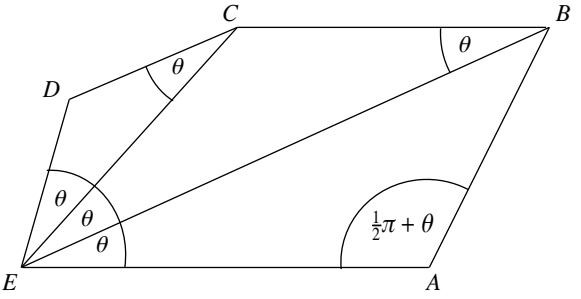
# Student Problems

Students up to the age of 19 are invited to send solutions to either or both of the following problems to Stan Dolan, 126A Harpenden Road, St Albans, Herts., AL3 6BZ.

Two prizes will be awarded – a first prize of £25, and a second prize of £20 – to the senders of the most impressive solutions for either problem. It is, therefore, not necessary to submit solutions to both. Solutions should arrive by May 20th 2018. Please give your School year, the name and address of your School or College, and the name of a teacher through whom the award will be made. Please print your own name clearly! The names of all successful solvers will be published in the July 2018 edition.

***Problem 2018.1*** **(Stan Dolan)**

The diagram shows some of the angles in pentagon *ABCDE*.




Find, in terms of $\theta$, the angle between lines *AD* and *BE*.

***Problem 2018.2*** **(Geoff Strickland)**

(a) Find three rational numbers such that their sum is 6, the sum of their squares is $\frac{25}{2}$ and the sum of their cubes is 27.

(b) Is your solution unique?

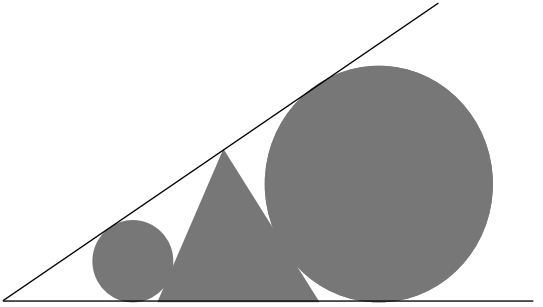
**Solutions to 2017.5 and 2017.6**

Many of the solutions to these two problems were of an exceptionally high standard. Problem 2017.5 was solved by Abigail Burn (South Wilts Grammar School), Callum Hobbis (The Sixth Form College Farnborough), Boris Litovski and Gi-Yian Yiu (King's College London Mathematics School). Problem 2017.6 was solved by Ayush Modi (The Sixth Form College Farnborough) and Simon Xu (Dulwich College). Both problems were solved by Elliot Gathercole (Cedars Upper School Leighton Buzzard), Sae Koyama (King's College London Mathematics School), Andy Li (Dulwich College), Andrew Ng (Tonbridge School) and Richard Zhuowen Su (Charterhouse).

https://doi.org/10.1017/mag.2018.36 Published online by Cambridge University Press

***Problem 2017.5 (Paul Stephenson)***

The diagram shows three shaded shapes fitting snugly between two lines.

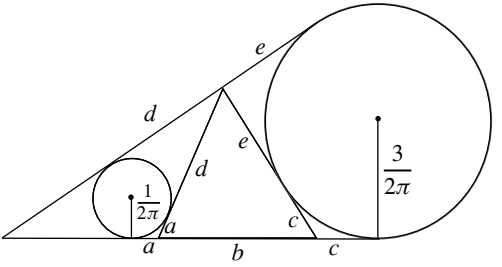


The perimeters of the small circle, triangle and large circle are 1, 2 and 3 units, respectively.

How far apart are the centres of the two circles?

*Solution*

Based upon Abigail's and Andrew's solutions, we can label various lengths as shown on the diagram.




By equality of tangents

$$a + b + c = d + e.$$

Perimeter of triangle

$$a + b + c + d + e = 2.$$

Then $a + b + c = 1$ and, applying Pythagoras, the distance between the centres is

$$\sqrt{1 + \left(\frac{1}{\pi}\right)^2} = \frac{\sqrt{1 + \pi^2}}{\pi}.$$

The method used by Elliot, Sae, Boris and Gi-Yuan followed similar lines. The latter two students sent in a joint solution which stood out for the high quality of its diagrams. Callum successfully adopted a different approach, first using similar triangles to find the angle between the original two lines.

https://doi.org/10.1017/mag.2018.36 Published online by Cambridge University Press

***Problem 2017.6 (Geoff Strickland)***

If $a$ and $b$ are positive real numbers with $a + b = 1$, then prove that

$$\left(a + \frac{1}{b}\right)^2 + \left(b + \frac{1}{a}\right)^2 \geqslant \frac{25}{2}.$$

*Solution*

Elliot, Richard and Simon used calculus to solve this problem and Ayush used a right-angled triangle. A non-calculus method was used by Sae, Andy and Andrew, along the following lines.

First note that

$$2\left(a^2 + b^2\right) = (a + b)^2 + (a - b)^2 \geqslant 1$$

$$4ab = (a + b)^2 - (a - b)^2 \leqslant 1.$$

Then

$$\left(a + \frac{1}{b}\right)^2 + \left(b + \frac{1}{a}\right)^2 = \left(a^2 + b^2\right)\left(1 + \frac{1}{ab}\right)^2 \geqslant \frac{25}{2}.$$

The first prize of £25 is awarded to Andrew Ng. The second prize of £20 is awarded to Abigail Burn.

10.1017/mag.2018.36 STAN DOLAN

*Note from the Editor*

For the time being, the first page of Student Problem Corner will be available on the Mathematical Association website.

*www.m-a.org.uk/the-mathematical-gazette*

https://doi.org/10.1017/mag.2018.36 Published online by Cambridge University Press