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Electromagnetic waves in self-gravitating, strongly coupled magnetized degenerate plasma

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Abstract. The linear propagation of the low-frequency (compared to the electron gyrofrequency) electromagnetic (EM) waves in a self-gravitating, strongly coupled magnetized plasma with ultra-relativistic degenerate electron fluid is investigated. It is found that the dispersion properties of the EM waves and stability criteria for such a degenerate plasma are significantly modified by the effects of the ultra-relativistic degenerate electron pressure, strong co-relation among extremely dense ion fluid, and the direction of the EM wave propagation relative to the ambient magnetic field direction. The relevance of our investigation to stability of white dwarf stars is briefly discussed. It is particularly seen here that the cores of such stars are stable for the class of gravito-electrodynamic waves that are analyzed for the characteristic ranges of relevant physical parameters.

1. Introduction

A class of stars of low luminosity and high surface emissivity, aptly referred to as white dwarf stars, are the compact bodies with masses that are typically $\leqslant 1\ M_\odot$ and radii that are typically $\leqslant 10^{-2}\ R_\odot$. Their average bulk densities are so high ($\sim 10^{30}\ \mathrm{cm}^{-3}$) that their deep interior is composed of strongly coupled ion lattices immersed in an ultra-relativistic, degenerate electron gas that follows the Fermi–Dirac statistics. The outward pressure that counterbalances the inward pull of the star's gravity is provided by the degenerate plasma, as first recognized by Chandrasekhar (1931a, 1931b) over 80 years ago. The degenerate pressure P_j associated with the plasma species j (where $j = e$ for the electron fluid and $j = i$ for the ion fluid) is given by

$$P_j = K_j N_j^{\gamma}, \tag{1.1}$$

where

$$\gamma = \frac{5}{3};\quad K_j = \frac{3}{5}\left(\frac{\pi}{3}\right)^{\frac{1}{3}}\frac{\pi\hbar^2}{m_j} \simeq \frac{3}{5}L_{cj}\hbar c \tag{1.2}$$

for the weakly relativistic limit (Shukla and Eliasson 2011), and

$$\gamma = \frac{4}{3};\quad K_j = \frac{3}{4}\left(\frac{\pi^2}{9}\right)^{\frac{1}{3}}\hbar c \simeq \frac{3}{4}\hbar c \tag{1.3}$$

for the ultra-relativistic limit (Shukla and Eliasson 2011). Here N_j is the number density of the plasma species j; m_j is the mass of the plasma species j; $L_{cj} = \pi\hbar/m_j c$; $\hbar$ is the Planck constant divided by 2π; and c is the speed of light in vacuum. We note that for the ultra-relativistic limit K_j is independent of the species.

Chandrasekhar (1931a) in his seminal paper considered the gravitational equilibrium of such stars, and showed in a very simple way that on account of $P_e \propto N_e^{4/3}$ there was a critical upper limit for the stable equilibrium of such stars. This limit M_c has since been called the Chandrasekhar limit, and has been subsequently revised upward from 0.91 $M_\odot$ to 1.44 $M_\odot$ (Chandrasekhar 1935). Chandrasekhar (1964) subsequently discussed the dynamic stability of white dwarf stars. Then it has been concluded that the general relativistic effects play an important role in the more massive stars close to the Chandrasekhar limit, and thereby decrease this limit slightly to $\sim 1.4\ M_\odot$ (Chandrasekhar and Topper 1964). Chandrasekhar, of course, did not consider the possible role of magnetic fields in the stability of white dwarf stars, because their existence was not known at that time. There is some observational evidence that white dwarf stars have variable surface magnetic fields (Musielak et al. 2003), and it has been recognized that white dwarf stars carry magnetic fields with the observed surface magnetic field ranging from a low of a few kilogauss to a high of about 300 megagauss (Libert 1980). The possibility that the internal magnetic field could be even larger, remains a possibility. Musielak (1987) investigated the magnetohydrodynamic (MHD) waves propagating in white dwarf photospheric layers and the associated damping processes that may be important for the propagation of the MHD waves (Musielak 1987).

Recently, Shukla et al. (2011) discussed the stability of a strongly magnetized white dwarf star (a more precisely the inner core thereof) assuming a uniform magnetic

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field of 300 megagauss. They assumed the white dwarf star to be of mass $\sim 1\ M_{\odot}$, consistent with prevailing observational estimates, and neglected the role of rotation because, unlike neutron stars, white dwarfs are slow rotators. For mathematical simplicity they also assumed the core to be of uniform density. Specifically, Shukla et al. (2011) did not deal with global oscillations of the configuration, but rather on the dispersive properties of a novel class of low-frequency gravito-electromagnetic waves that are almost equally effected by density and magnetic field perturbations, which may either be self-excited or may result from gravitational perturbations, such as from a binary companion star. Also, in order to simplify the analysis, the above authors confined their attention to waves propagating perpendicular to the magnetic field, and concluded that the star was stable against such perturbations, provided that their wavelengths were shorter than a critical value that was comfortably larger than the radius of the core (or indeed the star) for their assumed physical parameters. We, in our present work, will follow the approach similar to that of Shukla et al. (2011), and study the dispersive properties of the gravito-electromagnetic waves in a magnetized strongly coupled degenerate plasma containing ultra-relativistic degenerate electrons and strongly degenerate coupled ions. Our analysis, however, is not confined to a special case (where the plasma is weakly coupled and the wave propagation direction is perpendicular to the internal magnetic field, as considered by Shukla et al. (2011)), rather it is generalized to the strongly coupled degenerate plasma as well as to the propagation oblique to the magnetic field. We will also not confine our analysis to a single set of physical parameters (as considered by Shukla et al. (2011)), but rather to a significant range in parameter space.

The manuscript is organized as follows. The model equations are given in Sec. 2. The linear dispersion relation for the obliquely propagating gravito-electromagnetic waves is derived, and some limiting cases (viz. wave propagation along and across the internal magnetic field direction) are discussed in Sec. 3. A brief discussion of our findings, including their applications to white dwarf stars, is presented in Sec. 4.

2. Model equations

We consider a self-gravitating, strongly coupled, dense magnetoplasma composed of inertialess, ultra-relativistic, and weakly coupled degenerate electrons (Chandrasekhar 1931a; Mamun and Shukla 2010a, 2010b), and inertial, weakly relativistic, strongly coupled degenerate ions. We study the propagation of the low-frequency (compared to the electron gyro-frequency), low phase speed (in comparison with the speed of light in vacuum) EM waves in such a strongly coupled degenerate plasma. We assume the charge neutrality condition, i.e. $N_e = Z_i N_i$. The dynamics of the EM waves in this strongly coupled degenerate, ultra-cold (Mendonça 2010, 2011) electron–ion plasma is governed by the gravito-magnetohydrodynamic (G-MHD) equations (Ichimaru et al. 1987, Ichimaru and Tanaka 1986, Shukla et al. 2011),

$$\partial_t N_i + \vec{\nabla}\cdot(N_i\vec{u}_i) = 0, \tag{2.1}$$

$$(1+\tau_m D_t)\left[m_i N_i D_t \vec{u}_i - Z_i e N_i\left(\vec{E} + \frac{1}{c}\vec{u}_i \times \vec{B}\right) + \frac{3}{5}\hbar c L_{ci}\vec{\nabla}N_i^{\frac{5}{3}} + \vec{\nabla}\Psi_g\right] = \eta(\vec{\nabla}\cdot\vec{\nabla})\vec{u}_i + \left(\zeta + \frac{\eta}{3}\right)\vec{\nabla}(\vec{\nabla}\cdot\vec{u}_i), \tag{2.2}$$

$$0 = -Z_i e N_i\left(\vec{E} + \frac{1}{c}\vec{u}_e \times \vec{B}\right) - \frac{3}{4}Z_i^{\frac{4}{3}}\hbar c\vec{\nabla}N_i^{\frac{4}{3}}, \tag{2.3}$$

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\partial_t\vec{B}, \tag{2.4}$$

$$\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}Z_i e N_i(\vec{u}_i - \vec{u}_e), \tag{2.5}$$

$$\nabla^2\Psi_g = 4\pi G m_i N_i, \tag{2.6}$$

where $\partial_t = \partial/\partial t$; $D_t = \partial_t + \vec{u}_j\cdot\vec{\nabla}$; $\vec{u}_i$ ($\vec{u}_e$) is the ion (electron) fluid velocity; $\vec{E}$ is the electric field; $\vec{B}$ is the magnetic field; Ψ_g is the gravitational potential; m_i is the ion mass, G is the gravitational constant, e is the magnitude of the electron charge; τ_m is the viscoelastic ion relaxation time; and η (ζ) is the bulk (shear) viscosity coefficient. There are various approaches for calculating these transport coefficients. These have been widely discussed in the existing literature (Durisen 1973, Slattery et al. 1980, Ichimaru et al. 1987, Berkovsky 1992, Shukla 2010). We note here that (i) in (2.2) the effects of the external magnetic field on the transport coefficients ξ, η, and τ_m have been neglected (the modification of the transport coefficients ξ, η, and τ_m by the modified Deby–Hukkel potential in an external magnetic field are discussed by Nambu and Nitta (2002); (ii) in (2.3) and (2.5) the quasi-neutrality condition, $N_e = Z_i N_i$ has been used; (iii) the effect of rotation has been neglected, since unlike the much smaller and much denser neutron stars, white dwarf stars, in general, rotate very slowly (Gatewood and Gatewood 1978); (iv) the ambient magnetic field and the equilibrium plasma number density have been assumed to be uniform; (v) in (2.5) the contribution of the displacement current has been neglected, since the phase speed of the EM waves under consideration is much smaller than c. It is important to mention here that we are not interested in global oscillations of the configuration, rather we are concerned with the linear properties of the EM waves, which are affected by the degenerate electron pressure and gravitational force.

3. Dispersion relation

We assume that the amplitude of the EM waves is small enough to validate the linear theory. Thus, expressing

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all the dependent variables in terms of their equilibrium and perturbed parts, we can express (2.1)–(2.6) in the linearized form as

$$\partial_t n + \vec{\nabla}\cdot\vec{u}_i = 0, \tag{3.1}$$

$$d_\tau\left[\partial_t\vec{u}_i - V_A^2(\vec{\nabla}\times\vec{b})\times\hat{b}_0 + C_\hbar^2\vec{\nabla}n + \vec{\nabla}\Psi_g\right] = \eta'(\vec{\nabla}\cdot\vec{\nabla})\vec{u}_i + \xi\vec{\nabla}(\vec{\nabla}\cdot\vec{u}_i), \tag{3.2}$$

$$\partial_t\vec{b} = \vec{\nabla}\times(\vec{u}_i\times\hat{b}_0) - \delta_i^2\omega_{ci}\vec{\nabla}\times[(\vec{\nabla}\times\vec{b})\times\hat{b}_0], \tag{3.3}$$

$$\nabla^2\Psi_g = \omega_J^2 n, \tag{3.4}$$

where n is the perturbed part of N_i/n_{i0} (with n_{i0} being the equilibrium value of N_i); $\vec{b}$ is the perturbed part of $\vec{B}/B_0$ (with B_0 being the equilibrium value of $\vec{B}$); $V_A = B_0/\sqrt{4\pi\rho_{i0}}$ (with $\rho_{i0} = n_0 m_i$ being the equilibrium ion mass density); $\hat{b}_0(=\vec{B}_0/B_0)$ is the unit vector along $\vec{B}_0$; $\delta_i = V_A/\omega_{ci}$ is the ion skin-depth; $\omega_{ci} = Z_i eB_0/m_i c$ is the ion gyro-frequency; $\omega_J = \sqrt{4\pi G\rho_{i0}}$ is the Jeans frequency; $\partial_t = \partial/\partial t$; $d_\tau = 1 + \tau_m\partial_t$; $C_\hbar^2 = (Z_i^{3/2}\hbar c/m_i)n_{i0}^{1/3}(1+L_{ci}n_{i0}^{1/3})$; $\eta' = \eta/\rho_{i0}$; $\zeta' = \zeta/\rho_{i0}$; $\xi = \zeta' + \eta'/3$.

Now, assuming that all perturbed quantities are proportional to $\exp(-i\omega t + i\vec{k}\cdot\vec{r})$, where ω and $\vec{k}$ are the frequency and the propagation vector, respectively, we Fourier transform (3.1)–(3.4) to obtain

$$\vec{u}_i + \frac{V_A^2}{\omega}(\vec{k}\times\vec{b})\times\hat{b}_0 - \frac{k^2C_\hbar^2 - \omega_J^2}{\omega^2k^2}(\vec{k}\cdot\vec{u}_i)\vec{k} = \frac{\eta'k^2\vec{u}_i}{(i+\omega\tau_m)\omega} + \frac{\xi(\vec{k}\cdot\vec{u}_i)\vec{k}}{(i+\omega\tau_m)\omega}, \tag{3.5}$$

$$\vec{b} + \frac{\vec{k}\times(\vec{u}_i\times\hat{b}_0)}{\omega} - i\frac{\delta_i^2\omega_{ci}}{\omega}\vec{k}[(\vec{k}\times\vec{b})\times\hat{b}_0] = 0. \tag{3.6}$$

We finally assume that the EM waves (under consideration) propagate making an angle θ with the external magnetic field $\vec{B}_0$. This is equivalent to one of the two geometries: (i) $\vec{k} = \hat{x}k$ and $\vec{B}_0 = \hat{x}B_0\cos\theta + \hat{z}B_0\sin\theta$; (ii) $\vec{B}_0 = \hat{z}B_0$ and $\vec{k} = \hat{z}k\cos\theta + \hat{x}k\sin\theta$. It may be noted here that for both of these two geometries $\theta = 0$ ($\theta = 90^\circ$) correspond to the EM mode propagating parallel (perpendicular) to the external magnetic field. The consideration of any of these two geometries in (3.5) and (3.6) allows us to express a dispersion relation:

$$1 - \frac{k^2V_A^2\cos^2\theta}{\omega^2 - k^2V_\eta^2} - \frac{k^2V_A^2\sin^2\theta}{\omega^2 - \Omega_J^2} - \frac{k^4\delta_i^2V_A^2\cos^2\theta(\omega^2 - k^2V_\eta^2)}{\omega^2(\omega^2 - k^2V_A^2\cos^2\theta - k^2V_\eta^2)} = 0, \tag{3.7}$$

where $V_\eta^2 = \omega\eta'/(i+\omega\tau_m)$; $V_{\eta l}^2 = \omega\eta_l'/(i+\omega\tau_m)$; $\eta_l' = \eta_l/\rho_{i0}$; $\eta_l = \zeta + 4\eta/3$; $\Omega_J^2 = k^2(C_\hbar^2 + V_{\eta l}^2) - \omega_J^2$. Equation (3.7) represents the general dispersion relation for the obliquely propagating EM waves in the strongly coupled degenerate plasma under consideration. We assume that for a weakly coupled degenerate plasma $V_\eta \to 0$ and $V_\eta \to 0$, and that for a strongly coupled degenerate plasma $V_\eta \neq 0$ and $V_\eta \neq 0$, $\omega\tau_m$ is comparable to or larger than 1. It is obvious that $\omega\tau_m$ is involved only in V_η^2 and $V_{\eta l}^2$. Thus, the latter can be approximated as $V_\eta^2 \simeq \eta/\rho_{i0}\tau_m$ and $V_{\eta l}^2 = \eta_l/\rho_{i0}\tau_m$, which can be used for both weakly and strongly coupled degenerate plasmas under consideration. To examine the dispersion properties of these obliquely propagating EM waves in such a self-gravitating degenerate plasma, we now consider the following limiting cases.

3.1. The IMHD Limit ($k^2\delta_i^2 \ll 1$)

We first consider the ideal magnetohydrodynamic (IMHD) limit ($k^2\delta_i^2 \ll 1$), which allows us to write (3.7) as

$$\Omega_A^2\left[\omega^4 - \Omega_{kJ}^2\omega^2 + \Omega_J^2k^2\left(V_A^2\cos^2\theta + V_\eta^2\right) + k^4V_A^2V_\eta^2\sin^2\theta\right] = 0, \tag{3.8}$$

where $\Omega_A^2 = \omega^2 - k^2(V_A^2\cos^2\theta + V_\eta^2)$ and $\Omega_{kJ}^2 = k^2(V_A^2 + V_\eta^2) + \Omega_J^2$. It is obvious that (3.8) represents three types of EM waves. Their dispersion relations can be expressed in the form

$$\omega_1^2 = k^2\left(V_A^2\cos^2\theta + V_\eta^2\right), \tag{3.9}$$

$$\omega_\pm^2 = \frac{1}{2}\left[\Omega_{kJ}^2 \pm \sqrt{\Omega_{kJ}^4 - 4k^2\left\{\Omega_J^2(V_A^2\cos^2\theta + V_\eta^2) + k^2V_A^2V_\eta^2\sin^2\theta\right\}}\right]. \tag{3.10}$$

Equation (3.9) represents an obliquely propagating Alfvén-like mode (Khan and Saleem 2009) (ω_1, k), whereas (3.10) with + and − signs represent the fast (ω_+, k) and the slow (ω_-, k) magnetosonic-like modes, respectively. It is important to note that (i) $C_\hbar$ is a function of plasma particle number density; (ii) V_A is associated with the magnetic field and ion mass density; and (iii) V_η ($V_{\eta l}$) is the speed associated with the bulk (longitudinal) viscosity and ion mass density. To identify how the effects of plasma degeneracy and strong correlation in extremely dense ion fluid modify these modes [(ω_1, k) and ($\omega_\pm$, k)], we consider two special cases, namely the parallel and perpendicular propagation of the EM waves defined by (3.9) and (3.10).

Case 1: Parallel propagation $\theta = 0$

It is obvious from (3.9) and (3.10) that for the parallel propagation ($\theta = 0$), the plasma system under consideration supports two eigen-modes (since for parallel propagation (3.9) is identical to (3.10) with + sign). The dispersion relations for these two modes, therefore, become

$$\omega_1 = k\sqrt{V_A^2 + V_\eta^2} = k\sqrt{\frac{P_B + P_\eta}{\rho_{i0}}}, \tag{3.11}$$

$$\omega_- = \sqrt{k^2\left(C_\hbar^2 + V_{\eta l}^2\right) - \omega_J^2}. \tag{3.12}$$

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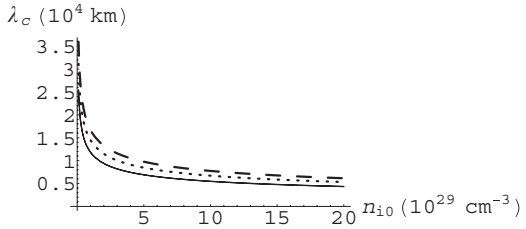


Figure 1. The variation of λ_c (defined by (3.13)) with n_{i0} for $\beta_\hbar = 0$ (solid curve), $\beta_\hbar = 0.5$ (dotted curve), and $\beta_\hbar = 1$ (dash curve).

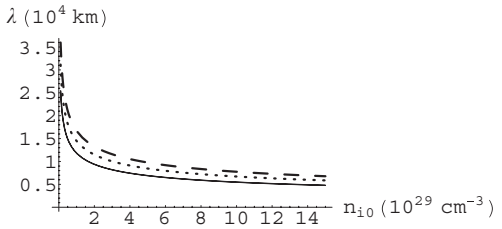


Figure 2. The variation of λ_c (defined by (3.16)) with n_{i0} for $B_0 = 3 \times 10^8$ G, $\beta_\hbar = 0$ (solid curve), $\beta_\hbar = 0.5$ (dotted curve), and $\beta_\hbar = 1$ (dash curve).

It is clear that (3.11) represents a stable Alfvén-like mode in which the sum of the magnetic pressure ($P_B = B_0^2/4\pi$), and the bulk viscous pressure ($P_\eta = \eta/\tau_m$), gives rise to the restoring force and the ion mass density ρ_{i0} provides the inertia. On the other hand, (3.12) represents an ion-acoustic-like mode in which the longitudinal viscosity plays the stabilizing role and the self-gravitating field plays the destabilizing role. The waves defined by (3.12) are unstable due to the self-gravitational field if and only if

$$\lambda > \lambda_c \equiv \frac{1}{\rho_{i0}\sqrt{G}}(P_\hbar + P_{\eta l})^{\frac{1}{2}}, \tag{3.13}$$

where $\lambda = 2\pi/k$ is the wavelength of the waves under present consideration; $P_\hbar = (\hbar c) Z_i^{4/3} n_{i0}^{4/3}(1 + L_{ci} n_{i0}^{1/3})$ is the degenerate plasma pressure associated with ultra-relativistic degenerate electrons and non-relativistic degenerate ions; and $P_{\eta l} = \eta_l/\tau_m$ is the pressure associated with the longitudinal viscosity. It is obvious from (3.13) that both the degenerate plasma pressure and strong correlation in the ion fluid play a very significant role in stabilizing the waves defined by (3.13). It is evident from (3.13) that the critical wavelength (λ_c) is inversely proportional to the ion mass (m_i). The variation of λ_c with n_{i0} and $\beta_\hbar (= V_\eta^2/C_\hbar^2)$ are obvious in Fig. 1. It is obvious from Fig. 1 that the critical wavelength (λ_c) decreases with the increase of the ion number density, but increases with the increase of the effect of the longitudinal viscosity coefficient (η_l).

Case 2: Perpendicular propagation $\theta = 90^\circ$

It is clear that for the perpendicular propagation ($\theta = 90^\circ$), the mode (ω_1, k_1) defined by (3.9) disappears, and the fast and slow magnetosonic-like modes ($\omega_\pm$, k) exist. The dispersion relations for the fast and slow magnetosonic-like modes ($\omega_\pm$, k) for $\theta = 90^\circ$ can be expressed as

$$\omega_+ = \sqrt{k^2\left(V_A^2 + C_\hbar^2 + V_{\eta l}^2\right) - \omega_J^2}, \tag{3.14}$$

$$\omega_- = k V_\eta. \tag{3.15}$$

The dispersion relation (3.15) represents a stable mode in which the bulk viscous pressure ($P_\eta = \eta/\tau_m$) gives rise to the restoring force, and the ion mass density $\rho_{i0} (= n_{i0} mi)$ provides the inertia. On the other hand, (3.14) represents a modified Alfvén mode in which the magnetic field, the plasma degenerate pressure, and the longitudinal viscosity play the stabilizing role, but the self-gravitating field plays the destabilizing role. The waves defined by (3.14) are unstable due to the self-gravitating field if and only if

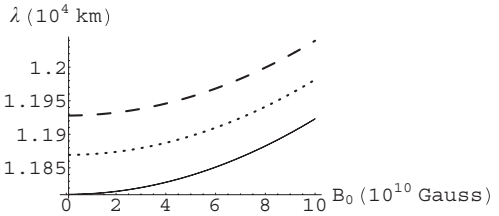


Figure 3. The variation of λ_c (defined by (3.16)) with B_0 for $n_{i0} = 10^{29}\,\mathrm{cm}^{-3}$, $\beta_\hbar = 0$ (solid curve), $\beta_\hbar = 0.01$ (dotted curve), and $\beta_\hbar = 0.02$ (dash curve).

$$\lambda > \lambda_c \equiv \frac{1}{\rho_{i0}\sqrt{G}}(P_B + P_\hbar + P_{\eta l})^{\frac{1}{2}}. \tag{3.16}$$

It is obvious from (3.16) that the magnetic pressure, degenerate plasma pressure, and strong correlation among ions play a very significant role in stabilizing the waves defined by (3.14). The variation of λ_c with n_{i0} and $\beta_\hbar$ for $B_0 = 3 \times 10^8$ G is shown in Fig. 2. We have also shown the variation of λ_c with B_0 and $\beta_\hbar$ for $n_{i0} = 6.5 \times 10^{29}\,\mathrm{cm}^{-3}$ (corresponding to $Z_i = 2$ and $n_{e0} = Z_i n_{i0} = n_0 = 2 \times 10^{29}\,\mathrm{cm}^{-3}$, where n_{e0} is the electron number density). This is displayed in Fig. 3. It is seen that $\lambda_c \simeq 10^4$ km for typical plasma parameters (viz. $n_0 = 1.3 \times 10^{30}\,\mathrm{cm}^{-3}$, $B_0 = 3 \times 10^8$ G, $\beta_\hbar = 0.1$, and $m_i = 4 \times 1.67 \times 10^{-24}$ gm) corresponding to the core of a white dwarf star of radius less than 10^4 km. We have also checked that if we neglect the effect of strong correlation among ions, i.e. set $\eta_l = 0$, the situation defined by (3.14) and (3.16) is identical to that considered and explained by Shukla et al. (2011).

3.2. Effects of Finite $k\delta_i$

It is obvious from (3.7) that the effects of finite $k\delta_i$ completely disappear for the perpendicular propagation of EM modes in a strongly coupled degenerate plasma under consideration, and thus the modes become identical to those defined by (3.14)–(3.16). The effects of finite $k\delta_i$ appear in the modes propagating parallel or oblique to the external magnetic field $\vec{B}_0$. For the parallel propagation, we can express the general

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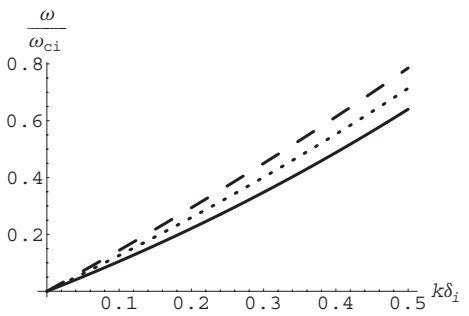


Figure 4. The dispersion properties of the accelerated Alfvén waves (represented by (3.17) with + sign) for $\beta_\eta = 0$ (solid curve), $\beta_\eta = 0.5$ (dotted curve), and $\beta_\eta = 1$ (dash curve).

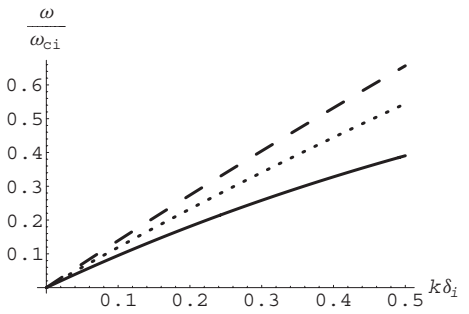


Figure 5. The dispersion properties of the retarded Alfvén waves (represented by (3.17) with − sign) for $\beta_\eta = 0$ (solid curve), $\beta_\eta = 0.5$ (dotted curve), and $\beta_\eta = 1$ (dash curve).

dispersion relation (3.7) in the form

$$\omega\left(\omega^2 - k^2V_A^2 - k^2V_\eta^2\right) = \pm k^2\delta_i V_A\left(\omega^2 - k^2V_\eta^2\right), \quad (3.17)$$

where + (−) sign represents accelerated (retarded) Alfvén waves modified by the bulk viscosity. The dispersion properties of the modified accelerated and retarded Alfvén waves (in which acceleration and retardation arise due to the effect of finite $k\delta_i$) defined by (3.17) are shown in Figs. 4 and 5, respectively. However, for $kV_\eta \ll \omega$ these two modes become

$$\omega = kV_A\left[\pm\frac{k\delta_i}{2} + \sqrt{1 + \left(\frac{k\delta_i}{2}\right)^2}\right]. \quad (3.18)$$

It is seen that the gravitational effect has no contribution in the dispersion properties of the accelerated and retarded Alfvén waves, and that they are completely stable for all wavelengths. It is also obvious that for the IMHD limit ($k\delta_i \ll 1$), (3.18) is identical to (3.9) with $\theta = 90^\circ$ and $\eta_l = 0$. It is obvious that both the acceleration and retardation of these modes are increased by increase in the values of $k\delta_i$.

4. Discussion

We have considered a self-gravitating degenerate magnetoplasma with ultra-relativistic degenerate electron fluid and non-relativistic degenerate ion fluid, and have studied the dispersion properties of the low-frequency (compared to the electron gyro-frequency), low phase velocity (compared to the speed of light in vacuum) EM waves. Specifically, our general theory has been used to examine the stability of self-gravitating white dwarf star cores ($M \leqslant 1\ M_\odot$) against EM perturbations. The results obtained from this investigation can be pointed out as follows:

The criteria (defined by (3.13) for parallel propagation and (3.16) for perpendicular propagation) for the instability of the EM modes (propagating parallel and perpendicular to the external magnetic field direction) and their graphical representation exhibit that (i) the critical wavelength (λ_c) is inversely proportional to the ion mass; (ii) λ_c increases with the plasma number density (n_0), but decreases with the external magnetic field strength (B_0) and longitudinal viscosity coefficient (η_l); (iii) $\lambda_c \simeq 10^4$ km for typical plasma parameters ($n_0 = 1.3 \times 10^{30}\,\mathrm{cm}^{-3}$, $B_0 = 3 \times 10^8$ G, $\beta_\eta = 0.1$, and $m_i = 4 \times 1.67 \times 10^{-24}$ gm) corresponding to the core of a white dwarf star of radius less than 10^4 km. This means that a self-gravitating white dwarf star of radius less than 10^4 km remains stable against EM perturbations.

The present investigation shows that the dispersion properties of the obliquely propagating EM modes in a degenerate, self-gravitating, magnetized plasma are completely different (in view of both the spatial and temporal scales) from those in the non-generate usual electron–ion plasma, and that the effects of ultra-relativistic degenerate electron pressure significantly modify the basic features of the EM waves. Our investigation reveals that (i) the EM perturbations propagate parallel to the external magnetic filed as two stable modes, namely the accelerated and retarded Alfveń waves; (ii) the acceleration and retardation arise due to the effect of finite $k\delta_i \ll 1$); (iii) both the acceleration and retardation are increased by the increase of $k\delta_i$.

We have used a wide range of values for the plasma parameters (plasma density and ambient magnetic field), they encompass the values used by Shukla et al. (2011) in their analysis of the stability of white dwarf stars. Our investigation confirms their finding that white dwarf cores are stable against a class of gravito-electromagnetic waves propagating perpendicular to the internal magnetic field for wavelengths that were comfortably smaller than the core radius; we find this to be the case for such waves propagating at all inclinations to the internal magnetic field. This includes the Alfvén-like waves propagating parallel to the magnetic field, in which case stability is maintained for any wavelength.

It is important to add here that we have followed Chandrasekhar (1931a, 1931b) by assuming that the white dwarf core is pure He. However, recently Koester (2002) has assumed that the core may be pure carbon or pure oxygen. The important point to note here is that whatever the 'chemical' composition of the core (pure He, pure C, or pure O) for a given mass, the total number of nucleons (protons plus neutrons) would be

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the same, and since the number of electrons per nucleon would also be the same, namely 1/2 (He: 4 nucleons, 2 electrons; C: 12 nucleons, 6 electrons; O: 16 nucleons, 8 electrons), we would end up with the same value of $n_0 = n_{e0} = Z_i n_{i0}$ for all compositions. This means that our theory is also valid for the recent assumption of Koester (2002). Our results have shown how the presence of ions of heavier elements C or O (instead of He) can modify the basic dispersion properties of the linear EM waves that propagate in our dense plasma with degenerate inertialess electron fluid and strongly coupled degenerate inertial ion fluid.

To simplify the mathematical analysis in this paper, as in the paper by Shukla et al. (2011), we have assumed a core of uniform density carrying a uniform magnetic field. But, in reality, the density within white dwarf stars varies with radius, which leads to electrons to be relativistically degenerate only within the inner core of the star (which accounts for over 90% of its mass), while being non-relativistically degenerate, in an outer mantle, which in turn is overlaid by a thin non-degenerate atmosphere. We propose to address this problem in a future paper, using a double 'polytrope' representation to model the realistic variation of electron density, and of the internal magnetic field within a white dwarf star.

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