 = (m_3 r_3 + m_6 a_3)\, g \cos\theta_3 - m_6 a_3 r_6 \dot{\theta}_6^2 \sin(\theta_6 + \beta_6 - \theta_3), \tag{27}$$

$$N_3 = (m_4 + m_5)\, g \sin\beta_1 - m_5 r_5 \dot{\theta}_5^2 \cos(\theta_5 - \beta_1), \tag{28}$$

$$N_4 = m_5 r_5 g \cos\theta_5, \tag{29}$$

$$N_5 = m_6 r_6 \left[a_3 \dot{\theta}_3^2 \sin(\theta_6 + \beta_6 - \theta_3) + g \cos(\theta_6 + \beta_6) \right]. \tag{30}$$

Here, m_i and I_i denote the mass and the centroidal moment of inertia of link i, g is the gravitational acceleration, and the mass centres, O_i's are located by $r_2 = CO_2$, $r_3 = CO_3$, $r_4 = EO_4$, $r_5 = EO_5$, $r_6 = DO_6$ and the angle β_6, which is measured in the counter-clockwise sense from the line DF to the line DO_6.

4.2. Singular positions

Noting that a_3 and a_5 are different than zero, type I singular configurations are given by $\theta_3 - \alpha_1 = \pm(2n-1)\frac{\pi}{2}$ or $\theta_5 - \beta_1 = \pm(2n-1)\frac{\pi}{2}$ for $n = 1, 2, 3, \cdots$, i.e. they are those where link 3 is perpendicular to link 2 or link 5 is perpendicular to link 4.

Noting that a_5 and a_6 are different than zero, type II singular configurations are, on the other hand, given by $\theta_5 - \theta_6 = \pm n\pi$ for $n = 0, 1, 2, \cdots$, i.e. they are those where the three points D, E and F lay on the same line.

4.3. Consistency condition

It can be shown that the consistency condition that the desired trajectory should meet at a type II singularity is

$$M_{43}\ddot{\zeta}_4 + M_{44}\ddot{\theta}_5 + N_4 \pm \frac{a_5}{a_6}\left(M_{51}\ddot{\zeta}_2 + M_{52}\ddot{\theta}_3 + M_{55}\ddot{\theta}_6 + N_5\right) = 0, \tag{31}$$

where, in the double sign, the upper sign is to be used for $\theta_5 = \theta_6$ and the lower sign is to be used for $\theta_5 = \theta_6 \pm \pi$.

4.4. Singularity robust balancing of the manipulator

After balanced for robust singularity passing, the dynamics of the manipulator can be expressed as in Eq. (7) where the $\boldsymbol{\psi} = [\,\psi_1\ \psi_2\ \psi_3\ \psi_4\ \psi_5\,]^T$ vector collects all the terms arising in the dynamics due to the applied balancing elements.

For the governing matrix Eq. (7), the new consistency condition to be met at the singularity is as follows:

$$M_{43}\ddot{\zeta}_4 + M_{44}\ddot{\theta}_5 + N_4 + \psi_4 \pm \frac{a_5}{a_6}\left(M_{51}\ddot{\zeta}_2 + M_{52}\ddot{\theta}_3 + M_{55}\ddot{\theta}_6 + N_5 + \psi_5\right) = 0. \tag{32}$$

By rearranging Eq. (32), one can obtain the singularity robust balance design equation as

$$\psi_4 \pm \frac{a_5}{a_6}\psi_5 = -\left[M_{43}\ddot{\zeta}_4 + M_{44}\ddot{\theta}_5 + N_4 \pm \frac{a_5}{a_6}\left(M_{51}\ddot{\zeta}_2 + M_{52}\ddot{\theta}_3 + M_{55}\ddot{\theta}_6 + N_5\right)\right]. \tag{33}$$

The numerical values of the balancing elements to be used should be determined by solving Eq. (33) at the singular position of the manipulator.

4.5. Numerical results

Let the numerical values of the manipulator parameters be as given in Table I. Referring to Fig. 1, consider a desired motion where link 6 maintains a constant orientation of $\theta_6 = 300^\circ$ while its end-effector point P performs a full counter-clockwise rotation around a circle centred at $(x_O, y_O) = (0.347, -0.011)$ m with a radius of $\rho = 0.060$ m. This motion is to be completed in $t_f = 2$ s, and the manipulator has to start and end at rest. The initial position (and hence the final position) of point P

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Table I. Numerical values of the manipulator parameters.

Linear dimensions (m)		Masses (kg)		Centroidal moments of inertia (kg.m^2)		Mass centre distances (m)		Angular dimensions (deg)	
a_1	1.2	m_2	1.5	I_3	0.02	r_2	0.10	α_1	70
a_3	0.6	m_3	1.0	I_5	0.02	r_3	0.20	β_1	160
a_5	0.6	m_4	1.5	I_6	0.05	r_4	0.10	α_6	25
a_6	0.5	m_5	1.0			r_5	0.20	β_6	10
b_6	0.7	m_6	2.5			r_6	0.30		

Table II. Joint displacements, velocities and accelerations at the singular configurations.

Joint displacements			Joint velocities			Joint accelerations		
q_j	$t_{sc_1} =$ 0.47 s	$t_{sc_2} =$ 0.65 s	$\dot{q}_j$	$t_{sc_1} =$ 0.47 s	$t_{sc_2} =$ 0.65 s	$\ddot{q}_j$	$t_{sc_1} =$ 0.47 s	$t_{sc_2} =$ 0.65 s
ζ_2 (m)	0.850	0.822	$\dot{\zeta}_2$ (m/s)	−0.057	−0.262	$\ddot{\zeta}_2$ (m/s^2)	−0.991	−1.100
θ_3 (rad)	3.788	3.707	$\dot{\theta}_3$ (rad/s)	−0.382	−0.493	$\ddot{\theta}_3$ (rad/s^2)	−1.110	0.132
ζ_4 (m)	1.531	1.570	$\dot{\zeta}_4$ (m/s)	0.249	0.165	$\ddot{\zeta}_4$ (m/s^2)	0.028	−1.044
θ_5 (rad)	5.236	5.236	$\dot{\theta}_5$ (rad/s)	0.147	−0.178	$\ddot{\theta}_5$ (rad/s^2)	−1.226	−2.275
θ_6 (rad)	5.236	5.236	$\dot{\theta}_6$ (rad/s)	0	0	$\ddot{\theta}_6$ (rad/s^2)	0	0

is given to be $(x_{P_i}, y_{P_i}) = (x_{P_f}, y_{P_f}) = (0.407, -0.011)$ m. Its position at time t can be expressed as follows:

$$\begin{bmatrix} x_P(t) \\ y_P(t) \end{bmatrix} = \begin{bmatrix} x_O + \rho \cos\gamma(t) \\ y_O + \rho \sin\gamma(t) \end{bmatrix}, \tag{34}$$

where $\gamma(t)$ is the angle of the line OP with the horizontal in the counter-clockwise direction. The prescribed task requirements then correspond to $\gamma(0) = 0$, $\dot{\gamma}(0) = 0$, $\gamma(2) = 2\pi$, $\dot{\gamma}(2) = 0$, and the $\gamma(t)$ function satisfying all of these requirements can be obtained as

$$\gamma(t) = \frac{3\pi}{2}t^2 - \frac{\pi}{2}t^3. \tag{35}$$

This motion is free of type I singularities. However, at $t_{sc_1} = 0.47$ s and $t_{sc_2} = 0.65$ s, type II singular configurations are encountered since θ_5 becomes equal to θ_6 (Therefore, in Eqs. (31)–(33), the upper part of the double sign is to be used). The joint displacements, velocities and accelerations at these instances are given in Table II. Since the consistency condition, i.e. Eq. (31), is not satisfied by the prescribed trajectory at any of these singular configurations, the required motor torque and actuator forces take unboundedly large values around them (Figs. 2 and 3), as expected from the results of refs. [8, 9, 17].

Referring to Fig. 4, some balancing elements that can be used are

- a mass, m_c with a centroidal moment of inertia of I_c, and with its centre of mass fixed at point O_c on link 6 where the point O_c is identified by $r_c = DO_c$ and the angle β_c, which is measured in the counter-clockwise sense from the line DF to the line DO_c,
- a linear torsional spring connected between links 4 and 5 with a spring constant of $k_{t,c}$, and an unstretched angle of θ_c^0,
- a linear torsional damper connected between links 3 and 6 with a damping coefficient of $b_{t,c}$.

Indeed, these are just some examples, and one can still use different balancing elements that are not enumerated above.

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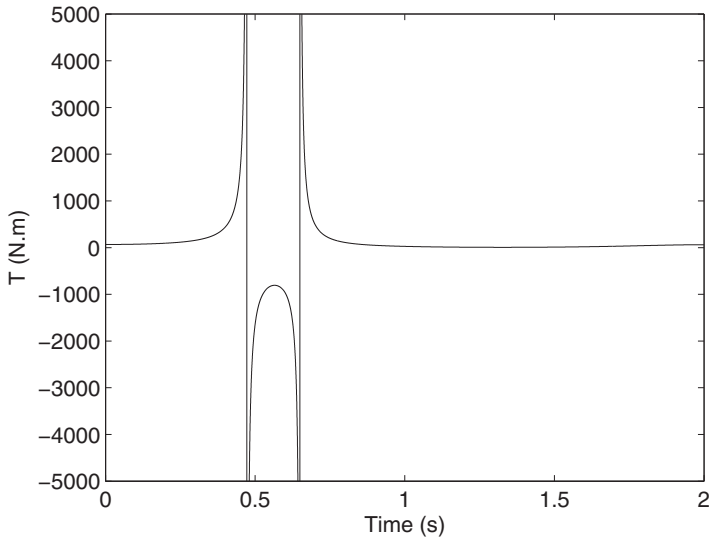


Fig. 2. Motor torque T for the inconsistent trajectory before singularity robust balancing.

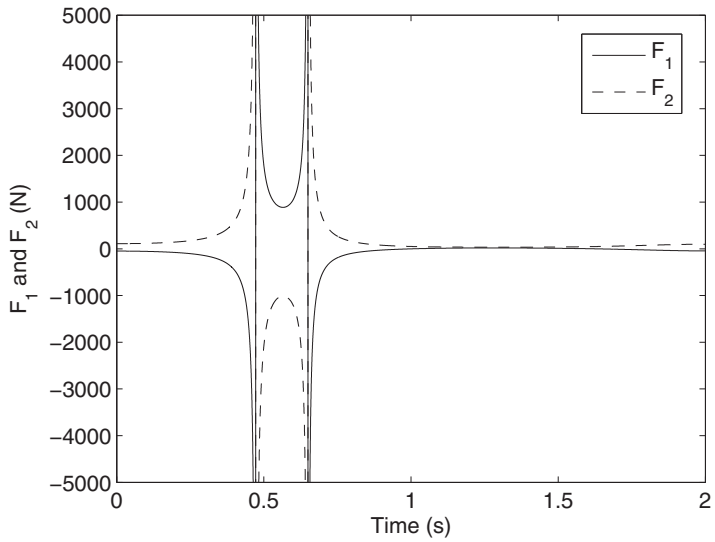


Fig. 3. Actuator forces F_1 and F_2 for the inconsistent trajectory before singularity robust balancing.

When all the above-listed balancing elements are employed together, the elements of the $\boldsymbol{\psi}$ vector are as follows:

$$\psi_1 = m_c \left\{ \ddot{\xi}_2 - a_3 \left[\ddot{\theta}_3 \sin(\theta_3 - \alpha_1) + \dot{\theta}_3^2 \cos(\theta_3 - \alpha_1) \right] - r_c \left[\ddot{\theta}_6 \sin(\theta_6 + \beta_c - \alpha_1) + \dot{\theta}_6^2 \cos(\theta_6 + \beta_c - \alpha_1) \right] + g \sin\alpha_1 \right\}, \tag{36}$$

$$\psi_2 = m_c a_3 \left\{ \ddot{\zeta}_2 \sin(\alpha_1 - \theta_3) + a_3 \ddot{\theta}_3 + r_c \left[\ddot{\theta}_6 \cos(\theta_6 + \beta_c - \theta_3) - \dot{\theta}_6^2 \sin(\theta_6 + \beta_c - \theta_3) \right] + g \cos\theta_3 \right\} - b_{t,c} \left(\dot{\theta}_6 - \dot{\theta}_3 \right), \tag{37}$$

$$\psi_3 = 0, \tag{38}$$

$$\psi_4 = k_{t,c} \left(\theta_5 - \theta_c^0 \right), \tag{39}$$

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Table III. Numerical values regarding the considered singularity robust balancing alternatives.

	Alternative 1	Alternative 2	Alternative 3	Alternative 4
m_c (kg)	2.0	1.0	3.0	2.5
r_c (m)	0.5179	0.1197	0.8953	0.5604
β_c (rad)	−3.0689	−3.0734	−3.3226	−3.3121
$k_{t,c}$ (N.m/rad)	–	10.0	–	5.0
θ_c^0 (rad)	–	$11\pi/6$	–	$11\pi/6$
$b_{t,c}$ (N.m.s/rad)	–	–	5.0	2.50

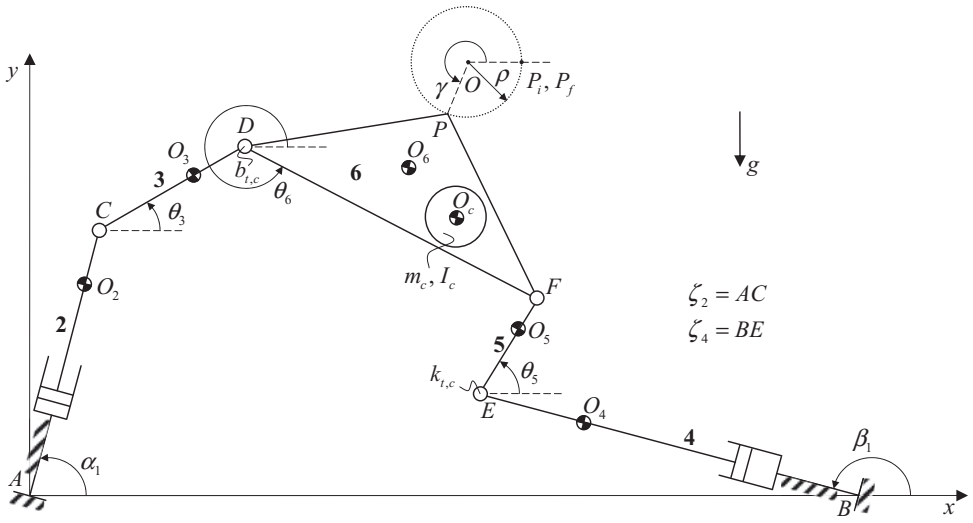


Fig. 4. Some singularity robust balancing options.

$$\psi_5 = m_c r_c \left\{ \ddot{\xi}_2 \sin(\alpha_1 - \theta_6 - \beta_c) + a_3 \left[\ddot{\theta}_3 \cos(\theta_6 + \beta_c - \theta_3) + \dot{\theta}_3^2 \sin(\theta_6 + \beta_c - \theta_3) \right] + r_c \ddot{\theta}_6 + g \cos(\theta_6 + \beta_c) \right\} + I_c \ddot{\theta}_6 + b_{t,c} \left(\dot{\theta}_6 - \dot{\theta}_3 \right). \tag{40}$$

On substituting Eqs. (20), (22)–(25), (29), (30), (39) and (40) together with the numerical values of the manipulator parameters as given in Table I, and the joint displacements, velocities and accelerations at the singular configurations as given in Table II, Eq. (33) becomes

$$\begin{aligned} -5.9139 = m_c r_c [& -1.1894 \sin(0.8727 + \beta_c) - 0.7991 \sin(\beta_c - 3.2644) \\ & - 0.1052 \cos(\beta_c - 3.2644) + 11.7679 \sin(0.5236 + \beta_c)] + 0.4587 b_{t,c} \\ & + k_{t,c} \left(5.2360 - \theta_c^0 \right) \end{aligned} \tag{41}$$

for $t_{sc_1} = 0.47$ s
and

$$\begin{aligned} -5.9145 = m_c r_c [& -1.3202 \sin(0.8727 + \beta_c) + 0.0953 \sin(\beta_c - 3.1834) \\ & - 0.1747 \cos(\beta_c - 3.1834) + 11.7679 \sin(0.5236 + \beta_c)] + 0.5911 b_{t,c} \\ & + k_{t,c} \left(5.2360 - \theta_c^0 \right) \end{aligned} \tag{42}$$

for $t_{sc_2} = 0.65$ s.

Four different balancing alternatives are considered. The numerical values regarding these scenarios are given in Table III. Indeed, these numerical values are selected such that Eqs. (41) and (42) are met simultaneously in each balancing scenario. As seen in Figs. 5–7, with all the considered balancing alternatives, the required motor torque and actuator forces remain bounded in

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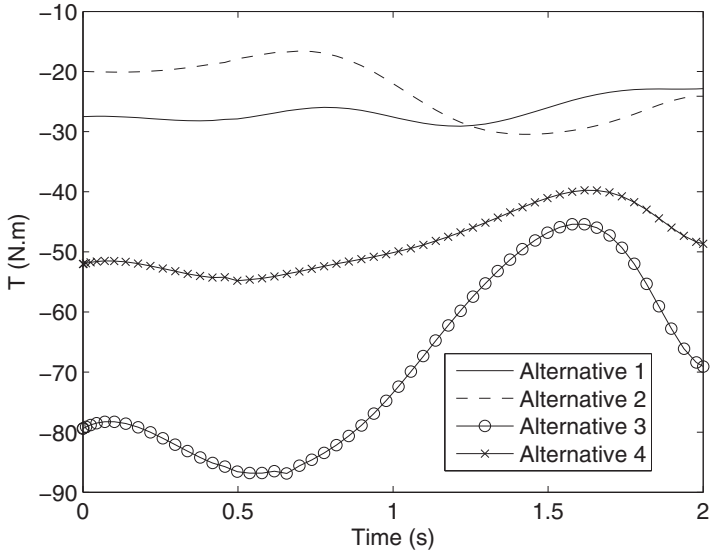


Fig. 5. Motor torque T for the inconsistent trajectory after singularity robust balancing.

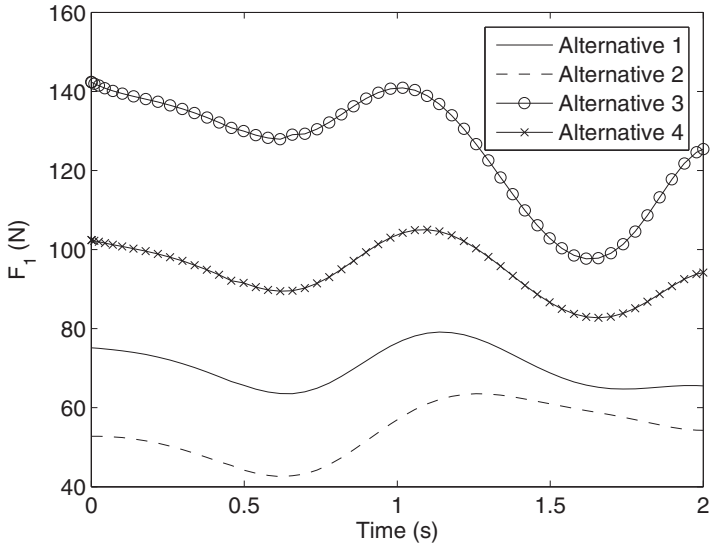


Fig. 6. Actuator force F_1 for the inconsistent trajectory after singularity robust balancing.

the neighbourhood of the singularities and at the singularities themselves. Hence, it is shown that a desired inconsistent trajectory that is originally unrealizable can be achieved by the manipulator after a proper balancing according to the suggested method and design equations in this paper.

5. Conclusions

To the known state of art, every path passing through a type II singular configuration can be tracked, if the desired trajectory along this path is modified such that the dynamic equations of the manipulator are consistent at this singular position. However, if the desired trajectory is inconsistent, it is concluded to be unrealizable.

This paper remedies this problem by introducing a singularity robust balancing technique. The basic idea behind it is to modify the dynamics of the manipulator such that the dynamic equations become consistent at the type II singularity. This can be achieved by employing balancing masses and/or springs and/or dampers. Thus, no modification is required in the desired trajectory.

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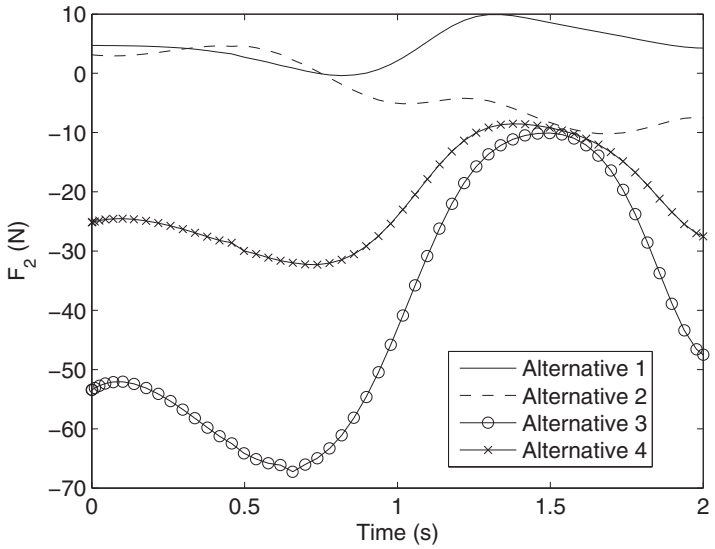


Fig. 7. Actuator force F_2 for the inconsistent trajectory after singularity robust balancing.

The design equations regarding the suggested technique are derived, and it is applied to a planar 3-DOF 2-PRR parallel manipulator. In all the considered balancing alternatives, the required motor torque and actuator forces remain finite while the manipulator is passing through its type II singular configurations on the chosen inconsistent trajectory.

The proposed technique in this study can lead to parallel manipulators that possess some specific balance connection points. Hence, the necessary singularity robust balancing elements can be applied without any permanent modification to the original structure of the manipulator. Another design concept for this purpose might be parallel manipulators that are equipped with adjustable (i.e. slidable) masses on their links, and/or springs and/or dampers between their links with adjustable spring and damping coefficients, respectively. A parallel manipulator having telescopic links will also be another useful idea since the telescopic structure will allow the required adjustments in the inertial parameters of the manipulator links for robust singularity passing while following an inconsistent trajectory.

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