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Ion-acoustic shock waves in a degenerate dense plasma

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Abstract. A theoretical investigation on the nonlinear propagation of ion-acoustic waves in a degenerate dense plasma has been made by employing the reductive perturbation method. The Burger's equation has been derived, and numerically analyzed. The basic features of electrostatic shock structures have been examined. It has been shown that the plasma system under consideration supports the propagation of electrostatic shock structures. The implications of our results (obtained from this investigation) in compact astrophysical objects have been briefly discussed.

1. Introduction

Degenerate matter in physics is a collection of free, non-interacting particles with a pressure and other physical characteristics (Goldberg and Scadron 1987; Carroll and Ostlie 2007). The degenerate state of matter arises at extraordinarily high density (in compact stars) or at extremely low temperatures in the lab (Andrew et al. 2001). Generally the extreme conditions of degenerate matter are caused by significant compression in which high density of degenerate matter is one of these extreme conditions.

There has been a great deal of interest in understanding the basic properties of degenerate or extremely dense matter because of its occurrence in many cosmic environments and compact astrophysical objects (Michel 1982; Rees 1983; Miller and Witta 1987; Tandberg-Hansen and Emslie 1988). Nowadays, most of the theoretical investigations are to understand the environment of the compact objects having their interiors supporting themselves via degenerate pressure. Generally the degenerate pressure, arising due to the combine effect of Pauli's exclusion principle and Heisenberg's uncertainty principle, depends only on the number density of constituent particles, but independent on its own temperature (Mamun and Shukla 2010a, 2010b; Roy et al. 2012). This degenerate pressure has an important role to study the electrostatic perturbation in matters that exist in extreme conditions (Chandrasekhar 1931a, 1931b, 1935; Koester 1990; Berro et al. 2010). Electron degenerate pressure will halt the gravitational collapse of a star if its mass is below the Chandrasekhar limit (i.e. 1.44 solar masses; Mazzali et al. 2007). This is the pressure that prevents a white dwarf star from collapsing. A star exceeding this limit and without usable nuclear fuel will continue to collapse to form either a neutron star or black hole because the degenerate pressure provided by the electrons is weaker than the inward pull of gravity. For stellar masses of less than about 1.44 solar masses (Chandrasekhar 1931a, 1931b, 1935), the energy from the gravitational collapse is not sufficient to produce the neutrons of a neutron star, so the collapse is halted by degenerate electron to form white dwarfs. This maximum mass for a white dwarf is called the Chandrasekhar limit (Chandrasekhar 1931a, 1931b, 1935)]. The observational evidence and the theoretical analysis imply that these compact objects support themselves against gravitational collapse by degenerate pressure. In case of such a compact object, the degenerate electron number density is so high (in white dwarfs it can be of the order of 10^{30} cm^{-3}, or even more; Mamun and Shukla 2010a, 2010b; Roy et al. 2012) that the electron Fermi energy is comparable to the electron mass energy, and as a result the electron speed becomes comparable to the speed of light in vacuum. For such interstellar compact objects the equation of state for degenerate electrons are mathematically explained by Chandrasekhar (1935) for two limits, named as non-relativistic and ultra-relativistic limits.

Recently, a number of authors (Hass 2007; Misra and Samanta 2008; Misra et al. 2010) have made a number of theoretical investigations on the nonlinear propagation of electrostatic waves in degenerate quantum plasma. These investigations are mainly based on the electron equation of state, which are only valid for the non-relativistic limit. Some investigations have been also made of the nonlinear propagation of electrostatic waves in a degenerate dense plasma which are mainly based on the degenerate electron equation of state valid for ultra-relativistic limit (Mamun and Shukla 2010a, 2010b; Roy et al. 2012). No theoretical investigation has been developed to study the extreme condition of matter for both non-relativistic and ultra-relativistic limits on the propagation of electrostatic shock wave. Therefore, we consider a degenerate dense plasma containing non-relativistic degenerate cold ion fluid and both non-relativistic and ultra-relativistic degenerate electron fluid to study the basic features of the electrostatic shock structures in such a degenerate dense plasma. The model

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is relevant to compact interstellar objects (i.e. white dwarf, neutron star, black hole, etc.).

2. Governing equations

We consider the propagation of electrostatic perturbation in a degenerate dense plasma containing non-relativistic degenerate cold ion and both non-relativistic and ultra-relativistic degenerate electron fluids. Thus, at an equilibrium condition we have $n_{i0} = n_{e0}$, where n_{i0} (n_{e0}) is the ion (electron) number density at equilibrium. The nonlinear dynamics of the electrostatic waves propagating in such a degenerate plasma is governed by

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0, \tag{2.1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \frac{\partial \phi}{\partial x} + \frac{K_1}{n_i}\frac{\partial n_i^{\gamma}}{\partial x} - \eta \frac{\partial^2 u_i}{\partial x^2} = 0, \tag{2.2}$$

$$n_e \frac{\partial \phi}{\partial x} - K_2 \frac{\partial n_e^{\gamma}}{\partial x} = 0, \tag{2.3}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n_i, \tag{2.4}$$

where n_i (n_e) is the ion (electron) number density normalized by its equilibrium value n_{i0} (n_{e0}), u_i is the ion fluid speed normalized by $C_i = (m_e c^2/m_i)^{1/2}$ with m_e (m_i) being the electron (ion) rest mass, c being the speed of light in vacuum, ϕ is the electrostatic wave potential normalized by $m_e c^2/e$ with e being the magnitude of the charge of an electron, the time variable (t) is normalized by $\omega_{pi} = (4\pi n_0 e^2/m_i)^{1/2}$, the space variable ($x$) is normalized by $\lambda_s = (m_e c^2/4\pi n_0 e^2)^{1/2}$, and the coefficient of viscosity η is a normalized quantity given by $\omega_{pi}\lambda_m^2 m_i n_{i0}$. The constants $K_1 = n_0^{\alpha-1}K_i/{m_i}^2{C_i}^2$ and $K_2 = n_0^{\gamma-1}K_e/m_i{C_i}^2$. The equations of the state used here are given by

$$P_i = K_i n_i^{\alpha}, \tag{2.5}$$

where

$$\alpha = \frac{5}{3}; \quad K_i = \frac{3}{5}\left(\frac{\pi}{3}\right)^{\frac{1}{3}}\frac{\pi\hbar^2}{m} \simeq \frac{3}{5}\Lambda_c \hbar c, \tag{2.6}$$

for the non-relativistic limit (where $\Lambda_c = \pi\hbar/mc = 1.2 \times 10^{-10}$ cm, and $\hbar$ is the Planck's constant divided by 2π). While for the electron fluid,

$$P_e = K_e n_e^{\gamma}, \tag{2.7}$$

where

$$\gamma = \alpha;\ K_e = K_i \text{ for non-relativistic limit, and} \tag{2.8}$$

$$\gamma = \frac{4}{3}; \quad K_e = \frac{3}{4}\left(\frac{\pi^2}{9}\right)^{\frac{1}{3}}\hbar c \simeq \frac{3}{4}\hbar c, \tag{2.9}$$

in the ultra-relativistic limit (Chandrasekhar 1931a, 1931b, 1935; Mamun and Shukla 2010a, 2010b)].

To examine electrostatic perturbations propagating in the ultra-relativistic degenerate dense plasma by analyzing the outgoing solutions of (2.1)–(2.4), we first introduce the stretched coordinates (Maxon and Viecelli 1974)

$$\xi = -\epsilon(x + V_p t), \tag{2.10}$$

$$\tau = \epsilon^2 t, \tag{2.11}$$

where V_p is the wave phase speed (ω/k with ω being angular frequency and k being the wave number of the perturbation mode), and ϵ is a smallness parameter measuring the weakness of the dispersion ($0 < \epsilon < 1$). We then expand n_i, n_e, u_i, and ϕ, in power series of ϵ:

$$n_i = 1 + \epsilon n_i^{(1)} + \epsilon^2 n_i^{(2)} + \cdots, \tag{2.12}$$

$$n_e = 1 + \epsilon n_e^{(1)} + \epsilon^2 n_e^{(2)} + \cdots, \tag{2.13}$$

$$u_i = \epsilon u_i^{(1)} + \epsilon^2 u_i^{(2)} + \cdots, \tag{2.14}$$

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots, \tag{2.15}$$

and develop equations in various powers of ϵ. To the lowest order in ϵ, (2.1)–(2.15) give $u_i^{(1)} = -3V_p\phi^{(1)}/(3V_p^2 - 2K_1')$, $n_i^{(1)} = 3\phi^{(1)}/(3V_p^2 - 2K_1')$, $n_e^{(1)} = \phi^{(1)}/\gamma K_2'$, and $V_p = \sqrt{\gamma K_2' + (2/3)K_1'}$, where $K_1' = \alpha/(\alpha - 1)$ and $K_2' = \gamma K_2/(\gamma - 1)$. The relation $V_p = \sqrt{\gamma K_2' + (2/3)K_1'}$ represents the linear dispersion relation for the ion-acoustic (IA)-type electrostatic waves in the degenerate plasma under consideration (the new constants introduced here are given by $K_1' = \alpha K_1/(\alpha - 1)$ and $K_2' = \gamma K_2/(\gamma - 1)$).

3. Derivation of Burger's equation

We are interested in studying the nonlinear propagation of these dissipative ion-acoustic-type electrostatic waves in a degenerate plasma. To the next higher order in ϵ, we obtain a set of equations

$$\frac{\partial n_i^{(1)}}{\partial \tau} - V_p \frac{\partial n_i^{(2)}}{\partial \xi} - \frac{\partial}{\partial \xi}[u_i^{(2)} + n_i^{(1)} u_i^{(1)}] = 0, \tag{3.1}$$

$$\frac{\partial u_i^{(1)}}{\partial \tau} - V_p \frac{\partial u_i^{(2)}}{\partial \xi} - u_i^{(1)}\frac{\partial u_i^{(1)}}{\partial \xi} - \frac{\partial \phi^{(2)}}{\partial \xi} - \eta \frac{\partial^2}{\partial \xi^2} u_i^{(1)} - K_1'\left[\frac{\partial}{\partial \xi}\frac{2n_i^{(2)}}{3} + \frac{(n_i^{(1)})^2}{9}\right] = 0, \tag{3.2}$$

$$\frac{\partial \phi^{(2)}}{\partial \xi} - \gamma K_2' \frac{\partial}{\partial \xi}\left[n_e^{(2)} + \frac{(\gamma - 2)}{2}(n_e^{(1)})^2\right] = 0, \tag{3.3}$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = n_e^{(2)} - n_i^{(2)}. \tag{3.4}$$

Now, combining (3.1)–(3.4) we deduce a Burger's equation

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} = C\frac{\partial^2 \phi^{(1)}}{\partial \xi^2}, \tag{3.5}$$

where

$$A = \frac{(3V_p^2 - 2K_1')^2}{18V_p}\left(\frac{1}{\gamma^2 K_2'^2} + \frac{3(27V_p^2 - 2K_1')}{(3V_p^2 - 2K_1')^3}\right), \tag{3.6}$$

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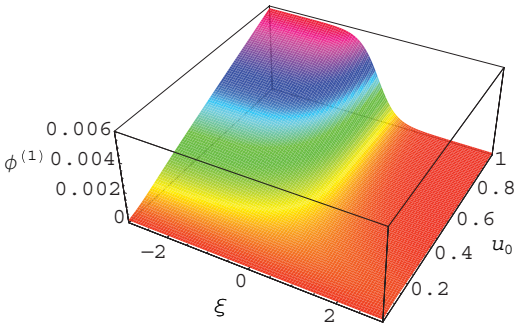


Figure 1. (Colour online) The effect of u_0 on shock profile (potential structure) for both electron–ion being non-relativistic degenerate.

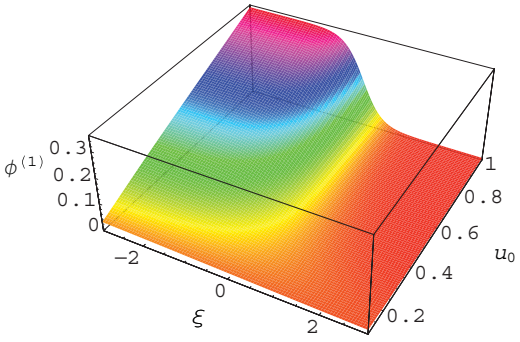


Figure 2. (Colour online) The effect of u_0 on shock profile (potential structure) for electrons being ultra-relativistic and ions being non-relativistic degenerate.

$$C = \frac{\eta}{2}. \tag{3.7}$$

The shock wave solution of (3.5) is

$$\phi^{(1)} = \phi_m \left[1 - tanh\left(\frac{\xi}{\delta}\right)\right], \tag{3.8}$$

where the special coordinate, $\xi = \zeta - u_0\tau$, the amplitude, $\phi_m = u_0/A$, and the width, $\delta = 2C/u_0$.

4. Numerical analysis

We have numerically solved (3.5), and have studied the effects of η and u_0 with ξ on electrostatic shock structures in both non-relativistic and ultra-relativistic degenerate electrons (ions always being non-relativistic degenerate). It is obvious from (3.6) and (3.8) that the degenerate plasma system under consideration supports compressive electrostatic shock waves which are associated with a positive potential, and the amplitude ($\phi^{(1)}$) of these shock structures depends on the Chandrasekhar limits (Chandrasekhar 1935), i.e. non-relativistic and ultra-relativistic limits.

The profiles of shock wave, caused by the balance between nonlinearity and dissipation, are depicted in Figs. 1–4. Moreover, the potential of the electrostatic profiles for non-relativistic degenerate ion fluid and ultra-relativistic degenerate electron fluid is different from that when both the particles follow the same limit.

In Figs. 1 and 2 we have observed the effect of u_0 on the potential, $\phi^{(1)}$ with the variation of ξ when η is

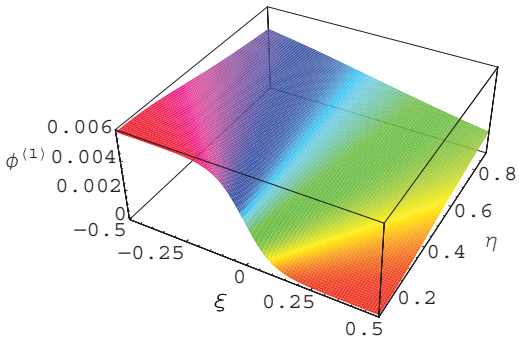


Figure 3. (Colour online) The effect of η on shock profile (potential structure) for both electron–ion being non-relativistic degenerate.

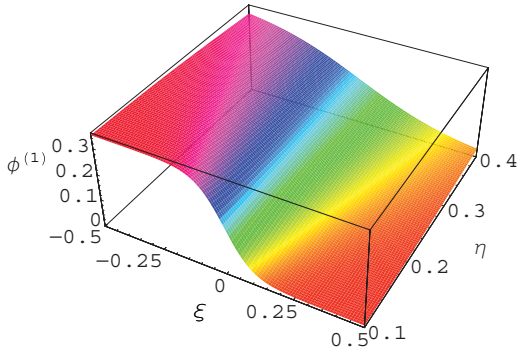


Figure 4. (Colour online) The effect η on on shock profile (potential structure) for electrons being ultra-relativistic and ions being non-relativistic degenerate.

equal to 0.5 in case of both non-relativistic and ultra-relativistic limits. It is also observed that in both cases the potential, $\phi^{(1)}$ increases with increasing value of u_0. Again, in Figs. 3 and 4 we have analyzed the effect of η on the potential with the variation of ξ when u_0 is equal to 1 in case of both limits. In these figures we have again observed that the potential, $\phi^{(1)}$ increases with increase of η.

From Figs. 1 to 4, we have studied the effect of the increasing value of u_0 and η on the potential, $\phi^{(1)}$ with the variation of ξ in both limits. And the potential, $\phi^{(1)}$ increases with the increasing values of u_0 and η. But we have also noted that the potential, $\phi^{(1)}$ comparably increases more rapidly for electrons being ultra-relativistic and ions being non-relativistic degenerate than for both electron–ion being non-relativistic degenerate.

It is to be noted here that we have taken all the parameters in normalized form, so all the ranges of parameters are taken arbitrarily.

5. Discussion

We have considered a degenerate dense plasma containing non-relativistic degenerate cold ion fluid and both non-relativistic and ultra-relativistic degenerate electron fluid, and have examined the basic features of the electrostatic shock structures that are found to exist in such degenerate dense plasma. Our present investigation is different from the related investigations (Mamun and

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Shukla 2010a, 2010b; Roy et al. 2012) in the way that we have considered the pressure of all the constituent particles (electrons and ions), as the whole system is degenerate and all the particles should follow the equations of the state (2.4)—(2.9) whatever is the limit (non-relativistic or ultra-relativistic). It is obvious that the shock profiles obtained from our present investigation are also quite different from the previous investigation (Mamun and Shukla 2010a, 2010b; Roy et al. 2012) in the sense that we have had electrostatic shock profiles, while the others have got to support electrostatic solitary profiles (Mamun and Shukla 2010a, 2010b; Roy et al. 2012)[and double layers (Mamun and Shukla 2010b; Roy et al. 2012). The degenerate dense plasma is found to support the shock structures whose basic features depend on the plasma number density. From this point of view our present investigation is more acceptable and the system constituents have made the validity of our investigations unique.

We note that in our numerical analysis we have used a wide range of degenerate plasma parameters (Chandrasekhar 1931a, 1931b, 1935; Mamun and Shukla 2010a, 2010b) that are relevant for many cosmic environments and compact astrophysical objects. The results of the present investigation are therefore expected to be useful in understanding the dispersion properties of electrostatic shock waves in such cosmic environments, compact astrophysical objects, and interstellar compact objects (Michel 1982; Shapiro and Teukolsky 1983; Shukla 1986; Yu et al. 1986; Miller and Witta 1987; Tandberg-Hansen and Emslie 1988; Silva et al. 1999; Hoyos et al. 2008; Asenjo et al. 2009, 2011).

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