Newton's Methodology and Mercury's Perihelion Before and After Einstein

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Newton's methodology is significantly richer than the hypothetico-deductive model. It is informed by a richer ideal of empirical success that requires not just accurate prediction but also accurate measurement of parameters by the predicted phenomena. It accepts theory-mediated measurements and theoretical propositions as guides to research. All of these enrichments are exemplified in the classical response to Mercury's perihelion problem. Contrary to Kuhn, Newton's method endorses the radical transition from his theory to Einstein's. The richer themes of Newton's method are strikingly realized in a challenge to general relativity from a new problem posed by Mercury's perihelion.

1. Introduction. Newton's method adds three distinctive features that go beyond the basic Hypothetico-Deductive (HD) model that dominated much of philosophy of science in the last century. On this familiar basic HD model, hypothesized principles are tested by experimental verification of observable consequences drawn from them. Empirical success is limited to accurate prediction of observable phenomena. For most scientists and philosophers of science such success is counted as confirmation taken to legitimate increases in probability accorded to the hypothesis.

One important addition afforded by Newton's method is the richer ideal of empirical success that is realized in his classic inferences from phenomena. This richer ideal of empirical success requires not just accurate prediction of phenomena. It requires, in addition, accurate measurement of parameters by the predicted phenomena. A second feature of Newton's method that goes beyond the HD method is to exploit, insofar as possible, theory-mediated measurements from phenomena so as to give empirical answers to theoretical questions. A third major feature is the acceptance of theoretical propositions as guides to research. All three of these features come together in a process of piecemeal, empirically guided,

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theory construction that is exemplified in Newton's basic argument for universal gravitation and its application to the system of the world. The propositions inferred at each stage are accepted as constraints on later stages of theory construction. They count as premises that can be appealed to in later inferences.

Newton's opening inferences to inverse square centripetal forces maintaining Jupiter's moons and the primary planets in their orbits are backed up by systematic dependencies that make orbital phenomena measure the centripetal direction and the inverse square variation with distance of those forces. Consider Newton's inferences to the centripetal direction of the force maintaining a planet or satellite in an orbit from its uniform description of areas by radii from the center of the body about which it orbits. According to Proposition 1 of Book 1, if the direction of the force is toward that center then the orbiting body will move in a plane in such a way that its description of areas by radii to that center will be uniform (Newton 1999, 444–445). According to the HD method, this conditional is what is needed to have the centripetal direction of the force count as a hypothesis that would explain the uniform description of areas.

Newton cites Proposition 2 of Book 1, which gives the converse conditional that if the description of areas is uniform then the force is centripetal.[1] These two conditionals follow from his application of his laws of motion to such orbital systems. He has proved, in addition, the corollaries that if the areal rate is increasing then the force is off center in a forward direction, and if the areal rate is decreasing then the force is off center in the opposite way (Newton 1999, 447). Taken together, these results show that the behavior of the rate at which areas are swept out by radii to the center depends systematically on the direction of the force maintaining a body in orbital motion with respect to that center. These systematic dependencies make the uniformity of the areal rate carry the information that the force is directed toward the center.

These inferences of Newton are far more compelling than the corresponding HD inferences. The systematic dependencies backing them up afford a far more compelling sort of explanation of the uniform description of areas by the centripetal direction of the forces than the HD model of explanation as a one way conditional with the hypothesis as antecedent and the phenomenon to be explained as consequent.

The systematic dependencies backing up Newton's inferences to centripetal forces from uniform description of areas make no assumptions about variation with respect to distance from the center. This allows fur-

1. See Newton 1999, 446 for Proposition 2, Book 1 and p. 802 for the inferences to centripetal forces for the moons of Jupiter and the primary planets in Propositions 1 and 2 of Book 3.

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ther systematic dependencies, which make the harmonic-law relation for a system of orbits (to have the periods be as the 3/2 power of the mean distances) measure the inverse square variation of the accelerations exhibited by those orbits at their mean distances.[2] Newton's inferences to inverse square centripetal forces are inferences to what Howard Stein (1970, 267–268; cf. Stein 2002, 284–287) has called 'fields of centripetal acceleration'. Consider any distance from the center of the sun great enough to be beyond its surface. Inverse square adjusting the estimates of the centripetal accelerations corresponding to the cited data for mean distances and periods of the planets affords agreement of measurements of what the centripetal acceleration toward the sun would be at that distance.[3] These agreeing measurements back up the inference to the acceleration toward the sun that a body would have at any such distance. Newton (1999, 806–808) also provides an impressive set of phenomena that all count as agreeing measurements of the equalities for all bodies of the ratios of their inertial masses to their inverse square adjusted weights toward the sun and each of the planets. These count as agreeing measurements of the equality of the corresponding inverse square adjusted accelerations any pair of freely falling bodies would exhibit toward the sun and toward any planet (Harper 2002, 187–189). They therefore count as measurements that back up the acceleration field character of the inverse square centripetal forces inferred by Newton.

Newton's fourth rule for doing natural philosophy articulates the nature of the acceptance his method accords to propositions inferred from phenomena:

> **Rule 4**. In experimental philosophy, propositions gathered from phenomena by induction should be considered either exactly or very nearly true notwithstanding any contrary hypotheses, until yet other phenomena make such propositions either more exact or liable to exceptions. (Newton 1999, 796)

He explicitly appeals to this rule to extend inverse square centripetal forces

2. Newton's basic theorems (Corollaries 6 and 7 of Proposition 4, Book 1, in Newton 1999, 451) are for uniform motion on concentric circular orbits. He proved, however, that at its mean distance, R, the centripetal acceleration of an elliptical orbit with force to a focus is

$$Acc = 4\mathrm{B}R/t^2,$$

which is the centripetal acceleration of a concentric circular orbit with radius R and period t. See Brackenridge 1995, 119–122.

3. For distance d the estimate of the centripetal acceleration afforded by the data for a planetary orbit with mean distance R and period t is $Acc = (4\mathrm{B}R/t^2)(R/d)^2$. See Myrvold and Harper 2002, S141–S147.

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of gravitation to planets without orbiting moons to measure them.[4] We may regard propositions assigning such inverse square gravities to planets without moons as gathered from phenomena by induction. This treats the measurements of centripetal directions and the inverse square variation of gravitation toward the sun and planets with moons as measurements of these properties of gravitation toward planets generally.

This rule tells us that contrary hypotheses are not to be allowed to undercut inferences to such propositions gathered from phenomena. The inferences we have been examining suggest that an alternative proposal is to be treated as a mere 'contrary hypothesis' unless it is sufficiently backed up by measurements from phenomena to count as a rival to be taken seriously.

One very important feature of the acceptance endorsed by this rule is that it explicitly makes room for accepting such propositions as approximations. We shall see that this is central to the way Newton's method was able to deal with the complexity of interactions in the solar system.

2. Successive Approximations. An especially important advantage offered by the systematic dependencies backing up Newton's inferences is that they make the inferences robust with respect to approximations in the phenomena. These systematic dependencies support nontrivial inferences to approximate values of theoretical parameters from approximate values of the corresponding phenomenal magnitudes. Newton's inferences from phenomena are informative even if the phenomena do not hold exactly.

One striking feature of Newton's argument is that its conclusion, his theory of universal gravitation, is actually incompatible with the Keplerian phenomena assumed as premises in his argument for it. The application of the third law of motion to the sun and planets leads to two-body corrections to the harmonic law. Newton expected that interactions between Jupiter and Saturn would sensibly disturb their orbital motions.

Newton's treatment of these deviations exemplify a method of successive approximations that informs applications of universal gravitation to motions of solar system bodies. On this method, deviations from the model developed so far count as new *theory mediated* phenomena to be exploited as carrying information to aid in developing a more accurate successor.

Smith (2002, 153–167) has argued that Newton developed this method in an effort to deal with the extreme complexity of solar system motions. He points to a striking passage, which he calls "the Copernican scholium," that Newton added to an intermediate augmented version of his *De Motu*

4. See Corollary 1 and the Scholium to Proposition 5, Book 3 (Newton 1999, 806).

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tract, before it grew into the *Principia* (Smith 2002, 153–154; cf. Smith 1999, 46–51):

> By reason of the deviation of the Sun from the center of gravity, the centripetal force does not always tend to that immobile center, and hence the planets neither move exactly in ellipses nor revolve twice in the same orbit. There are as many orbits of a planet as it has revolutions, as in the motion of the Moon, and the orbit of any one planet depends on the combined motion of all the planets, not to mention the action of all these on each other. But to consider simultaneously all these causes of motion and to define these motions by exact laws admitting of easy calculation exceeds, if I am not mistaken, the force of any human mind. (Quoted in Wilson 1989, 253)

It appears that shortly after articulating this daunting complexity problem, Newton was hard at work developing resources for responding to it with successive approximations.

The development and applications of perturbation theory, from Newton through Laplace (1798–1825) at the turn of the 19th century and on through much of the work of Simon Newcomb (1895) at the turn of the 20th, led to successive, increasingly accurate, corrections of Keplerian planetary orbital motions. At each stage, discrepancies from motion in accord with the model developed so far counted as theory-mediated phenomena carrying information about further interactions.[5] These successive corrections led to increasingly precise specifications of solar system phenomena backed up by increasingly precise measurements of the masses of the interacting solar system bodies.

3. The Classical Mercury Perihelion Problem. The famous Mercury perihelion problem, which led to the demise of Newton's theory, illustrates the continuing relevance of Newton's method to the transition from his theory to Einstein's and, especially, to the later development and application of testing frameworks for relativistic theories of gravity.

Newton claims that the inverse square variation with distance from the sun of the forces maintaining the planets in their orbits is "proved with the greatest exactness" from the fact that the aphelia are at rest. He cites Corollary 1 of Proposition 45, Book 1, according to which "precession is p degrees per revolution if and only if the centripetal force f is as $(360/360 + p)^2 - 3$ power of distance." If a planet, in going from aphelion to return to it, makes an angular motion against the fixed stars of 360 +

5. See Smith 2001 for an illuminating contrast with Newton's less successful attempts to deal with resistance forces in *Principia*, Book 2.

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p degrees, then the aphelion is precessing forward with p degrees per revolution. The aphelion is the farthest point from the sun. According to this corollary, zero precession is equivalent to having the centripetal force be as the -2 power of distance; forward precession is equivalent to having the centripetal force fall off faster than the inverse square; and backward precession is equivalent to having the centripetal force fall off slower than the inverse square power of distance.[6]

In his *System of the World*, an earlier version of Book 3, which he tell us was composed "in a popular method that it might be read by many" (Newton 1999, 793), Newton points out: "But now, after innumerable revolutions, hardly any such motion has been perceived in the orbits of the circumsolar planets. Some astronomers affirm there is no such motion; others reckon it no greater than what may easily arise from causes hereafter to be assigned, which is of no moment to the present question" (1999, 561). Any precession that can be accounted for by perturbation due to forces toward other bodies can be ignored in using stable apsides to measure inverse square variation of the centripetal force toward the sun maintaining planets in their orbits.

In 1859 Le Verrier found 38 arc seconds per century of Mercury's precession to be not accounted for by Newtonian perturbations (Roseveare 1982, 20–23). In 1882, Newcomb gave a revised corrected value of 43 arc seconds/century (1882, 473). This extra perihelion precession was a departure from the developing Newtonian theory, which resisted attempts to account for it by interactions with known bodies.

In 1894, Hall, appealing to a formula of Bertrand's which is equivalent to Newton's precession theorem (see Valluri et al. 1997, 13–27), proposed to account for the extra 43 arc seconds per century by revising the inverse square to the -2.00000016 power that would be measured by it: "Applying Bertrand's formula to the case of *Mercury* I find, taking Newcomb's value for the motion, or $43''$, that the perihelion would move as the observations indicate by taking $n = -2.00000016$" (Hall 1894, 49). In 1895, Newcomb (1895, 110–118), by then the major doyen of predictive astronomy, cited Hall's proposal as "provisionally not inadmissible" after rejecting all the other accounts he had considered. Newcomb (1895, 174) also introduced precession in accord with Hall's hypothesis into his predictive theory.

In 1903, Brown refined the Hall-Brown theory of the lunar orbit with sufficient precision to rule out Hall's hypothesis and to empirically con-

6. Newton's Proposition 45, Book 1, and its corollaries are proved for orbits that are very nearly circular. The results, however, can be extended to orbits of arbitrarily great eccentricity. Indeed, orbital eccentricity increases the sensitivity of absence of unaccounted for precession as a null experiment measuring inverse square variation of a centripetal force. See Valluri et al. 1997.

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strain departures, denoted by 'δ', from the inverse square to less than 0.00000004: "If the new theoretical values of the motion of the Moon's perigee and node are correct, the greatest difference between theory and observation is only $0''.3$, making $\delta < .00000004$. Such a value for δ is quite insufficient to explain the outstanding deviation in the motion of the perihelion of *Mercury*. It appears, then, that this assumption must be abandoned for the present, or replaced by some other law of variation which will not violate the conditions existing at the distance of the moon" (Brown 1903, 396–397).

This is an example of all three of the features that make Newton's method richer than merely HD method. It is a striking realization of Newton's richer ideal of empirical success, as accurate measurement of parameters by phenomena. It successfully turns a theoretical question into one that can be answered empirically by measurement from phenomena, and is one where the outcome of that measurement is accepted as a guide to further research. The more precise measurement of the -2 power provided by Brown rules out Hall's hypothesis as a solution to the problem of Mercury's perihelion.

4. Einstein's Solution as Support for General Relativity. Kuhn writes: "Like the choice between competing political institutions, that between competing paradigms proves to be a choice between incompatible modes of community life. Because it has that character, the choice is not and cannot be determined by the evaluative procedures characteristic of normal science, for these depend in part upon a given paradigm, and that paradigm is at issue. When paradigms enter, as they must, into a debate about paradigm choice, their role is necessarily circular. Each group uses its own paradigm to argue in that paradigm's defense" (Kuhn 1970, 94). Contrary to Kuhn, the revolutionary change to General Relativity (GR) is in accordance with the evaluative procedures of Newton's methodology. The successful account of this extra precession (Einstein 1915; see also Earman and Janssen 1993; Roseveare 1982), together with the Newtonian limit, which allowed it to recover the empirical successes of Newtonian perturbation theory (including the account of the other 531 arc seconds per century of Mercury's perihelion precession), made GR do better than Newton's theory on Newton's own ideal of empirical success. Pais indicates Einstein's extreme excitement over this result:

> The first result was that his theory 'explains . . . quantitatively . . . the secular rotation of the orbit of mercury, discovered by Le Verrier, . . . without the need of any special hypotheses'. This discovery was, I believe, by far the strongest emotional experience in Einstein's scientific life, perhaps in all his life. Nature had spoken to him. He had

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> to be right. 'For a few days, I was beside myself with joyous excitement'. Later, he told Fokker that his discovery had given him palpitations of the heart. What he told de Haas is even more profoundly significant: when he saw that his calculations agreed with the unexplained astronomical observations, he had the feeling that something actually snapped in him. (Pais 1982, 253)

This excitement was entirely appropriate to this implication of it. Einstein's theory does better than Newton's theory on Newton's own method. The successful account of Mercury's perihelion motion provides more than just a successful prediction. For example, the result about Mercury's perihelion adds a new agreeing measurement of the mass of the sun.

Since its initial development, GR has continued to improve upon what Newton's methodology counts as its clear advantage over Newtonian gravitation theory (Will 1993, 320–352). In addition, the appeal to the Newtonian limit, required to have GR account for the 531″ per century of precession accounted for by Newtonian perturbations, offers support for counting Newton's theory as an approximation recovered by Einstein's. This raises additional difficulties for Kuhn's treatment of pre- and post-revolutionary theories as incommensurable.

5. The Dicke-Goldenberg Challenge and Shapiro's Measurement. In 1966 Robert Dicke and H. Mark Goldenberg carried out measurements of solar oblateness that suggested a rapidly rotating inner core of the sun that would account for about 4 of the extra 43″ of Mercury's perihelion precession (Will 1993, 181–182). This would allow an alternative relativistic gravitational theory, the Brans-Dicke theory, to do better than GR. In the Brans-Dicke theory, the parameter γ representing the amount of space curvature per unit rest mass is given by $\gamma = (1 + \omega)/(2 + \omega)$, where ω is an extra parameter. The Brans-Dicke theory differs from GR in postulating an extra scalar field that represents the contribution of distant masses to local curvature. The adjustable constant ω represents this extra scalar field (Brans and Dicke 1961; reprinted in Dicke 1965, 77–96). In GR, the value of γ is fixed so that $\gamma = 1$. This makes GR unable to accommodate alternatives to the 43″ extra precession for Mercury. In contrast, setting its adjustable parameter ω at 5 would let the Brans-Dicke theory account for 39″ of Mercury's perihelion precession (Dicke and Goldenberg 1974). Ironically, Mercury's perihelion, the first big success of GR, now threatened to be its undoing.

In 1964, Irwin Shapiro (1964; see also Shapiro et al. 1971) proposed radar time delay as a test of GR. On a relativistic gravitation theory there is round trip time delay for radar ranging to planets, due to the gravitational potential of the sun along the path of the radiation, when that

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path passes close to the sun. This round trip time delay δt for radar ranging to planets measures γ, the parameter representing space curvature at issue above:

$$\delta t = (2r_0/c)(1 + \gamma/2) \ln (r_e + r_p + R)(r_e + r_p - R),$$

where $r_0 = 2GM_\odot/c^2$, $M_\odot$ is the solar mass, r_e and r_p are respectively the distances of the earth and the target planet from the sun, and R is the distance of the target planet from the earth (Reasenberg et al. 1979). By 1979, such measurements were precise enough to rule out any versions of the Brans-Dicke theory with $\omega < 500$ (Reasenberg et al. 1979). It was some time later, perhaps into the late 1980s, before further investigation yielded measurements indicating that effects of solar rotation are not great enough to undercut GR's account of Mercury's perihelion motion (Will 1993, 334).

As in the classical case, a proposed alternative to the accepted gravitational theory was ruled out as a solution before an adequate solution was found. In the null experiment generated by Brown's lunar theory, the bound (< 0.3 sec/year) on lunar precession not accounted for by perturbation counts as a phenomenon measuring bounds of $\delta < .00000004$ for divergences from the -2 power for orbits of the earth at the lunar distance.

In the positive detection of Shapiro's time delay, the exhibited pattern counts as a phenomenon. In the 1979 Viking experiment with Mars, this time delay phenomenon was established with sufficient precision to measure $\gamma = 1 \pm .002$ (Reasenberg et al. 1979). The development and applications of testing frameworks for relativistic gravitation theories is very much an illustration of Newton's method. The many successful tests of GR measure parameters that constrain alternative theories to approximate GR for scales and field strengths similar to those explored by the phenomena exhibited in those tests. In addition to the famous three basic tests, there are now a great many post-Newtonian corrections required by the more precise data made available by such new observations as radar ranging to planets and laser ranging to the moon. These provide not just predictions but also measurements of parameters, such as those of the PPN testing framework, which support GR (Will 1993, 86–115).

6. Conclusions. Earlier we saw that Newton's inferences are robust with respect to approximations in the phenomena. His method was able to deal with the daunting complexity of solar system motions by successive corrections to the phenomena in which discrepancies from motion in accord with the model developed so far counted as theory-mediated phenomena carrying information to be exploited. Our review of the Mercury perihelion problem suggests that Newton's method informs the radical theoretical transformation to Einstein's theory and continues to inform

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the development and application of solar system testing frameworks for relativistic theories of gravitation today. All three of the features that make Newton's method richer than the usual HD model discussed by philosophers are evident in our story. The richer notion of empirical success as accurate measurement of parameters by phenomena, rather than just prediction alone, is achieved in Shapiro's measurement of the space-time curvature parameter, just as it was exhibited in Brown's measurement-limiting deviations from the inverse square. The exploitation of theory mediated measurements from phenomena to give empirical answers to theoretical questions is at the heart of the enterprise of developing and applying testing frameworks for relativistic gravitation theories. Finally, the provisional acceptance of theory as a guide to further research is evident both in the role of the background assumptions of the testing frameworks, which play a role somewhat analogous to the role the laws of motion played for Newton, and in the role of GR in guiding research in cosmology, which is very much analogous to the role Newton's theory of gravity played in guiding research into the structure and details of solar system motions.

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