A conceptual wing-box weight estimation model for transport aircraft

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ABSTRACT

This paper presents an overview of an advanced, conceptual wing-box weight estimation and sizing model for transport aircraft. The model is based on linear thin-walled beam theory, where the wing-box is modelled as a simple, swept tapered multi-element beam. It consists of three coupled modules, namely sizing, aeroelastic analysis, and weight prediction. The sizing module performs generic wing-box sizing using a multi-element strategy. Three design cases are considered for each wing-box element. The aeroelastic analysis module accounts for static aeroelastic requirements and estimates their impact on the wing-box sizing. The weight prediction module estimates the wing-box weight based on the sizing process, including static aeroelastic requirements. The breakdown of the models into modules increases its flexibility for future enhancements to cover complex wing geometries and advanced aerospace materials. The model has been validated using five different transport aircraft. It has shown to be sufficiently robust, yielding an error bandwidth of ±3%, an average error estimate of -0·2%, and a standard error estimate of 1·5%.

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NOMENCLATURE

A	area, m^2
b_s	structural span, m
d	wing-box effective depth, m
D	drag, N
E_t	tangent Young's modulus, N/m^2
F	efficiency factor of skin-stringer structural panel (–)
{**F**}	nodal loads vector, N
FS	factor of safety (–)
h	wing-box height at a local chordwise location, m
I_x	second moment of area in the spanwise direction, m^4
J	polar moment of inertia, m^4
(**K**)	stiffness matrix, N/m^2
L	lift, N
l	rib pitch, m
M	bending moment, Nm
P	limit compressive load intensity (N/m)
Q	shear flow in the webs N/m
S_w	wing platform area, m^2
T	torsional moment, Nm
t	thickness, m
{**U**}	nodal displacement vector, m or rad
V	speed, ms^{-1}
F	shear force, N
w	average width of an element, m
W	component weight, kg
η	wing-box cross-section bending efficiency factor (–)
ρ	ambient density, kg/m^3
σ	stress, N/m^2

Subscripts

b	allowable bending stress
config	configuration
crit	critical
DD	design dive
div	divergence
ec	enclosed
ES	equivalent skin
FS	front spar
l	lower panel
lim	limit load
max	maximum
np	non-optimum
r	rib
ref	reference
rev	reversal

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RS rear spar
s allowable shear
SW spar web
Tot total
u upper panel
ucs ultimate compression strength
v vertical
w web
WB wing-box

1.0 INTRODUCTION

The use of Multidisciplinary Design Optimisation (MDO) during aircraft pre-concept, conceptual, and preliminary design has grown significantly due to the nature and complexity of the trade studies involved. Coupled with the need to investigate aircraft life cycle costs, key technical decisions early in the design process must take place[1,2].

MDO methodologies for aircraft design require various wing structural parameters to be controlled or varied depending on the sensitivity of the objective function to these parameters. This might have a significant impact on the wing-box weight[2], which necessitates a wing-box weight estimation and sizing model that permits computationally inexpensive analyses. In addition it must be capable of capturing the correct sensitivities of the wing structural driver variables that may occur during the optimisation process, and generates a good measure of conformity in the absolute result.

The majority of existing wing-box weight estimation methods/models available in literature can be classified into two main categories: semi-empirical and finite element. Semi-empirical models are based on data from similar existing aircraft; therefore the robustness of these models depends on the similarities (size, configuration, and technology (systems, structural efficiency, and materials)) between the aircraft under investigation and the aircraft that have been used in the derivation process of these models[2,3]. Semi-empirical models are limited to early design stages where detailed geometric, structural, and aerodynamic knowledge are typically unavailable[3]. Furthermore, these models are not entirely compatible with being integrated within an MDO environment, because the designer cannot guarantee that they are capable of capturing the correct sensitivities to various changes that may occur during the optimisation process and which may fall outside the bounds of the database used to derive these methods. In addition, semi-empirical models are incapable of handling complex wing-box geometries that may be generated by the optimiser, and they are usually limited to conventional aerospace materials such as light alloys[4]. Torenbeek compared results from semi-empirical methods and his linear beam theory method for different scales of aircraft, and he demonstrated the limitations of these semi-empirical methods[5]. On the other hand, time consuming Finite Element Models (FEMs) are not suitable for integration within an MDO environment during the conceptual-preliminary design phase because they require detailed knowledge of the internal geometry and aerodynamics that are usually unavailable at this early design stage[3]. Therefore, it is desirable to formulate a quasi-analytical model that can share the advantages of both categories, in other words, an advanced conceptual (low fidelity) model. The importance of advanced low fidelity models for MDO applications during early design stages was discussed by Piperni *et al*[6], Kafyeke[7], and Viana[8]. These models provide a good level of conformity while minimising both the computational time and the level of detailed knowledge required.

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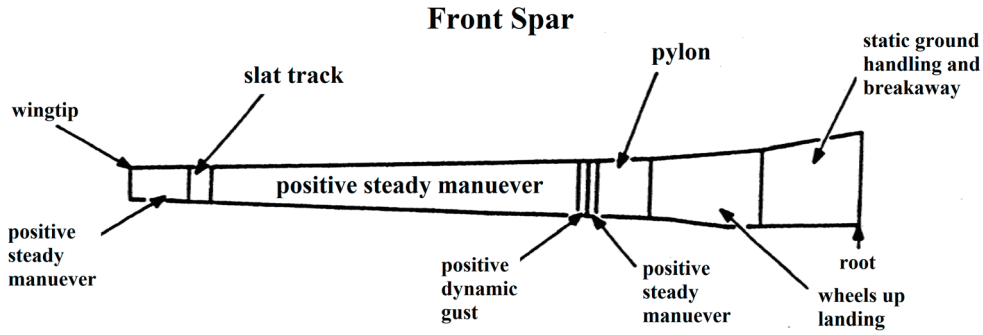
This paper discusses the features of an advanced quasi-analytical Wing-box Weight Estimation and Sizing (WWES) model that is sufficiently flexible to be integrated within an MDO environment. The model is a compromise between low fidelity semi-empirical models that are based purely on statistical data from existing aircraft, and high fidelity time consuming FEM. It is based on thin-walled Euler-Bernoulli linear beam theory, where the wing-box is modelled as a linear tapered swept beam, and employs a novel multi-element sizing strategy based on a local load factor. A generic sizing of each element is performed based on strength, stiffness, and aeroelastic requirements. The weight of each element is estimated, and then summed to provide a detailed breakdown of the overall wing-box weight. The model consists of three coupled expert design modules; this increases its flexibility, broadens its range of applications, and simplifies its integration within the MDO environment. The model has been validated comprehensively to guarantee the credibility of predictions using five transport aircraft of different configurations, sizes, and mission roles. These five aircraft are the Airbus A320-200, the Fairchild Dornier 728, the Saab AB 2000, the Bombardier Aerospace Global Express, and the Bombardier Aerospace CRJ900.

2.0 FEATURES OF THE MODEL

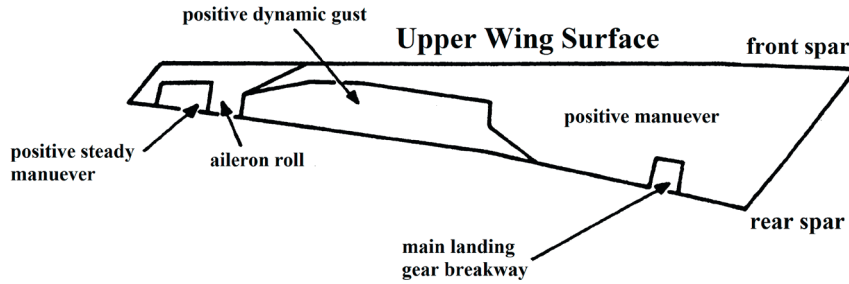
The WWES model belongs to the family of quasi-analytical methods; hence it shares some of the advantages of both low fidelity and high fidelity methods. It is a linearised thin-walled structural analysis code written in MATLAB™, and based on linear beam theory. MATLAB™ was selected as the tool to develop this model because it has a wide range of toolboxes that can be integrated with this model to perform various multidisciplinary design studies, and the aerodynamic model used in this study is already available in MATLAB™. The WWES model offers a variety of benefits in comparison with the existing wing-box weight estimation models that are available in literature. These benefits can be summarised as follows:

- It executes with an excellent level of conformity, i.e. it has a good level of accuracy per computational expense when compared to the accuracy per computational expense of high fidelity models. This excellent level of conformity (accuracy per computational expense) allows the model to be effectively integrated with a conceptual MDO suite.
- It employs a multi-element sizing strategy based on local load factor for each element, and hence avoids over-designing or under-designing some wing-box elements based on a global load factor as most low fidelity methods do[9,10].
- It considers three design cases for each wing-box element in accordance with FAR 23/25 and EASA CS-23/25 regulations for the transport aircraft category. The sizing cases are: symmetrical manoeuvres, discrete gusts, and roll manoeuvres[9,10].
- It is capable of handling all traditional aircraft configurations, e.g. aircraft with aft fuselage-mounted or wing-mounted engines[9,10].
- It is capable of handling highly complex wing geometries, i.e. multiple dihedral and sweep[9,10].
- It is flexible enough to allow future enhancements and modifications in order to investigate wide range of aerospace materials, thereby facilitating the study of heterogeneous wing-boxes.
- It accounts for inertial relief provided by the wing-box primary structure, wing secondary structures, wing systems, fuel, powerplant, and landing gear.
- It estimates the flexural properties of the wing-box at different spanwise locations, which allows the wing deflections under external loads to be estimated.
- It estimates the wing deflections under the external loads using quasi-static aeroelastic analysis.

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(a) Critical design loadings on the front spar



(b) Critical design loadings on the upper wing surface

Figure 1. Spanwise variation of critical design loads on different wing-box components for typical large transport aircraft[5].

- Unlike most low fidelity models, it accounts for static aeroelastic considerations in the sizing process and accounts for their localised structural reinforcement analytically.
- It has a high level of flexibility and a good level of sensitivity allowing efficient integration within an MDO environment.
- It defines rib pitch and orientation to prevent overall buckling of the upper wing panels.

2.1 Details of the model

During flight, the wing-box is subject to a variety of critical design loads and the importance of these loads varies across the wing span as shown in Fig. 1.

Therefore to avoid over-designing or under-designing any component, the model discretises the wing-box into elements, and identifies a limit load factor on each element based on three design scenarios, namely discrete gust loads, symmetric manoeuvre loads, and aileron roll loads. To continue, the highest of the three load factors corresponding to the design scenarios is used to perform generic sizing of the element. A similar multi-element sizing strategy was described by Kelm *et al*[11].

Element sizing includes equivalent skin representation (produced by summing the upper and lower skin panels, stringers, and spar flanges) and the spar webs, and defining the rib pitch and orientation to prevent overall buckling of the upper wing panel. The equivalent upper and lower panels are sized based on bending, torsion, and aeroelastic requirements while the spar webs are sized to withstand vertical shear and torsional loads. On the other hand, the rib pitch is defined to support the upper wing panels against buckling instabilities.

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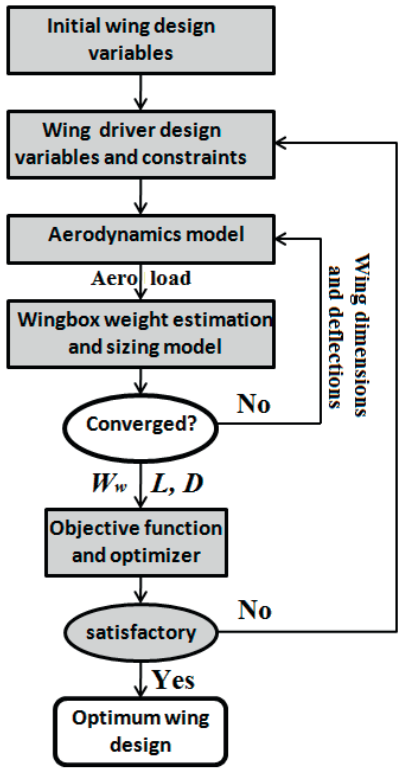


Figure 2. Integration of the wing-box weight estimation and sizing model into an MDO environment[9,10].

In fact, the majority of wing weight and sizing prediction methods available in literature employ a global limit load factor (usually symmetric manoeuvre limited to 2·5g for large transport aircraft[5]) for the entire wing. This may result in over-designing or under-designing some elements or components of the wing-box. Examples of these methods are the General Dynamics (GD) method given in[12,13] and the C_1 method given in Ref 4. This problem can be eliminated with the multi-element sizing strategy employed by the WWES model.

The discretisation process (generation of elements) is harmonised with the number of spanwise aerodynamic panels generated by the aerodynamic model. Increasing the number of elements, i.e. reducing their nodal spans, yields better conforming results, but increases the computational time for the model. The elements can be of equal span (linear distribution), or their span can vary from root to tip in a cosine or sine distribution. Cosine or sine distributions produce more accurate results with higher computational expense. The choice between one distribution and the other depends on the baseline geometry of the wing and the change in aerodynamic loads due to aeroelastic deformations that might occur around the tip region. This harmonisation with the aerodynamic model can be uncoupled depending on the user

Table 1
The three modules and their corresponding tasks

Module	Task
Sizing	● Discretising the wing-box into elements. ● Estimating the load factor on each element. ● Sizing the wing-box elements. ● Accounting for inertial relief effects. ● Accounting for aeroelastic requirements. ● Estimating spanwise flexural properties of the wing-box ● Sizing the wing-box elements.
Aeroelastic analysis	● Checking for static divergence. ● Checking for aileron reversal. ● Estimating nodal wing deflections.
Weight prediction	● Estimating the weight of wing-box components. ● Providing a detailed breakdown of the wing-box weight. ● Determining the sensitivity of the wing-box weight to the size and configuration of the aircraft.

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intent. After performing the generic sizing of the elements, the weight of each element is estimated using a combination of semi-empirical and analytical equations. As mentioned in the introductory section, the model is split into three expert design modules: sizing, aeroelastic analysis, and weight prediction. This breakdown increases the flexibility of the model and simplifies its integration within an MDO environment as shown in Fig. 2. These modules are linked with respect to their physical dependencies by an iterative algorithm. Each module performs a variety of design tasks that are listed in Table 1.

The details of the MDO suite are beyond the scope of this paper, however Smith *et al*[9,10] provides extensive details about the MDO suite, objective functions, design constraints, and computational expenses.

The Tornado Vortex Lattice Model (VLM)[14-16] is the aerodynamic model that has been coupled with the wing-box weight estimation and sizing model to estimate the aerodynamic loads on the wing-box. Briefly, the Tornado VLM software implements the 3D Vortex Lattice Method (VLM), with a flexible wake to solve linear aerodynamics problems with a very high computational speed. The lifting surfaces are modelled as thin plates, and a variety of wing geometries can be handled including camber profiles of aerofoil sections. Tornado VLM is coded in MATLABTM, and is limited to global angles of attack of 8-10° prior to the onset of viscous effects. Furthermore Tornado VLM is capable of accurately predicting the transonic lift curve slope using the Prandtl-Glauert compressibility correction[16].

2.2 The wing-box sizing process

The wing-box sizing is a very crucial endeavour, since its accuracy determines the precision of the weight estimation process and the correct wing deflections. The sizing process, as shown in Fig. 3, includes sizing the upper and lower equivalent panels (including stringers and spar webs), front and rear spar webs, and defining the rib pitch. Throughout the sizing process the following assumptions are employed:

- The wing-box resists all of the external loads, i.e. no contribution from wing secondary structures;
- The equivalent upper and lower wing panels resist bending and torsional loadings[17];
- The spar webs resist the vertical shear and torsional loads;
- The rib pitch is defined to support the equivalent upper wing panels against overall buckling[5,17];
- The internal pressure loads and landing gear critical loads are neglected throughout the sizing process;
- The impact of low speed flutter on the wing-box sizing and weight is neglected: The majority of the aircraft studied here are highly swept (above 15° quarter chord sweep). Ward *et al*[18] showed that aileron reversal is the critical aeroelastic case at the outboard region for a highly swept wing. However, the Saab AB 2000 aircraft has low quarter chord sweep (3·6°), but it has a thick aerofoil and thus a high structural stiffness, so that flutter instabilities are unlikely to occur; and,
- The sizing process is performed iteratively in order to account for inertial relief effects.

2.2.1 Determination of local load factor

On each wing-box element, the typical critical three sizing scenarios are considered in accordance with FAR 23/25 and EASA CS-23/25 regulations for transport aircraft, and a limit load factor corresponding to each of the three scenarios is estimated. These scenarios are as follows:

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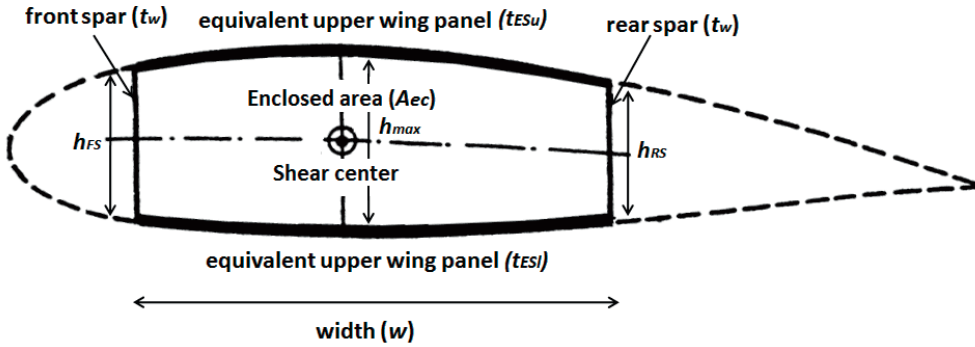


Figure 3. A representative wing-box cross-section perpendicular to the elastic axis.

- Discrete gust loads: where the aircraft is assumed to be subjected to symmetrical vertical and lateral gusts loads in level flight. The loads on each element of the wing-box must be determined using a dynamic analysis while accounting for unsteady aerodynamic characteristics and all the significant structural degrees of freedom including rigid body motions.
- Symmetric manoeuvre limit loads: where the aircraft is assumed to be subjected to symmetrical manoeuvres resulting in the limit manoeuvring load factors. Pitching velocities appropriate to the corresponding pull-up and steady turn manoeuvres must be taken into account.
- Aileron roll loads: the aircraft must be designed to withstand loads resulting from roll manoeuvres. Different combinations of speeds and aileron deflections (except that the deflections may be limited by pilot effort for aircraft with fully manually flight control for the ailerons, for example the Saab AB 2000) must be considered in combination with an airplane load factor of zero and of two-thirds of the positive manoeuvring factor used in design.

The local load factor on each element is estimated using the local lift coefficient instead of the overall lift coefficient of the wing, the local lift force generated by each element is used. Then, the highest of the three load factors is considered for sizing purposes. Those local load factors with the normal lift distribution are used to generate a new lift distribution that is used in the sizing process. The model is run at different points of the flight envelope to find the critical flight condition. The critical flight condition is the start of cruise for all of the aircraft studied here. Throughout the

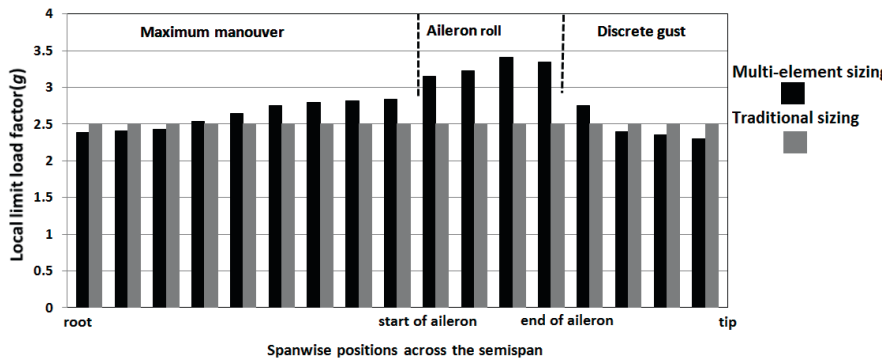


Figure. 4 Local load factor at different spanwise positions of a representative wing-box semi-span of a narrow body aircraft.

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sizing process, a factor of safety *FS* of 1·5 is employed to account for uncertainties in loadings, material properties, and geometric irregularities[17,19]. Figure 4 illustrates the spanwise variation of the limit load factor on each element of a conventional representative wing-box from root to the tip.

Figure 4 shows that symmetric manoeuvre is the critical design case from the wing-box elements from the root to the start of the aileron, while aileron roll is the critical design case across the aileron, and discrete gust is the critical design case at the wing tip region.

2.2.2 Sizing equivalent upper and lower wing panels

The equivalent upper and lower wing panels are designed to withstand bending and torsional loads[5,17,19,20]. The upper wing panel is assumed to be under compression, while the lower wing panel is under tension. The ultimate stress at the i^{th} wing-box element is given by:

$$\sigma_{ult\,(i)} = \frac{FS\, M_{lim}(i) d(i)}{2 I_x\,(i)} \quad \ldots (1)$$

where FS is the factor of safety, M_{lim} is the limit bending moment (Nm) at the i^{th} element, d is the effective depth (m) of the wing-box at the i^{th} element, and I_x is the second moment of inertia (m^4) at the i^{th} element. The effective depth of the aerofoil, d, can be approximated as:

$$d(i) = \eta h_{max}(i) \quad \ldots (2)$$

where η is the bending efficiency factor and h_{max} is the maximum height (m) of the aerofoil at the i^{th} element. The second moment of inertia I_x at the i^{th} element can be approximated as follows:

$$I_x(i) \approx \frac{t_{ES}(i)\, w(i)\, d(i)^2}{2} \quad \ldots (3)$$

where w is the average width (m) of the i^{th} element, t_{ES} is the equivalent skin thickness (equivalent upper or lower panels). Equation (3) is applicable if and only if the thin-walled assumption is valid and the thicknesses of the equivalent upper and lower panels are identical. The thickness of the equivalent upper or lower panel required to withstand the bending loads can be estimated by:

$$t_{ES_{bending}}(i) = \frac{FS\, M_{lim}\,(i)}{w(i)\, d(i)\, \sigma_b(i)} \quad \ldots (4)$$

where η is the bending efficiency factor, and σ_b is the allowable bending stress (N/m^2) at the i^{th} element. According to Howe[17], the use of the allowable compression stress (σ_b) for preliminary sizing of the equivalent upper and lower wing panels simplifies the sizing process without introducing an undue error. The bending efficiency factor (η) is employed throughout the sizing of the equivalent upper and lower wing panels to represent the actual skin-stringer panels with equivalent upper and lower panels. Torenbeek[5] presents an approximate equation to estimate the value of the bending efficiency factor:

$$\eta = \frac{1}{3}\left[1 + \left(\frac{h_{FS}(i)}{h_{max}(i)}\right)^2 + \left(\frac{h_{RS}(i)}{h_{max}(i)}\right)^2\right] - \eta_{ref} \quad \ldots (5)$$

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where h_{FS} is the effective height of the front spar (m) at the i^{th} element, and h_{RS} is the effective height (m) of the rear spar at the i^{th} element, and η_{ref} is a statistical constant equal to 0·025. The use of Equation (5) requires detailed knowledge of the spar positions, depth, and aerofoil section. If these details of the wing-box section are unavailable, η can be approximated to be equal to 80%[5]. The thickness of the equivalent upper or lower panel required to withstand the torsional loads can be estimated by:

$$t_{ES_{torsion}}(i) = \frac{FS\, T_{lim}(i)}{2A_{ec}(i)\sigma_s(i)} \quad \ldots (6)$$

where T_{lim} is the limit torque (Nm) at the i^{th} wing-box element, A_{ec} is the enclosed area (m^2) of the i^{th} element, and σ_s is the allowable shear stress (N/m^2). Equation (6) is derived from the Bredt-Batho formula and it is only valid when the thin-walled assumption is valid. The total thickness of the equivalent upper or lower panels is the sum of the thicknesses of the material required to withstand bending loads and the material required to resist torsional loads. Thus the minimum thickness of the upper or lower panel is given by:

$$t_{ES}(i) = t_{ES_{bending}}(i) + t_{ES_{torsion}}(i) \quad \ldots (7)$$

It should be noted that the model assumes that the thickness of the equivalent upper and lower panel is constant over the span and the width of each element.

2.2.3 Sizing the spar webs

At the level of conceptual-preliminary design, both vertical shear loads and torsional loads define the required thicknesses of the spar webs. Three major steps are followed to estimate the required thickness of the webs[17]:

i. First, estimate the shear flow in the webs of the i^{th} element due to vertical shear loads:

$$Q_v(i) = \frac{FS\, Fv_{lim}(i)}{h_{Tot}(i)} \quad \ldots (8)$$

where Fv_{lim} is the limit shear force (N) at the i^{th} element, h_{Tot} is the total effective height (m) of the spars at the i^{th} element. Equation (8) assumes a unidirectional loading case where the vertical shear force is applied across the shear centre, and also it assumes a uniform thickness for each of the spar webs. The total effective height of the spars (h_{Tot}) at the i^{th} element can be estimated as:

$$h_{Tot}(i) = h_{FS}(i) + h_{RS}(i) \quad \ldots (9)$$

where h_{FS} is the effective height (m) of the front spar at the i^{th} element, and h_{RS} is the effective height (m) of the rear spar at the i^{th} element.

ii. Second, estimate the shear flow in the webs at the i^{th} element due to torsional loads:

$$Q_T(i) = \frac{FS\, T_{lim}(i)}{2A_{ec}(i)} \quad \ldots (10)$$

where T_{lim} is the limit torque (Nm) at the i^{th} element and A_{ec} is the enclosed area (m^2) of the

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wing-box of the i^{th} element.

iii. Finally, the web thickness is given by;

$$t_w(i) = \frac{Q_v(i) + Q_T(i)}{\sigma_s} \quad \ldots (11)$$

where (σ_s) is the allowable shear stress (N/m^2) in the webs.

2.2.4 Defining the rib pitch and orientation

Ribs are designed to fulfil a multitude of tasks. These tasks can be summarised as follows[5,17]:

- Stabilise the skin-stringer panels under compression;
- Contain the fuel;
- Transfer concentrated loads into the wing-box; and,
- Act as a beam to transfer distributed air loads to the spars.

At the level of conceptual-preliminary design, the multi-tasking nature of the ribs makes their sizing process a tedious one[5]. They require a detailed knowledge of the loading path, wing-box geometry, material distribution, and manufacturing process. Since this detailed knowledge is typically unavailable, the model focuses on defining the rib pitch across the wing-box to stabilise the upper wing panel against compressive loads rather than sizing the ribs. The rib pitch (l) (m) can be estimated using the following equation given in Equation (5):

$$\sigma_{crit}(i) = F\sqrt{\frac{FS\ P(i)\ E_t}{l(i)}} \leq \sigma_{ucs} \quad \ldots (12)$$

where F is the efficiency factor of a stiffened skin panel, P is the limit compressive load intensity (N/m) in the upper panel at the i^{th} element, E_t is the tangent modulus (N/m^2), and σ_{ucs} is the ultimate compression strength (N/m^2) of the equivalent upper panels. The efficiency factor (F) depends on the details of the skin-stringer panel such as the type of stringers and the ratio of skin to stringer material. Since these details are unavailable at the level of conceptual-preliminary design, F can be approximated as 0·8 for Z-stringers and 0·9 for Hat-stringers[5]. Furthermore the WWES model allows defining the rib orientation relative to the airflow. Two rib orientations can be handled: the first orientation is the one where the ribs are parallel to the airflow, and the second orientation is the one where the ribs are perpendicular to the wing leading edge.

2.2.5 Inertial relief

The wing self-weight (structural and systems), fuel, powerplant and landing gear modifies the bending and shear load distribution across the wing, which can have a significant impact on the sizing and weight prediction processes. The wing-box components are sized to withstand ultimate aerodynamic loads, but due to inertial relief effects the influence of these loads reduces. The reduction depends on the aircraft configuration, wing geometry, and fuel management. The model accounts for inertial relief effects iteratively with an initial mass distribution of the wing self-weight that is obtained using the semi-empirical equations. This initial mass distribution is assumed while sizing the wing, and then a new wing weight and mass distribution are obtained. These are then used again to perform a new wing sizing. The mass distribution and the sizing process continue to be refined until convergence is achieved.

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3.0 AEROELASTIC ANALYSIS

Aeroelastic considerations can have a significant impact on the wing-box sizing process. They often require stiffer wing-boxes with tailored mass distributions, which can result in significant weight penalties ranging from 2-5% of the total wing weight[5]. The majority of the low fidelity wing-box weight estimation models available in literature focus on strength and stiffness considerations, while neglecting aeroelastic requirements or accounting for them using statistical data from existing designs. In this paper, a more advanced analysis and sizing strategy is employed in order to account for static and quasi-static aeroelastic considerations.

3.1 Static aeroelastic considerations

Static divergence and aileron reversal checks have been embedded in the aeroelastic analysis module. After the wing-box is sized based on strength and stiffness requirements, the divergence and reversal speeds of the wing are estimated[21,22] and are compared to the critical speed (V_{crit}) of the aircraft, which can be approximated as 125% of the design dive speed (V_{DD}) of the aircraft[17]. If the divergence and reversal speeds are below the critical speed, then the wing suffers from static aeroelastic problems. From a structural design point of view, these static aeroelastic problems can be resolved by increasing the torsional rigidity of the wing-box, which can be achieved in using a variety of structural design methods. However for the sake of simplicity, the torsional rigidity in this model is increased by resizing the corresponding equivalent upper and lower panels (adding thickness) to increase the divergence and reversal speeds to be greater than or equal to the critical speed. The resizing of the panels to increase the torsional rigidity of the wing-box has a direct impact on its structural weight. Taking stock of the abovementioned considerations, the thickness of the i^{th} equivalent upper or lower panels can be approximated by:

$$t_{ES}(i) = t_{ES_{bending}}(i) + t_{ES_{torsion}}(i) + \max\left(\Delta t_{div}(i),\ \Delta t_{rev}(i)\right) \quad \ldots (13)$$

where $t_{ES_{bending}}$ (m) and $t_{ES_{torsion}}$ (m) are the thicknesses of the equivalent upper or lower skin panels required to resist bending and torsional loads at the i^{th} element respectively, Δt_{div} is the thickness increment (m) at the i^{th} required to increase the divergence speed up to the critical speed, and Δt_{rev} is the thickness increment (m) at the i^{th} element required to increase the reversal speed up to the critical speed.

3.2 Quasi-static aeroelastic analysis

Wing deformations under external loads can distort the aerodynamic performance by modifying the lift distribution and drag (profile and induced). The model must be capable of capturing this performance augmentation and feed it back to the objective function of the optimisation process (Fig. 2). In order to predict the deformed wing shape under external loadings, a quasi-static aeroelastic loop was embedded in the aeroelastic analysis module, where the wing-box is modelled as a one dimensional linear Euler beam. The beam is discretised into elements (different from the discretisation for sizing purposes) with each element having two nodes at its extremities. Each node has six degrees of freedom: three in translation and three in rotation. The stiffness matrix for each element is determined using the geometric and mechanical properties of the element. The loads (forces and moments) on each node are then estimated using the Tornado VLM, and hence, the wing nodal deflections can be estimated using the following relation:

$$\{\boldsymbol{F}\} = [\boldsymbol{K}]\{\boldsymbol{U}\} \quad \ldots (14)$$

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where {**F**} is the nodal load vector (N or Nm), [**K**] is the global stiffness matrix (N/m or Nm), and {**U**} is the nodal displacement vector (m or rad). In the calculation of the stiffness matrix the Bredt-Batho formula was employed to estimate the polar moment of inertia (J). The quasi-static aeroelastic loop requires an iterative solution to estimate the wing deformations, while accounting for the strong interaction between the aerodynamics and the structure. The iterative solution continues until equilibrium between aerodynamic loads and structural deformations is achieved.

4.0 WING-BOX WEIGHT

The wing-box weight estimation is a bottom-up process where the weights of various wing-box components are estimated and summed to give the overall weight. The various wing-box components can be listed as follows:

- Equivalent upper and lower panels (skins, stringers, and spar flanges)
- Spar webs
- Ribs

$$W_{WB} = W_{ES} + W_{SW} + W_r + W_{np} \quad \ldots (15)$$

where W_{WB} is the total wing-box weight (kg), W_{ES} is the weight of the equivalent upper and lower wing panels (kg), W_{SW} is the weight of the front and rear spar webs (kg), W_r is the weight of the ribs (kg), and W_{np} is the weight of non-optimum effects (kg).

4.1 Weight of equivalent upper and lower panels

The weight of the upper and lower panels is regarded as the weight of the material required to withstand bending and torsional loads and prevent static aeroelastic problems. The thickness of the equivalent upper and lower panels is given in Equation (13). From the thicknesses of the equivalent upper and lower panels at different spanwise locations, the weight of the material required to resist bending and torsion can be estimated as:

$$W_{ES} = \rho_{ES_u} \sum_{i=1}^{i=n} t_{ES_u}(i) w(i) b_s(i) + \rho_{ES_l} \sum_{i=1}^{i=n} t_{ES_l}(i) w(i) b_s(i) \quad \ldots (16)$$

where ρ_{ES_U} and ρ_{ES_l} are the material mass densities (kg/m^3) of the upper and lower equivalent skin panels at the i^{th} element respectively, w is the average width (m) of the i^{th} element, b_s is the structural span (m) of the i^{th} element, t_{ES_U} and t_{ES_l} are the equivalent thicknesses (m) of the upper and lower panel respectively at the i^{th} element.

4.2 Weight of spar webs

The required thickness of the spar web to resist shear and torsional loads is derived in Equation (11). Given the required thickness of the spar webs at different spanwise locations, the weight of the webs can be estimated as:

$$W_r = k_{config} k_r \rho_r S \left(\frac{(d_r + d_t)}{2} + d_{ref} \right) \quad \ldots (17)$$

where ρ_{FS} and ρ_{RS} are the material mass densities (kg/m^3)of the front and rear spar webs respectively, h_{FS} and h_{RS} are the heights of the front and rear spar webs (m) at the i^{th} element respectively, and t_{FS} and t_{RS} are the thicknesses of the front and rear spar webs (m) at the i^{th} element respectively.

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4.3 Rib weight

As discussed above, ribs are designed to fulfil a multitude of functions and tasks[17]; hence at the level of conceptual-preliminary design, it is difficult to estimate the ribs weight analytically. Torenbeek[5] states that the weight of the ribs is proportional to the following geometrical and mechanical parameters of the wing:

- Rib material;
- Wing area (or structural span);
- Wing depth at the root and at the tip.

The WWES model employs a modification of Torenbeek's semi-empirical equation to estimate the rib weight[5]:

where k_{config} is an empirical correction factor to account for the change in aircraft configuration, k_r is an empirical constant whose value can be approximated to 5×10^{-4}, ρ_r is the material density (kg/m^3) of the ribs, S_w is the platform wing area(m^2), d_r and d_t are the effective depth (m) of the wing-box at the root and tip respectively, and d_{ref} is a statistical reference depth (m) equivalent to 1m. This semi-empirical equation accounts for the wing-to-fuselage interconnection (pressure bulkheads) and fuel tank boundaries. The original Tornbeek's equation does not contain the correction factor (k_{config}), and this was introduced in this model. Tornbeek's original equation showed that it is limited to conventional transport aircraft where the engines are mounted on the wing. This results in signicant error when applied to conventional aircraft configurations but with the engines mounted on the aft fuselage, such as the Bombardier Aerospace CRJ 900 and Bombardier Aerospace Global Express aircraft. Torenbeek's semi-empirical equation does not account for the changes in bending moment and shear force distributions due to the inertial relief provided by mounting the engine on the wing. Therefore in this model, a correction factor k_{config} has been introduced into the equation to account for the change in aircraft configuration (wing mounted engines or aft fuselage mounted engines). The correction facor k_{config} is 1·00 for a wing mounted engines (Tornebeek's orginal equation) and 2·15 for fuselage mounted engine. The 2·15 correction factor was obtained by matching actual rib weight of rear-mounted aircraft with the rib weight estimated by Tornbeek's equation for these aircraft.

4.4 Non-optimum effects

After the estimation of the wing primary weight using the maximum stress levels, the model corrects this estimation by accounting for the weight of non-optimum effects. These effects can be summarised as follows:

- Thickness variation, joints, and large cut-outs;
- Mountings and connections; and,
- Torsion loads

The model uses Torenbeek's method[5] to estimate the weight non-optimum effects.

4.5 Inertial relief

Fuel, wing-mounted engines, wing-mounted landing gears, wing secondary structures, wing systems, and the wing-box self-weight modify the loading distribution (bending and shear) across the wing and provide inertial relief. The reduction in wing weight due inertial relief depends on the size and the configuration of the aircraft. Accounting for inertial relief analytically throughout the sizing process is not a trivial task and requires an iterative solution. The solution starts with an initial mass distribution that is continuously refined until convergence is achieved. The initial

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mass distribution of the wing-box is obtained by sizing it assuming only aerodynamic loads and ignoring the effect of inertial relief.

Fuel and wing empty weight (structural and systems) are distributed masses, so their contribution to inertial relief depends on their distribution, which is concentrated in the inboard section of the wing. The wing-box initial mass distribution is generated through sizing the wing-box using aerodynamic loads. In contrast, wing-mounted engines and wing-mounted landing gear are concentrated masses, which results in a linear decrease in the bending moment and shear forces between their corresponding positions and the wing root. Outboard of their location, they do not contribute to any inertial relief. According to Torenbeek[5], concentrated masses such as powerplant and landing gear can affect the flutter characteristics of the wing depending on the location of their centres of gravity with respect to the elastic axis. This may result in a weight reduction or weight penalty. A prediction of the impact of inertial relief on the wing structural weight is presented in Table 2 for the five validated transport aircraft.

The magnitude of inertial relief estimated by the WWES has been validated by comparing its magnitude to the magnitude of inertial relief provided by Howe[4]. The validation process is presented in Table 2. It demonstrates that the values of inertial relief estimated by the WWES are in the same order of magnitude to the inertial relief values provide by Howe[4]. The magnitude of inertial relief depends on the size, mission, and configuration of the aircraft. For instance, the Bombardier Aerospace Global Express aircraft has a high value of inertial relief (48%) because of the large amount of fuel stored in the wing to meet the ultra-long range requirements of the aircraft.

Table 2
The impact of inertial relief on the wing-box weight using two different methods

Transport aircraft	Impact of inertial relief on wing-box weight (%) using the model	Impact of inertial relief on wing-box (%) using Howe semi-empirical equations[4]
Airbus A320-200	37·8	41·4
Fairchild Dornier 728	31·5	31·0
Saab AB 2000	36·2	35·0
Bombardier Aerospace Global Express	48·0	56·0
Bombardier Aerospace CRJ900	25·0	29·0

he model predicts the impact of inertial relief iteratively using the actual weight distribution of the primary and secondary structures, fuel, powerlant, and landing gear. In contrast, Howe's semi-empirical method uses various correction factors to account for the different weights. In Table 2, Howe's semi-empirical method predicts higher impact for the inertial relief on the wing-box especially for the aircraft with highly swept and slender wings. Howe's method tends to be more conservative. Moreover, Howe's method is incapable of capturing the spanwise variation of various wing geometric parameters which can have significant impact on the magnitude of inertial relief, whereas the WWES model has more flexibility to handle various wing geometries and structural details and accounts for their spanwise variations.

4.6 Weight of the wing secondary structures

Studies on existing aircraft show that the secondary wing structures contribute approximately 30% of the total structural wing weight[5,11]. Therefore, they can have a significant effect on the magnitude

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of inertial relief. The weight of secondary wing structures is estimated using semi-empirical equations from Torenbeek[5]. These secondary structures include the following components:

- Control surfaces (primary and secondary);
- High-lift devices at the trailing and leading edges;
- Leading and trailing edge structure; and;
- Landing gear attachment and associated fittings.

The contribution of the secondary wing structures to the total structural wing weight for the five validated aircraft is estimated using Torenbeek's semi-empirical Equations (5) and is listed in Table 3.

Table 3
Predicted contribution of secondary wing structures to the total structural wing weight

Aircraft	Weight of secondary wing structures (% of structural wing weight)
Airbus A320-200	30
Fairchild Dornier 728	34
Saab AB 2000	34
Bombardier Aerospace Global Express	27
Bombardier Aerospace CRJ 900	37

4.7 Model validation

The WWES model has been comprehensively validated in order to guarantee credibility of predictions. The validation process covered five transport aircraft of different sizes, configurations, and mission roles. A list of these aircraft is given in Table 4.

The validation process is summarised in the residual plot presented in Fig. 5. The validation process yields an error bandwidth of ±3%, average error estimate of –0·2%, and a standard error estimate of 1·5% for the five tranport aircrafts.

Table 4
Design parameters of the validated transport aircraft

Aircraft	Mission role	Wing geometry	Configuration	Mach at cruise	Wing loading (MTOW/S) [kg/m^2]
Airbus A320-200	mid-range, narrow body aircraft	mid sweep, high aspect ratio	under wing-podded engines	0·78	645
Fairchild Dornier 728	short-range, regional jet	mid sweep, high aspect ratio	under wing-podded engines	0·78	470
Saab AB 2000	turboprop regional aircraft	low sweep, high aspect ratio	wing mounted turboprops	0·55	395
Bombardier Aerospace Global Express	ultra long-range business high speed jet	high sweep, moderate aspect ratio	aft fuselage mounted engines	0·85	834
Bombardier Aerospace CRJ900	Short-range regional jet	mid sweep high aspect, ratio	aft fuselage mounted engines	0·78	559

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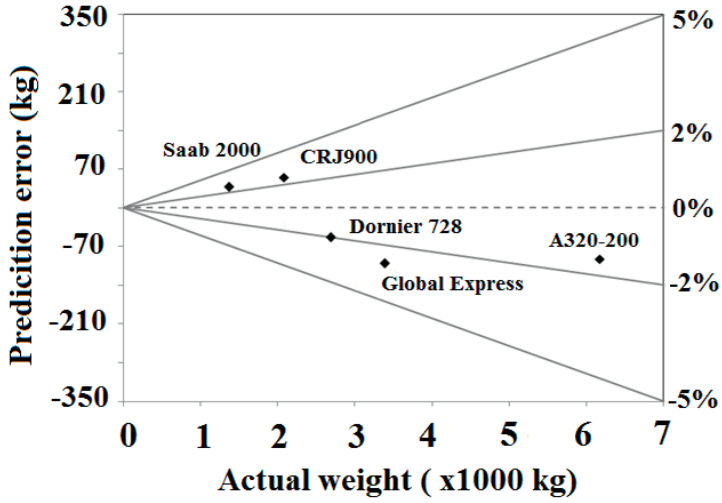


Figure 5. Validation of the model using different transport aircraft.

Typically, aileron reversal is the critical aeroelastic design case for highly swept wings, whereas flutter tends to be the critical aeroelastic design case for low swept wings[18]. However, Fig. 5 illustrates that the assumption of neglecting the impact of low speed flutter is tolerable even for low swept wing such as the Saab AB 2000 where the prediction error is +2·75%. This again demonstrates the robustness and the efficiency of the model and its ability to handle aircraft from different sizes, configuration, and mission role.

5.0 CONCLUSIONS

The development and application of linearised conceptual wing-box weight estimation and sizing model, has been presented. The model clearly demonstrates the benefits of linearised low fidelity models at the stage of pre-concept, conceptual, and preliminary design. The model provides a variety of novel features compared to existing state-of-the art models. These novel features include: multi-element sizing strategy;

- spanwise check for critical load factor;
- definition of rib pitch and orientation;
- the ability to handle complex wing geometries, and all traditional aircraft configurations;
- the ability to investigate a wider range of aerospace materials; and,
- quasi-static aeroelastic analysis to estimate the spanwise wing deflections.

This quasi-analytical model offers various advantages when compared to both semi-empirical models and finite element models (FEMs). It requires less knowledge of the geometry and structure, and it can run within a relatively short-time compared to FEM. Furthermore, it can be integrated with an MDO environment, as it is capable of capturing the correct sensitivities to various changes that may occur during the optimisation process. The model allows an optimisation study at the conceptual design level with a turnaround time between 3 to 4 hours with a good level of conformity[9,10], which in not possible with any FEM.

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The model consists of three expert core design modules: sizing, aeroelastic analysis, and weight prediction. This increases its flexibility and simplifies its coupling with an aerodynamic model and integration within an MDO environment. An extensive validation of the model using five transport aircraft, of different sizes, configurations, and mission roles, has been conducted. The validation demonstrates the effectiveness and robustness of the model yielding an error bandwidth of ±3%, average error estimate of -0·2%, and a standard error of 1·5%.

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